Effective and temperature-dependent viscosities in a hydrodynamically-expanding QCD plasma

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August 16, 2019

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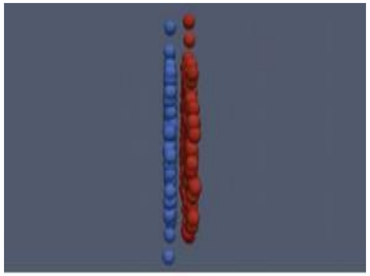
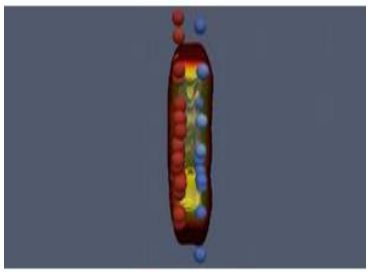
Many body QCD from heavy ion collisions



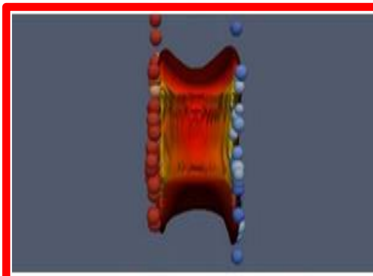
$\tau = "0^{+}"$: Nuclei collide

- Large energy deposition



$\tau \sim 0.1$ fm: “Pre-equilibrium phase”

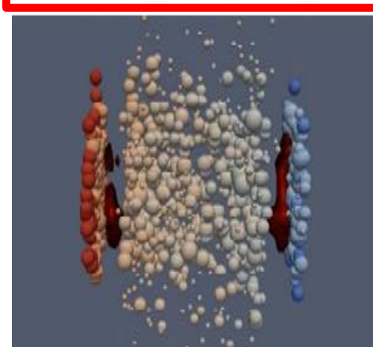
- Saturation, “hydrodynamization”, attractors, …



$\tau \sim 1$ fm: Beginning of “hydrodynamic phase”

- Coarse-grained description of energy-momentum tensor evolution
- Strongly-coupled quark-gluon plasma proper

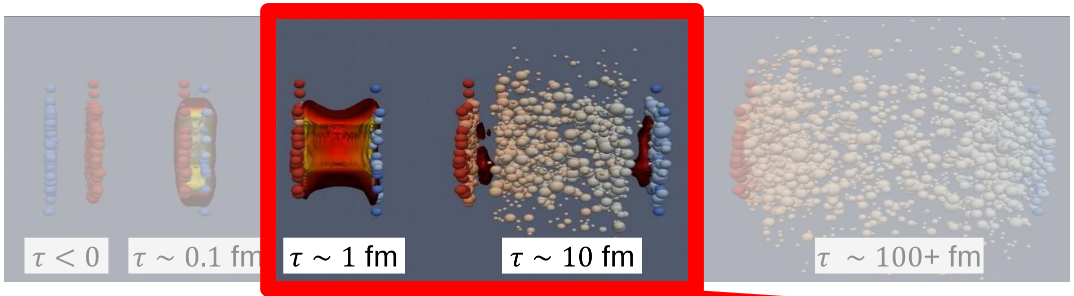
[Connection with (near-) equilibrium properties of QCD]

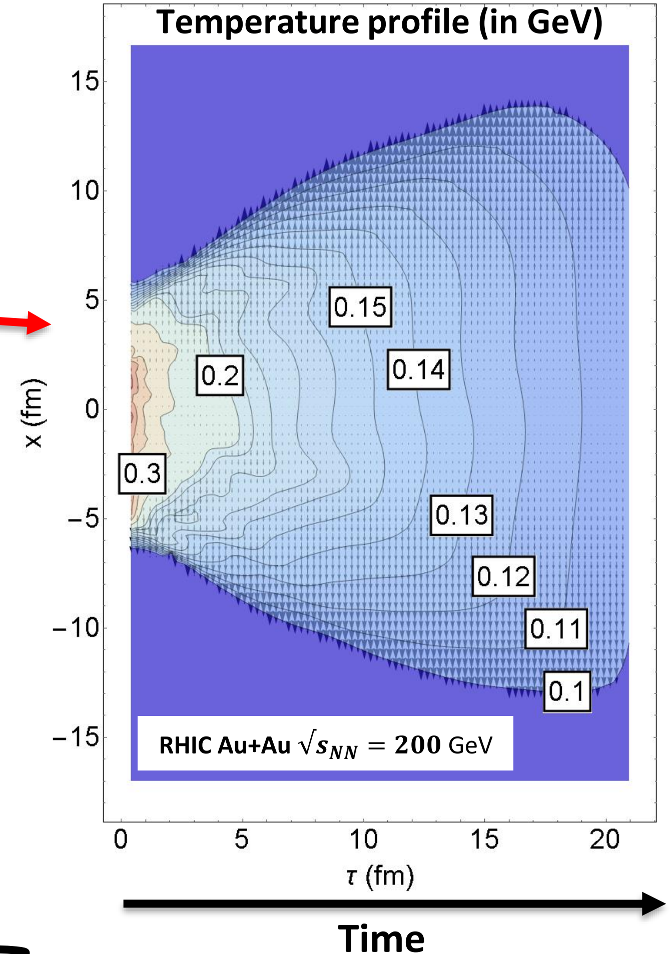


$\tau \sim 10$ fm: End of “hydrodynamic phase”

- Fluid converted to particles (hadrons)
- Hadronic interactions continue until density is sufficiently low

Strongly-coupled quark-gluon plasma (sQGP)





<u>Spacetime evolution of the plasma:</u>

- Evolution of energy-momentum tensor $T^{\mu\nu}$ described with **relativistic viscous hydrodynamics**

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

$$\boxed{\text{Conservation of energy/momentum: } \partial_\nu T^{\mu\nu} = 0}$$

Equations of motion for shear tensor $\pi^{\mu\nu}$ and bulk pressure Π:

$$\left[\begin{array}{l} \tau_\Pi \dot{\Pi} + \Pi = -\zeta\, \theta + (2^{\text{nd}} \text{ order terms}) \\ \tau_\pi \Delta^{\mu\nu}_{\alpha\beta} \pi^{\alpha\beta} + \pi^{\mu\nu} = 2\, \eta\, \sigma^{\mu\nu} + (2^{\text{nd}} \text{ order terms}) \end{array}\right]$$

