FCNC Top Quark Production Via Anomalous Couplings

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Introduction

 $\begin{array}{c} gu \rightarrow tZ\\ gu \rightarrow t\gamma\\ gu \rightarrow tg\\ \text{Concluding Remarks} \end{array}$

FCNC Top Production Soft Gluon Corrections

Effective Lagrangians

Effective Lagrangians

$$\Delta \mathcal{L}_1 = \frac{1}{\Lambda} e \kappa_{tqV} \bar{t} \sigma_{\mu\nu} F_V^{\mu\nu} + h.c., \quad \Delta \mathcal{L}_2 = \frac{g_s \kappa_{qgt}}{\Lambda} \bar{t} \sigma^{\mu\nu} T^a \chi q G^a_{\mu\nu} + h.c.$$

where:

- Λ is an effective scale, e is the elementary charge and g_s is the strong coupling
- κ_{tqV} & κ_{tqg} are the anomalous couplings
- $\sigma_{\mu\nu} = \frac{i}{2} \gamma_{[\mu} \gamma_{\nu]}$ and γ_{μ} are the dirac matrices and T^a are the Gell-Mann matrices
- $\chi = f^L P_L + f^R P_R$ where $P_L(P_R)$ is the left(right) hand projection operator
- V is either a Z or a photon
- q is either a c or a u
- $F_V^{\mu\nu}$ is the field tensor for $V, G^a_{\mu\nu}$ is the field tensor for g

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FCNC Top Production Soft Gluon Corrections

LHC as FCNC Probe

The LHC has been used to examine FCNC in the top-quark sector.

- Historically, HERA and Tevatron looked for these processes.
- ATLAS set limits of $\kappa_{ugt}/\Lambda < 6.9 \cdot 10^{-3} \text{TeV}^{-1}$ and $\kappa_{cgt}/\Lambda < 1.5 \cdot 10^{-2} \text{TeV}^{-1}$ and $\text{BR}(t \to Zq) < 0.73\%$ [Arxiv:1302.3698v1]
- We note that t-u-Z and t-u- γ dominate
- And charm contributions are small

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FCNC Top Production Soft Gluon Corrections

Soft Gluon Corrections

Scale variations at LO produce large uncertainties, but higher order corrections stabilize the cross sections.

- Soft gluons important since the virtual and real diagrams don't cancel completely
- We define $s_4 = s + t + u \sum m^2$ and $s_4 \to 0$ threshold

• We consider logarithmic corrections $\left[\frac{\ln^l(s_4/m^2)}{s_4}\right]_+$

- For α_s^n , LL l = 2n 1 and NLL l = 2n 2
- We calculate NLO at NLL



Analytical

$gu \rightarrow tZ$ - Diagram



Figure 1 : Tree level diagrams for $gu \to tZ$

$$g(p_g) + u(p_u) \to t(p_t) + Z(p_Z)$$

$$s = (p_g + p_u)^2, \ t = (p_g - p_t)^2, \ u = (p_u - p_t)^2$$

$$s_4 = s + t + u - m^2 - m_Z^2$$



The Born cross section for $gu \to tZ$ is given by:

 $\begin{aligned} \frac{d^2 \hat{\sigma}_B^{gu \to tZ}}{dt du} \\ &= \frac{2\pi \alpha \alpha_s \kappa_Z}{3m^2 s^3 (m^2 - t^2)^2} \{ 2m^8 - m^6 (2m_Z^2 + 4s + 2t) \\ &- t \left[2m_Z^6 - 2m_Z^4 (s + t) - 4st (s + t) + m_Z^2 (s + t) \right] \\ &+ m^4 \left[2m_Z^4 - m_Z (2s + t) + 2(s^2 + 4st + t^2) \right] \\ &+ m^2 \left[2m_Z^6 - 4m_Z^4 t + m_Z^2 (s + t) (s + 5t) - 2t (2s^2 + 6t + t^2) \right] \end{aligned}$



$gu \to tZ$ - NLO

The NLO-NLL partonic cross section is given by:

$$\frac{d^2 \hat{\sigma}_{gu \to tZ}^{(1)}}{dt du} = F_B^{gu \to tZ} \frac{\alpha_s(\mu_R^2)}{\pi} \left(c_3 \left[\frac{\ln(s_4/m^2)}{s_4} \right]_+ + c_2 \left[\frac{1}{s_4} \right]_+ + c_1 \delta(s_4) \right)$$

where

$$c_{1} = \left[C_{F}\ln\left(\frac{-t+m_{Z}^{2}}{m^{2}}\right) + C_{A}\ln\left(\frac{-u+m_{Z}^{2}}{m^{2}}\right) - \frac{3}{4}C_{F} - \frac{\beta_{0}}{4}\right]\ln\left(\frac{\mu_{F}^{2}}{m^{2}}\right) + \frac{\beta_{0}}{4}\ln\left(\frac{\mu_{R}^{2}}{m^{2}}\right)$$

