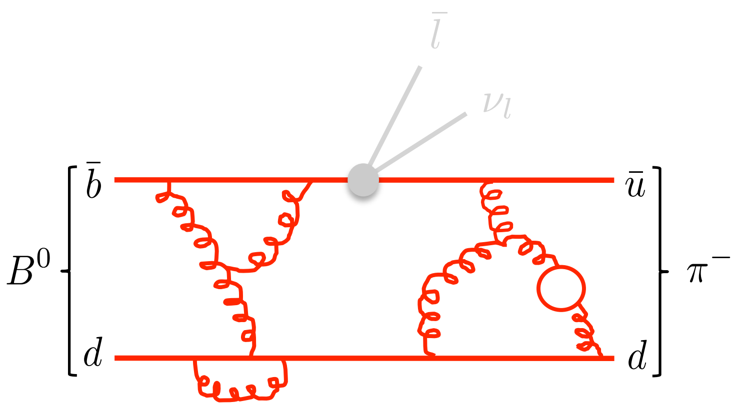
Flavor Physics and Lattice QCD

Chris Bouchard, Ohio State

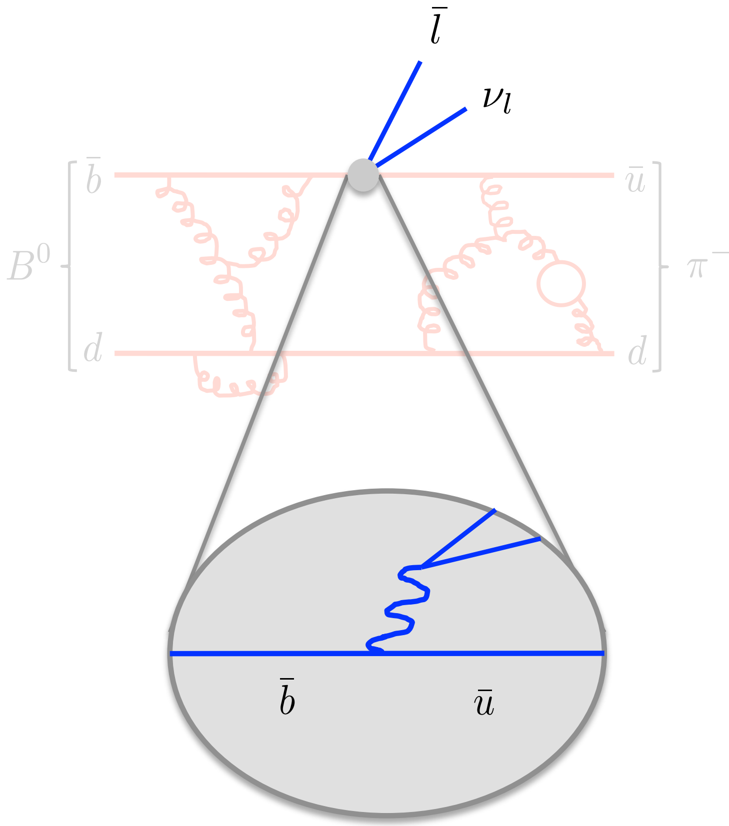
- Role of LQCD
  - different scales
  - lattice basics
  - lattice errors
- “Gold plated” quantities
  - 3 examples
- Harder (but still being done)
- The future

— Different Scales —



$\Lambda_{\rm QCD} \sim \text{few} \times 100\ \text{MeV}$

— Different Scales —



$\Lambda_{\mathrm{QCD}} \sim \text{few} \times 100\ \mathrm{MeV}$

$\Lambda_{\mathrm{EW}} \sim 100\ \mathrm{GeV}$

— Different Scales —

- community flavor effort

$$\text{observable} = \sum_i \; C_i(\mu)\; \mathrm{ME}_i(\mu) \;\; + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\Lambda_{\mathrm{EW}}}\right)^2$$

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UV IR $< 0.01\%$

- cleanly factorizes under OPE
  - short distance, perturbative $\longrightarrow C_i(\mu)$ Wilson coefficients
  - long distance, nonperturbative $\longrightarrow \text{ME}_i(\mu)$ hadronic matrix elements
    * lattice the only first principles method (QFT on a computer)
    * other methods (e.g. sum rules) often complementary

— Lattice Basics —

A correlation function (PI, Euclidean, Berezin integration, continuum)...

$$\langle O \rangle = \frac{\int [dG] \; O[G, (\not{D} + m)^{-1}] \; e^{-S[G] + \ln \det(\not{D} + m)}}{\int [dG] \; e^{-S[G] + \ln \det(\not{D} + m)}}$$

O is a composite operator

$$O = (\text{final state}) \times (\text{interaction}) \times (\text{initial state})^\dagger$$

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$$= \quad \pi(T) \qquad V_\mu(t) \qquad B^\dagger(0)$$

interpolating operators
& flavor-changing current

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$$= \quad (\bar{u}\gamma_5 d)_T \qquad (\bar{b}\gamma_\mu u)_t \qquad (\bar{d}\gamma_5 b)_0$$

Wick contract

— Lattice Basics —

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O is a composite operator

$$\begin{aligned}
O &= (\text{final state}) \times (\text{interaction}) \times (\text{initial state})^\dagger \\
&= \quad \pi(T) \qquad\qquad V_\mu(t) \qquad\qquad B^\dagger(0) && \text{interpolating operators} \\
& && \text{\& flavor-changing current} \\
&= \quad (\bar{u}\gamma_5 d)_T \qquad (\bar{b}\gamma_\mu u)_t \qquad (\bar{d}\gamma_5 b)_0 && \text{Wick contract} \\
&= \gamma_5\ (\not{D}+m_d)^{-1}_{T,0}\ \gamma_5\ (\not{D}+m_u)^{-1}_{t,T}\ \gamma_\mu\ (\not{D}+m_b)^{-1}_{0,t}
\end{aligned}$$

— Lattice Basics —

Numerically evaluate

$$\langle O \rangle = \frac{\int [dG] \; O[G, (\not{D} + m)^{-1}] \; e^{-S[G] + \ln\det(\not{D}+m)}}{\int [dG] \; e^{-S[G] + \ln\det(\not{D}+m)}}$$

discretize S and $\not{D}$

- $\displaystyle\int d^4x \to \sum_{x,y,z,t}$
- $\displaystyle\partial_\mu f(x) \to \frac{f(x + a\hat{\mu}) - f(x)}{a}$
- and much more...

— Lattice Basics —

Numerically evaluate

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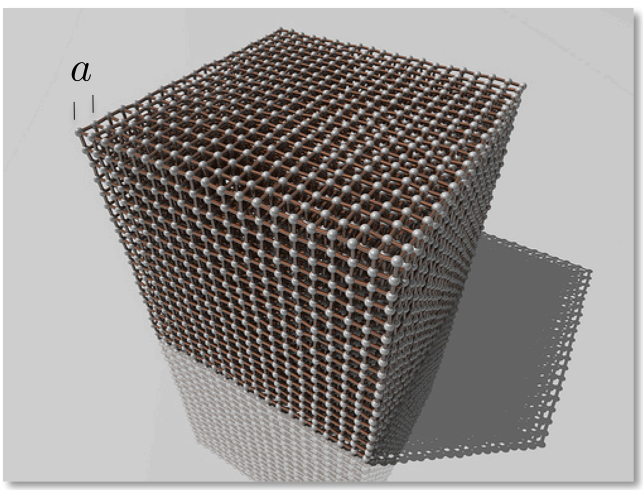

importance sampling

generate $\{G_n\}$ with probability distribution $e^{-S[G]+\ln\det(\not{D}+m)}$

$$\langle O \rangle^{\text{latt}}(a) = \frac{1}{N} \sum_{n=1}^{N} O[G_n, (\not{D} + m)^{-1}_n]$$

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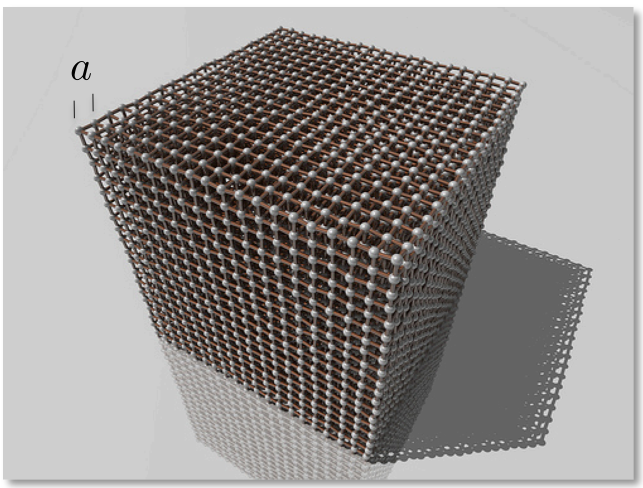

<u>importance sampling</u>

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$$\langle O \rangle^{\mathrm{latt}}(a) = \frac{1}{N}\sum_{n=1}^{N} O[G_n, (\not{D}+m)_n^{-1}]$$

$$\langle O \rangle^{\overline{\mathrm{MS}}}(\mu) = Z(\mu, a)\ \langle O \rangle^{\mathrm{latt}}(a)$$

— Lattice Errors —

$$\langle O\rangle^{\overline{\mathrm{MS}}}(\mu) = Z(\mu, a)\,\frac{1}{N}\sum_{n=1}^{N} O[G_n, (\not{D} + m)_n^{-1}]$$

matching

statistics

input parameters

discretization

chiral extrapolation

scale setting

isospin/EM effects

correlation function fits

charm sea

— Lattice Errors —

- small, generally $< 1\%$

$$\langle O\rangle^{\overline{\text{MS}}}(\mu) = Z(\mu,a)\,\frac{1}{N}\sum_{n=1}^{N} O[G_n, (\not{D}+m)_n^{-1}]$$

