Threshold Resummation and Determinations of Parton Distribution Functions

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- e.g. A massive state produced at high rapidity would require knowledge of PDFs at high x, where resummation effects are known to be large.

$$\sigma_{H_1+H_2}(x) = \sum_{a,b} \iiint \mathrm{d}x_1\, \mathrm{d}x_2\, \mathrm{d}z\, f_{a/H_1}(x_1) f_{b/H_2}(x_2) \hat{\sigma}_{ab}(z) \delta(x - x_1 x_2 z)$$

- Observables for hadron-initiated processes comprise a convolution of two parts:
  - Parton distribution functions (PDFs) $f_{a/H}(x)$
  - Hard-scattering cross section $\hat{\sigma}_{ab}(x)$

- PDFs are not calculable using perturbation theory; their forms are inferred by comparing data to theoretical predictions of observables.

- $\hat{\sigma}_{ab}$ is calculated using perturbation theory.

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- When the kinematics is constrained (as occurs in differential cross sections), logarithms associated with soft gluons become large.
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$$\int_0^1 \mathrm{d}z\, z^{N-1} \int_0^1 \mathrm{d}x\, f(x) \int_0^1 \mathrm{d}y\, g(y)\, \delta(z - xy) =$$
$$\int_0^1 \mathrm{d}x\, x^{N-1}\, f(x) \int_0^1 \mathrm{d}y\, y^{N-1}\, g(y) = \tilde{f}(N)\, \tilde{g}(N)$$

- Threshold logarithms in Mellin space manifest in powers of $\ln N$.

Organization of Threshold Logarithms

	LL	NLL	$\cdots$
NLO	$\alpha_S \ln^2 N$	$\alpha_S \ln N$	$\cdots$
NNLO	$\alpha_S^2 \ln^4 N$	$\alpha_S^2 \ln^3 N$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\ddots$

- Threshold (or soft-gluon) resummation is summing the logarithms to all orders in $\alpha_S^m \ln^n N$ for all $n = 2m$ (LL), $2m - 1 \leq n \leq 2m$ (NLL), etc.

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- It is known that this sum is an exponential in Mellin space.

- Two processes being considered:
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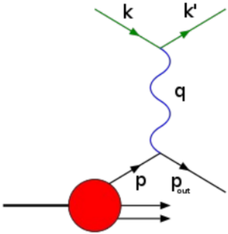
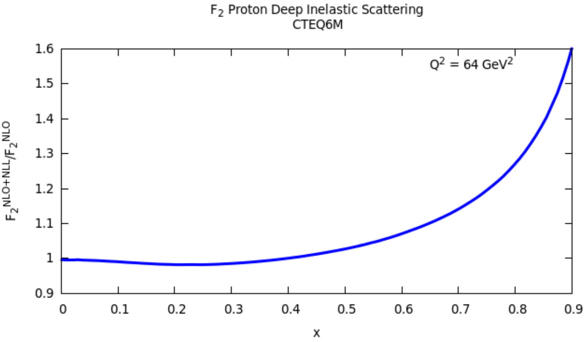
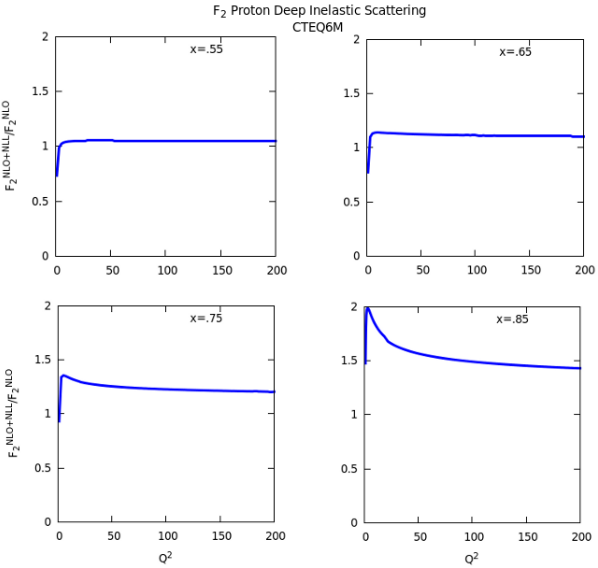
- These two processes are primary sources of information on PDFs.
  - DIS is used to constrain valence PDFs ($F_2 \sim 4u + d$)
  - LPP is used to constrain antiquark PDFs ($\sigma \sim u\bar{u}, d\bar{d}$)

DIS Kinematics



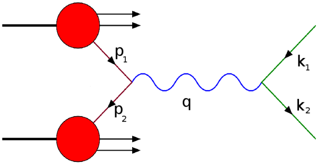
- The squared mass of the final hadronic state in DIS is given by $W^2 = M^2 + Q^2(\frac{1}{x} - 1)$
- Threshold occurs at $W^2 = M^2$.
- This corresponds to $x = 1$.


