Conformal Field Theories in 3.99 Dimensions

in collaboration with
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Outline

The Wilson-Fisher fixed point

Simplest example of fixed point: $\lambda\phi^4$ interaction in $4-\epsilon$ dimensions

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However at Wilson-Fisher fixed point: $\gamma_{\phi^2} \simeq \sqrt{12\gamma_\phi}$

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- infinite towers of states (or operators) with increasing, equally spaced, dimensions.
- Lower state is called Primary: $K_\mu|s>=0$.
- Other states, called Descendants, obtained applying P_μ
- representation totally characterized by scaling dimension and spin of the primary

The Operator Product Expansion

Completeness of the Hilbert space of states $\Leftrightarrow$ OPE:

$$\mathcal{O}_{\Delta_1}(x) \times \mathcal{O}_{\Delta_2}(y) = \frac{1}{|x-y|^{\Delta_1+\Delta_2}} \sum_{\mathcal{O}} C_{12\mathcal{O}} C_{\mu_1\ldots\mu_l}(y, \partial^\nu) \mathcal{O}_\Delta^{\mu_1\ldots\mu_l}(y)$$

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$C_{12\mathcal{O}}$ are called OPE coefficients and define completely the theory.

The power of conformal invariance

Two point function of primaries: completely fixed

$$\langle \mathcal{O}_i(x_1)\mathcal{O}_j(x_2)\rangle = \frac{\delta_{ij}}{x_{12}^{2\Delta}} \qquad x_{12} \equiv |x_1 - x_2|$$

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Use OPE to reduce higher point functions to smaller ones

Four point functions

Recalling the OPE

$$\mathcal{O}(x_1) \times \mathcal{O}(x_2) = \sum_{\mathcal{O}'} \frac{C_{\mathcal{O}'}}{x_{12}^{2d-\Delta}} (\mathcal{O}'_{\Delta,l} + \text{descendants})$$

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$$\langle \underbrace{\mathcal{O}(x_1)\mathcal{O}(x_2)}\underbrace{\mathcal{O}(x_3)\mathcal{O}(x_4)}\rangle = \frac{u^{-d}}{(x_{13}^{2d} x_{24}^{2d})} \sum_{O'_{\Delta,l}} C^2_{\mathcal{O}'} \underbrace{(\langle O'_{\Delta,l} O'_{\Delta,l}\rangle + \text{descendants})}_{\text{function of } u,\, v \text{ only by conformal symmetry}}$$

$$u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2} \qquad v = \frac{x_{14}^2 x_{23}^2}{x_{13}^2 x_{24}^2}$$

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Conformal Blocks

$$g_{\Delta,l}(u,v) \equiv \langle O'_{\Delta,l} O'_{\Delta,l} \rangle + \text{descendants}$$

They sum up the contribution of an entire representation

More on Conformal Blocks

Old idea (70's) but none could use them for long time, until..

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- ('13: El-Showk, Paulos, Poland, Rychkov, Simmons-Duffin, AV): efficient method to compute Taylor coefficients of conformal block in any dimension. (See David Simmons-Duffin's talk).

The Bootstrap program

Which expansion is the right one?

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Crossing symmetry ⇒ Sum Rule

$$\sum_{\Delta,l} C^2_{\Delta,l} \underbrace{\frac{v^d g_{\Delta,l}(u,v) - u^d g_{\Delta,l}(v,u)}{u^d - v^d}}_{F_{d,\Delta,l}} = 1$$

[Rattazzi,Rychkov,Tonni, AV]

- $F_{d,\Delta,l}$ known functions
- $C^2_{\Delta,l}$ unknown coefficients

Geometric interpretation

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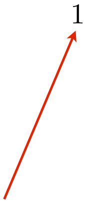

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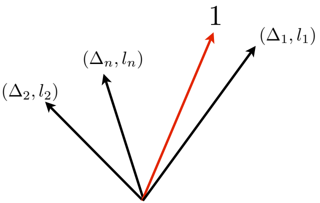