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— Lattice Errors —

- small, generally $< 1\%$
- improves with statistics

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— Lattice Errors —

- small, generally $< 1\%$
- improves with statistics
- beating down the errors

$$\langle O \rangle^{\overline{\mathrm{MS}}}(\mu) = Z(\mu, a)\ \frac{1}{N} \sum_{n=1}^{N} O[G_n, (\not{D} + m)_n^{-1}]$$

two-loop,
non-perturbative,
RGI matrix element

matching

statistics

input parameters

$a \approx 0.03$ fm

discretization

chiral extrapolation

physical m_q^{light}

new methods

scale setting

correlation function fits

isospin/EM effects

charm sea

including in simulations

— Gold Plated Quantities —

SM parameters
$$\begin{cases} m_{ud},\, m_u/m_d,\, m_s \\ m_c,\, m_b \\ \alpha_s \end{cases}$$

SM tree-level
$$\begin{cases} \pi \to l\nu \\ K \to l\nu,\, K \to \pi l\nu \\ D_{(s)} \to l\nu,\, D \to \pi l\nu,\, D \to K l\nu \\ B_{(s)} \to l\nu,\, B \to \pi l\nu,\, B_s \to K l\nu,\, B_{(s)} \to D_{(s)} l\nu,\, B \to D^* l\nu \\ \Lambda_b \to p l\nu \end{cases}$$

rare
$$\begin{cases} \left[K^0 - \bar{K}^0\right]_{\text{s.d.}},\, \left[D^0 - \bar{D}^0\right]_{\text{s.d.}},\, \left[B^0_{(s)} - \bar{B}^0_{(s)}\right]_{\text{s.d.}} \\ B \to \pi ll,\, B \to K ll \\ \Lambda_b \to \Lambda ll \end{cases}$$

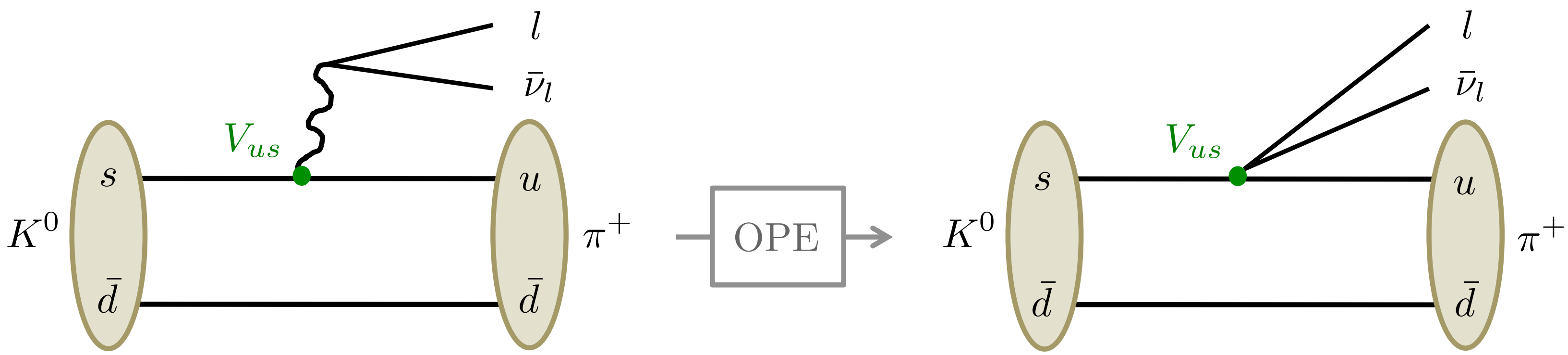
— Gold Plated Quantities — 3 examples...

SM parameters
- m_{ud}, m_u/m_d, m_s
- m_c, m_b
- α_s

SM tree-level
- $\pi \to l\nu$
- $K \to l\nu$, $K \to \pi l\nu$
- $D_{(s)} \to l\nu$, $D \to \pi l\nu$, $D \to K l\nu$
- $B_{(s)} \to l\nu$, $B \to \pi l\nu$, $B_s \to K l\nu$, $B_{(s)} \to D_{(s)} l\nu$, $B \to D^* l\nu$
- $\Lambda_b \to p l\nu$

rare
- $[K^0 - \bar{K}^0]_{\text{s.d.}}$, $[D^0 - \bar{D}^0]_{\text{s.d.}}$, $[B^0_{(s)} - \bar{B}^0_{(s)}]_{\text{s.d.}}$
- $B \to \pi ll$, $B \to K ll$
- $\Lambda_b \to \Lambda ll$

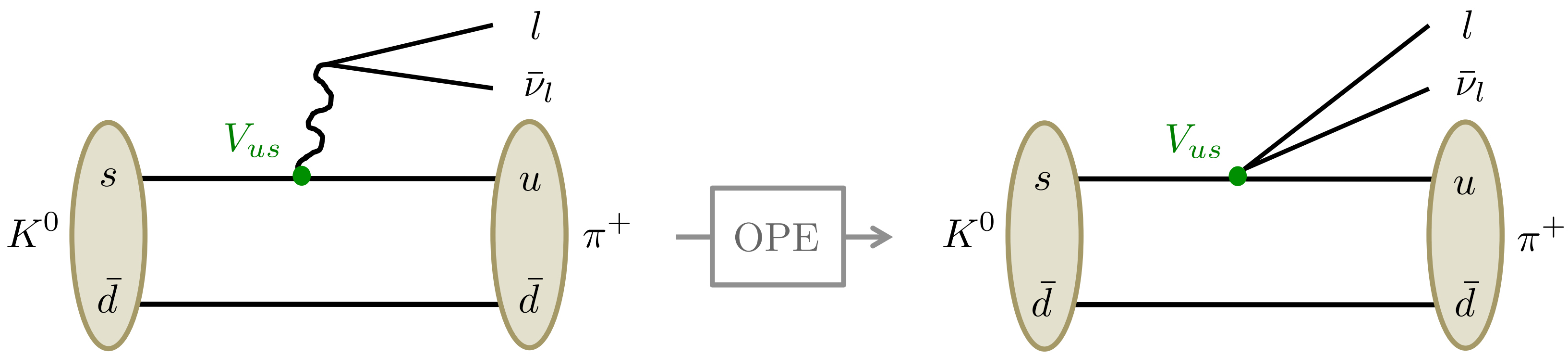
$K \to \pi l \bar{\nu}$



$$\frac{\Gamma(K \to \pi l \bar{\nu})}{S_{\mathrm{EW}}\ (1 + \delta^{Kl}_{\mathrm{EM}} + \delta^{K\pi}_{\mathrm{SU(2)}})}\ \frac{128\pi^3}{G_F^2 M_K^5 C_K^2} = I^{(0)}_{Kl}\ |V_{\mathrm{us}}|^2\ f_+^2(0)$$

[Cirigliano et al, RMP 84, 399 (2012)]

$K \to \pi l \bar{\nu}$



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[Cirigliano et al, RMP 84, 399 (2012)]

know/measure

extract

LQCD

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[Cirigliano et al, RMP 84, 399 (2012)]

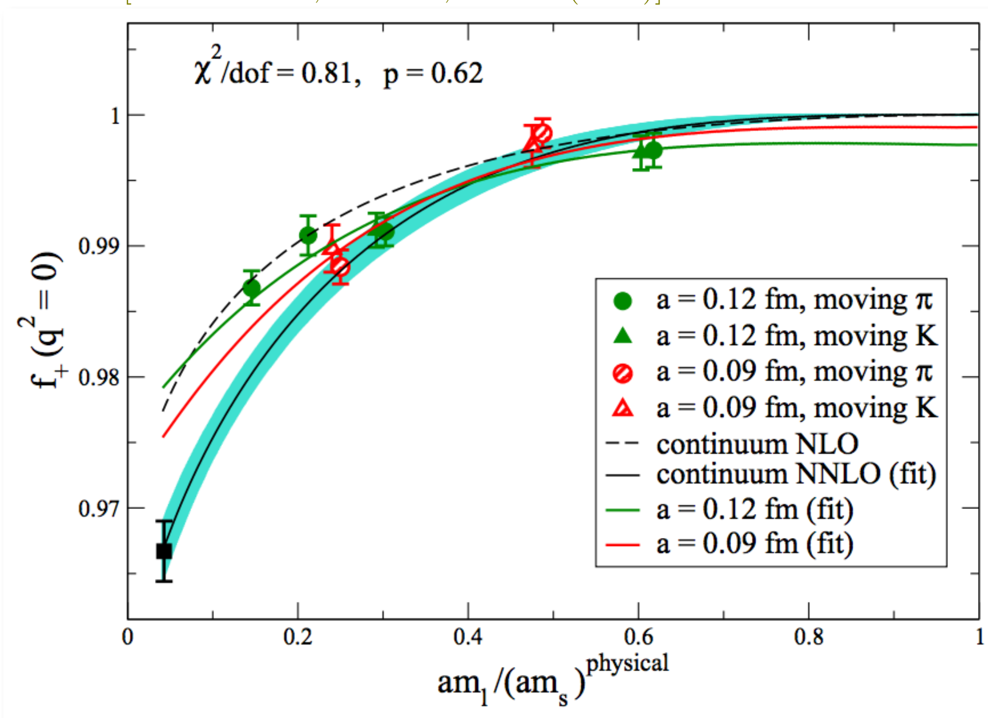
kinematic constraint: $f_+(0) = f_0(0)$

$$f_0(q^2) = \frac{m_s - m_l}{M_K^2 - M_\pi^2} \langle \pi | S^{\rm latt} | K \rangle_{q^2}$$

$\longrightarrow$ absolutely normalized

Source of uncertainty	Error $f_+(0)$ (%)
Statistics	0.24
Chiral extrapolation & fitting	0.3
Discretization	0.1
Scale	0.06
Finite volume	0.1
Total Error	0.42

[FNAL-MILC, PRD 87, 073012 (2012)]



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[Gámiz, Lattice '13]

