FCNC Top Quark Production Via Anomalous Couplings

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Effective Lagrangians

Effective Lagrangians

$$\Delta\mathcal{L}_1 = \frac{1}{\Lambda} e \kappa_{tqV} \bar{t} \sigma_{\mu\nu} F_V^{\mu\nu} + h.c., \;\; \Delta\mathcal{L}_2 = \frac{g_s \kappa_{qgt}}{\Lambda} \bar{t} \sigma^{\mu\nu} T^a \chi q G^a_{\mu\nu} + h.c.$$

where:

- Λ is an effective scale, e is the elementary charge and g_s is the strong coupling
- κ_{tqV} & κ_{tqg} are the anomalous couplings
- $\sigma_{\mu\nu} = \frac{i}{2}\gamma_{[\mu}\gamma_{\nu]}$ and γ_μ are the dirac matrices and T^a are the Gell-Mann matrices
- $\chi = f^L P_L + f^R P_R$ where $P_L(P_R)$ is the left(right) hand projection operator
- V is either a Z or a photon
- q is either a c or a u
- $F_V^{\mu\nu}$ is the field tensor for V, $G^a_{\mu\nu}$ is the field tensor for g

LHC as FCNC Probe

The LHC has been used to examine FCNC in the top-quark sector.

- Historically, HERA and Tevatron looked for these processes.
- ATLAS set limits of $\kappa_{ugt}/\Lambda < 6.9 \cdot 10^{-3}\text{TeV}^{-1}$ and $\kappa_{cgt}/\Lambda < 1.5 \cdot 10^{-2}\text{TeV}^{-1}$ and BR$(t \to Zq) < 0.73\%$ [Arxiv:1302.3698v1]
- We note that t-u-Z and t-u-γ dominate
- And charm contributions are small

Soft Gluon Corrections

Scale variations at LO produce large uncertainties, but higher order corrections stabilize the cross sections.

- Soft gluons important since the virtual and real diagrams don't cancel completely
- We define $s_4 = s + t + u - \sum m^2$ and $s_4 \to 0$ threshold
- We consider logarithmic corrections $\left[\frac{\ln^l(s_4/m^2)}{s_4}\right]_+$
- For α_s^n, LL $l = 2n - 1$ and NLL $l = 2n - 2$
- We calculate NLO at NLL

$gu \to tZ$ - Diagram

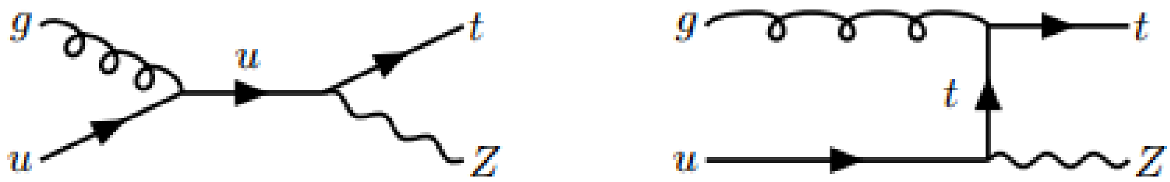


Figure 1 : Tree level diagrams for $gu \to tZ$

$$g(p_g) + u(p_u) \to t(p_t) + Z(p_Z)$$

$$s = (p_g + p_u)^2,\ t = (p_g - p_t)^2,\ u = (p_u - p_t)^2$$

$$s_4 = s + t + u - m^2 - m_Z^2$$

$gu \to tZ$ - LO

The Born cross section for $gu \to tZ$ is given by:

$$\frac{d^2\hat{\sigma}_B^{gu\to tZ}}{dtdu}$$
$$= \frac{2\pi\alpha\alpha_s\kappa_Z}{3m^2s^3(m^2-t^2)^2}\{2m^8 - m^6(2m_Z^2+4s+2t)$$
$$-t\left[2m_Z^6-2m_Z^4(s+t)-4st(s+t)+m_Z^2(s+t)\right]$$
$$+m^4\left[2m_Z^4-m_Z(2s+t)+2(s^2+4st+t^2)\right]$$
$$+m^2\left[2m_Z^6-4m_Z^4t+m_Z^2(s+t)(s+5t)-2t(2s^2+6t+t^2)\right]$$

$gu \to tZ$ - NLO

The NLO-NLL partonic cross section is given by:

$$\frac{d^2\hat{\sigma}^{(1)}_{gu\to tZ}}{dt\,du} = F_B^{gu\to tZ}\frac{\alpha_s(\mu_R^2)}{\pi}\left(c_3\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c_2\left[\frac{1}{s_4}\right]_+ + c_1\delta(s_4)\right)$$

where

$$c_1 = \left[C_F\ln\left(\frac{-t+m_Z^2}{m^2}\right) + C_A\ln\left(\frac{-u+m_Z^2}{m^2}\right) - \frac{3}{4}C_F - \frac{\beta_0}{4}\right]\ln\left(\frac{\mu_F^2}{m^2}\right) + \frac{\beta_0}{4}\ln\left(\frac{\mu_R^2}{m^2}\right)$$

$$c_2 = 2\mathrm{Re}\Gamma_S^{(1)} - C_F - C_A - 2C_F\ln\left(\frac{-t+m_Z^2}{m^2}\right) - 2C_A\ln\left(\frac{-u+m_Z^2}{m^2}\right) - (C_F+C_A)\ln\left(\frac{\mu_F^2}{s}\right)$$

$$c_3 = 2(C_F + C_A)$$

and $C_F = (N_c^2-1)/(2N_c)$, $C_A = N_c$, $\beta_0 = (11C_A - 2n_f)/3$.
n_f is the number of light quark flavors and N_c the number of colors

