Threshold Resummation and Determinations of Parton Distribution Functions

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An accurate knowledge of parton distribution functions (PDFs) is vital to precision phenomenology.
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- e.g. A massive state produced at high rapidity would require knowledge of PDFs at high $x$, where resummation effects are known to be large.
Quantum Chromodynamics Overview

\[ \sigma_{H_1+H_2}(x) = \sum_{a,b} \int \int \int \, dx_1 \, dx_2 \, dz \, f_{a/H_1}(x_1) f_{b/H_2}(x_2) \hat{\sigma}_{ab}(z) \delta(x - x_1 x_2 z) \]

- Observables for hadron-initiated processes comprise a convolution of two parts:
  - Parton distribution functions (PDFs) \( f_{a/H}(x) \)
  - Hard-scattering cross section \( \hat{\sigma}_{ab}(x) \)

- PDFs are not calculable using perturbation theory; their forms are inferred by comparing data to theoretical predictions of observables.

- \( \hat{\sigma}_{ab} \) is calculated using perturbation theory.
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  ▶ Final state gluons are soft
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These “threshold logarithms” appear at every order beyond LO in a predictable manner.
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\[
\int_0^1 dz \ z^{N-1} \int_0^1 dx \ f(x) \int_0^1 dy \ g(y) \delta(z - xy) = \\
\int_0^1 dx \ x^{N-1} f(x) \int_0^1 dy \ y^{N-1} g(y) = \tilde{f}(N) \tilde{g}(N)
\]

Threshold logarithms in Mellin space manifest in powers of ln \( N \).
Threshold (or soft-gluon) resummation is summing the logarithms to all orders in $\alpha_s^m \ln^n N$ for all $n = 2m$ (LL), $2m - 1 \leq n \leq 2m$ (NLL), etc.
Threshold (or soft-gluon) resummation is summing the logarithms to all orders in $\alpha^m_S \ln^n N$ for all $n = 2m$ (LL), $2m - 1 \leq n \leq 2m$ (NLL), etc.

- It is known that this sum is an exponential in Mellin space.
Two processes being considered:

- Deep inelastic scattering (DIS): \( l + H \rightarrow l + X \)
- Lepton pair production (LPP): \( H_1 + H_2 \rightarrow l + l + X \)
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- Deep inelastic scattering (DIS): $l + H \rightarrow l + X$
- Lepton pair production (LPP): $H_1 + H_2 \rightarrow l + l + X$

These two processes are primary sources of information on PDFs.

- DIS is used to constrain valence PDFs ($F_2 \sim 4u + d$)
- LPP is used to constrain antiquark PDFs ($\sigma \sim u\bar{u}, d\bar{d}$)
The squared mass of the final hadronic state in DIS is given by

\[ W^2 = M^2 + Q^2 \left( \frac{1}{x} - 1 \right) \]

Threshold occurs at \( W^2 = M^2 \).

This corresponds to \( x = 1 \).
F_2 Proton Deep Inelastic Scattering
CTEQ6M

Q^2 = 64 GeV^2
Resummation Effects

Threshold Resummation and PDFs

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F$_2$ Proton Deep Inelastic Scattering

CTEQ6M

$Q^2$
**LPP Kinematics**

- $x_1$ and $x_2$ are the momentum fractions that the partons take from the parent hadrons.
- Threshold occurs when $x_1 x_2 = \tau = \frac{Q^2}{S}$.
- LPP data is often in the form of the $x_F$ distribution, where $x_F = \frac{2p_t}{\sqrt{S}}$. 
LPP Kinematics

- At NLO, $x_1$ and $x_2$ are integrated, implying that threshold can occur at many values of $x_F$.

- Threshold kinematics requires that at large $x_F$, $x_1$ is large and $x_2$ is small. (And vice-versa)
- The PDFs fall rapidly at large $x$, so the largest contribution comes from the threshold region.
- Therefore, threshold corrections dominate at high $|x_F|$. 
for Proton-Proton LPP $x_F$ Distribution

$Q = 8 \text{ GeV}$
$
\sqrt{s} = 38.76 \text{ GeV}$

CTEQ6M
Others have found similar results:

Figure 15 from
Bonvini, M.; Forte, S. & Ridolfi, G.
Soft gluon resummation of Drell-Yan rapidity distributions: theory and phenomenology
Figure 2 from
Corcella, G. & Mitov, A. D.
Soft-Gluon Resummation for Heavy Quark Production in Charged-Current Deep Inelastic Scattering
Figure 3 from
Aicher, M.; Schafer, A.; & Vogelsang, W.
Soft-gluon resummation and the valence parton distribution function of the pion
The recent CJ12 PDF set was fit including data from the high $x$ and moderate $Q^2$ kinematic regions:
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Figure 1 from Owens, J. F.; Accardi, A. & Melnitchouk, W. Global parton distributions with nuclear and finite-$Q^2$ corrections. Phys.Rev. D87 (2013) 094012.
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Preliminary results of the global fit are currently being investigated.