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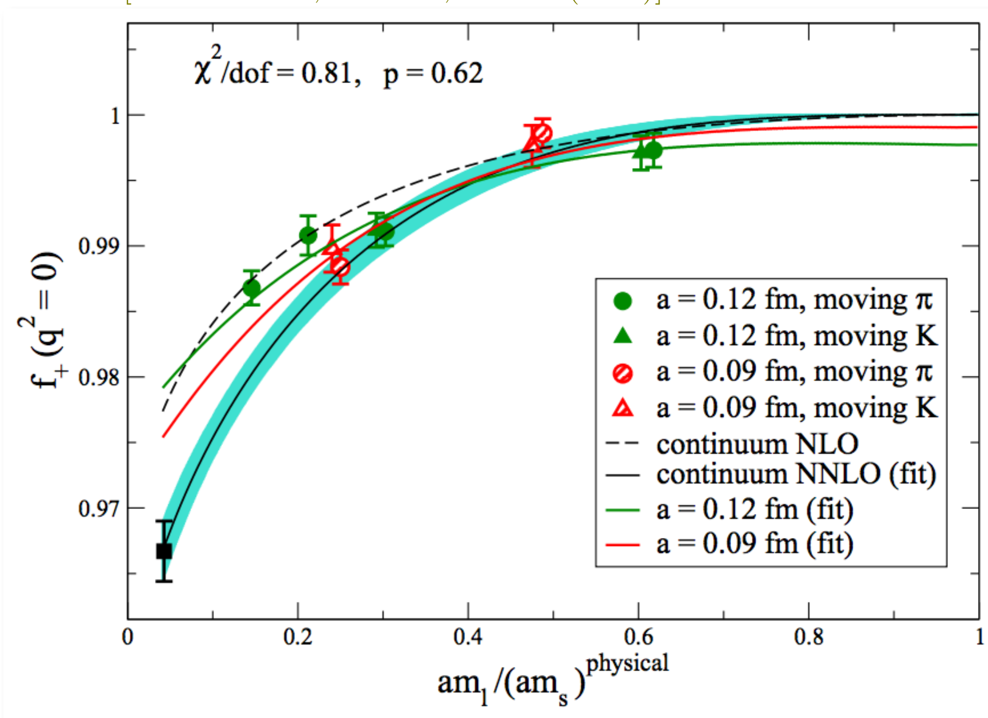

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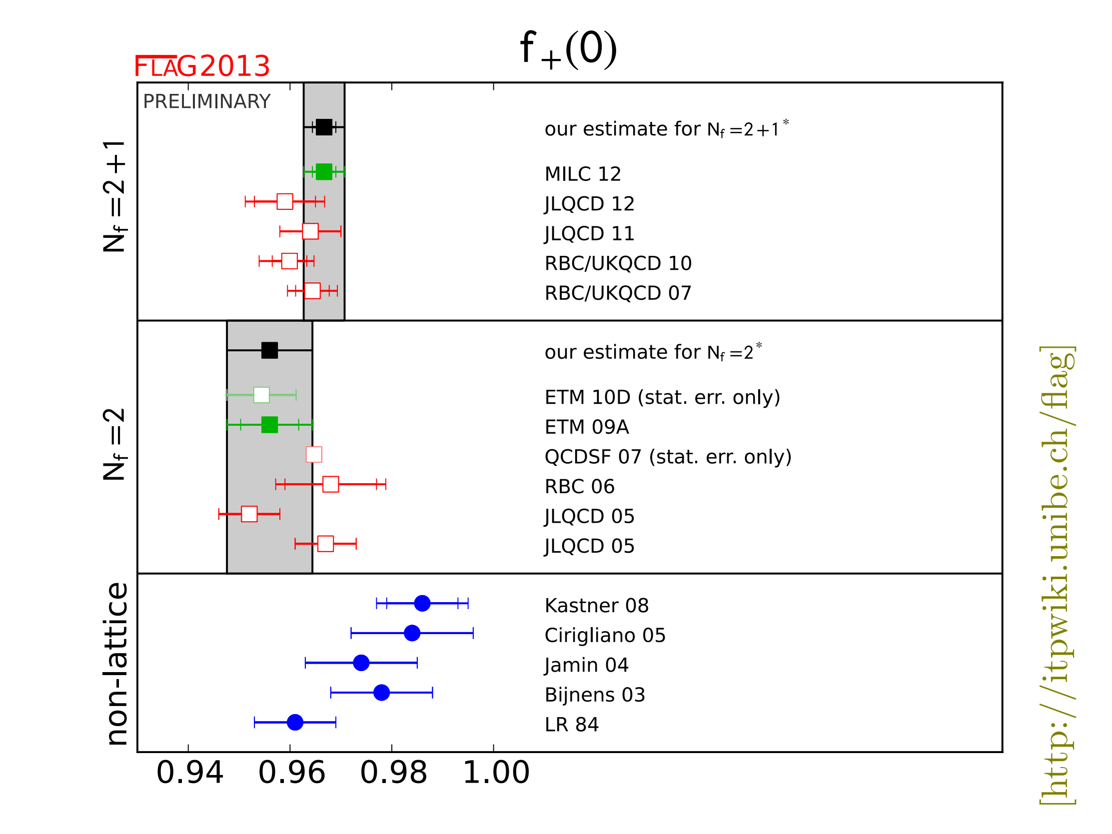
[Cirigliano et al, RMP 84, 399 (2012)]

Flavor **L**attice **A**veraging **G**roup

- worldwide
- heavy & light quark quantities
- average lattice results
  - color code
  - correlations



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- π, K precision due, in part, to intelligent choice of matrix elements

$$\langle \pi | S^{\text{latt}} | K \rangle, \quad \langle 0 | A_0 | K, \pi \rangle$$

- effective theory would ruin this ...

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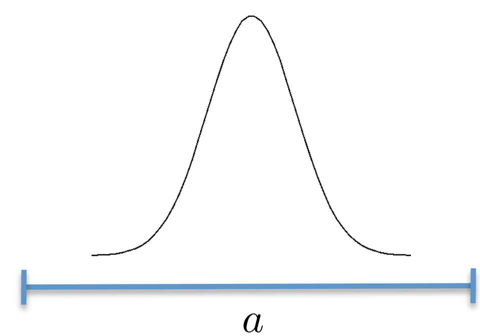
— Heavy quarks (c,b) —

- $\lambda_C \sim 1/m$, if smaller than a problematic
  - heavy quarks "fall through the lattice"
  - discretization errors $(am)^n$



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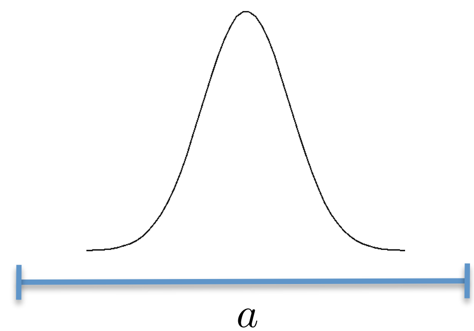
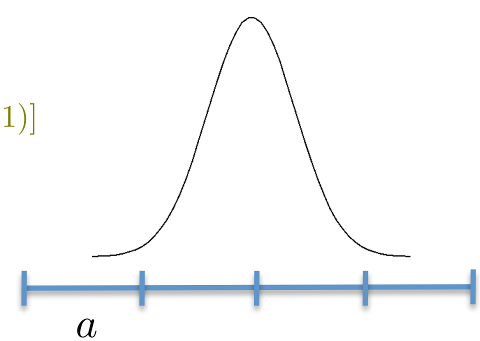
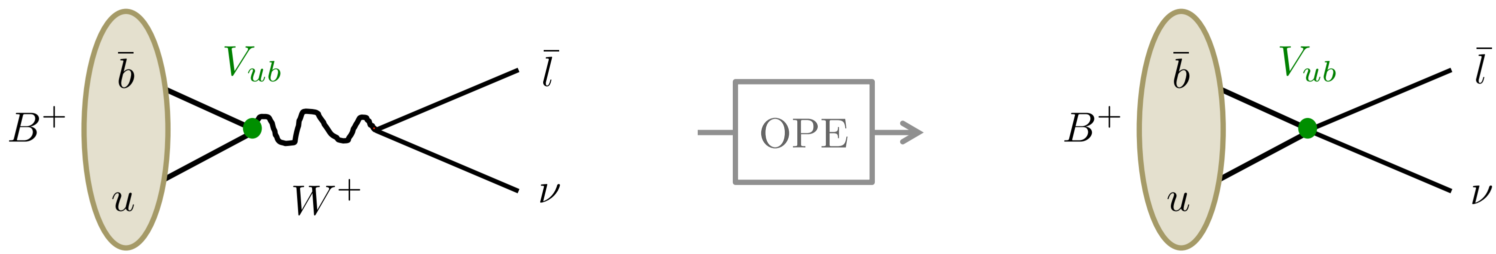


- effective theory for heavy quark (matching error $\sim$ few%)
- small a so that $am < 1$
  - c quarks [HPQCD, PRD 82, 114506 (2010); 114504 (2010); 84, 114505 (2011)]
  - b quarks, *almost* (0.03 fm should do it)
  - b and u, d quarks problematic