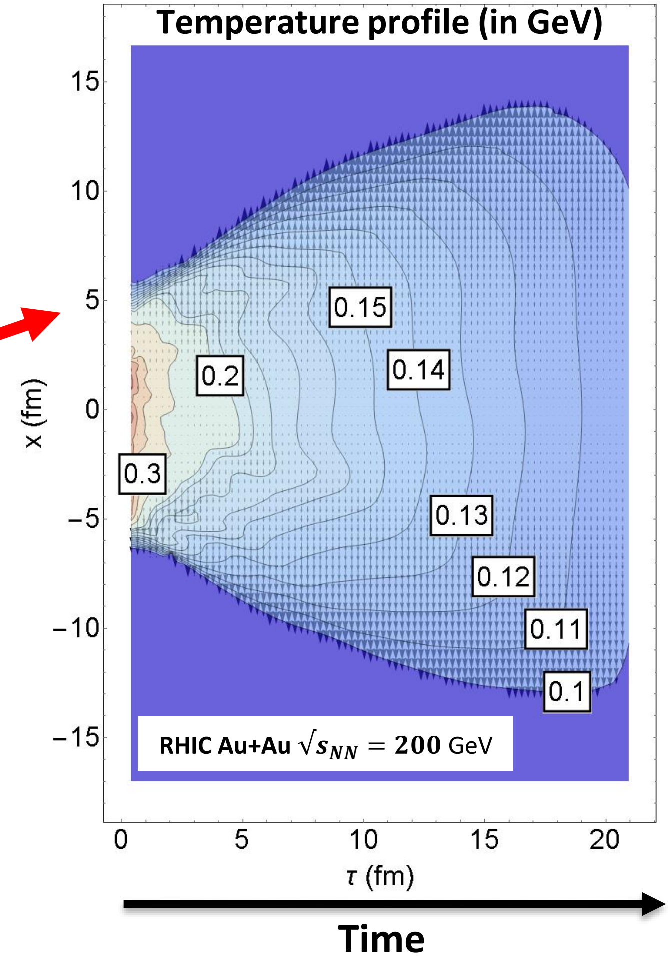
Equilibrium QCD

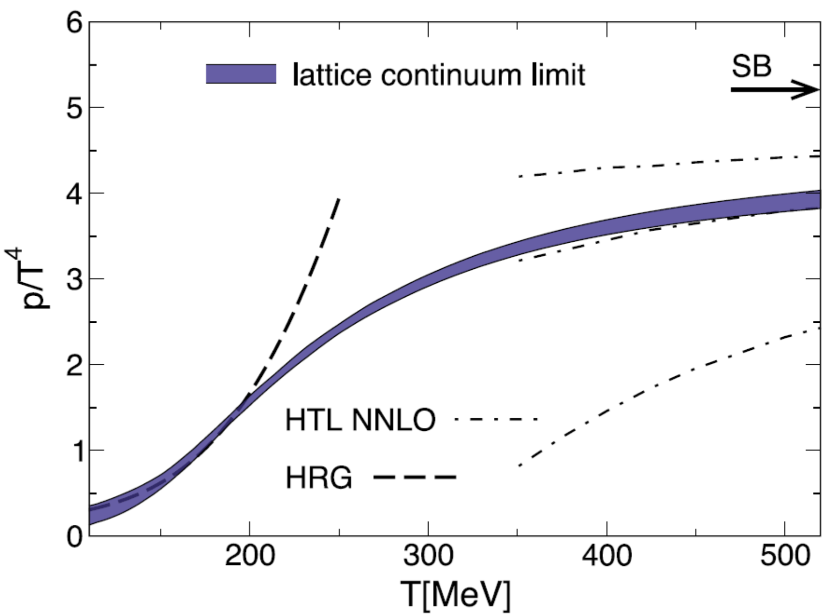
<u>Spacetime evolution of the plasma:</u>

- Evolution of energy-momentum tensor $T^{\mu\nu}$ described with relativistic viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P(\epsilon) + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

Equation of state





Ref.: Borsányi et al (2014) PLB 730, 99

Near-equilibrium QCD

Spacetime evolution of the plasma:

- Evolution of energy-momentum tensor $T^{\mu\nu}$ described with relativistic viscous hydrodynamics

$$T^{\mu\nu} = \epsilon u^\mu u^\nu - (P(\epsilon) + \Pi)(g^{\mu\nu} - u^\mu u^\nu) + \pi^{\mu\nu}$$

Conservation of energy/momentum: $\partial_\nu T^{\mu\nu} = 0$

Relaxation equations for shear tensor $\pi^{\mu\nu}$:

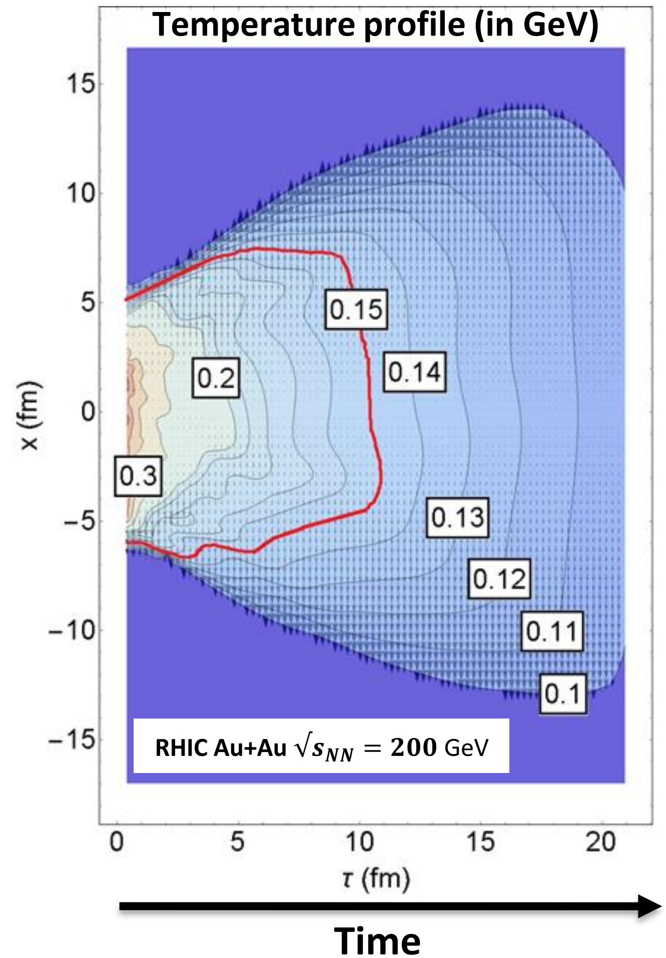
$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta}\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\,\eta\,\sigma^{\mu\nu} + (2^{\text{nd}}\text{ order terms})$$

