CALIFORNIA STATE UNIVERSITY, FRESNO

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The Covariant, time-dependent Aharonov-Bohm Effect*

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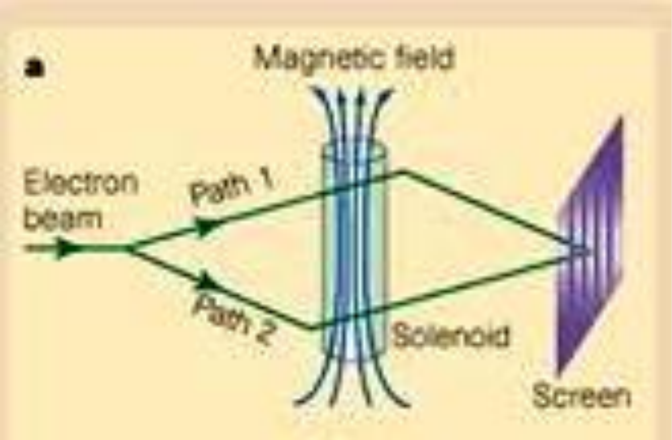
*D. Singleton and E. Vagenas, _Phys.Lett. B_**_723_** _(2013) 241-244 [arXiv:1305.1498 (hep-th)]_

Static Magnetic AB Effect

- The magnetic AB effect is the QM two-slit experiment with an infinite solenoid placed between the slits



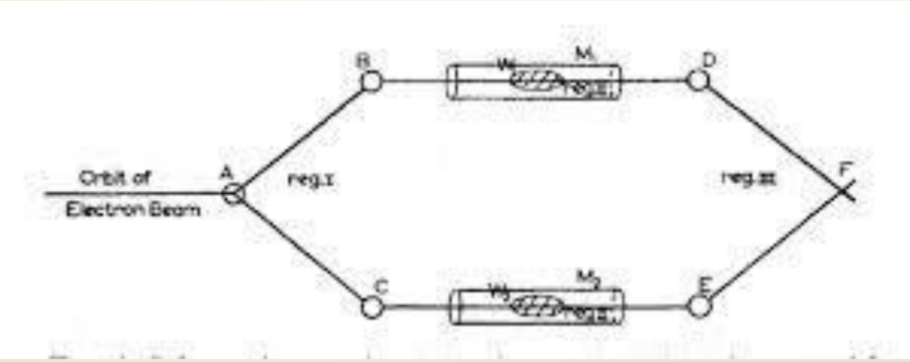
- There is a phase difference between the two paths given by

$$\delta\alpha_B = \alpha_{B_1} - \alpha_{B_2} = \frac{e}{\hbar}\oint_{2-1} \mathbf{A}\cdot d\mathbf{x} = \frac{e}{\hbar}\int \mathbf{B}\cdot d\mathbf{S} = \frac{e}{\hbar}\Phi_0$$

- Leading to a shift in the phase pattern $\Delta x = L\lambda(\delta\alpha_B)/2\pi d$

Static Electric AB Effect

- In the electric AB effect electrons travel along two paths with different potentials and **E**=0



- When the electrons recombine there is a phase difference

$$\delta\alpha_E = \frac{e}{\hbar}\int_{t_1}^{t_2} \Delta\phi dt = \frac{e}{\hbar}\int_{t_1}^{t_2}\int \mathbf{E}\cdot d\mathbf{x}\ dt$$

Common description of AB effects

- The common form of the phase shift is

$$\delta(Phase) \propto (Field) \times (Area)$$

- For the electric case the “area” has one temporal “leg”

$$\delta\alpha_E = \frac{e}{\hbar}(\mathbf{E} \cdot \Delta\mathbf{x})\Delta t$$

Covariant versions of AB phase

- In terms of the four-potential the AB phase shift can be written as

$$\delta\alpha_{EB} = \frac{e}{\hbar}\oint A_\mu dx^\mu = \frac{e}{\hbar}\left[\int_{t_1}^{t_2} \Delta\phi dt - \oint \mathbf{A}\cdot d\mathbf{x}\right]$$

- In terms of the **E** and **B** fields the AB phase shift can be written in terms of differential forms, dx^μ ,wedge product, ^, and the Faraday two form, F.

$$\delta\alpha_{EB} = -\frac{e}{2\hbar}\int F_{\mu\nu} dx^\mu \wedge dx^\nu = \frac{e}{\hbar}\int F$$

Recovery of Static Results

- The static magnetic and electric AB results can be recovered from the Faraday two form expression given above and expanded below.