F2 Proton Deep Inelastic Scattering
CTEQ6M
Q2 = 64 GeV2
F2NLO+NLL/F2NLO
x
1.6
1.5
1.4
1.3
1.2
1.1
1
0.9
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9



F_2 Proton Deep Inelastic Scattering
CTEQ6M
x=.55
x=.65
x=.75
x=.85
$F_2^{NLO+NLL}/F_2^{NLO}$
Q^2
0
0.5
1
1.5
2
0
50
100
150
200

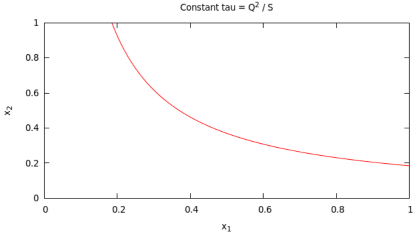
LPP Kinematics



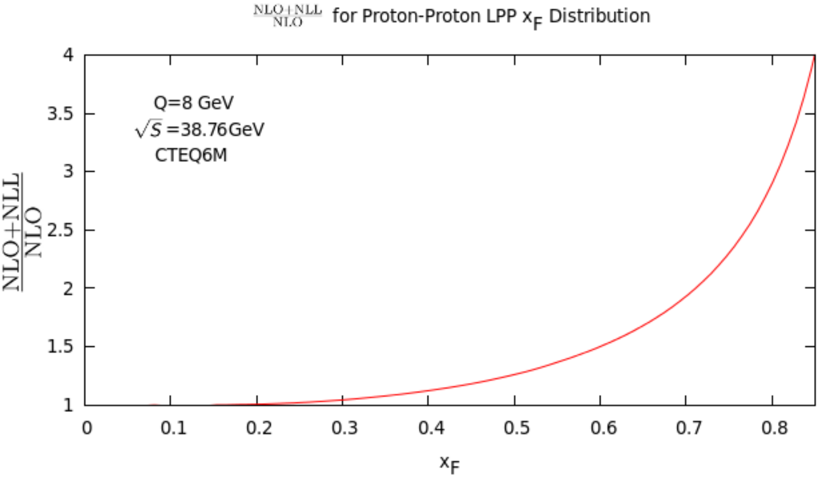
- x_1 and x_2 are the momentum fractions that the partons take from the parent hadrons.
- Threshold occurs when $x_1 x_2 = \tau = \frac{Q^2}{S}$.
- LPP data is often in the form of the x_F distribution, where $x_F = \frac{2p_L}{\sqrt{S}}$.

LPP Kinematics

- At NLO, x_1 and x_2 are integrated, implying that threshold can occur at many values of x_F.



- Threshold kinematics requires that at large x_F, x_1 is large and x_2 is small. (And vice-versa)
- The PDFs fall rapidly at large x, so the largest contribution comes from the threshold region.
- Therefore, threshold corrections dominate at high $|x_F|$.



$\frac{NLO+NLL}{NLO}$ for Proton-Proton LPP x_F Distribution
Q=8 GeV
$\sqrt{S}$ =38.76GeV
CTEQ6M
$\frac{NLO+NLL}{NLO}$
x_F
4
3.5
3
2.5
2
1.5
1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8

- Others have found similar results:

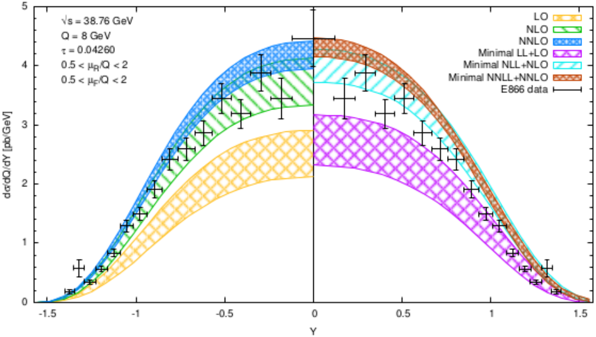


Figure 15 from
Bonvini, M.; Forte, S. & Ridolfi, G.
Soft gluon resummation of Drell-Yan rapidity distributions: theory and phenomenology
Nucl.Phys. **B847**:93-159, 2011