Shear viscosity

Relaxation equations for bulk pressure Π:

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\,\theta + (2^{\text{nd}}\text{ order terms})$$

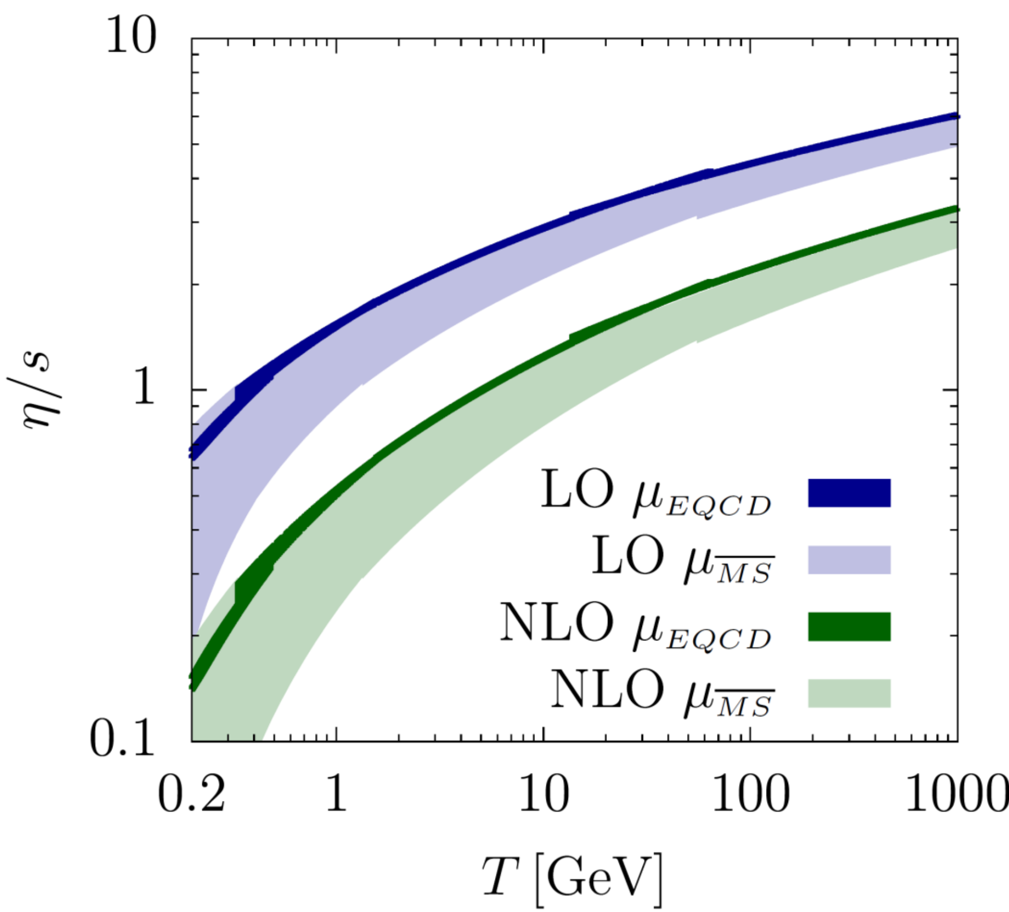
Bulk viscosity



Transport coefficients of QCD from theory

Shear viscosity from finite-temperature pQCD

Ref.: Ghiglieri, Moore, Teaney (2018) JHEP 1803 179



- Poor convergence even at high T
- "Uncertainties" (from renorm. scale, running, …) do not overlap for LO & (almost) NLO

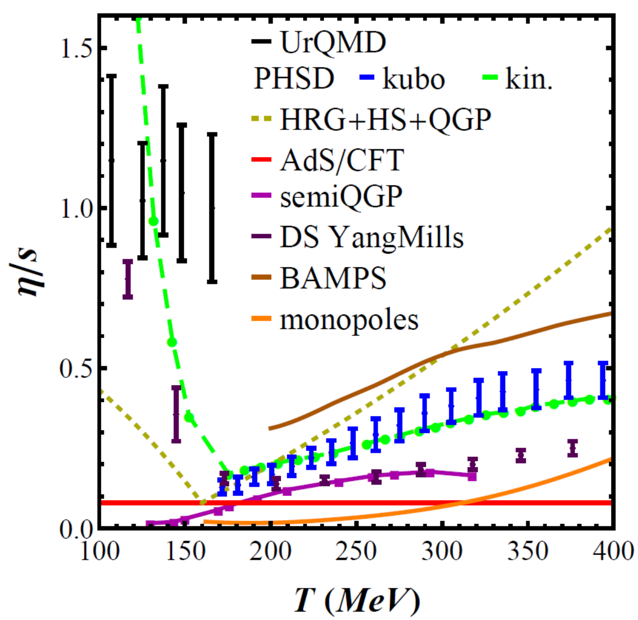
Transport coefficients of QCD from theory