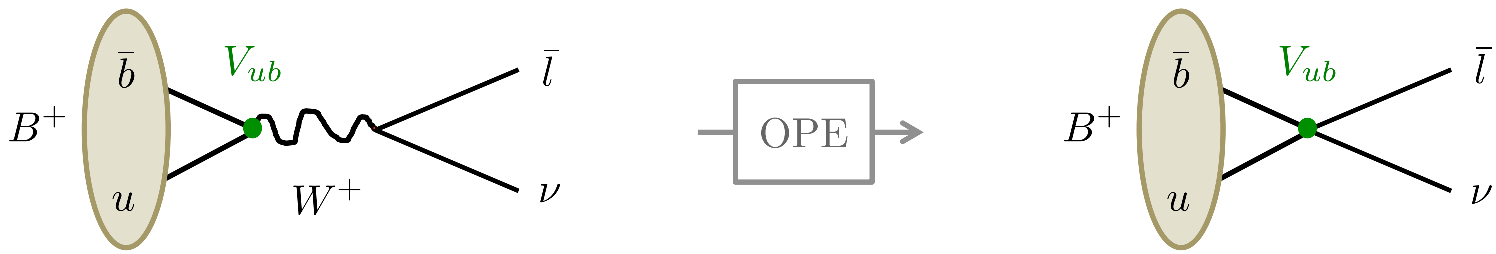


$B \to \bar{l}\nu$



$$\Gamma(B \to \bar{l}\nu) = \frac{M_B G_F^2}{8\pi}\; m_l^2 \left(1 - \frac{m_l^2}{M_B^2}\right)\; f_B^2\; |V_{\mathrm{ub}}|^2$$

$$B \to \bar{l}\nu$$



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know/measure

LQCD

extract

$B \to \bar{l}\nu$

[Buras, PLB 566, 115 (2003)]

$$\underbrace{\boxed{\Gamma(B \to \bar{l}\nu) = \frac{M_B G_F^2}{8\pi}\, m_l^2 \left(1 - \frac{m_l^2}{M_B^2}\right)}}_{18\%} \boxed{f_B^2}\, |V_{\text{ub}}|^2$$

$$\frac{\mathcal{B}(B_d \to \mu\bar{\mu})}{\mathcal{B}(B_s \to \mu\bar{\mu})} = \underbrace{\boxed{\frac{\tau_{B_d}}{\tau_{B_s}} \frac{M_{B_d}}{M_{B_s}} \left|\frac{V_{\text{td}}}{V_{\text{ts}}}\right|^2}}_{3\%} \boxed{\frac{f_{B_d}^2}{f_{B_s}^2}}$$

- LQCD calculation: $\langle 0|A_0|B_{(s)}\rangle = M_{B_{(s)}} f_{B_{(s)}}$
- absolutely normalized
  - B: b and u too expensive
  - B_s: b and s doable

$B \to \bar{l}\nu$

[Buras, PLB 566, 115 (2003)]

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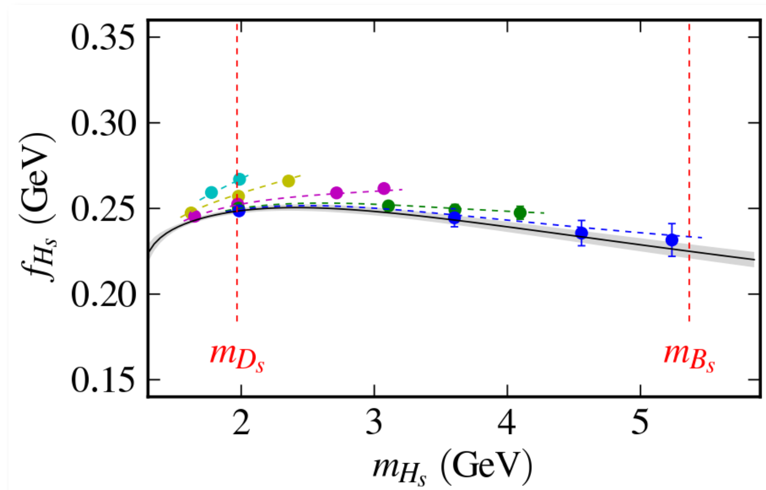


(no matching error, but not quite at $m_{B_s}^{\rm phys}$)

($a \approx 0.15, 0.12, 0.09, 0.06, 0.045$ fm)

	f_{B_s}
Monte Carlo statistics	1.30%
$m_{H_s} \to m_{B_s}$ extrapolation	0.81
r_1 uncertainty	0.74
$a^2 \to 0$ extrapolation	0.63
$m_{\eta_s} \to m_{\eta_s,\rm phys}$ extrapolation	0.13
r_1/a uncertainties	0.12
Total	1.82%

[HPQCD, 1110.4510]

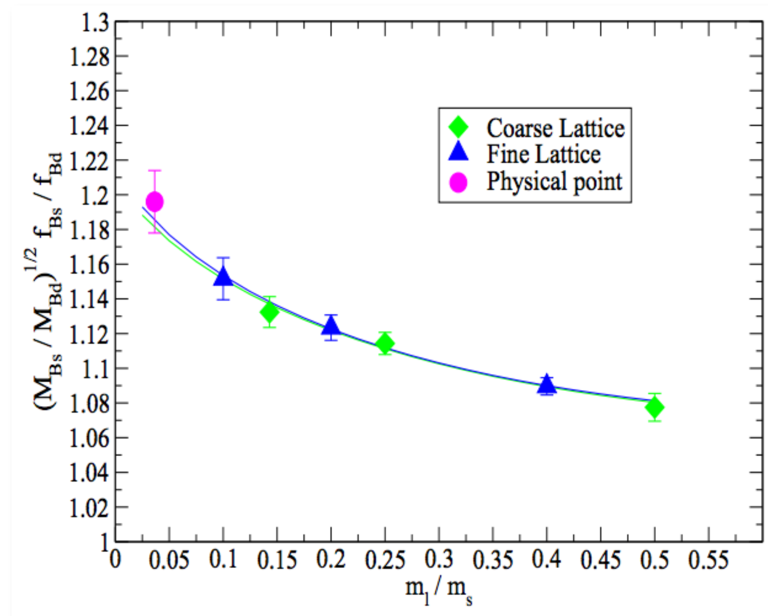
$B \to \bar{l}\nu$

EFT for b quark, but ...

calculate a ratio f_B/f_{B_s}:

- (most of) matching error cancels
- combine with precise f_{B_s}
- get precise f_B

[HPQCD, PRD 86, 034506 (2012)]

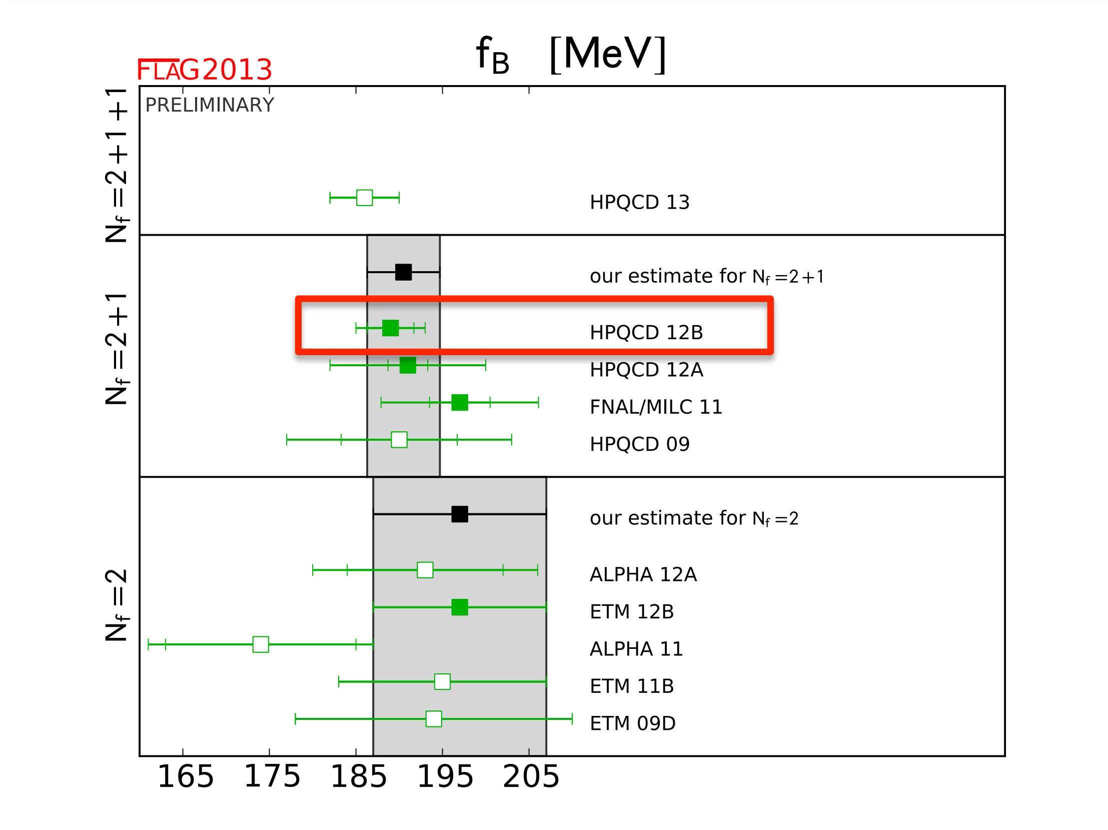


Source	f_{B_s} (%)	f_B (%)	f_{B_s}/f_B (%)
statistical	0.6	1.2	1.0
scale $r_1^{3/2}$	1.1	1.1	—
discret. corrections	0.9	0.9	0.9
chiral extrap. & $g_{B^*B\pi}$	0.2	0.5	0.6
mass tuning	0.2	0.1	0.2
finite volume	0.1	0.3	0.3
relativistic correct.	1.0	1.0	0.0
operator matching	4.1	4.1	0.1
Total	4.4	4.6	1.5

($a \approx 0.12, 0.09$ fm)