$gu \to tZ$ - NNLO

The NNLO-NLL partonic cross section is given by:

$$\frac{d^2\hat{\sigma}^{(2)}_{gu\to tZ}}{dt\,du} = F_B^{gu\to tZ}\frac{\alpha_s^2(\mu_R^2)}{\pi^2}\left\{\frac{1}{2}\left(c_3^{gu\to tZ}\right)^2\left[\frac{\ln^3(s_4/m^2)}{s_4}\right]_+ \right.$$
$$+\left[\frac{3}{2}c_3^{gu\to tZ}\,c_2^{gu\to tZ}-\frac{\beta_0}{4}c_3^{gu\to tZ}\right]\left[\frac{\ln^2(s_4/m^2)}{s_4}\right]_+$$
$$+\left[c_3^{gu\to tZ}\,c_1^{gu\to tZ}+(C_F+C_A)^2\ln^2\left(\frac{\mu_F^2}{m^2}\right)-2(C_F+C_A)T_2^{gu\to tZ}\ln\left(\frac{\mu_F^2}{m^2}\right)\right.$$
$$\left.+\frac{\beta_0}{4}c_3^{gu\to tZ}\ln\left(\frac{\mu_R^2}{m^2}\right)-\zeta_2\left(c_3^{gu\to tZ}\right)^2\right]\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+$$
$$+\left[-(C_F+C_A)\ln\left(\frac{\mu_F^2}{m^2}\right)c_1^{gu\to tZ}-\frac{\beta_0}{4}(C_F+C_A)\ln\left(\frac{\mu_F^2}{m^2}\right)\ln\left(\frac{\mu_R^2}{m^2}\right)\right.$$
$$\left.\left.+(C_F+C_A)\frac{\beta_0}{8}\ln^2\left(\frac{\mu_F^2}{m^2}\right)-\zeta_2\,c_2^{gu\to tZ}\,c_3^{gu\to tZ}+\zeta_3\left(c_3^{gu\to tZ}\right)^2\right]\left[\frac{1}{s_4}\right]_+\right\}$$

Figure 2 : $\sqrt{s}=7$ where the ζs are the Riemann zeta function of the subscript.

$pp \to tZ$ - Hadronic Cross Section

The partonic cross sections for NLO(1) and NNLO(2), $\frac{d^2\sigma^{(1,2)}}{dtdu}$, convolute into the hadronic cross sections by:

$$\sigma^{FCNC}_{pp\to tZ} = \int_{T_{min}}^{T_{max}} dT \int_{-S-T+m^2+m_Z^2}^{m^2+m^2 S/(T-m^2)} dU \int_{(m_Z^2-T)/(S+U-m^2)}^{1} dx_b \int_0^{x_b(S+U-m^2)+T-m_Z^2} ds_4$$
$$\times \frac{x_a x_b}{x_b S + T - m^2} \phi(x_a)\phi(x_b) \frac{d^2\hat{\sigma}^{(1,2)}_{gu\to tZ}}{dtdu}$$

where T, S, and U are the Mandelstam variables for the hadronic process.

σ vs. m at 7 TeV

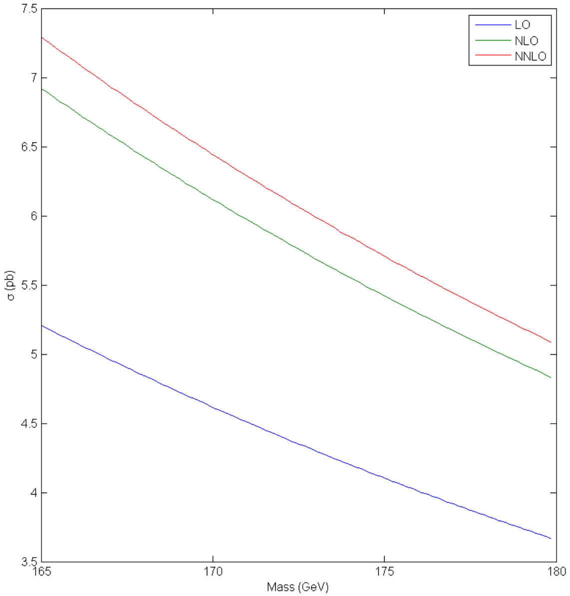


Figure 3 : $\sqrt{s} = 7$ TeV cross section for $gu \to tZ$ as a function of mass.

σ vs. m at 8 TeV

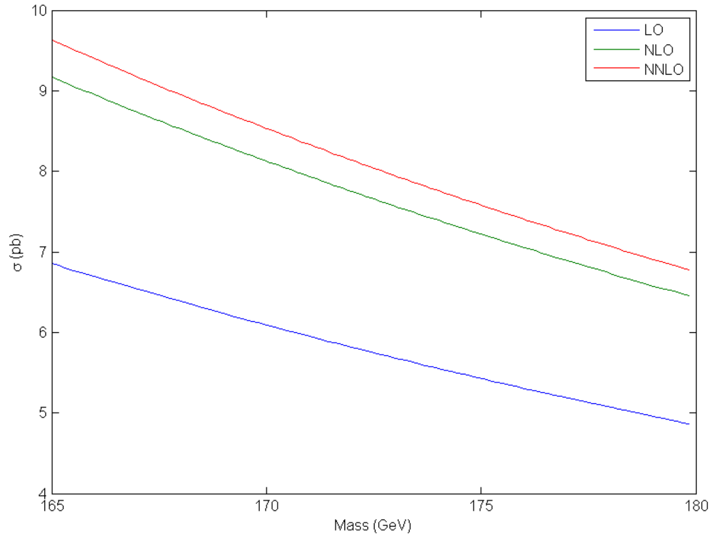


Figure 4 : $\sqrt{s} = 8$ TeV cross section for $gu \to tZ$ as a function of mass.