$$c_{2} = 2\operatorname{Re}\Gamma_{S}^{(1)} - C_{F} - C_{A} - 2C_{F}\ln\left(\frac{-t+m_{Z}^{2}}{m^{2}}\right) - 2C_{A}\ln\left(\frac{-u+m_{Z}^{2}}{m^{2}}\right) - (C_{F} + C_{A})\ln\left(\frac{\mu_{F}^{2}}{s}\right)$$

$$c_{3} = 2(C_{F} + C_{A})$$

and $C_F = (N_c^2 - 1)/(2N_c)$, $C_A = N_c$, $\beta_0 = (11C_A - 2n_f)/3$. n_f is the number of light quark flavors and N_c the number of colors



$gu \rightarrow tZ$ - NNLO

The NNLO-NLL partonic cross section is given by:

$$\begin{aligned} \frac{d^2 \hat{\sigma}_{gu \to tZ}^{(2)}}{dt \, du} &= F_B^{gu \to tZ} \frac{\alpha_s^2(\mu_R^2)}{\pi^2} \left\{ \frac{1}{2} \left(c_3^{gu \to tZ} \right)^2 \left[\frac{\ln^3(s_4/m^2)}{s_4} \right]_+ \right. \\ &+ \left[\frac{3}{2} c_3^{gu \to tZ} \, c_2^{gu \to tZ} - \frac{\beta_0}{4} c_3^{gu \to tZ} \right] \left[\frac{\ln^2(s_4/m^2)}{s_4} \right]_+ \\ &+ \left[c_3^{gu \to tZ} \, c_1^{gu \to tZ} + (C_F + C_A)^2 \ln^2 \left(\frac{\mu_F^2}{m^2} \right) - 2(C_F + C_A) T_2^{gu \to tZ} \ln \left(\frac{\mu_F^2}{m^2} \right) \right. \\ &+ \left. \frac{\beta_0}{4} c_3^{gu \to tZ} \ln \left(\frac{\mu_R^2}{m^2} \right) - \zeta_2 \left(c_3^{gu \to tZ} \right)^2 \right] \left[\frac{\ln(s_4/m^2)}{s_4} \right]_+ \\ &+ \left[-(C_F + C_A) \ln \left(\frac{\mu_F^2}{m^2} \right) c_1^{gu \to tZ} - \frac{\beta_0}{4} (C_F + C_A) \ln \left(\frac{\mu_F^2}{m^2} \right) \ln \left(\frac{\mu_R^2}{m^2} \right) \right. \\ &+ \left. \left(C_F + C_A \right) \frac{\beta_0}{8} \ln^2 \left(\frac{\mu_F^2}{m^2} \right) - \zeta_2 \, c_2^{gu \to tZ} \, c_3^{gu \to tZ} + \zeta_3 \left(c_3^{gu \to tZ} \right)^2 \right] \left[\frac{1}{s_4} \right]_+ \end{aligned}$$

Figure 2 : $\sqrt{s} = 7$ where the ζ s are the Riemann zeta function of the subscript.

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 $\begin{array}{c} \text{Introduction} \\ gu \rightarrow tZ \\ gu \rightarrow t\gamma \\ gu \rightarrow tg \\ \text{Concluding Remarks} \end{array} \begin{array}{c} \text{Analytical} \\ \sigma \text{ vs. } m \\ \sigma \text{ vs. } \mu \end{array}$

$pp \rightarrow tZ$ - Hadronic Cross Section

The partonic cross sections for NLO(1) and NNLO(2), $\frac{d^2\sigma^{(1,2)}}{dtdu}$, convolute into the hadronic cross sections by:

$$\begin{split} \sigma_{pp \to tZ}^{FCNC} &= \int_{T_{min}}^{T_{max}} dT \int_{-S-T+m^2+m_Z^2}^{m^2+m^2S/(T-m^2)} dU \int_{(m_Z^2-T)/(S+U-m^2)}^{1} dx_b \int_{0}^{x_b(S+U-m^2)+T-m_Z^2} ds_4 \\ &\times \frac{x_a x_b}{x_b S+T-m^2} \phi(x_a) \phi(x_b) \frac{d^2 \hat{\sigma}_{gu \to tZ}^{(1,2)}}{dt du} \end{split}$$

where T, S, and U are the Mandelstam variables for the hadronic process.

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σ vs. m at 7 TeV



Figure 3 : $\sqrt{s} = 7$ TeV cross section for $gu \to tZ$ as a function of mass.

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σ vs. m at 8 TeV



Figure 4 : $\sqrt{s} = 8$ TeV cross section for $gu \to tZ$ as a function of mass.