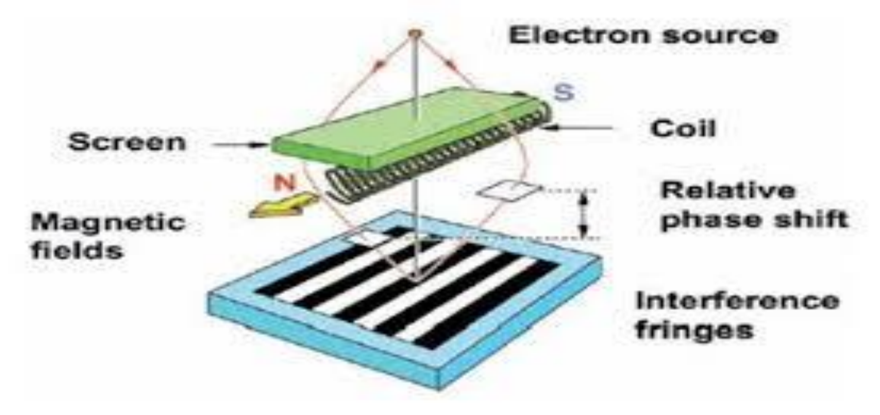
$$
\begin{aligned}
F &= -\frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu \\
&= (E_x dx + E_y dy + E_z dz) \wedge dt + B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy
\end{aligned}
$$

- If **E**=0 the last three terms are just the infinitesimal magnetic phase shift.

- If **B**=0 and there is no time dependence (∂_t**A**=0) the first three terms are the infinitesimal electric phase shift.

Time dependent AB phase 1.

- These two covariant expressions can be used to study the time-dependent AB effect e.g. a solenoid with time varying flux (ϕ=0 for this situation)



- The Faraday two form version of the phases gives exactly cancelling results i.e.

$$\frac{e}{\hbar}\int [B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy] = \frac{e}{\hbar}\int \mathbf{B}(\mathbf{x}, t) \cdot d\mathbf{S}$$

Cancels

$$\frac{e}{\hbar}\int [(E_x dx + E_y dy + E_z dz) \wedge dt] = -\frac{e}{\hbar}\oint \mathbf{A} \cdot d\mathbf{x} = -\frac{e}{\hbar}\int \mathbf{B}(\mathbf{x}, t) \cdot d\mathbf{S}$$

$$\mathbf{E} = -\partial_t \mathbf{A}$$

Time dependent AB phase 2

- The expression four-vector potential expression gives the same result. Consider a time dependent current and vector potential

$$\mathbf{A} = \frac{kI(t)}{r}\hat{\theta}$$

- Taylor expand the current $I(t_i + \Delta t/2) \approx I(t_i) + I'(t_i)\frac{\Delta t}{2} + \ldots$

- This gives two contributions to the phase difference

$$\delta\alpha_B = \frac{ekI(t_i)\Delta\theta}{\hbar}$$

$$\delta\alpha_{A(t)} = \frac{ekI'(t)\Delta t\Delta\theta}{2\hbar}$$

Time dependent AB phase 2

- Now due to **E**=-∂_t **A** there is an electric field outside the solenoid which contributes to the phase difference

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{kI'(t)}{r}\hat{\theta}$$

$$\mathbf{a} = \frac{e\mathbf{E}}{m} = -\frac{ekI'(t)}{mr}\hat{\theta}$$

- This leads to a change in distance of

$$\Delta d = \frac{1}{2}a\Delta t^2 = -\frac{ekI'(t)}{2mr}\frac{r\Delta\theta}{v}\Delta t = -\frac{ekI'(t)\Delta\theta\Delta t}{2mv}$$

- And a phase difference of

$$\delta\alpha_{E-field} = \frac{\Delta d}{\lambda/(2\pi)} = -\frac{ekI'(t)\Delta t\Delta\theta}{2\hbar}$$

- Which cancels the $\delta\alpha_{A(t)}$ contribution from the previous slide.

Prediction for time dependent AB phase

- Both the Faraday two-form expression and the four-vector expression for the phase difference give no *time dependent* phase shift due to the time variation of the flux in the solenoid.

- This is easier to calculate/see using the Faraday two-form expression.

- The four-vector expression shows this result is due to a cancellation between the usual AB phase and a non-AB phase coming from the electric field external to the solenoid.

Experimental Status of Time-Dependent AB Effect

- Not much work has been done on the time-dependent AB effect

B. Lee, E. Yin, T. K. Gustafson, and R. Chiao, "Analysis of Aharonov-Bohm effect due to time-dependent vector potentials", *Phys. Rev. A* ***45**, 4319 (1992).*

A. N. Ageev, S. Yu. Davydov, and A. G. Chirkov, ``Magnetic Aharonov-Bohm Effect under Time-Dependent Vector Potential" *Tech. Phys. Lett.* ***26**, 392 (2000).*

- These two papers predict a time-changing phase shift but ignore the external electric field.

- The second paper reports on an experiment that did find no time-dependent phase shift but this was attributed to a bad choice of parameter (time rate of change of **A** was to rapid).

Summary and Conlusions

- Two covariant expressions for AB phase difference were given.
- These expression allow one to study the time-dependent AB effect.
- The result for a time varying version of the usual AB set-up is a prediction of no time dependent phase shift.
- This is due to a cancellation between a standard AB phase shift and a non-AB shift due to the external electric field.

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