(effective theory)

$$B \to \bar{l}\nu$$



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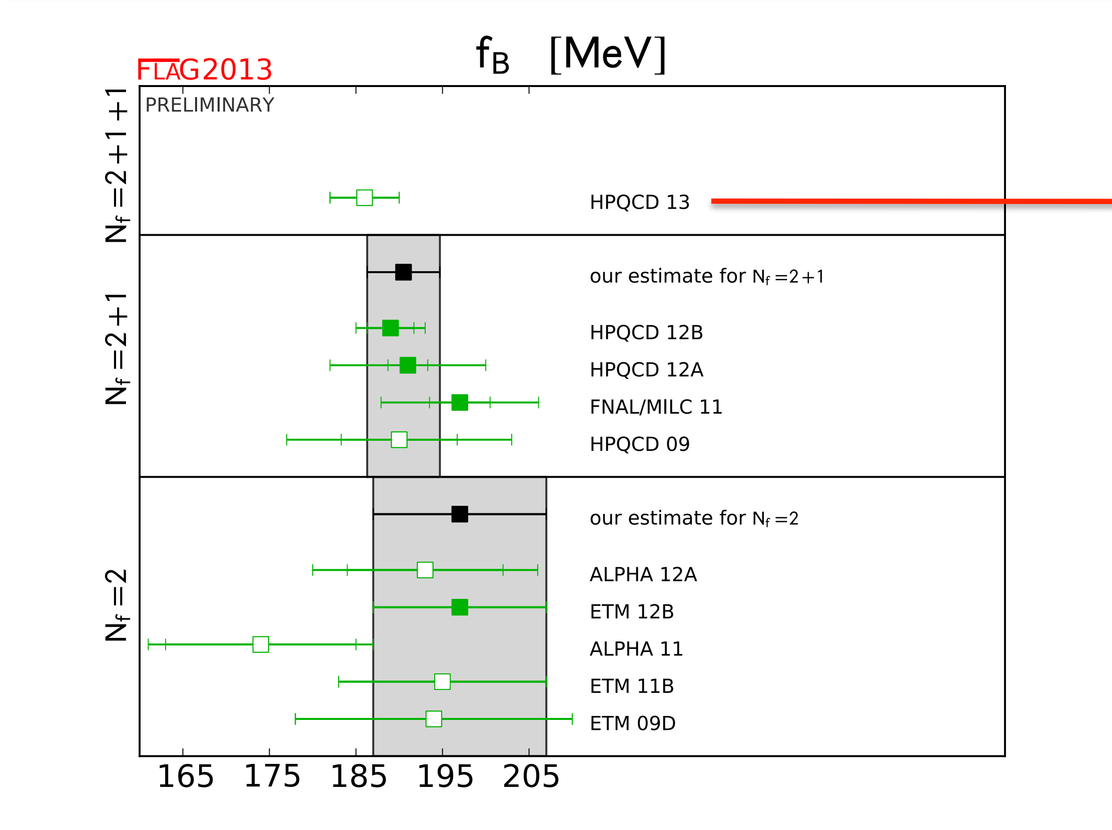
$B \to \bar{l}\nu$



[HPQCD, PRL 110, 222003 (2013)]

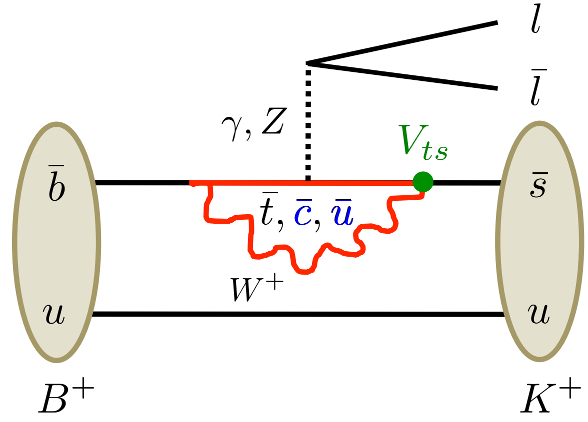
- physical light m_q
- improved EFT treatment of b quark
- small matching error

[http://itpwiki.unibe.ch/flag]

$$B \to K\, l\, \bar{l}$$

SM suppressed: loop, GIM, Cabibbo

$\longrightarrow$ detectable NP effects?



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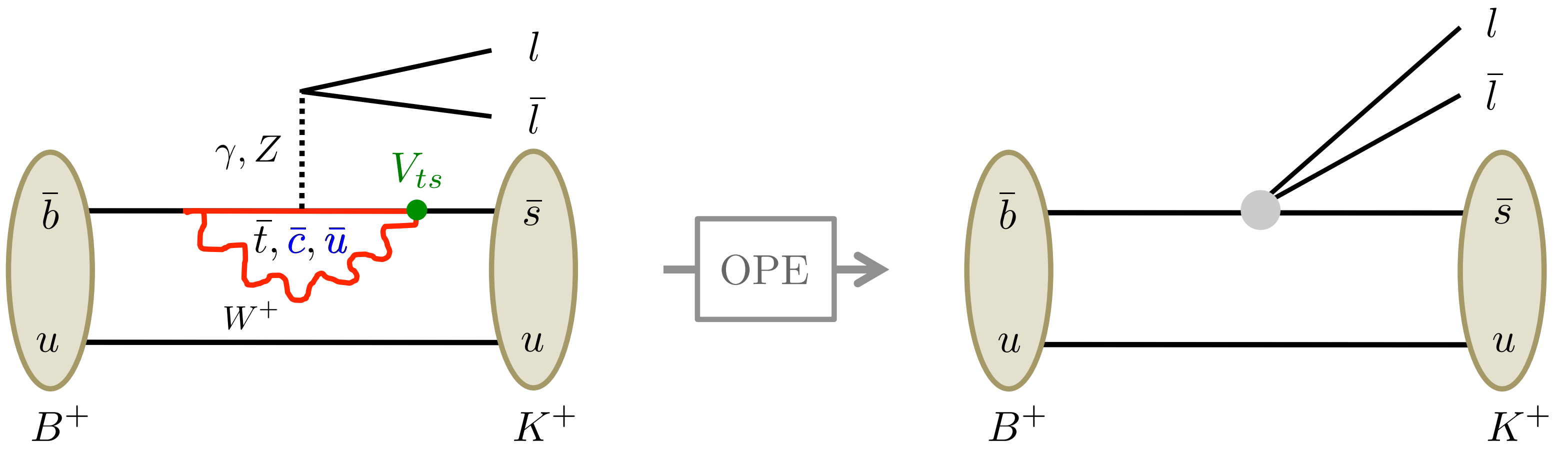


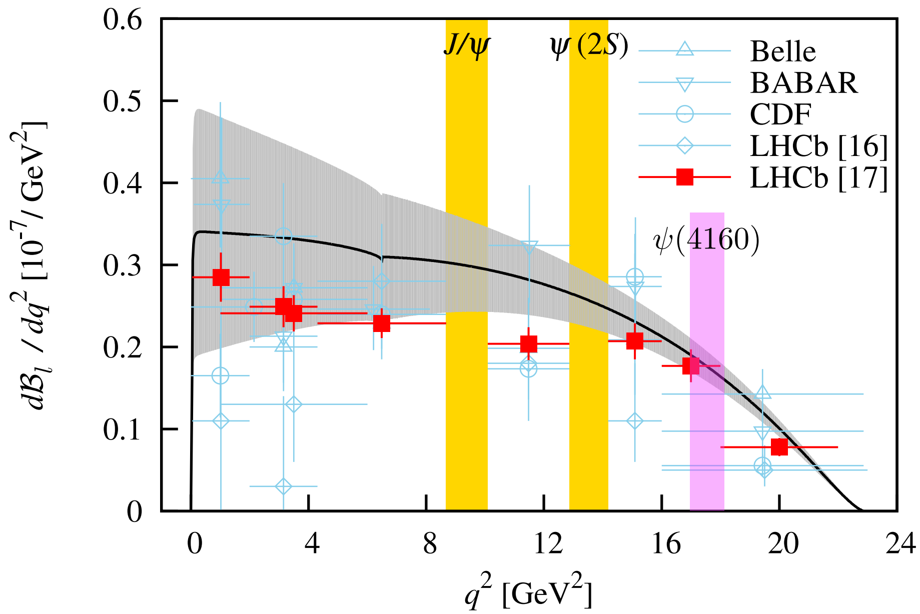
LQCD

$$\frac{d\mathcal{B}(B \to K \ell \bar{\ell})}{dq^2} = C(q^2)\left\{\left[\boxed{c_1}\, f_0(q^2) + \boxed{c_2}\, f_+(q^2)\right]^2 + \boxed{c_3}\, f_T^2(q^2) + \boxed{c_4}\, f_+(q^2) f_T(q^2)\right\}$$

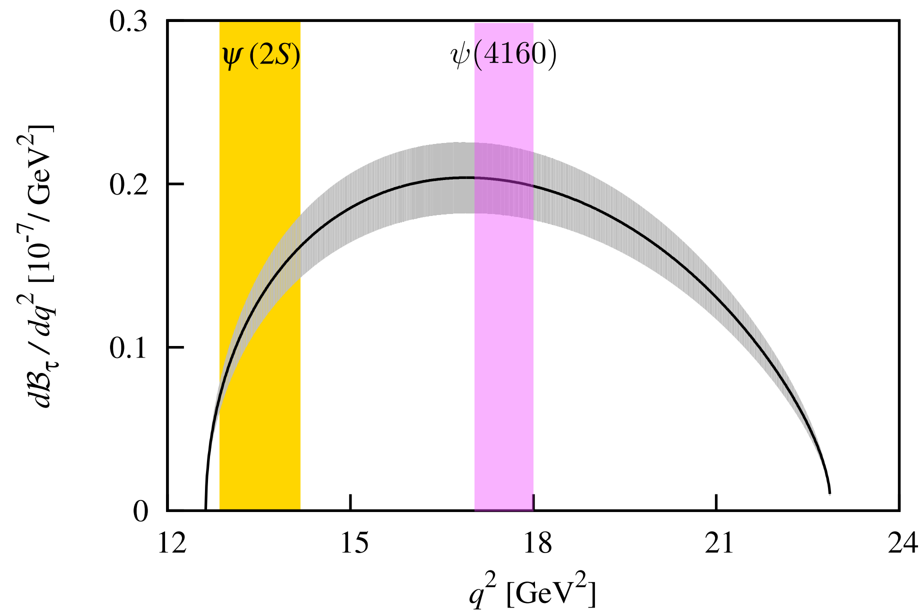
know/measure

constrain model

SM vs. experiment for light dileptons



SM prediction for ditau



[HPQCD, 1306.2384; 1306.0434]