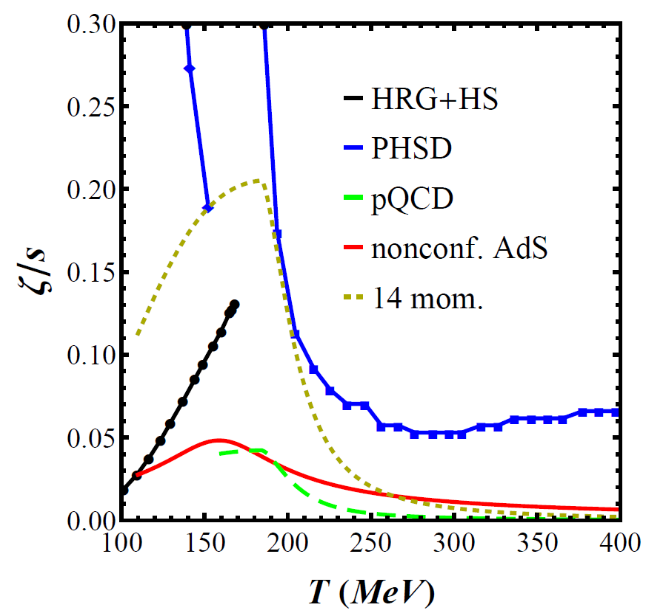
Low temperature:

- Hadronic transport (e.g. UrQMD, SMASH), holography (e.g. AdS/CFT), effective models, …

High temperature:

- Weak-coupling, holography, effective models, …

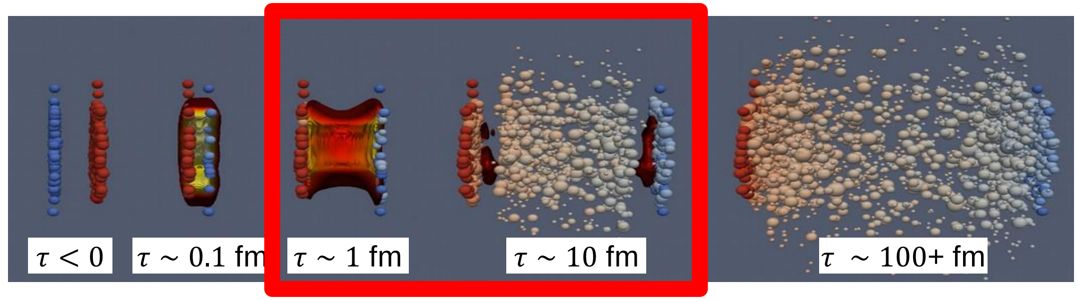




Ref.: Noronha-Hostler, arXiv:1512.06315

Shear viscosity from heavy ion collisions

1) Initialize $T^{\mu\nu}$ of hydrodynamic at an early time ($\tau \sim 0.1 - 1$ fm)



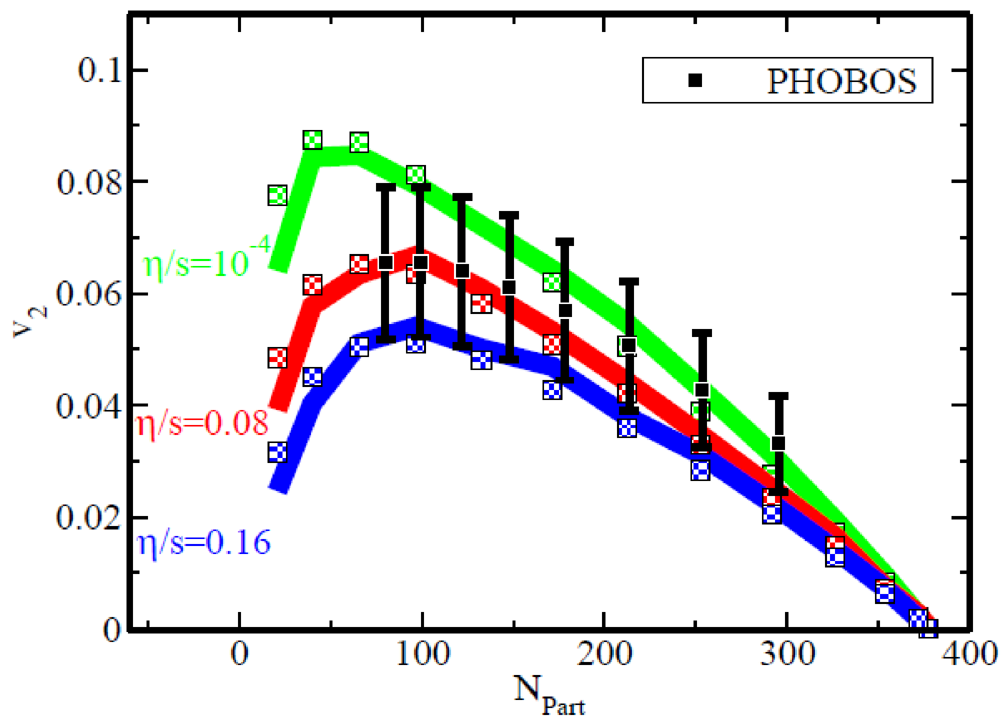
2) Solve hydrodynamic equations with fixed choice of shear viscosity:

$$\partial_\nu T^{\mu\nu} = 0$$
$$\tau_\pi \Delta^{\mu\nu}_{\alpha\beta}\pi^{\alpha\beta} + \pi^{\mu\nu} = 2\,\boldsymbol{\eta}\,\sigma^{\mu\nu} + \ \ldots$$

3) Convert fluid to hadrons

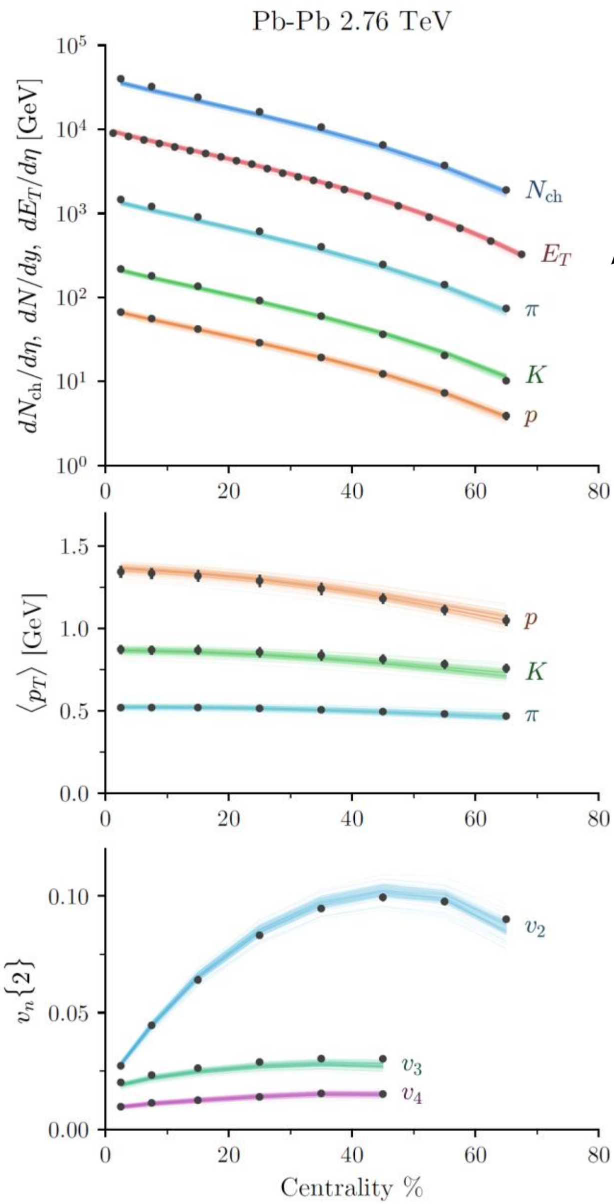
4) Evaluate observable and compare with data