σ vs. m at 14 TeV



Figure 5 : $\sqrt{s} = 14$ TeV cross section for $gu \to tZ$ as a function of mass.



σ vs. m at NNLO for 7,8, & 14 TeV



Figure 6 : $\sqrt{s} = 7, 8$, & 14 TeV cross sections for $gu \to tZ$ as a function of mass.



σ vs. μ at 7 TeV



Figure 7 : $\sqrt{s} = 7$ TeV cross section for $gu \to tZ$ scale dependence.



σ vs. μ at 8 TeV



Figure 8 : $\sqrt{s} = 8$ TeV cross section for $gu \to tZ$ scale dependence.



σ vs. μ at 14 TeV



Figure 9 : $\sqrt{s} = 14$ TeV cross section for $gu \to tZ$ scale dependence.



σ vs. μ at NNLO for 7,8, & 14 TeV



Figure 10 : $\sqrt{s} = 7, 8, \& 14 \text{ TeV cross sections for } gu \to tZ \text{ as a}$ function of scale.



$$gu \to t\gamma$$



Figure 11 : Tree level diagrams for $gu \to t\gamma$

$$g(p_g) + u(p_u) \to t(p_t) + \gamma(p_\gamma)$$

$$s = (p_g + p_u)^2, \ t = (p_g - p_t)^2, \ u = (p_u - p_t)^2$$

$$s_4 = s + t + u - m^2$$



The Born cross section for $gu \to t\gamma$ is given by:

$$\frac{d^2 \hat{\sigma}_B^{gu \to t\gamma}}{dt du} = \frac{4\pi \alpha \alpha_s \kappa_\gamma (m^2 - s - t) \left[m^6 - m^4 s - 2st^2 + m^2 t(3s + t)\right]}{3m^2 s^3 (m^2 - t^2)^2} \delta(s_4)$$

where $\alpha \& \alpha_s$ are as noted above and $\kappa_{\gamma} = \kappa_{tg\gamma}$



$gu \to t\gamma$ - NLO

The NLO-NLL partonic cross section is given by:

$$\frac{d^2 \hat{\sigma}_{gu \to t\gamma}^{(1)}}{dt du} = F_B^{gu \to t\gamma} \frac{\alpha_s(\mu_R^2)}{\pi} \left(c_3 \left[\frac{\ln(s_4/m^2)}{s_4} \right]_+ + c_2 \left[\frac{1}{s_4} \right]_+ + c_1 \delta(s_4) \right)$$

where

$$c_{1} = \left[C_{F}\ln\left(\frac{-t}{m^{2}}\right) + C_{A}\ln\left(\frac{-u}{m^{2}}\right) - \frac{3}{4}C_{F} - \frac{\beta_{0}}{4}\right]\ln\left(\frac{\mu_{F}^{2}}{m^{2}}\right) + \frac{\beta_{0}}{4}\ln\left(\frac{\mu_{R}^{2}}{m^{2}}\right)$$

$$c_{2} = 2\operatorname{Re}\Gamma_{S}^{(1)} - C_{F} - C_{A} - 2C_{F}\ln\left(\frac{-t}{m^{2}}\right) - 2C_{A}\ln\left(\frac{-u}{m^{2}}\right) - (C_{F} + C_{A})\ln\left(\frac{\mu_{F}^{2}}{s}\right)$$

$$c_{3} = 2(C_{F} + C_{A})$$

Note that this is equivalent to the result for $gu \to tZ$ in the limit of $m_Z^2 \to 0$.

 $\begin{array}{c} \text{Introduction} \\ gu \rightarrow tZ \\ gu \rightarrow t\gamma \\ gu \rightarrow tg \\ \text{Concluding Remarks} \end{array} \begin{array}{c} \text{Analytical} \\ \sigma \text{ vs. } m \\ \sigma \text{ vs. } \mu \end{array}$

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The partonic cross sections for NLO(1) and NNLO(2), $\frac{d^2\sigma^{(1,2)}}{dtdu}$, convolute into the hadronic cross sections by:

$$\begin{split} \sigma_{pp \to t\gamma}^{FCNC} &= \int_{0}^{m^{2}-S} dT \int_{-S-T+m^{2}}^{m^{2}+m^{2}S/(T-m^{2})} dU \int_{-T/(S+U-m^{2})}^{1} dx_{b} \int_{0}^{x_{b}(S+U-m^{2})+T} ds_{4} \\ &\times \frac{x_{a}x_{b}}{x_{b}S+T-m^{2}} \phi(x_{a})\phi(x_{b}) \frac{d^{2}\hat{\sigma}_{gu \to t\gamma}^{(1,2)}}{dtdu} \end{split}$$

where T, S, and U are the Mandelstam variables for the hadronic process.



σ vs. m at 7 TeV



Figure 12 : $\sqrt{s} = 7$ TeV cross section for $gu \to t\gamma$ as a function of mass.