σ vs. m at 14 TeV

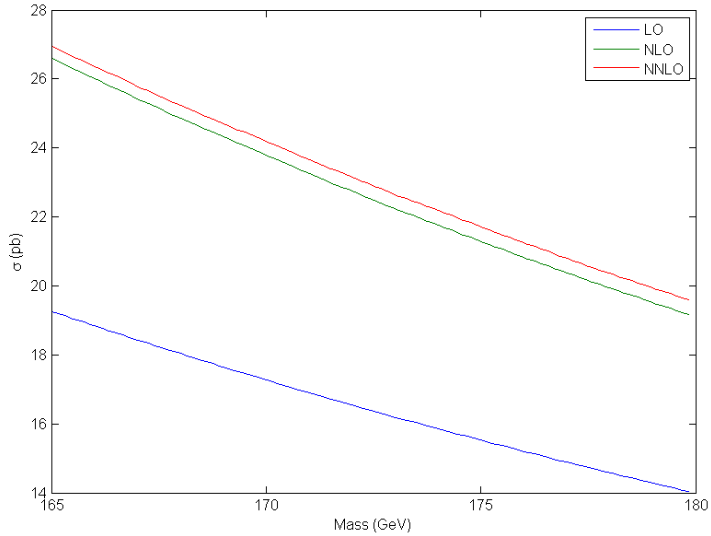


Figure 5 : $\sqrt{s} = 14$ TeV cross section for $gu \to tZ$ as a function of mass.

σ vs. m at NNLO for 7,8, & 14 TeV

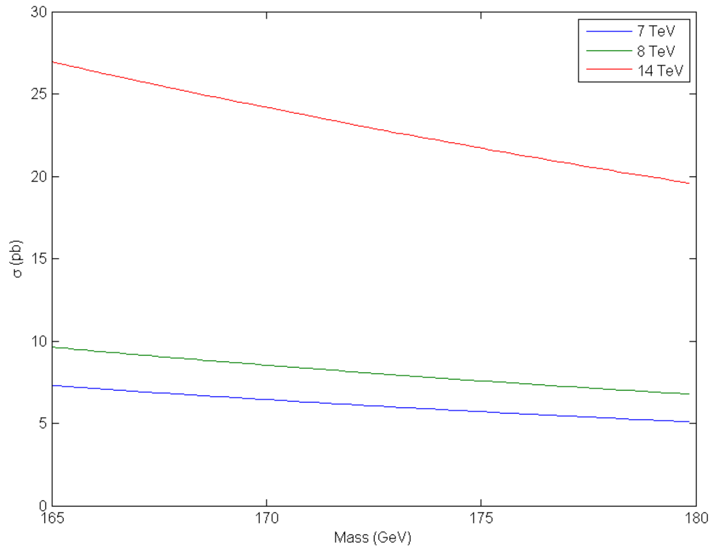


Figure 6 : $\sqrt{s} = 7, 8,$ & 14 TeV cross sections for $gu \to tZ$ as a function of mass.

σ vs. μ at 7 TeV

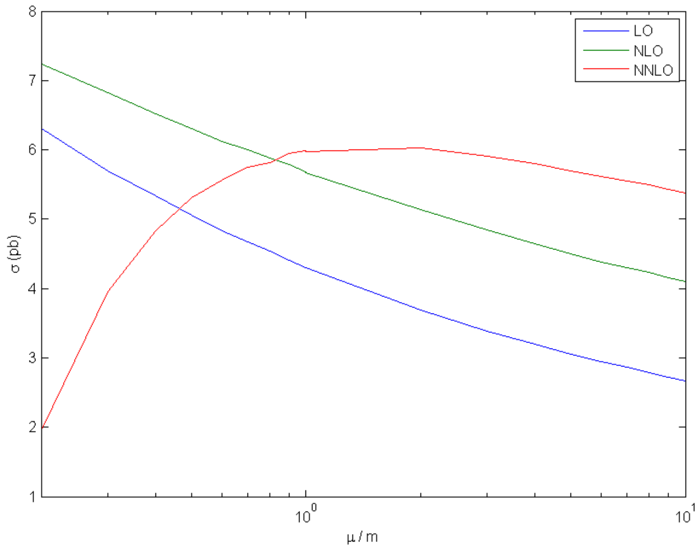


Figure 7 : $\sqrt{s} = 7$ TeV cross section for $gu \to tZ$ scale dependence.