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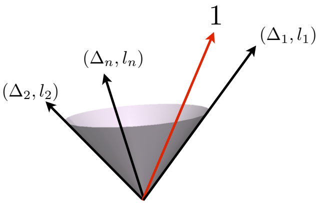

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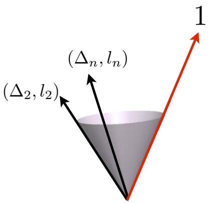

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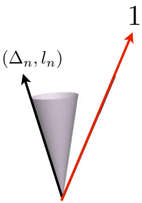

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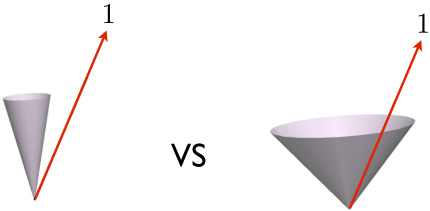
- A cone too narrow can't satisfy crossing symmetry: inconsistent spectrum

Geometric interpretation

How can we distinguish feasible spectra from unfeasible ones?



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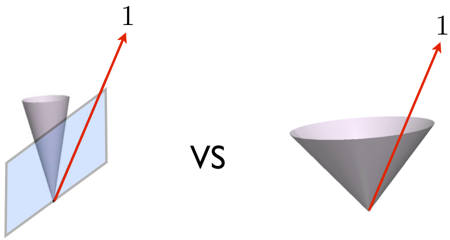

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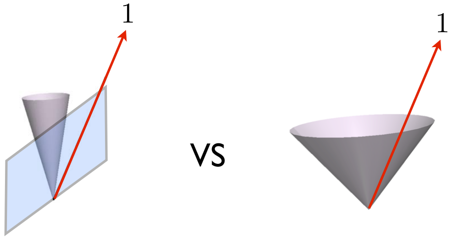

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More formally...

Look for a Linear functional

$$\Lambda[F_{d,\Delta,l}] \equiv \sum_{n,m}^{N_{\rm max}} \lambda_{mn} \partial^n \partial^m F_{d,\Delta,l}$$

such that

$$\Lambda[F_{d,\Delta,l}] > 0 \qquad \text{and} \qquad \Lambda[1] < 0$$

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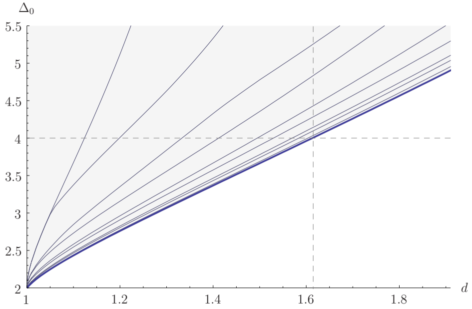

When $d \lesssim 1.6$, no CFT exists without relevant operator in $\phi \times \phi$
[Poland,Simmons-Duffin, AV]

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- Conformal descendants

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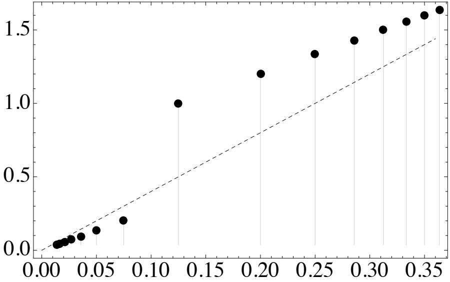

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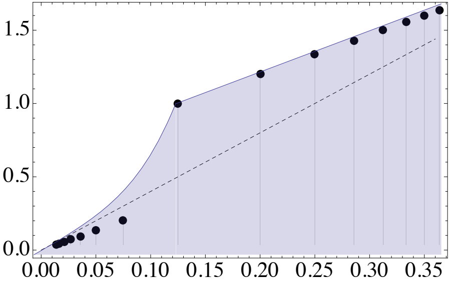