σ vs. m at 8 TeV



Figure 13 : $\sqrt{s} = 8$ TeV cross section for $gu \to t\gamma$ as a function of mass.



σ vs. m at 14 TeV



Figure 14 : $\sqrt{s} = 14$ TeV cross section for $gu \to t\gamma$ as a function of mass.

 $qu \to tZ$ $gu \to t\gamma$ σ vs. mConcluding Remarks

σ vs. m at NNLO for 7,8, & 14 TeV



Figure 15 : $\sqrt{s} = 7, 8, \& 14 \text{ TeV cross sections for } gu \to t\gamma \text{ as a}$ function of mass.



σ vs. μ at 7 TeV



Figure 16 : $\sqrt{s} = 7$ TeV cross section for $gu \to t\gamma$ scale dependence.



σ vs. μ at 8



Figure 17 : $\sqrt{s} = 8$ TeV cross section for $gu \to t\gamma$ scale dependence.



σ vs. μ at 14



Figure 18 : $\sqrt{s} = 14$ TeV cross section for $gu \to t\gamma$ scale dependence.

IntroductionAnalytica $gu \rightarrow tZ$ σ vs. m $gu \rightarrow t\gamma$ σ vs. μ Concluding Remarks σ vs. μ

σ vs. μ at NNLO for 7,8, & 14 TeV



Figure 19 : $\sqrt{s} = 7, 8$, & 14 TeV cross sections for $gu \to t\gamma$ as a function of scale.

Analytical

 $gu \rightarrow tg$ - Tree Level Diagrams

Tree level diagrams for $gu \to tg$ $g(p_{q1}) + u(p_u)$ $\rightarrow t(p_t) + g(p_{g2})$ $s = (p_{q1} + p_u)^2,$ $t = (p_{q1} - p_t)^2$ $u = (p_u - p_t)^2$ $s_4 = s + t + u - m^2$



 $gu \rightarrow tg$ - Progress

Still a work in progress

- Born cross section calculated...but not useable.
- Poles isolated...also not useable.
- Color factors calculated, actually useable!
- Conversion to meaningful representation and numerics to follow soon!

Analytical

Analytical

$gu \rightarrow tg$ - LO

${\rm Mess...}$

$ \int_{\mathbb{R}^{2}} \frac{1}{ $	*	$ \begin{array}{c} t \\ \text{Blagram.} & 3 \mbox{ in pulpercess } 1 \\ & & & & & & & & & & & & & & & & & &$	(* Dispram 9 in subprocess 1 	Diagram 15 in subprocess 1 0 (-1) $(-$
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Analytical

 $gu \rightarrow tg$ - Example Pole Extraction



- 1. Apply Eikonal rules
- 2. Calculate diagram
- 3. ????
- 4. Profit!

Analytical

 $gu \rightarrow tg$ - Example Pole Extraction

Begin with the integral,

$$I_{exp} = g_s^2 \int \frac{d^n k}{(2\pi)^n} \frac{(-i)g^{\mu\nu}}{k^2} \frac{v_i^{\mu}}{v_i \cdot k} \frac{(-v_j^{\nu})}{(-v_j \cdot k)}$$

Feynman parametrize,

$$= -2ig_s^2 \frac{v_i \cdot v_j}{(2\pi)^n} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{(xk^2 + yv_i \cdot k + (1-x-y)v_j \cdot k)^3}$$

Analytical

$gu \rightarrow tg$ - Example Pole Extraction (Cont.)

Integrate over k,

$$I_{exp} = g_s^2 v_i \cdot v_j 2^{6-2n} \pi^{-n/2} \Gamma\left(3 - \frac{n}{2}\right) \int_0^1 x^{3-n} dx \int_0^{1-x} dy$$
$$\times \left(-y^2 v_i^2 - (1 - x - y)^2 v_j^2 - 2y v_i \cdot v_j (1 - x - y)\right)^{n/2 - 3}$$

and then take the other integrals in the $n = 4 - \epsilon$ limit and include the relevant kinematics

$$= -\frac{\alpha_s}{\pi} \ln\left(\frac{\sqrt{2}v_i \cdot v_j}{\sqrt{v_j^2}}\right) \frac{1}{\epsilon}$$

in the case where one of the outgoing quarks is massless

Summary & Conclusions

- We presented NNLO-NLL calculations for $pp \to tZ$ and $pp \to t\gamma$ at 7, 8, and 14 TeV
- We find that the corrections at NLO are large, mostly between 25% & 37%; NNLO corrections were on the order of $\sim {\rm few}\%$
- And scale dependence is only minimally improved by NLO, but is less pronounced at higher energies; NNLO scale dependence is bad at low scale factors, but evens out for around *m* and above.
- Our immediate next step will be the completion of the $gu \rightarrow tg$ cross section
- Afterwards? TBD

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