σ vs. μ at 8 TeV

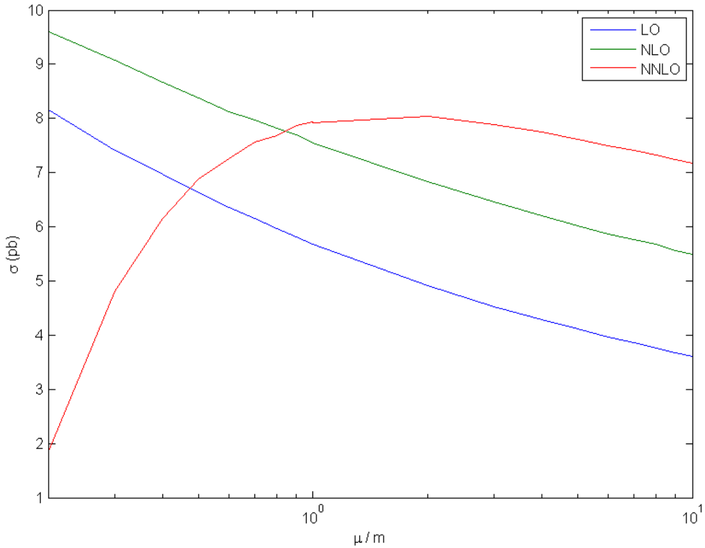


Figure 8 : $\sqrt{s} = 8$ TeV cross section for $gu \to tZ$ scale dependence.

σ vs. μ at 14 TeV

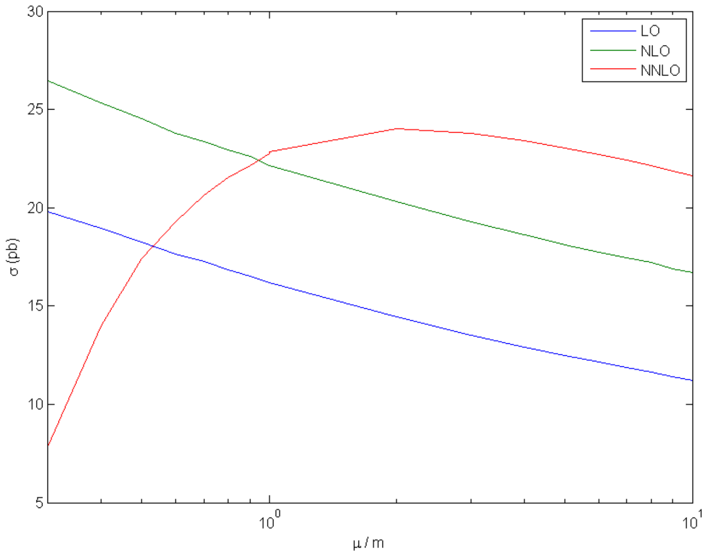


Figure 9 : $\sqrt{s} = 14$ TeV cross section for $gu \to tZ$ scale dependence.

σ vs. μ at NNLO for 7,8, & 14 TeV

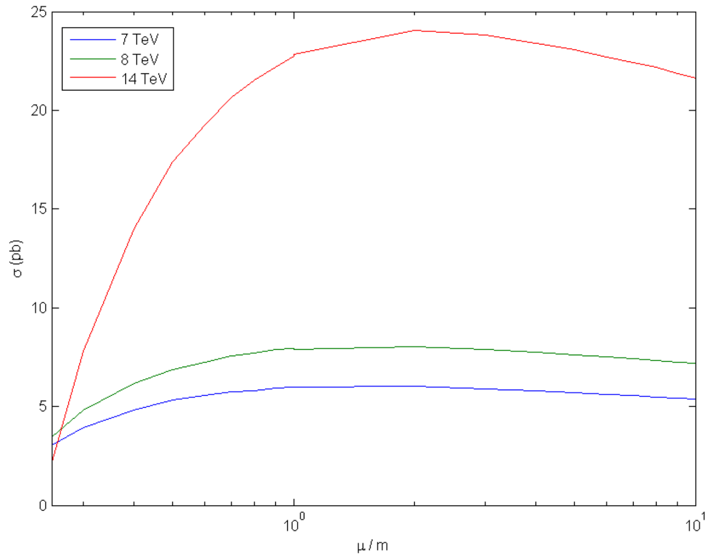


Figure 10 : $\sqrt{s} = 7, 8,$ & 14 TeV cross sections for $gu \to tZ$ as a function of scale.

$gu \to t\gamma$

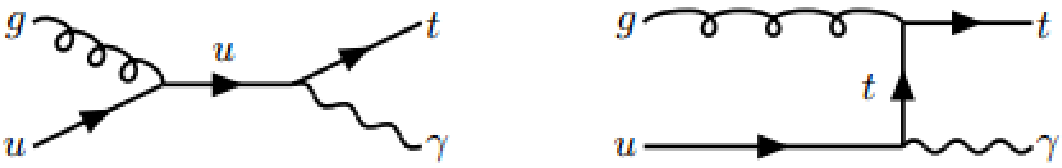


Figure 11 : Tree level diagrams for $gu \to t\gamma$

$$g(p_g) + u(p_u) \to t(p_t) + \gamma(p_\gamma)$$

$$s = (p_g + p_u)^2,\ t = (p_g - p_t)^2,\ u = (p_u - p_t)^2$$

$$s_4 = s + t + u - m^2$$

$gu \to t\gamma$ - LO

The Born cross section for $gu \to t\gamma$ is given by:

$$\frac{d^2\hat{\sigma}_B^{gu\to t\gamma}}{dt\,du} = \frac{4\pi\alpha\alpha_s\kappa_\gamma(m^2-s-t)\left[m^6-m^4s-2st^2+m^2t(3s+t)\right]}{3m^2s^3(m^2-t^2)^2}\delta(s_4)$$

where α & α_s are as noted above and $\kappa_\gamma = \kappa_{tg\gamma}$

$gu \to t\gamma$ - NLO

The NLO-NLL partonic cross section is given by:

$$\frac{d^2\hat{\sigma}^{(1)}_{gu\to t\gamma}}{dt\,du} = F_B^{gu\to t\gamma}\frac{\alpha_s(\mu_R^2)}{\pi}\left(c_3\left[\frac{\ln(s_4/m^2)}{s_4}\right]_+ + c_2\left[\frac{1}{s_4}\right]_+ + c_1\delta(s_4)\right)$$

where

$$c_1 = \left[C_F \ln\left(\frac{-t}{m^2}\right) + C_A \ln\left(\frac{-u}{m^2}\right) - \frac{3}{4}C_F - \frac{\beta_0}{4}\right]\ln\left(\frac{\mu_F^2}{m^2}\right) + \frac{\beta_0}{4}\ln\left(\frac{\mu_R^2}{m^2}\right)$$

$$c_2 = 2\mathrm{Re}\Gamma_S^{(1)} - C_F - C_A - 2C_F\ln\left(\frac{-t}{m^2}\right) - 2C_A\ln\left(\frac{-u}{m^2}\right) - (C_F + C_A)\ln\left(\frac{\mu_F^2}{s}\right)$$

$$c_3 = 2(C_F + C_A)$$

Note that this is equivalent to the result for $gu \to tZ$ in the limit of $m_Z^2 \to 0$.

$pp \to t\gamma$ - Hadronic Cross Section

The partonic cross sections for NLO(1) and NNLO(2), $\frac{d^2\sigma^{(1,2)}}{dtdu}$, convolute into the hadronic cross sections by:

$$\sigma^{FCNC}_{pp\to t\gamma} = \int_0^{m^2-S} dT \int_{-S-T+m^2}^{m^2+m^2S/(T-m^2)} dU \int_{-T/(S+U-m^2)}^{1} dx_b \int_0^{x_b(S+U-m^2)+T} ds_4$$
$$\times \frac{x_a x_b}{x_b S + T - m^2} \phi(x_a)\phi(x_b) \frac{d^2\hat{\sigma}^{(1,2)}_{gu\to t\gamma}}{dtdu}$$

where T, S, and U are the Mandelstam variables for the hadronic process.

σ vs. m at 7 TeV

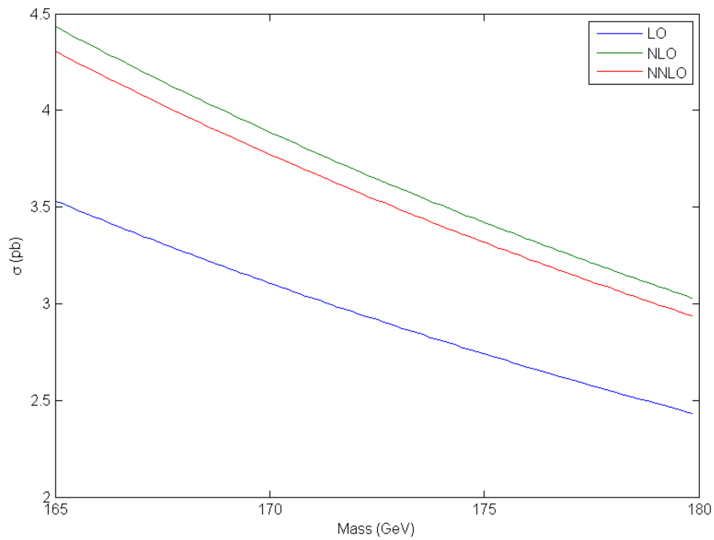


Figure 12 : $\sqrt{s} = 7$ TeV cross section for $gu \to t\gamma$ as a function of mass.

σ vs. m at 8 TeV

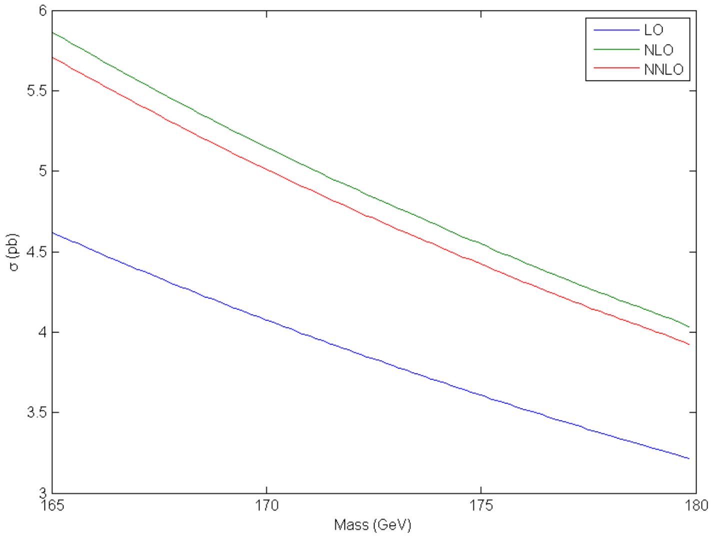


Figure 13 : $\sqrt{s} = 8$ TeV cross section for $gu \to t\gamma$ as a function of mass.