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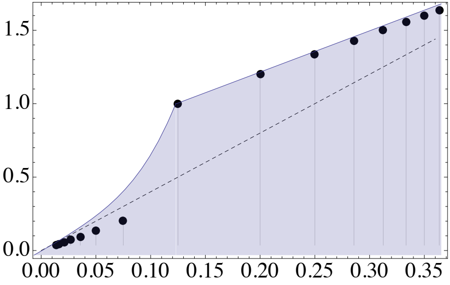

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A kink signals the presence of the Ising Model

A proliferation of kinks

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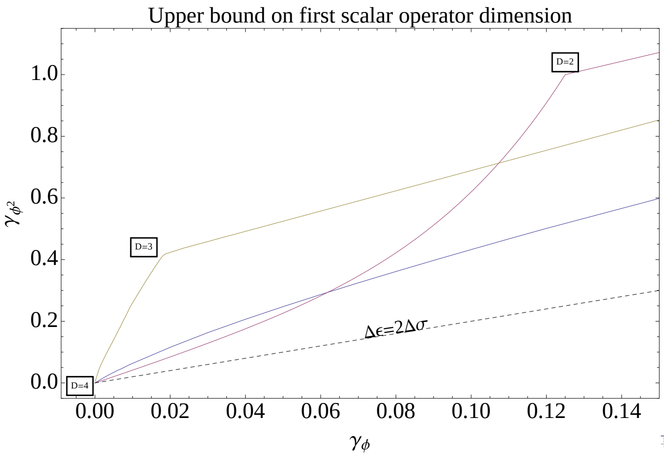

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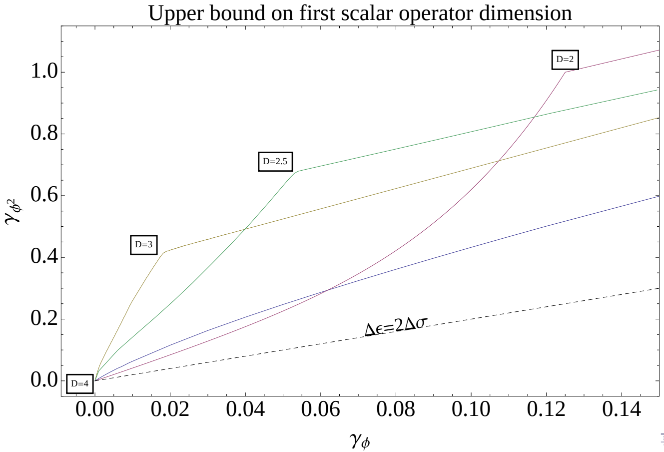

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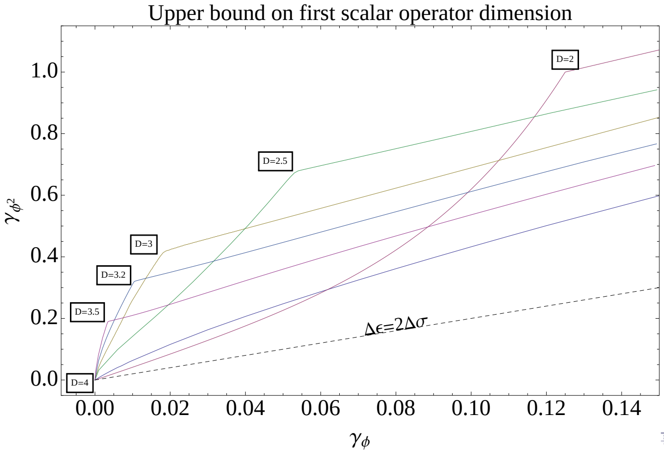
Compare bounds on the anomalous dimensions for various D:

$$\gamma_\phi = \Delta_\phi - \frac{(D-2)}{2} \qquad \gamma_{\phi^2} = \Delta_{\phi^2} - (D-2)$$