— Harder (but still being done) —

$\left[K^0 - \bar{K}^0\right]_{\text{l.d.}}$ [RBC/UKQCD, 1212.5931]

$K \to \pi\pi$ [Chris Kelly's talk: Fri @ 10:30 in Cowell 131]

$D \to \pi\pi,\ D \to KK$ [Hansen and Sharpe, 1211.0511]

$B \to K^* l\bar{l}$ [Liu et al, 1101.2726]

$\langle \bar{n} | \mathcal{O}_{\Delta \mathrm{B}=2} | n \rangle$ [Buchoff et al, 1207.3832]

$\langle \pi, K, \ldots | \mathcal{O}_{\Delta \mathrm{B}=1} | p \rangle$ [RBC, 1304.7424]

$(g-2)_\mu$ HVP [Blum, PRL 91, 052001 (2003); QCDSF, NPB 688, 135 (2004); Aubin & Blum, PRD 75, 114502 (2007); ETMC, PRL 107, 081802 (2011); Boyle et al, PRD 85, 074505 (2012); Della Morte et al, JHEP 1203, 055 (2012)]

$(g-2)_\mu$ HLbL [Hayakawa et al, PoS LAT2005, 353 (2006); Blum et al, PoS LAT2012, 022 (2012); Chowdhury, PhD Thesis (2009); JLQCD, PRL 109, 182001 (2012)]

$\langle N | s\bar{s} | N \rangle$ for Mu2e [Junnarkar & Walker-Loud, PRD 87, 114510 (2013)]

$F_A(Q^2)$ for ν cross-sections [Bhattacharya et al, PRD 84, 073006 (2011); Capitani et al, PRD 86, 074502 (2012); QCDSF, 1302.2233]

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additional lattice efforts...

"Project X: Physics Opportunities"

Kronfeld and Tschirhart (editors)

1306.5009

$(g-2)_\mu$ HVP [Blum, PRL 91, 052001 (2003); QCDSF, NPB 688, 135 (2004); Aubin & Blum, PRD 75, 114502 (2007); ETMC, PRL 107, 081802 (2011); Boyle et al, PRD 85, 074505 (2012); Della Morte et al, JHEP 1203, 055 (2012)]

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— The Future —

Push all Gold-plated quantities toward 1%

Long distance contributions ($D^0 - \bar{D}^0$, $K \to \pi\, l\, \bar{l}$, ...)

Multi-hadron final states ($B \to \pi\pi$, ...)

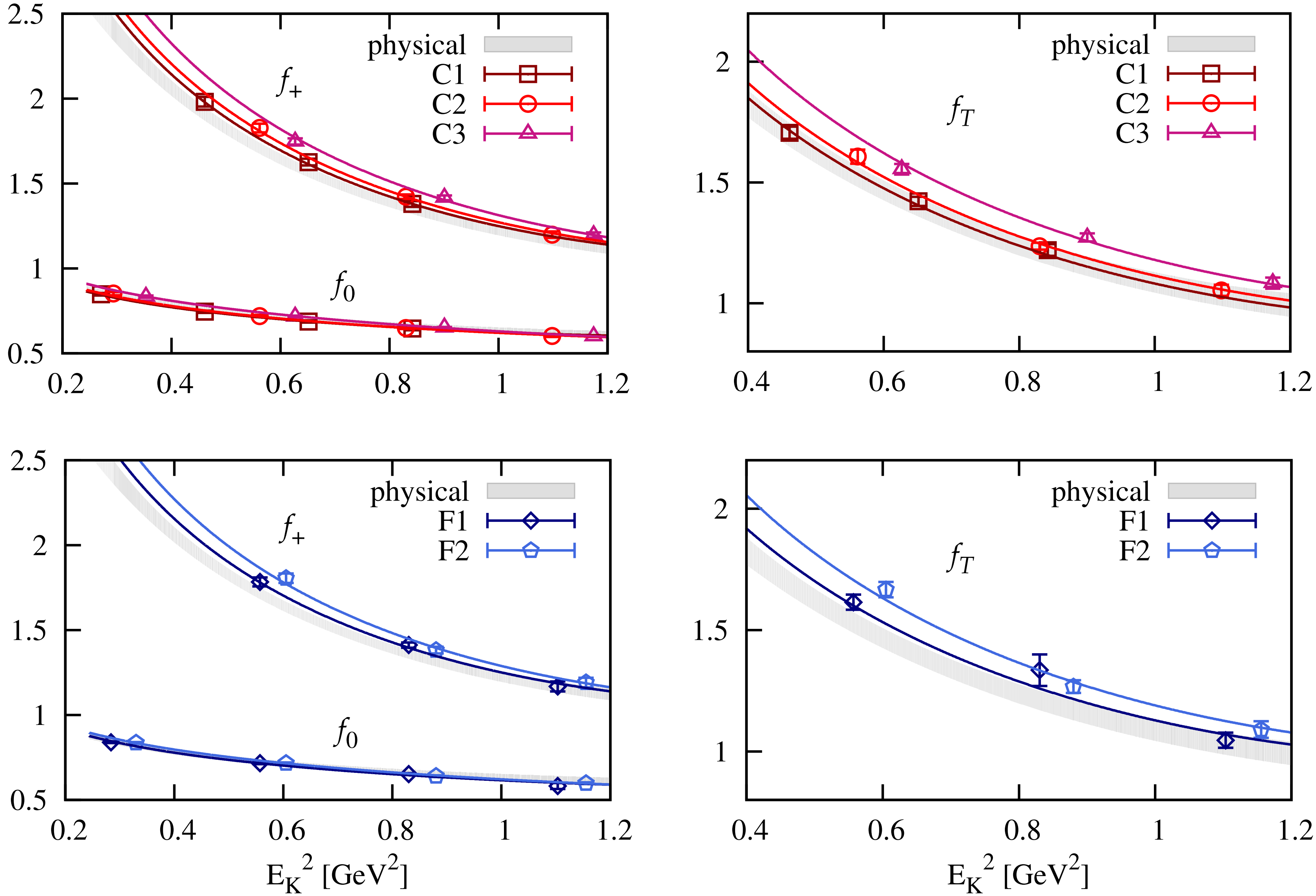
Hadronically unstable states ($B \to \rho l \bar{\nu}$, ...)

Additional rare decays (K, D) . . .

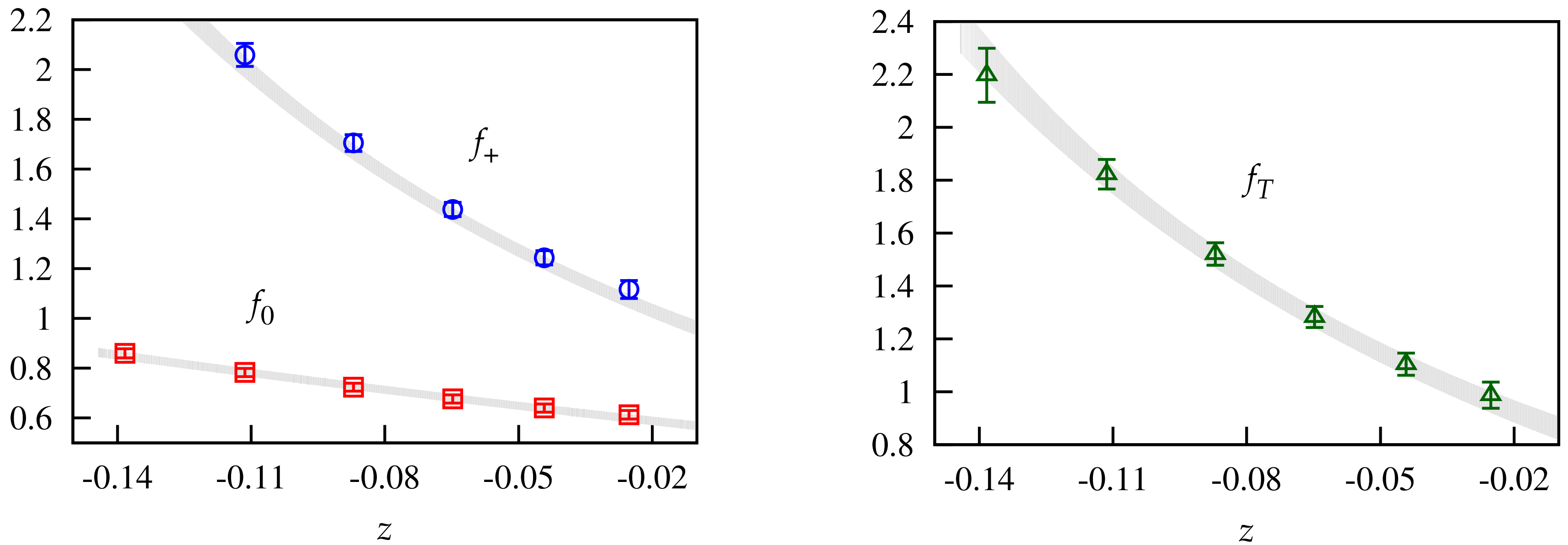
Are there other leptonic opportunities?

. . .

