Charge and Colour Breaking Constraints in the MSSM

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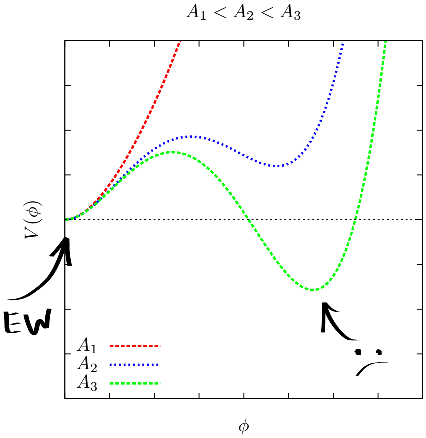
Supersymmetry and Stability

- Supersymmetry is good: naturalness, gauge unification, dark matter
- SM fermions have charged and colored scalar partners ⇒ more complicated scalar potential
- Quantum tunneling can destabilize the electroweak vacuum

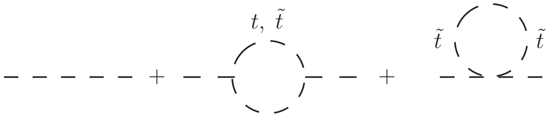


Higgs Mass in the MSSM

LHC measured $m_h \approx 126$ GeV

- Strong constraint on the MSSM, since at tree level $\lambda \sim g^2 + g'^2$

$$m_h^2 \;=\; m_Z^2 \cos^2 2\beta \;+\; \text{(s)tops!} \;+\; \cdots$$



- Loop corrections needed to bring m_h up to physical value, depend (primarily) on stop parameters

Higgs Mass in the MSSM

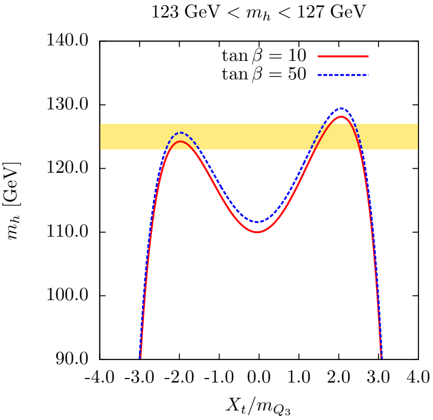
Stop mass matrix:

$$M_{\tilde{t}}^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + \Delta_{u_L} & m_t X_t \\ m_t X_t^* & m_{u_3}^2 + m_t^2 + \Delta_{u_R} \end{pmatrix}$$

- X_t - stop mixing parameter
- $M_S = (m_{Q_3} m_{u_3})^{1/2}$ - SUSY scale
- Fine-tuning minimized for light stops, i. e. small M_S

Hall, Pinner & Ruderman JHEP **1204**

Draper, Meade, Reece & Shih PRD**85**



Supersymmetric Scalar Potential

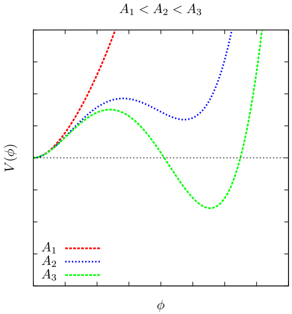
Stop mixing

$$X_t = A_t^* - \mu/\tan\beta \approx A_t^*$$

A_t is the cubic coupling in the potential:

$$V \supset A_t \tilde{t}_R^\dagger \tilde{t}_L H_u^0 + \text{h.c.}$$

Light stops $\Rightarrow$ Large mixing X_t $\Rightarrow$ Potentially destabilized EW vacuum



Charge and Colour Breaking (CCB) Minima

- Electroweak (EW) vacuum: $\langle H_u^0,\, H_d^0 \rangle \neq 0$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to SU(3)_C \times U(1)_{\text{EM}}$$

- Large $A_t \Rightarrow \langle H_u^0,\, H_d^0,\, \tilde{t}_L,\, \tilde{t}_R \rangle \neq 0$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \to$$ 

Global minimum of the theory (true groundstate) in general breaks $SU(3)_C$ and $U(1)_{\text{EM}}$

$$\Downarrow$$

Can we exclude parameters that generate a shallow EW and global CCB minima?

Fate of the False Vacuum

Can we exclude parameters that generate a shallow EW and global CCB minima?