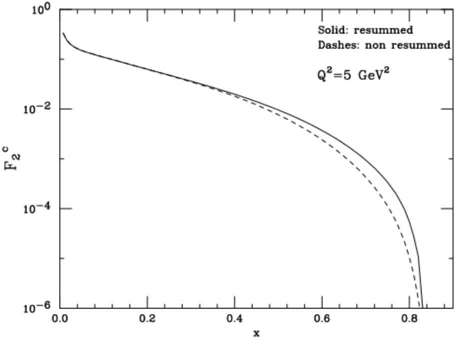


Figure 2 from
Corcella, G. & Mitov, A. D.
Soft-Gluon Resummation for Heavy Quark Production in Charged-Current Deep Inelastic Scattering
Nucl.Phys. **B676**:346-364, 2004

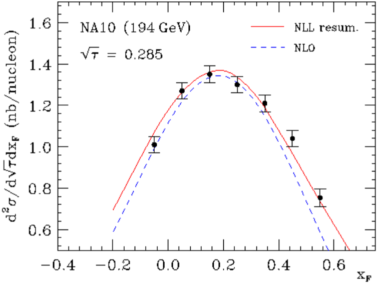


Figure 3 from
Aicher, M.; Schafer, A.; & Vogelsang, W.
Soft-gluon resummation and the valence parton distribution function of the pion
Phys.Rev.Lett. **105**, 252003, 2010

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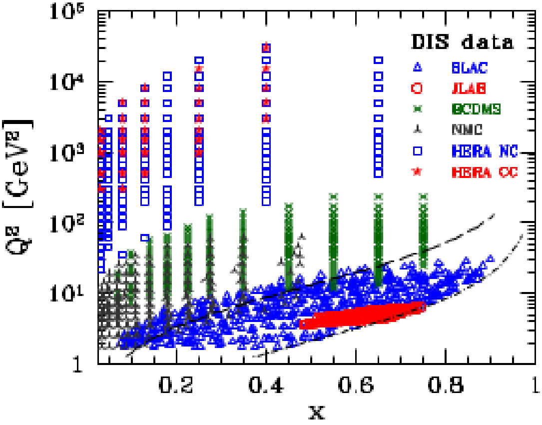

Figure 1 from Owens, J. F.; Accardi, A. & Melnitchouk, W. Global parton distributions with nuclear and finite-Q^2 corrections. Phys.Rev. D87 (2013) 094012.

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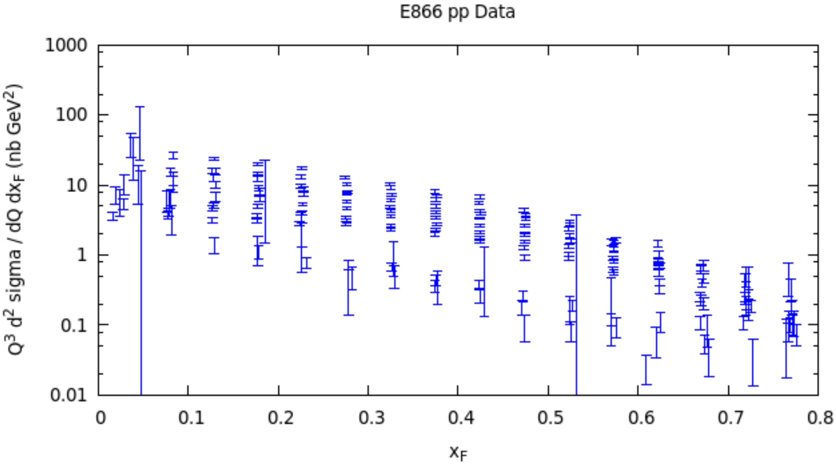

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  - PDFs are sensitive to resummation effects at different values of x for DIS and LPP.
- Preliminary results of the global fit are currently being investigated.