Charge and Colour Breaking Constraints in the MSSM

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Supersymmetry and Stability

- Supersymmetry is good: naturalness, gauge unification, dark matter
- SM fermions have charged and colored scalar partners $\Rightarrow$ more complicated scalar potential
- Quantum tunneling can destabilize the electroweak vacuum
LHC measured $m_h \approx 126$ GeV

- Strong constraint on the MSSM, since at tree level $\lambda \sim g^2 + g'^2$

$$m_h^2 = m_Z^2 \cos^2 2\beta + (s)\text{tops}! + \cdots$$

- Loop corrections needed to bring $m_h$ up to physical value, depend (primarily) on stop parameters
Higgs Mass in the MSSM

Stop mass matrix:

\[
M_t^2 = \begin{pmatrix} m_{Q_3}^2 + m_t^2 + \Delta_{uL} \\ m_tX_t^* \\ m_{u_3}^2 + m_t^2 + \Delta_{uR} \end{pmatrix}
\]

- \( X_t \) - stop mixing parameter
- \( M_S = (m_{Q_3}m_{u_3})^{1/2} \) - SUSY scale
- Fine-tuning minimized for light stops, i.e. small \( M_S \)

Hall, Pinner & Ruderman JHEP 1204
Draper, Meade, Reece & Shih PRD 85

123 GeV < \( m_h \) < 127 GeV

\( \tan \beta = 10 \)
\( \tan \beta = 50 \)
Supersymmetric Scalar Potential

Stop mixing

\[ X_t = A_t^* - \mu / \tan \beta \approx A_t^* \]

\( A_t \) is the cubic coupling in the potential:

\[ V \supset A_t \tilde{t}_R^* \tilde{t}_L H_u^0 + \text{h.c.} \]

Light stops \( \Rightarrow \) Large mixing \( X_t \Rightarrow \) Potentially destabilized EW vacuum
Charge and Colour Breaking (CCB) Minima

- Electroweak (EW) vacuum: \( \langle H^0_u, H^0_d \rangle \neq 0 \)

\[
SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow SU(3)_C \times U(1)_{EM}
\]

- Large \( A_t \Rightarrow \langle H^0_u, H^0_d, \tilde{t}_L, \tilde{t}_R \rangle \neq 0 \)

\[
SU(3)_C \times SU(2)_L \times U(1)_Y \rightarrow \left[ \begin{array}{c}
\end{array} \right]
\]

Global minimum of the theory (true groundstate) in general breaks \( SU(3)_C \) and \( U(1)_{EM} \)

\[
\downarrow
\]

Can we exclude parameters that generate a shallow EW and global CCB minima?
Fate of the False Vacuum

Can we exclude parameters that generate a shallow EW and global CCB minima?

Not if the EW vacuum is metastable:

\[ \tau_{\text{EW}} > t_0 \sim 10^{10} \text{ years} \]

- Lifetime is determined by the rate of quantum tunneling \( \Gamma \).
- Unstable state \( \Rightarrow \) energy acquires imaginary part such that

\[ \Gamma = -\frac{2}{\hbar} \text{Im} E_{\text{EW}} \]
Vacuum Decay Rate

Decay rate per unit volume:

\[ \frac{\Gamma}{V} = C \exp(-S_E[\bar{\phi}]/\hbar), \]

Metastability requires \( \Gamma^{-1} > t_0 \)

\[ S_E[\bar{\phi}]/\hbar > \log(t_0^4 C) \approx 400 \]

Computing \( S_E[\bar{\phi}] \): Coleman PRD 15, Coleman & Callan PRD 16

- Single field - shooting method (special to 1D boundary value problems) ✓
- Multiple fields:
  1. Path deformation - implemented in CosmoTransitions (by Max Wainwright at UCSC) ✓
  2. Constrained or improved potential with dimensional deformation - algorithms outlined in Konstandin & Huber JCAP 0606 and Park JCAP 1102 - in progress.
The Bounce

\( \bar{\phi} \)

\[ V(\phi) \]

\[ \rho \]

\( \phi_1 \)

\( \phi_2 \)

\[ \bar{\phi}(\rho) \]
Previous Stability Constraints

**Analytic:** Kounnas, Lahanas & Nanopoulos NPB 236

- Assume VEVs are all equal

$$\langle H_u^0 \rangle = \langle \tilde{t}_L \rangle = \langle \tilde{t}_R \rangle$$

