The Covariant, time-dependent Aharonov-Bohm Effect*

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Static Magnetic AB Effect

• The magnetic AB effect is the QM two-slit experiment with an infinite solenoid placed between the slits

• There is a phase difference between the two paths given by

\[ \delta \alpha_B = \alpha_{B_1} - \alpha_{B_2} = \frac{e}{\hbar} \int_{2-1} A \cdot dx = \frac{e}{\hbar} \int B \cdot dS = \frac{e}{\hbar} \Phi_0 \]

• Leading to a shift in the phase pattern \( \Delta x = L \lambda (\delta \alpha_B) / 2\pi d \)
Static Electric AB Effect

• In the electric AB effect electrons travel along two paths with different potentials and $E=0$

• When the electrons recombine there is a phase difference

\[ \delta \alpha_E = \frac{e}{\hbar} \int_{t_1}^{t_2} \Delta \phi dt = \frac{e}{\hbar} \int_{t_1}^{t_2} \int E \cdot dx \, dt \]
Common description of AB effects

• The common form of the phase shift is

\[ \delta(\text{Phase}) \propto (\text{Field}) \times (\text{Area}) \]

• For the electric case the “area” has one temporal “leg”

\[ \delta \alpha_E = \frac{e}{\hbar} (E \cdot \Delta x) \Delta t \]
Covariant versions of AB phase

- In terms of the four-potential the AB phase shift can be written as

\[ \delta \alpha_{EB} = \frac{e}{\hbar} \oint A_\mu \, dx^\mu = \frac{e}{\hbar} \left[ \int_{t_1}^{t_2} \Delta \phi \, dt - \oint A \cdot dx \right] \]

- In terms of the E and B fields the AB phase shift can be written in terms of differential forms, dx^\mu, wedge product, ^, and the Faraday two form, F.

\[ \delta \alpha_{EB} = -\frac{e}{2\hbar} \int F_{\mu\nu} \, dx^\mu \wedge dx^\nu = \frac{e}{\hbar} \int F \]
Recovery of Static Results

- The static magnetic and electric AB results can be recovered from the Faraday two form expression given above and expanded below.

\[
F = -\frac{1}{2} F_{\mu\nu} dx^\mu \wedge dx^\nu \\
= (E_x dx + E_y dy + E_z dz) \wedge dt + B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy
\]

- If \( E = 0 \) the last three terms are just the infinitesimal magnetic phase shift.

- If \( B = 0 \) and there is no time dependence (\( \partial_t A = 0 \)) the first three terms are the infinitesimal electric phase shift.
Time dependent AB phase 1.

- These two covariant expressions can be used to study the time-dependent AB effect e.g. a solenoid with time varying flux (φ=0 for this situation)

- The Faraday two form version of the phases gives exactly cancelling results i.e.

$$\frac{e}{\hbar} \int [B_x dy \wedge dz + B_y dz \wedge dx + B_z dx \wedge dy] = \frac{e}{\hbar} \int \mathbf{B}(x, t) \cdot dS$$

Cancels

$$\frac{e}{\hbar} \int [(E_x dx + E_y dy + E_z dz) \wedge dt] = -\frac{e}{\hbar} \int \mathbf{A} \cdot dx = -\frac{e}{\hbar} \int \mathbf{B}(x, t) \cdot dS$$

\[ E = -\partial_t \mathbf{A} \]
**Time dependent AB phase 2**

- The expression four-vector potential expression gives the same result. Consider a time dependent current and vector potential

\[ A = \frac{k I(t) \hat{\theta}}{r} \]

- Taylor expand the current

\[ I(t_i + \Delta t/2) \approx I(t_i) + I'(t_i) \frac{\Delta t}{2} + \ldots \]

- This gives two contributions to the phase difference

\[ \delta \alpha_B = \frac{ek I(t_i) \Delta \theta}{\hbar} \]

\[ \delta \alpha_{A(t)} = \frac{ek I'(t) \Delta t \Delta \theta}{2\hbar} \]
Time dependent AB phase 2

- Now due to $\mathbf{E}=-\partial_t \mathbf{A}$ there is an electric field outside the solenoid which contributes to the phase difference.

$$E = -\frac{\partial \mathbf{A}}{\partial t} = -\frac{k I'(t)}{r} \dot{\theta}$$

$$a = \frac{eE}{m} = -\frac{ek I'(t)}{mr} \dot{\theta}$$

- This leads to a change in distance of

$$\Delta d = \frac{1}{2} a \Delta t^2 = -\frac{ek I'(t) r \Delta \theta}{2mr} \Delta t = -\frac{ek I'(t) \Delta \theta \Delta t}{2mv}$$

- And a phase difference of

$$\delta \alpha_{E-field} = \frac{\Delta d}{\lambda/(2\pi)} = -\frac{ek I'(t) \Delta t \Delta \theta}{2\hbar}$$

- Which cancels the $\delta \alpha_{A(t)}$ contribution from the previous slide.
Prediction for time dependent AB phase

• Both the Faraday two-form expression and the four-vector expression for the phase difference give no *time dependent* phase shift due to the time variation of the flux in the solenoid.

• This is easier to calculate/see using the Faraday two-form expression.

• The four-vector expression shows this result is due to a cancellation between the usual AB phase and a non-AB phase coming from the electric field external to the solenoid.
Experimental Status of Time-Dependent AB Effect

• Not much work has been done on the time-dependent AB effect


• These two papers predict a time-changing phase shift but ignore the external electric field.

• The second paper reports on an experiment that did find no time-dependent phase shift but this was attributed to a bad choice of parameter (time rate of change of $A$ was too rapid).
Summary and Conclusions

• Two covariant expressions for AB phase difference were given.

• These expressions allow one to study the time-dependent AB effect.

• The result for a time varying version of the usual AB set-up is a prediction of no time dependent phase shift.

• This is due to a cancellation between a standard AB phase shift and a non-AB shift due to the external electric field.

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