$$E\frac{d^3N}{d^3p} = \frac{1}{2\pi}\frac{dN}{p_T dp_T dy}\left[1 + 2\sum_{n=1}^{\infty} v_n \cos n(\phi - \Psi_n)\right].$$



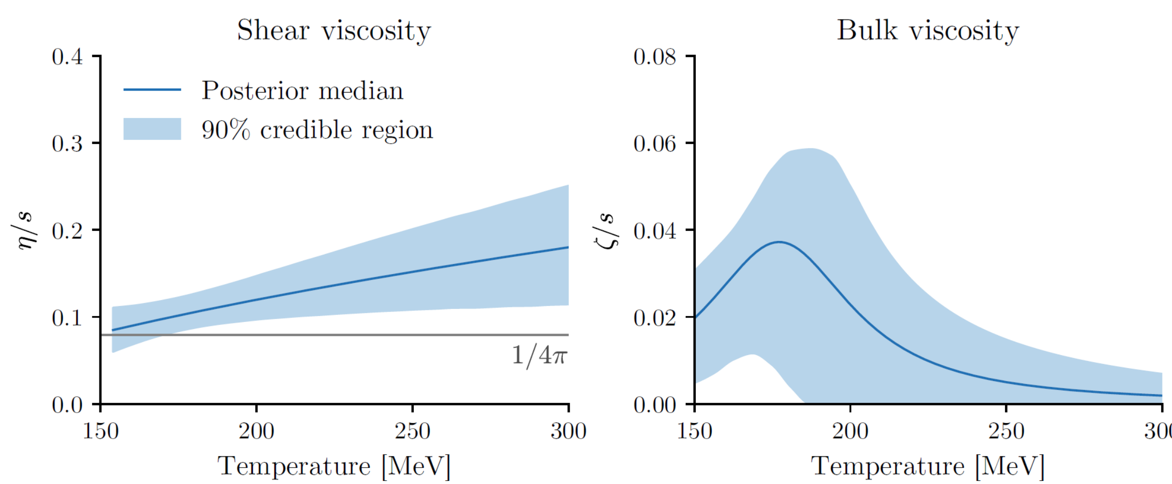
Ref.: Luzum & Romatschke (2008) PRC78, 034915

Modern approach: Bayesian analysis



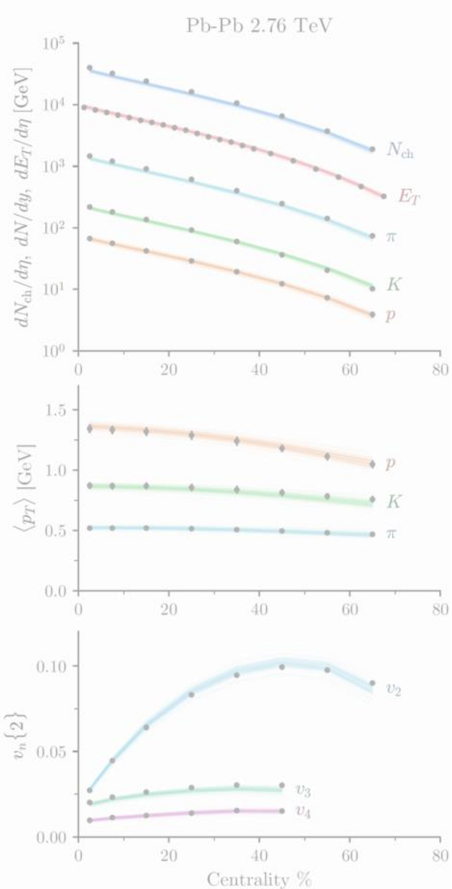
Systematic approach:

- Compare to large number of measurements simultaneously
- Systematic propagation of uncertainties (data & theory) to shear/bulk viscosity constraints



Refs.: Bernhard (2018) PhD Thesis, arXiv:1804.06469
Bernhard, Moreland & Bass, Nature Physics (2019)

Modern approach: Bayesian analysis, revisited



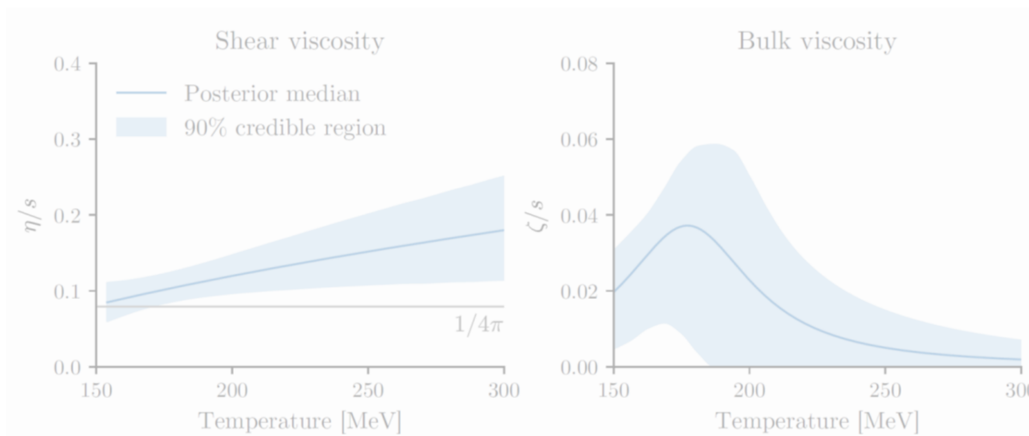
Ref.: Bernhard (2018)
PhD Thesis,
arXiv:1804.06469