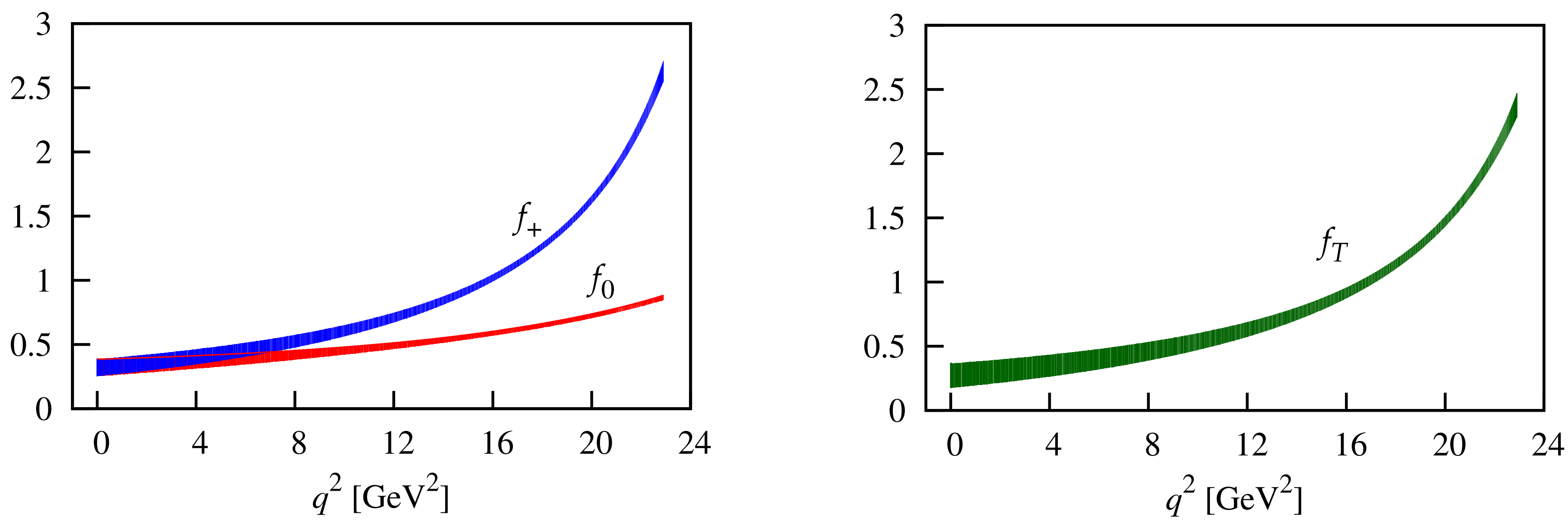
- simultaneous extrapolation for f_0, f_+, f_T with $\chi^2/\mathrm{dof} = 35.1/50$



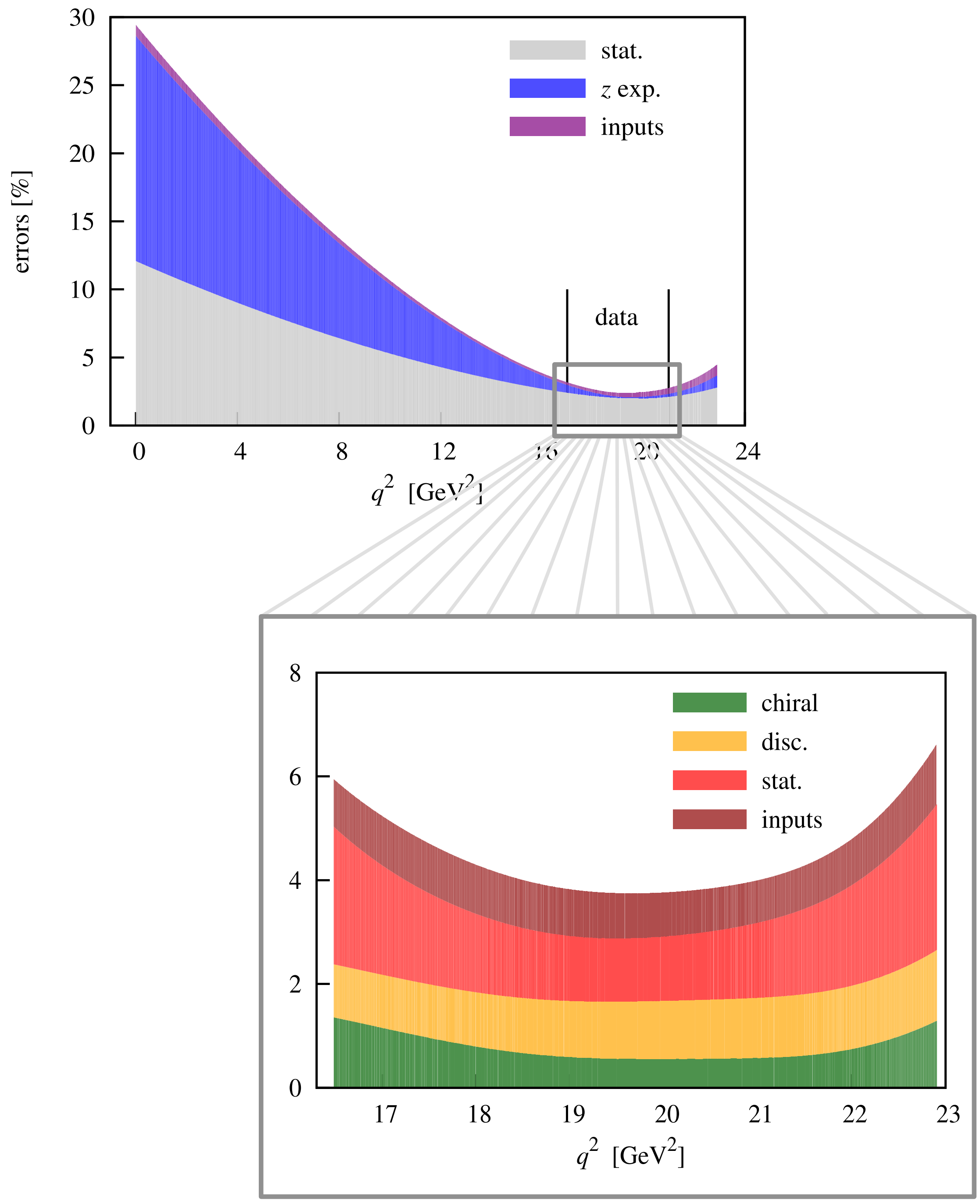
- simultaneous extrapolation for f_0, f_+, f_T with $\chi^2/\mathrm{dof} = 8.58/11$



- extrapolated curves over full range of q^2

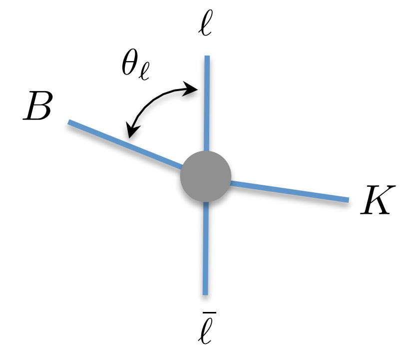


Errors for $f_+(q^2)$



- angular distribution of differential decay rate ($\Gamma_\ell = \mathcal{B}_\ell/\tau_B$)

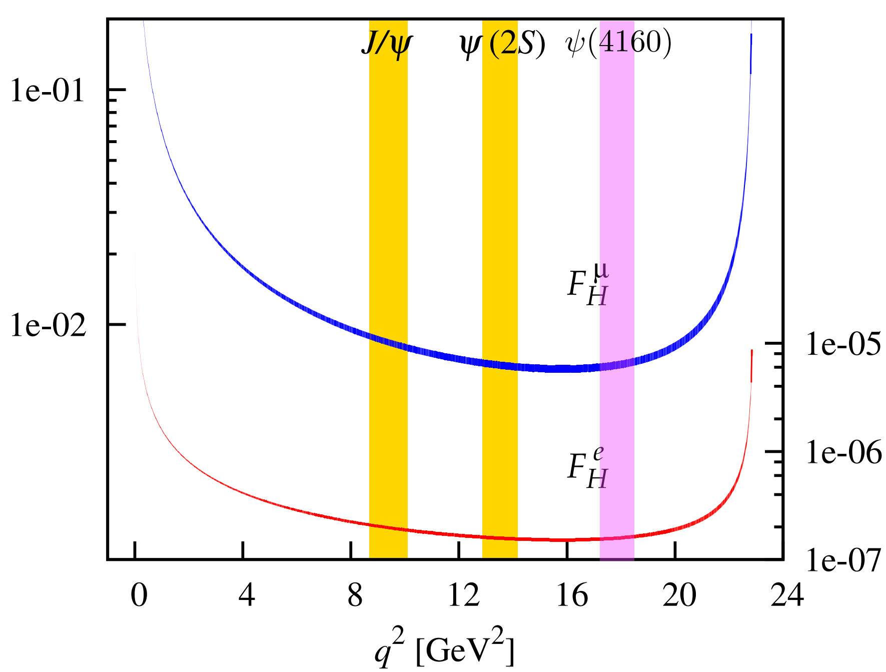
$$\frac{1}{\Gamma_\ell}\frac{d\Gamma_\ell}{d\cos\theta_\ell} = \frac{1}{2}F_H^\ell + A_{FB}^\ell\cos\theta_\ell + \frac{3}{4}(1-F_H^\ell)(1-\cos^2\theta_\ell)$$