σ vs. m at 14 TeV

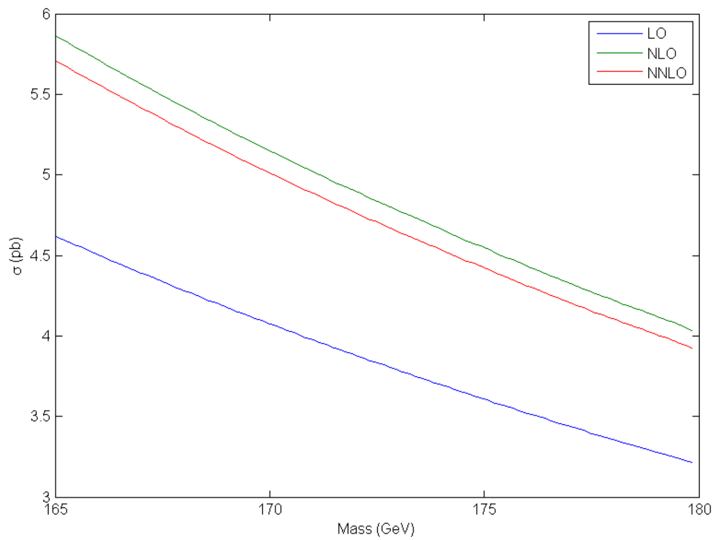


Figure 14 : $\sqrt{s} = 14$TeV cross section for $gu \to t\gamma$ as a function of mass.

σ vs. m at NNLO for 7,8, & 14 TeV

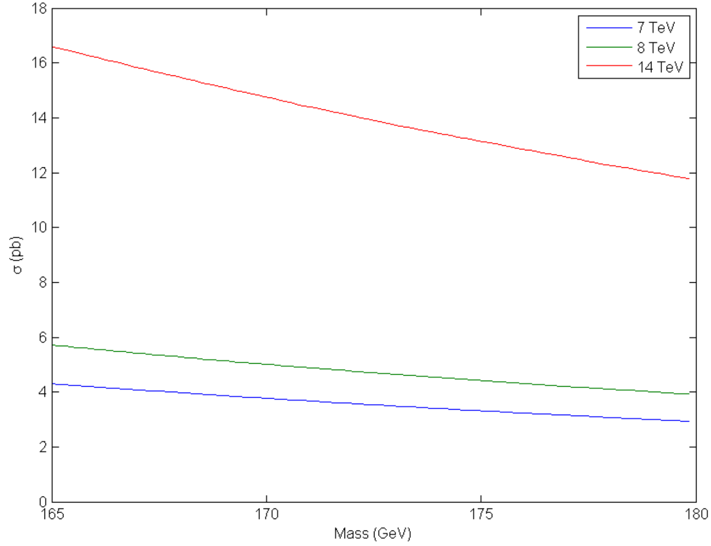


Figure 15 : $\sqrt{s} = 7, 8,$ & 14 TeV cross sections for $gu \to t\gamma$ as a function of mass.

σ vs. μ at 7 TeV

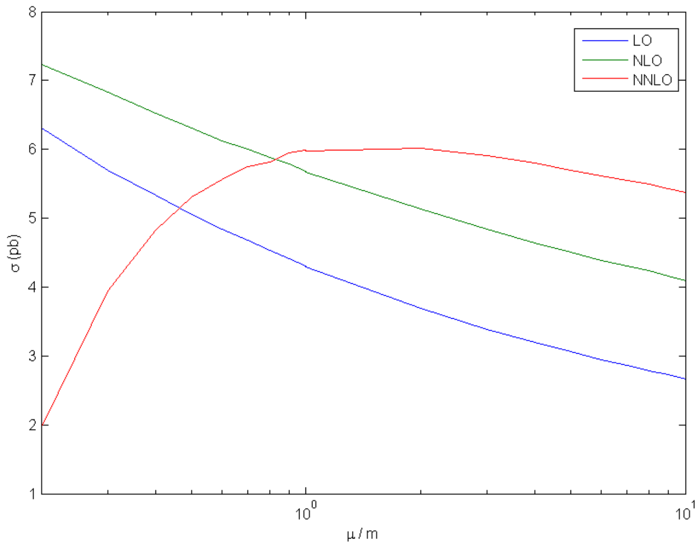


Figure 16 : $\sqrt{s} = 7$ TeV cross section for $gu \to t\gamma$ scale dependence.

σ vs. μ at 8

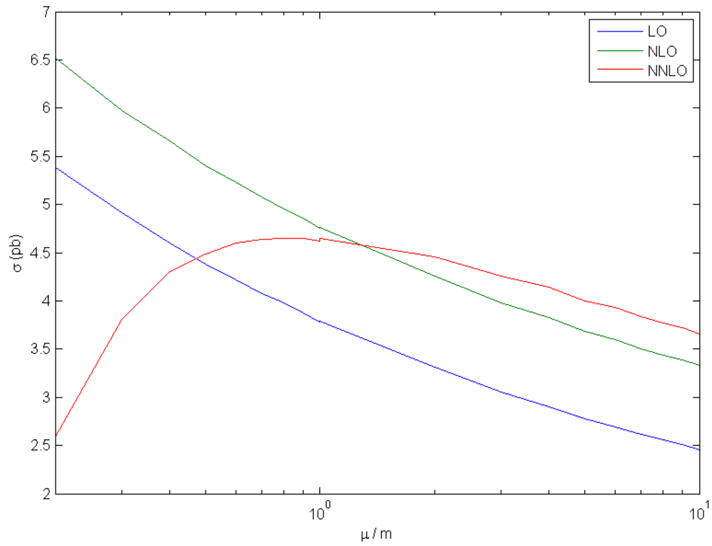


Figure 17 : $\sqrt{s} = 8$ TeV cross section for $gu \to t\gamma$ scale dependence.