Potential now is a function of 1 VEV, easy to minimize by hand

- Demand *absolute* stability (SM-like minimum is global):

$$V_{CCB} > V_{SML}$$

$$\downarrow$$

$$A_t^2 < 3(m_2^2 + m_{Q3}^2 + m_{\tilde{t}_R}^2)$$

*This is neither necessary nor sufficient.* More sophisticated analyses by Casas, Lleyda & Muñoz: NPB 471, PLB 380, PLB 389
Previous Metastability Constraints

Numeric

- Scan MSSM parameters
- If $\exists$ global CCB minimum, find $S_E[\phi]$
- If $S_E < 400$ parameters are excluded
- Empirical inequality:

$$A_t^2 + 3\mu^2 < 7.5(m_{Q3}^2 + m_{tR}^2)$$
Previous Metastability Constraints

Why do another analysis?

- $m_h$ has been measured. What does metastability imply for the Higgs parameter space?
- Bounds on stop parameters for direct (LHC) and indirect searches ($b \rightarrow s\gamma$, ...)
- Loop corrections should be included
- More reliable numerics

123 GeV < $m_h$ < 127 GeV

Analytic Bound
Empirical Bound
Preliminary Results - No Higgs Mass Constraint

- SM metastable, ● - SM unstable

- Empirical bound completely invalid

- Analytic result surprisingly robust (except for some extreme values of parameters)
Preliminary Results - Higgs

- SM absolutely stable,
- SM metastable,
- SM unstable

CCB minima appear for $|X_t| \gtrsim 1$ TeV

Most CCB points $X_t \gtrsim 1$ TeV not metastable $\Rightarrow$ excluded
Conclusion

Summary:
- Large values of the stop cubic term $A_t$ lead to appearance of CCB minima
- Models with global CCB minima ruled out if lifetime of SM-like vacuum too short
- Metastability constrains the Higgs parameter space in the MSSM

To do:
- Recompute bounce using independent method
- Include quantum corrections
Backup
Fate of the False Vacuum

- $E_{EW}$ extracted from the matrix element

$$\langle \phi_+ | \exp(-HT/\hbar) | \phi_+ \rangle = \int [\mathcal{D}\phi] \exp(-S_E[\phi]/\hbar)$$

$\phi_+$ is the false vacuum.

- RHS evaluated semi-classically by expanding

$$S_E[\phi] = S_E[\bar{\phi}] + \frac{1}{2} (\phi - \bar{\phi}) \frac{\delta^2 S_E}{\delta \phi^2} (\phi - \bar{\phi}) + \ldots$$

- $\bar{\phi}$ is a classical solution such that

$$\frac{\delta S_E}{\delta \phi}[\bar{\phi}] = 0 \Rightarrow \partial^2 \phi = U'(\phi), \text{ BCs: } \lim_{t,|\vec{x}|\to\pm\infty} \bar{\phi}(t, \vec{x}) = \phi_+.$$
Pre-exponential Factor

- Performing the path integral gives

\[ \Gamma / V = C \exp(-S_E[\phi]/\hbar), \]

where

\[ C = \left( \frac{S_E[\phi]}{2\pi} \right)^2 \left| \frac{\det' \left[ -\partial^2 + U''(\phi) \right]}{\det \left[ -\partial^2 + U''(\phi_+) \right]} \right|^{-1/2} \]

- \( \det' \) omits translational zero modes
- Prefactor usually estimated as

\[ [C] = M^4 \Rightarrow C \approx (100 \text{ GeV})^4 \]

- Numerical computation of the prefactor described in

Min JPA 39, Dunne & Min PRD 72
Loop Corrections

Groundstate of the quantum theory given by the minimum of the effective potential (in DR)

\[ V_{\text{eff}}(Q) = V_0(Q) + \Delta V_1(Q), \quad \Delta V_1(Q) = \frac{1}{64\pi^2} \text{Str} \left[ \mathcal{M}^4 \left( \ln \frac{\mathcal{M}^2}{Q^2} - \frac{3}{2} \right) \right] \]

- Typical approach: choose \( Q \) s. t. \( \Delta V_1 \approx 0 \Rightarrow \) log corrections reabsorbed into running couplings in \( V_0 \).
- **Issue 1:** \( V_0(Q) \) is now very sensitive to choice of \( Q \): Gamberini, Ridolfi & Zwirner NPB 331
- **Issue 2:** \( V_{\text{eff}}(Q) \) is gauge-dependent: Patel & Ramsey-Musolf JHEP 1107