Not if the EW vacuum is metastable:

$$\tau_{\mathrm{EW}} > t_0 \sim 10^{10} \text{ years}$$

- Lifetime is determined by the rate of quantum tunneling Γ.
- Unstable state $\Rightarrow$ energy acquires imaginary part such that

$$\Gamma = -\frac{2}{\hbar} \mathrm{Im} E_{\mathrm{EW}}$$

Vacuum Decay Rate

Decay rate per unit volume:

$$\Gamma/V = C \exp(-S_E[\bar{\phi}]/\hbar),$$

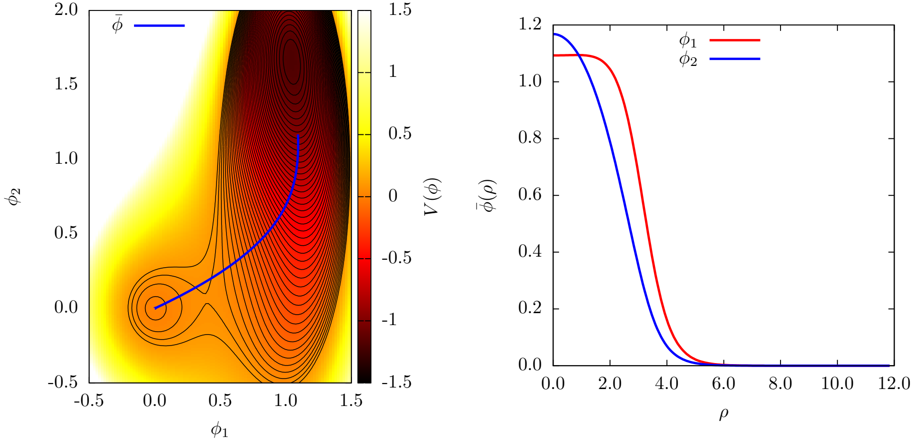
Metastability requires $\Gamma^{-1} > t_0$

$$\boxed{S_E[\bar{\phi}]/\hbar > \log(t_0^4 C) \approx 400}$$

Computing $S_E[\bar{\phi}]$: Coleman PRD **15**, Coleman & Callan PRD **16**

- Single field - shooting method (special to $1D$ boundary value problems) ✓
- Multiple fields:
  1. Path deformation - implemented in CosmoTransitions (by Max Wainwright at UCSC) ✓
  2. Constrained or improved potential with dimensional deformation - algorithms outlined in Konstandin & Huber JCAP 0606 and Park JCAP 1102 - in progress.

The Bounce



$\bar{\phi}$
2.0
1.5
1.0
0.5
0.0
-0.5
ϕ_2
-0.5
0.0
0.5
1.0
1.5
ϕ_1
1.5
1
0.5
0
-0.5
-1
-1.5
$V(\phi)$
ϕ_1
ϕ_2
1.2
1.0
0.8
0.6
0.4
0.2
0.0
$\bar{\phi}(\rho)$
0.0
2.0
4.0
6.0
8.0
10.0
12.0
ρ

Previous Stability Constraints

Analytic: Kounnas, Lahanas & Nanopoulos NPB **236**

- Assume VEVs are all equal

$$\langle H_u^0 \rangle = \langle \tilde{t}_L \rangle = \langle \tilde{t}_R \rangle$$

  Potential now is a function of 1 VEV, easy to minimize by hand

- Demand *absolute* stability (SM-like minimum is global):

$$V_{\mathrm{CCB}} > V_{\mathrm{SML}}$$

$$\Downarrow$$

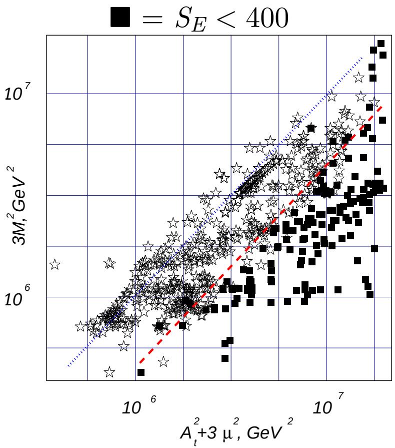
$$A_t^2 < 3(m_2^2 + m_{Q_3}^2 + m_{\tilde{t}_R}^2)$$

This is neither necessary nor sufficient. More sophisticated analyses by Casas, Lleyda & Muñoz: NPB **471**, PLB **380**, PLB **389**

Numeric

- Scan MSSM parameters
- If $\exists$ global CCB minimum, find $S_E[\bar{\phi}]$
- If $S_E < 400$ parameters are excluded
- *Empirical* inequality:

$$A_t^2 + 3\mu^2 < 7.5(m_{Q_3}^2 + m_{\tilde{t}_R}^2)$$

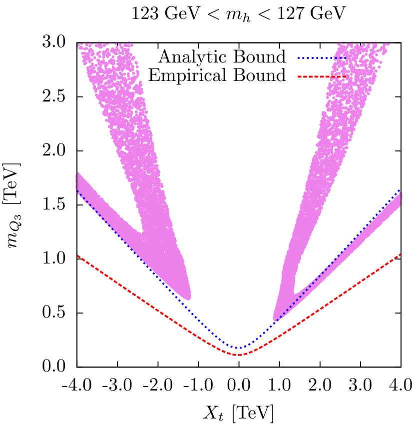


Kusenko, Langacker & Segre PRD **54**

Previous Metastability Constraints

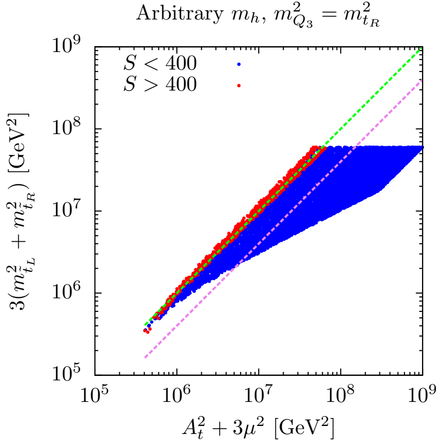
Why do another analysis?