Multi-institution effort by JETSCAPE Collaboration

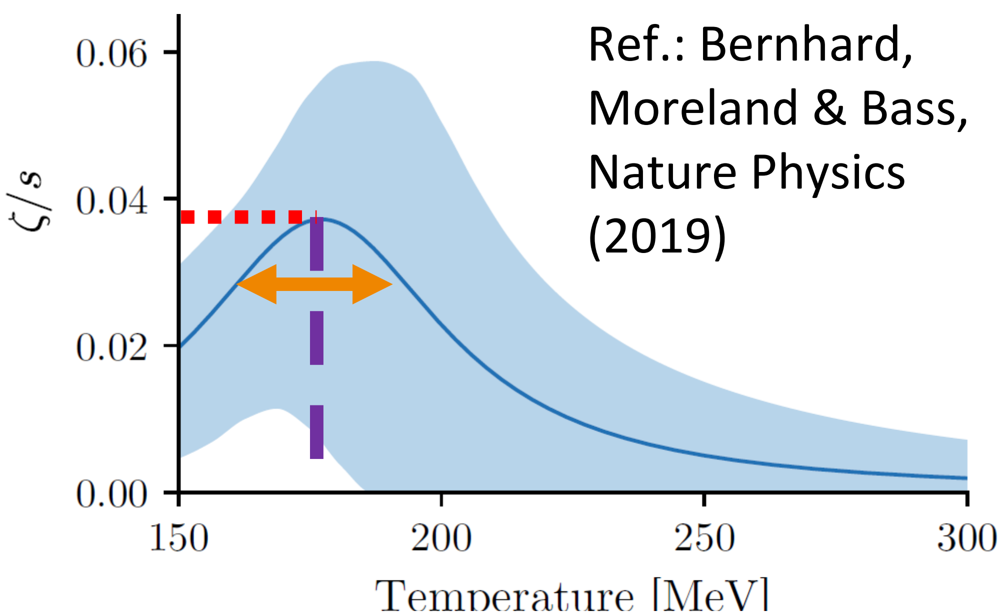
(BNL, Duke, OSU, McGill, Wayne State, USP)

Revisiting this analysis:

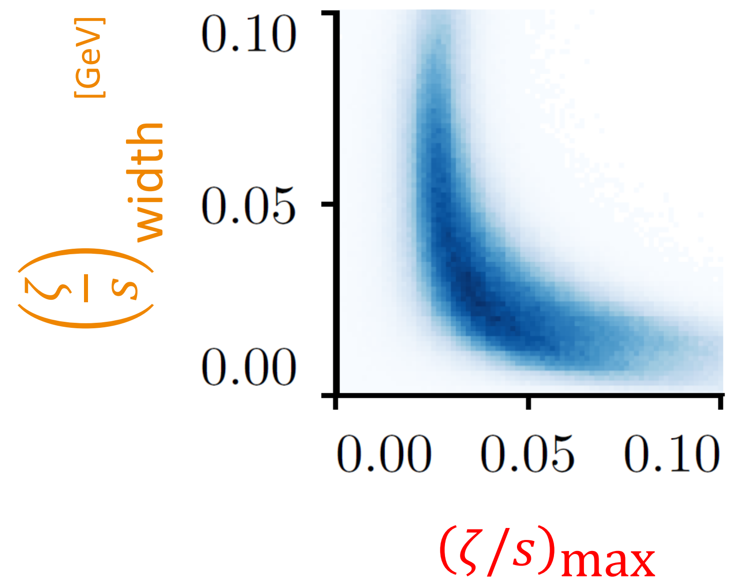
- More collision systems (Xe-Xe @ $\sqrt{s_{NN}} = 5.44$ TeV, Pb-Pb @ $\sqrt{s_{NN}} = 2.76$ & 5.02 TeV, Au-Au @ $\sqrt{s_{NN}} = 0.2$ TeV)
- More flexible model (e.g. transport coefficients)
- Quantify known systematic uncertainties



Result from Bayesian analysis: a closer look

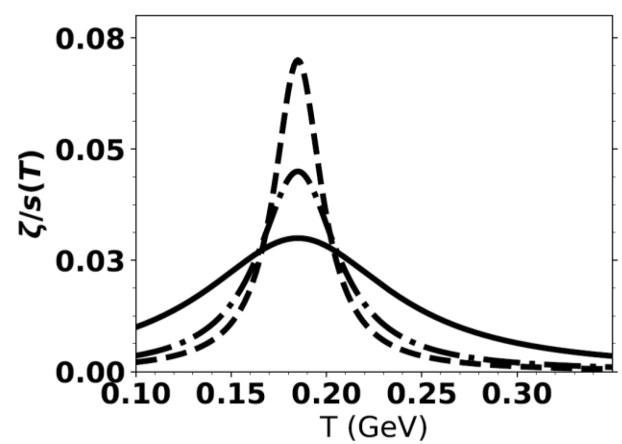


Ref.: Bernhard, Moreland & Bass, Nature Physics (2019)



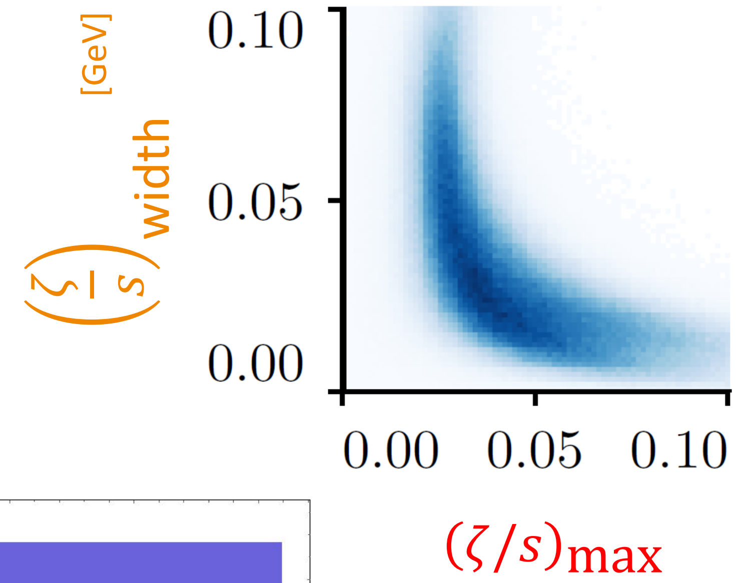
$$\left(\frac{\zeta}{s}\right)(T) = \frac{\left(\frac{\zeta}{s}\right)_{\text{max}}}{1+\left(\frac{T-T_{peak}}{\text{width}}\right)^2}$$