σ vs. μ at 14

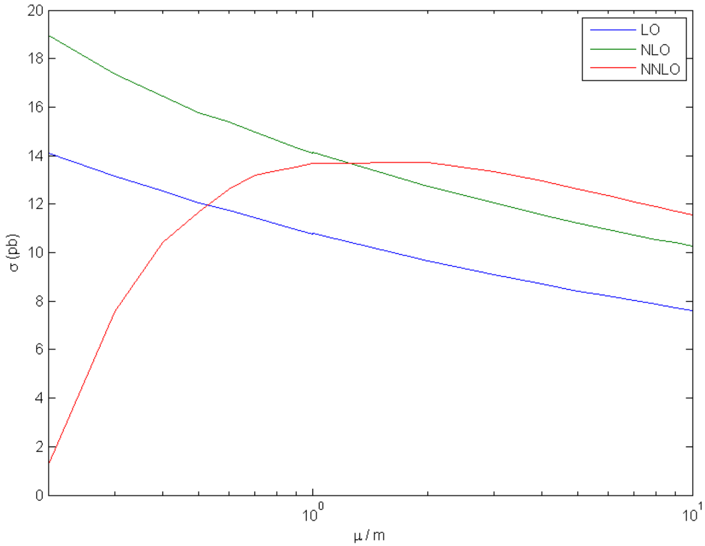


Figure 18 : $\sqrt{s} = 14$ TeV cross section for $gu \to t\gamma$ scale dependence.

σ vs. μ at NNLO for 7,8, & 14 TeV

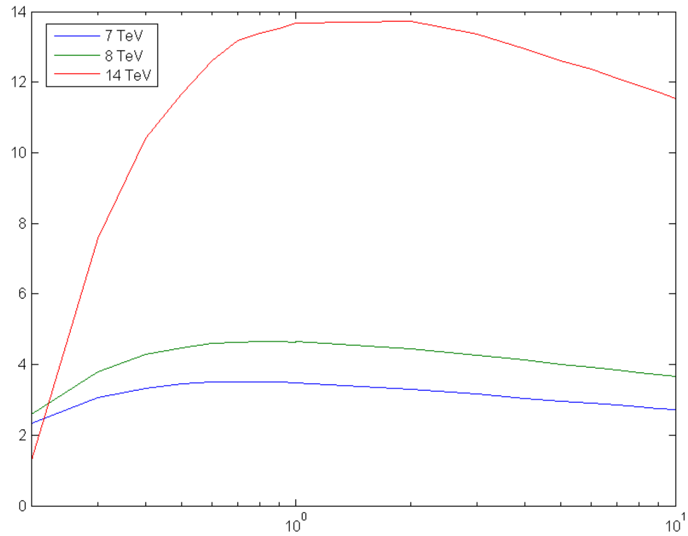


Figure 19 : $\sqrt{s} = 7, 8,$ & 14 TeV cross sections for $gu \to t\gamma$ as a function of scale.