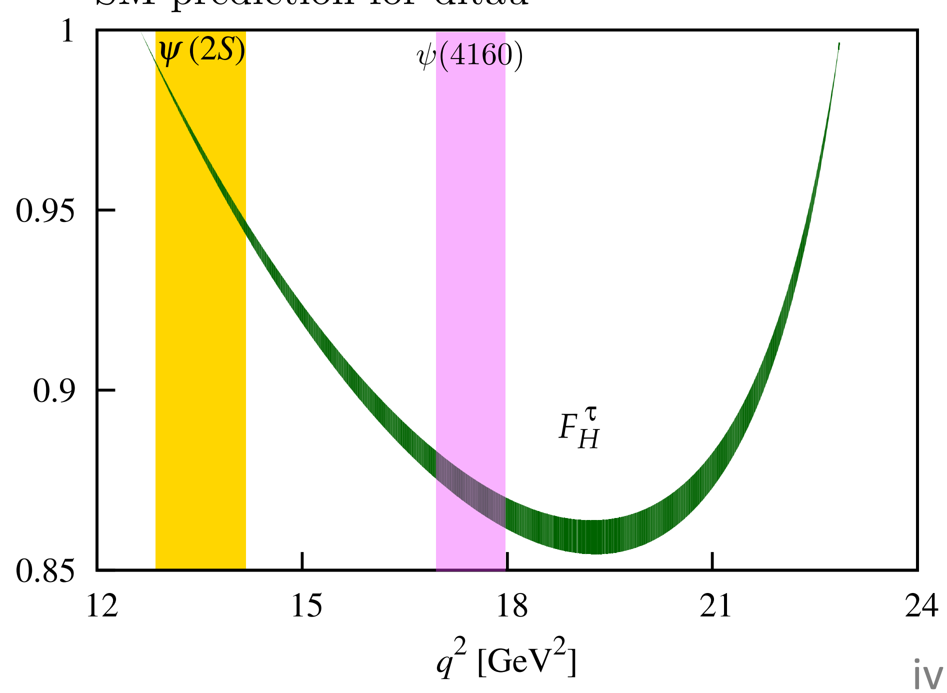
in dilepton rest frame

"flat term": $\quad F_H^\ell(q^2) = \dfrac{a_\ell + c_\ell}{a_\ell + \frac{1}{3}c_\ell}$

SM prediction for light dileptons



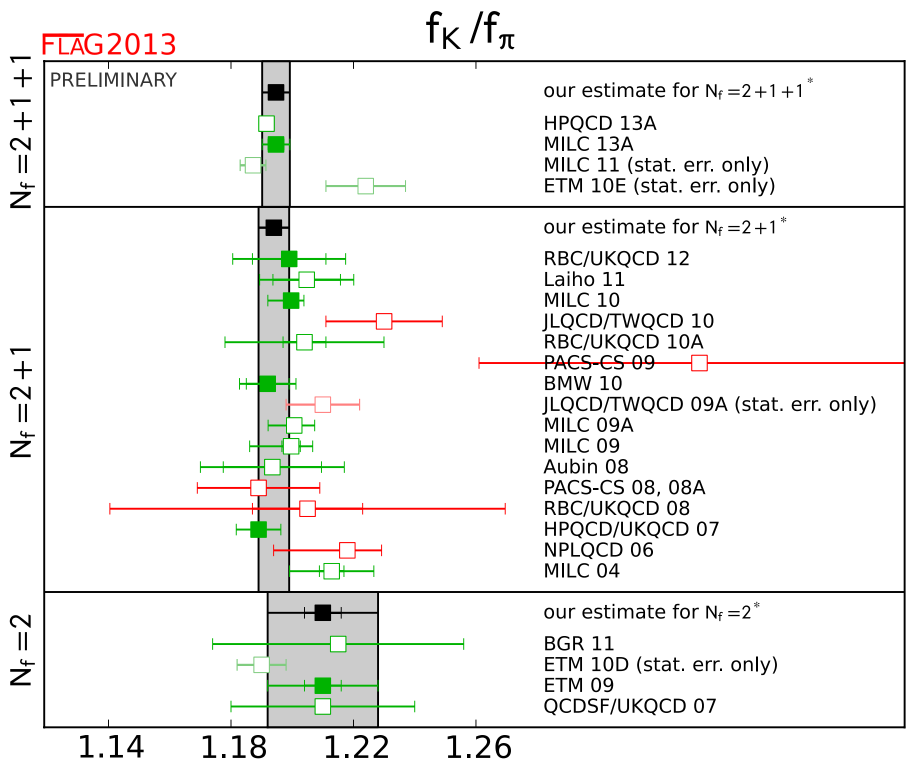
SM prediction for ditau



$$\frac{\Gamma(K \to l\bar{\nu}_l)}{\Gamma(\pi \to l\bar{\nu}_l)} = |V_{\rm us}|^2 \, \frac{1}{|V_{\rm ud}|^2} \, \underbrace{\frac{M_K(1-m_l^2/M_K^2)^2}{M_\pi(1-m_l^2/M_\pi^2)^2}}_{0.006\%} \, \frac{f_K^2}{f_\pi^2}$$

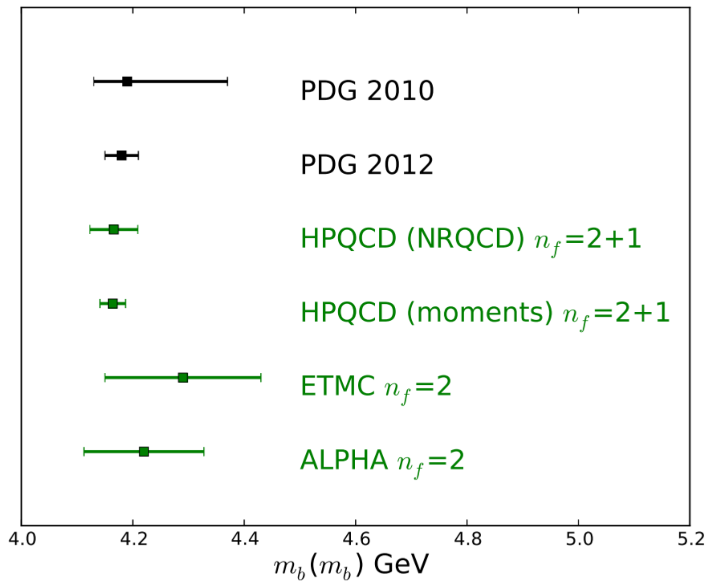
[Marciano, PRL 93, 231803 (2004)]

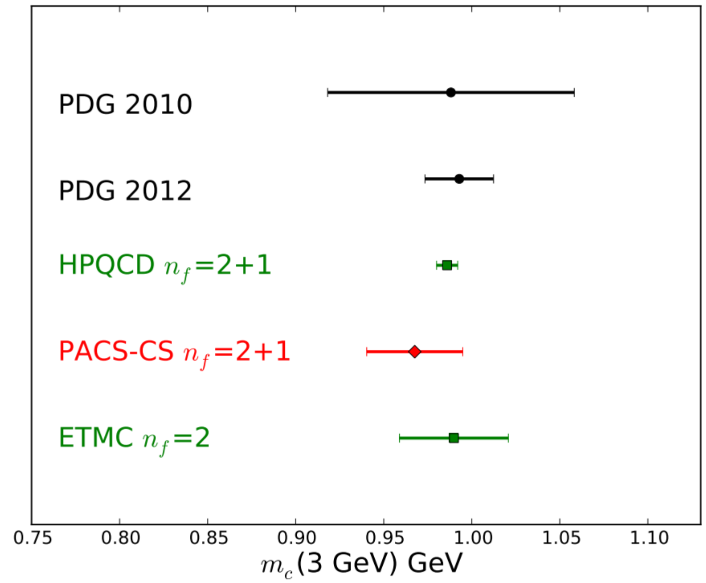
0.2%   0.03%   0.006%   0.8%



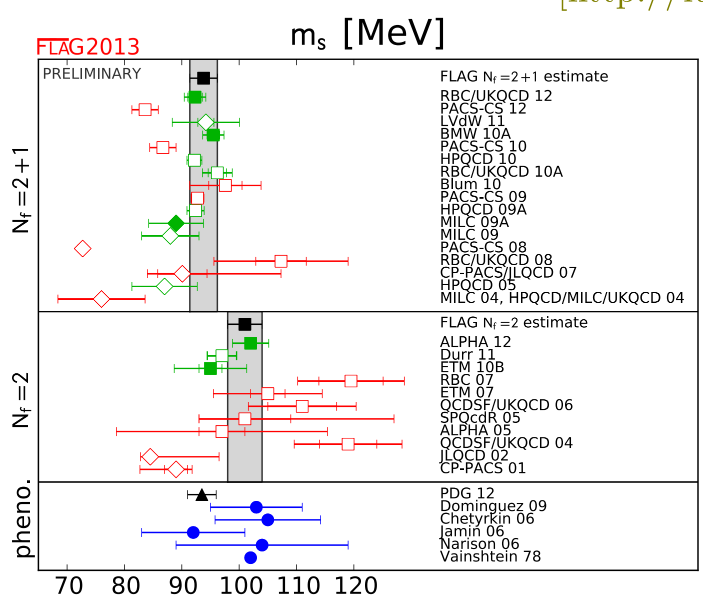
[http://itpwiki.unibe.ch/flag]

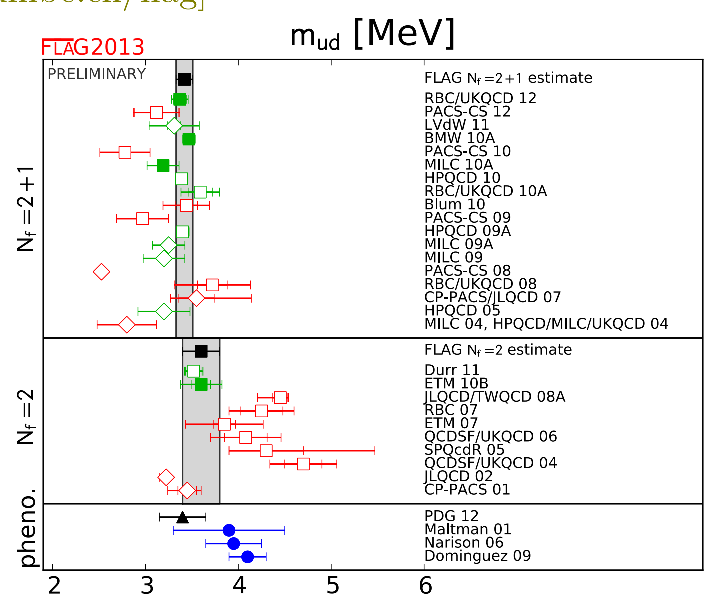
[C. McNeile, 1306.3326]





[http://itpwiki.unibe.ch/flag]





[C. McNeile, 1306.3326]