Anticorrelation between maximum and width of ζ/s

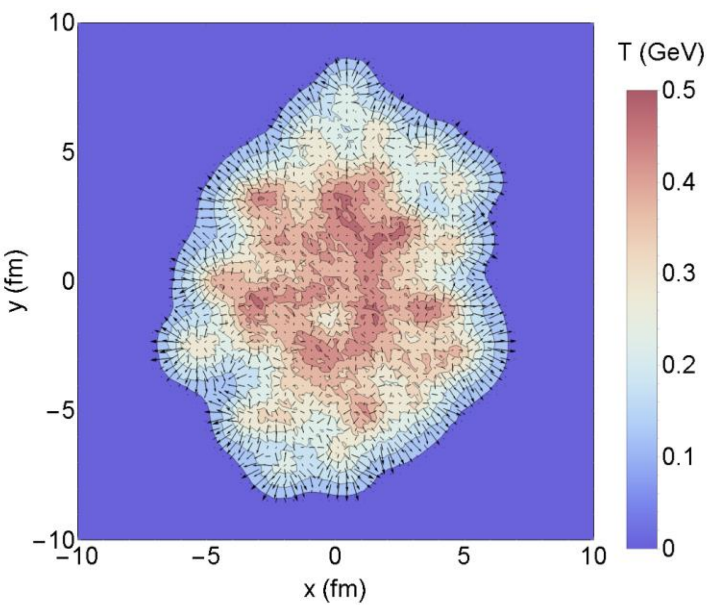


Origin of degeneracy?

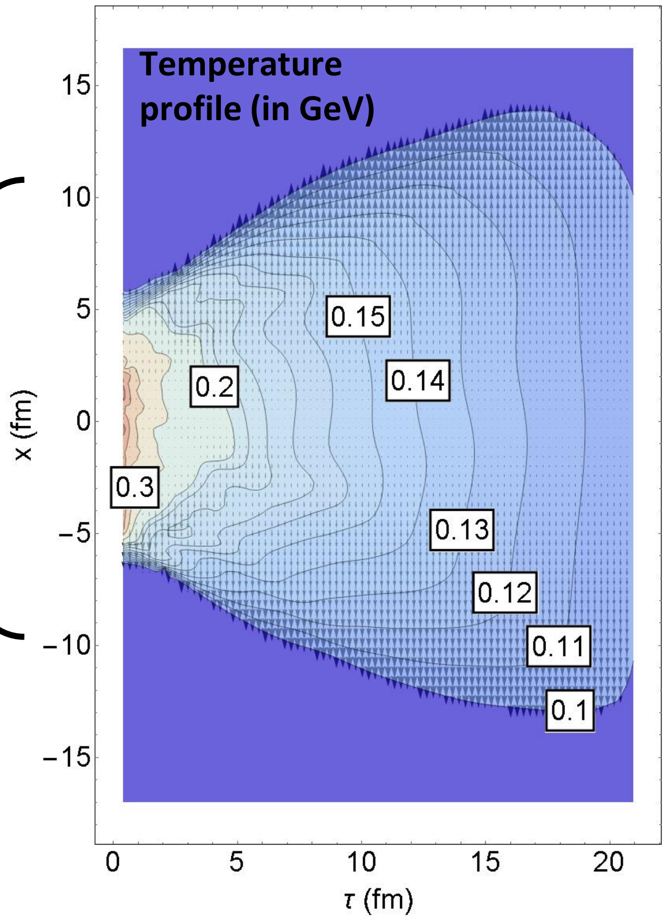
- Not enough measurements?
- Measurements not sensitive enough?



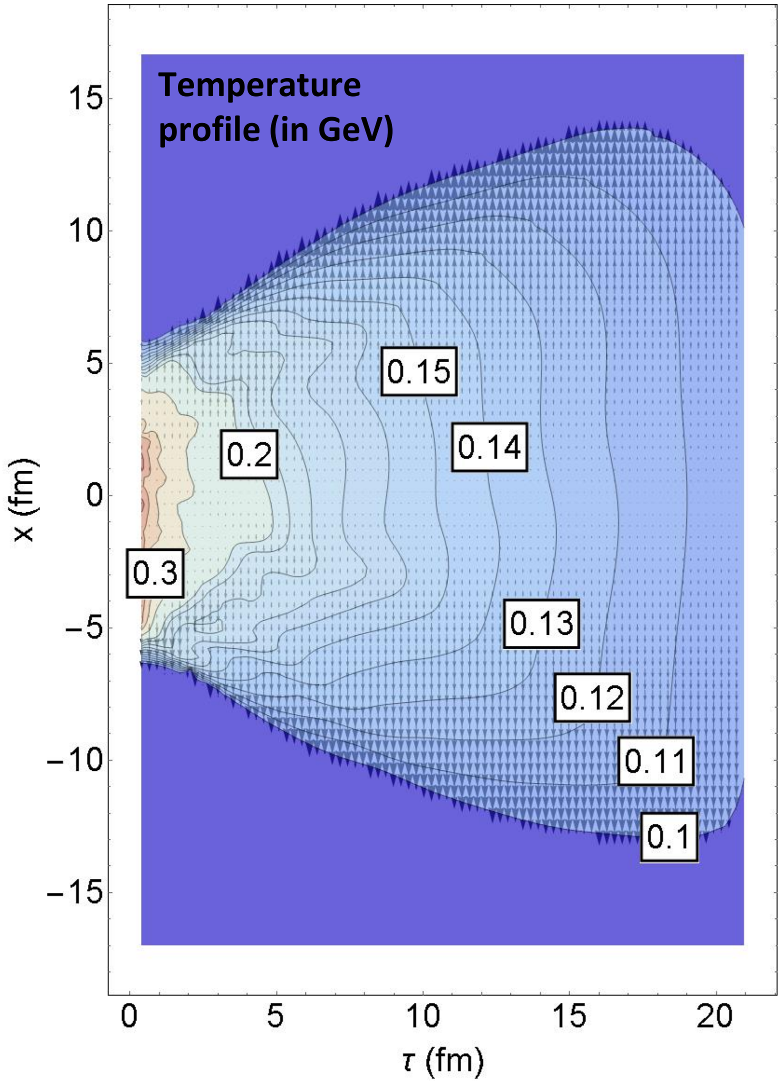
Symmetries?