$gu \to tg$ - Tree Level Diagrams

Tree level diagrams for $gu \to tg$

$$g(p_{g1}) + u(p_u)$$

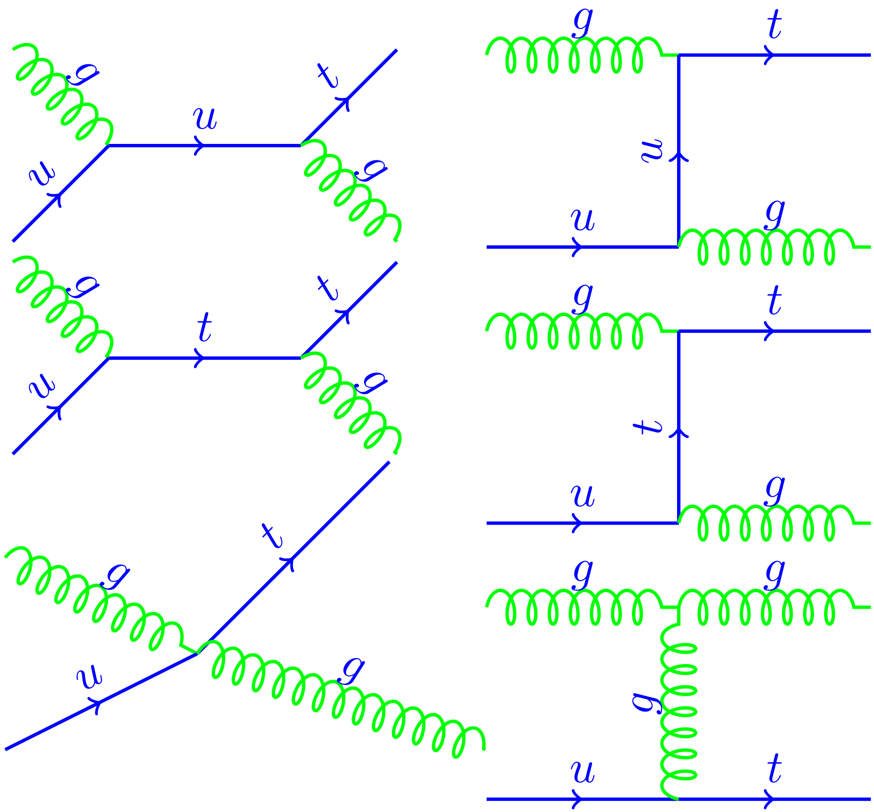
$$\to t(p_t) + g(p_{g2})$$

$$s = (p_{g1} + p_u)^2,$$

$$t = (p_{g1} - p_t)^2$$

$$u = (p_u - p_t)^2$$

$$s_4 = s + t + u - m^2$$



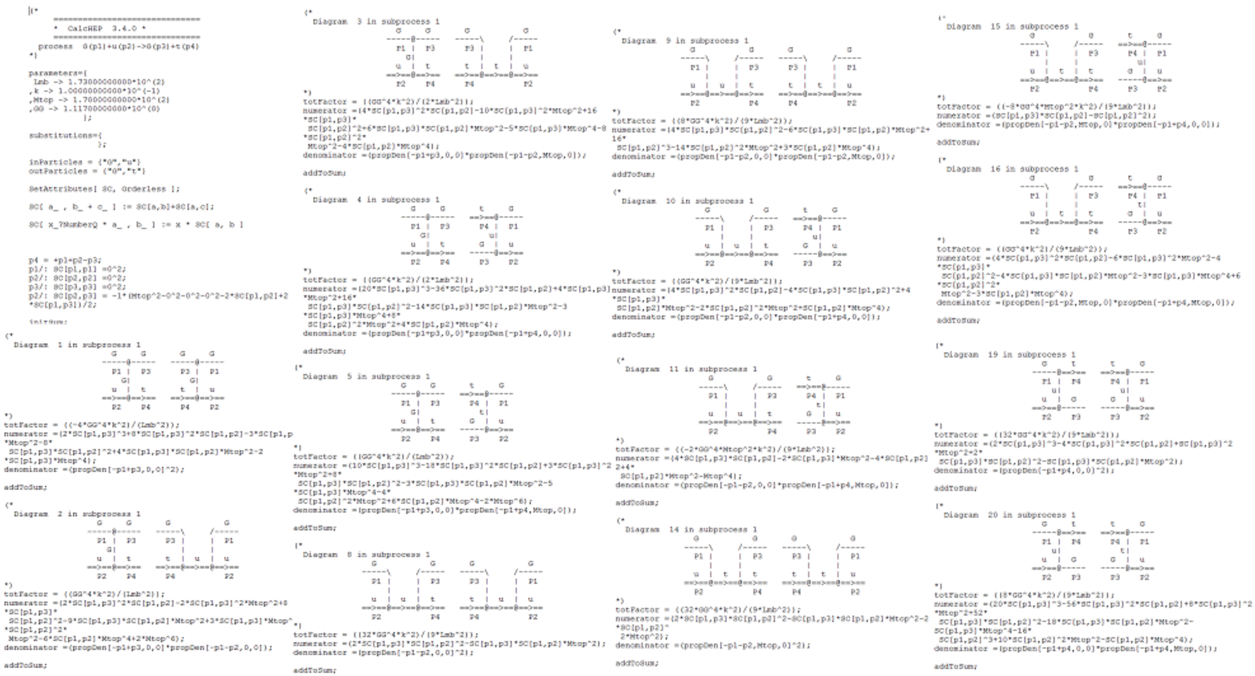
$gu \to tg$ - Progress

Still a work in progress

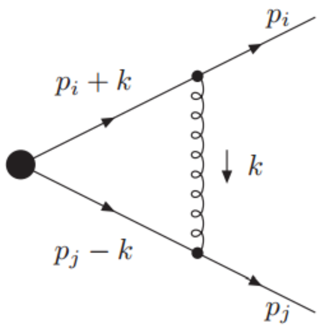
- Born cross section calculated...but not useable.
- Poles isolated...also not useable.
- Color factors calculated, actually useable!
- Conversion to meaningful representation and numerics to follow soon!

$gu \to tg$ - LO

Mess...



$gu \to tg$ - Example Pole Extraction



- 1. Apply Eikonal rules
- 2. Calculate diagram
- 3. ????
- 4. Profit!

$gu \to tg$ - Example Pole Extraction

Begin with the integral,

$$I_{exp} = g_s^2 \int \frac{d^n k}{(2\pi)^n} \frac{(-i)g^{\mu\nu}}{k^2} \frac{v_i^\mu}{v_i \cdot k} \frac{(-v_j^\nu)}{(-v_j \cdot k)}$$

Feynman parametrize,

$$= -2ig_s^2 \frac{v_i \cdot v_j}{(2\pi)^n} \int_0^1 dx \int_0^{1-x} dy \int \frac{d^n k}{(xk^2 + yv_i \cdot k + (1-x-y)v_j \cdot k)^3}$$

$gu \to tg$ - Example Pole Extraction (Cont.)

Integrate over k,

$$I_{exp} = g_s^2 v_i \cdot v_j 2^{6-2n} \pi^{-n/2} \Gamma\left(3 - \frac{n}{2}\right) \int_0^1 x^{3-n} dx \int_0^{1-x} dy$$
$$\times \left(-y^2 v_i^2 - (1-x-y)^2 v_j^2 - 2y v_i \cdot v_j (1-x-y)\right)^{n/2-3}$$

and then take the other integrals in the $n = 4 - \epsilon$ limit and include the relevant kinematics

$$= -\frac{\alpha_s}{\pi} \ln\left(\frac{\sqrt{2} v_i \cdot v_j}{\sqrt{v_j^2}}\right) \frac{1}{\epsilon}$$

in the case where one of the outgoing quarks is massless

Summary & Conclusions

- We presented NNLO-NLL calculations for $pp \to tZ$ and $pp \to t\gamma$ at 7, 8, and 14 TeV
- We find that the corrections at NLO are large, mostly between 25% & 37%; NNLO corrections were on the order of $\sim$ few%
- And scale dependence is only minimally improved by NLO, but is less pronounced at higher energies; NNLO scale dependence is bad at low scale factors, but evens out for around m and above.
- Our immediate next step will be the completion of the $gu \to tg$ cross section
- Afterwards? TBD

Acknowledgements

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