- m_h has been measured. What does metastability imply for the Higgs parameter space?
- Bounds on stop parameters for direct (LHC) and indirect searches ($b \to s\gamma$, ...)
- Loop corrections should be included
- More reliable numerics



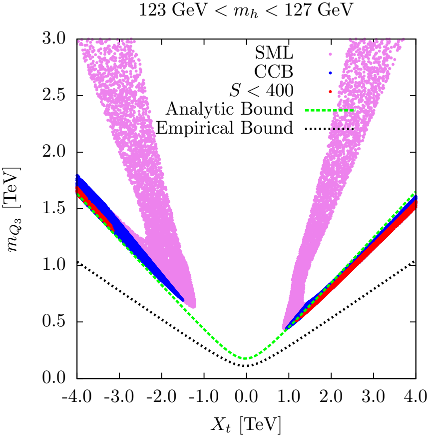
Preliminary Results - No Higgs Mass Constraint

- ● - SM metastable, ● - SM unstable
- Empirical bound completely invalid
- Analytic result surprisingly robust (except for some extreme values of parameters)



Preliminary Results - Higgs

- • - SM absolutely stable, • - SM metastable, • - SM unstable
- CCB minima appear for $|X_t| \gtrsim 1$ TeV
- Most CCB points $X_t \gtrsim 1$ TeV not metastable ⇒ excluded



Conclusion

Summary:

- Large values of the stop cubic term A_t lead to appearance of CCB minima
- Models with global CCB minima ruled out if lifetime of SM-like vacuum too short
- Metastability constrains the Higgs parameter space in the MSSM

To do:

- Recompute bounce using independent method
- Include quantum corrections

Backup

Fate of the False Vacuum

- E_{EW} extracted from the matrix element

$$\langle\phi_+|\exp(-HT/\hbar)|\phi_+\rangle = \int[\mathscr{D}\phi]\exp(-S_E[\phi]/\hbar)$$

  ϕ_+ is the false vacuum.

- RHS evaluated semi-classically by expanding

$$S_E[\phi] = S_E[\bar{\phi}] + \frac{1}{2}(\phi-\bar{\phi})\frac{\delta^2 S_E}{\delta\phi^2}(\phi-\bar{\phi}) + \ldots$$

- $\bar{\phi}$ is a classical solution such that

$$\frac{\delta S_E}{\delta\phi}[\bar{\phi}] = 0 \;\Rightarrow\; \partial^2\phi = U'(\phi),\; \mathrm{BCs}:\; \lim_{t,|\vec{x}|\to\pm\infty}\bar{\phi}(t,\vec{x}) = \phi_+.$$

Coleman PRD **15**, Coleman & Callan PRD **16**

Pre-exponential Factor

- Performing the path integral gives

$$\Gamma/V = C\exp(-S_E[\bar{\phi}]/\hbar),$$

where

$$C = \left(\frac{S_E[\bar{\phi}]}{2\pi}\right)^2 \left|\frac{\det'\left[-\partial^2 + U''(\bar{\phi})\right]}{\det\left[-\partial^2 + U''(\phi_+)\right]}\right|^{-1/2}$$

- $\det'$ omits translational zero modes
- Prefactor usually estimated as

$$[C] = M^4 \;\Rightarrow C \approx (100\text{ GeV})^4$$

- Numerical computation of the prefactor described in

Min JPA **39**, Dunne & Min PRD **72**

Loop Corrections

Groundstate of the quantum theory given by the minimum of the effective potenial (in $\overline{\mathrm{DR}}$)

$$V_{\mathrm{eff}}(Q) = V_0(Q) + \Delta V_1(Q),\ \Delta V_1(Q)\frac{1}{64\pi^2}\mathrm{Str}\left[\mathscr{M}^4\left(\ln\frac{\mathscr{M}^2}{Q^2} - \frac{3}{2}\right)\right]$$

- Typical approach: choose Q s. t. $\Delta V_1 \approx 0 \Rightarrow$ log corrections reabsorbed into running couplings in V_0.
- Issue 1: $V_0(Q)$ is now very sensitive to choice of Q: Gamberini, Ridolfi & Zwirner NPB **331**
- Issue 2: $V_{\mathrm{eff}}(Q)$ is gauge-dependent: Patel & Ramsey-Musolf JHEP **1107**