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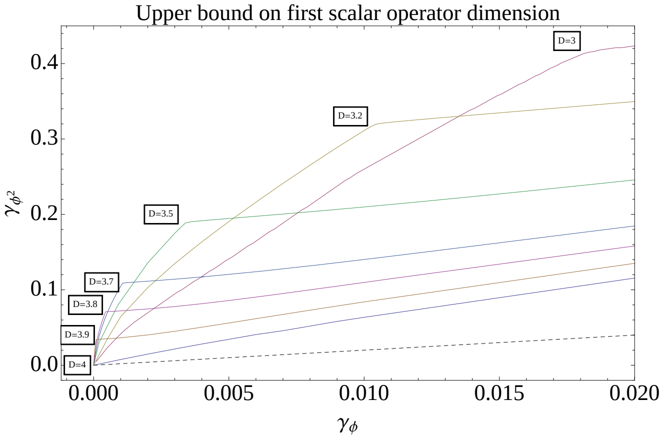
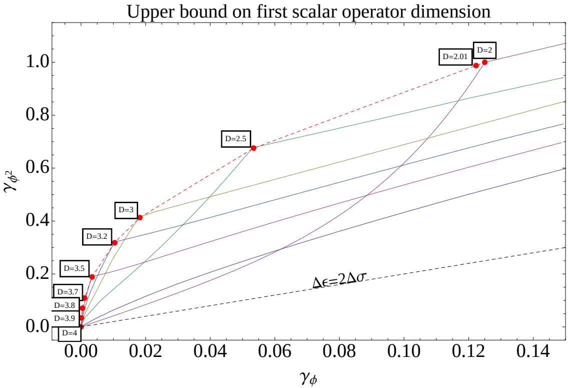


A family of CFT's



- Bounds smoothly interpolate from 4D to 2D
- Kinks lie on a smooth curve
- Kinks easy to identify for $D \geq 3.2$ and $D \leq 2.5$ (Ising 3D: the hardest..)

Epsilon Expansion: $D = 4 - \epsilon$

$$\gamma_\phi = \frac{(N+2)\epsilon^2}{4(N+8)^2} - \frac{(N+2)\left(N^2 - 56N - 272\right)\epsilon^3}{16(N+8)^4} + O(\epsilon^3)$$

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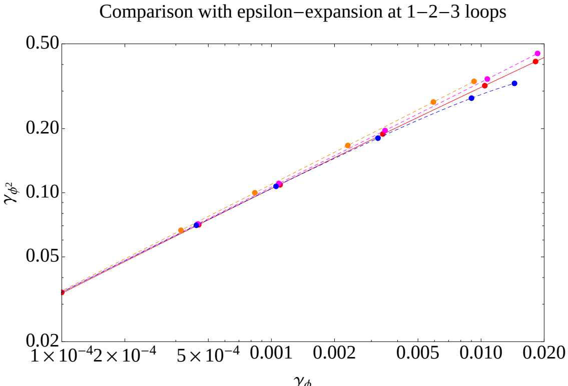

- Our Kinks
- 1-loop
- 2-loop
- 3-loop

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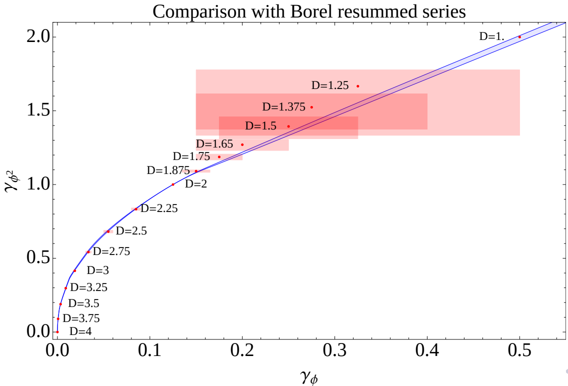

- - Our prediction
- Borel resumed series: central values and errors [Guillou,Zinn-Justin]

Conclusions and Future directions

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... stay tuned for updates!