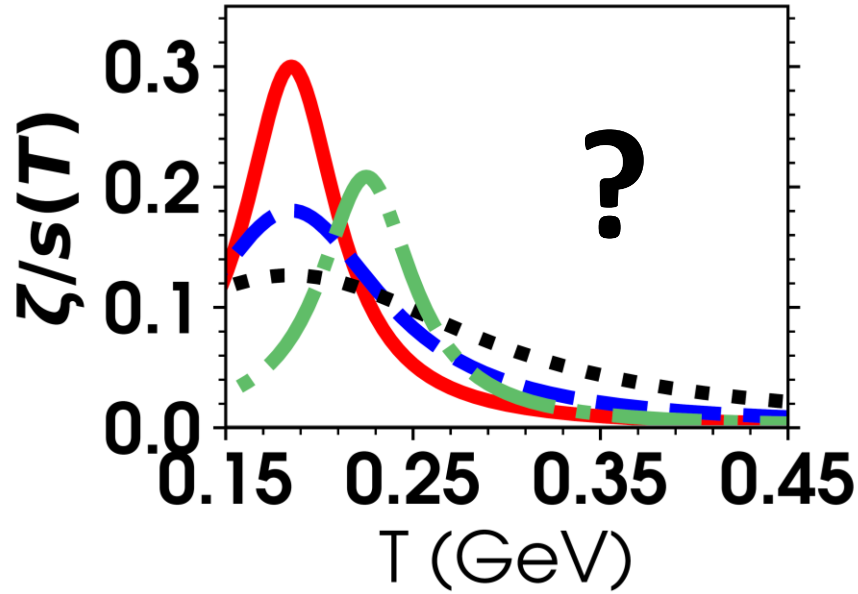


Initial temperature profile



(Approximate) Degeneracy possible?

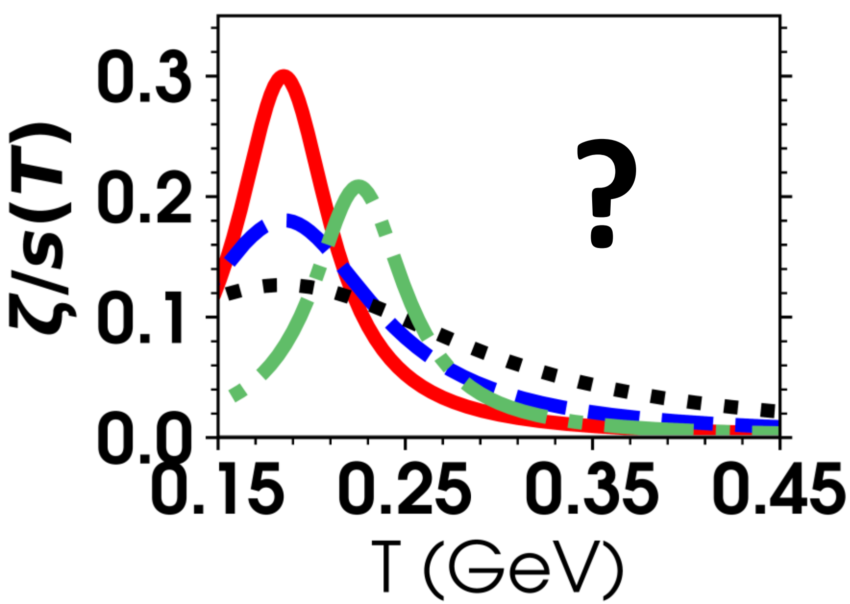




Approach:

What kind of degeneracies are possible in simpler hydrodynamics settings?

Viscosity degeneracies in Bjorken hydrodynamics



Bjorken hydrodynamics & viscosities

Assumed fluid symmetries (0+1D system):

- Boost-invariant in one dimension
- Uniform in the other dimensions

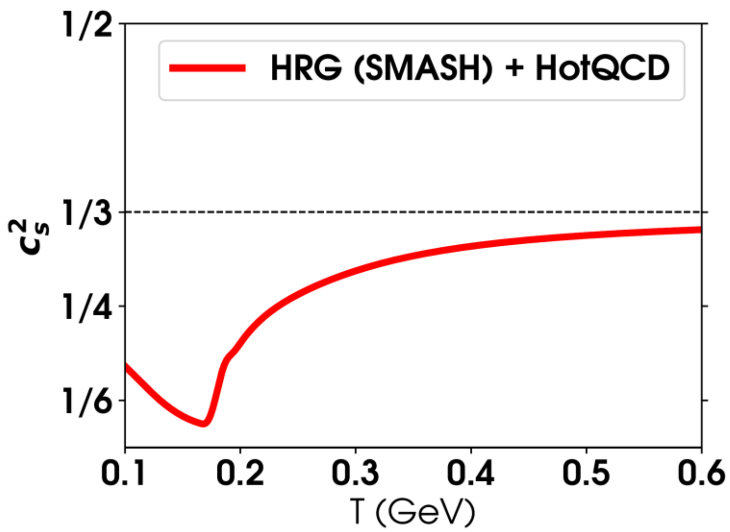
["Zeroth-order" approximation in heavy ion collisions]

Hydrodynamic equations:

a) First order (Navier-Stokes)
b) Second order (Israel-Stewart)

Equation of states:

I. Conformal or constant speed of sound
II. QCD



Bjorken hydrodynamics: Navier-Stokes

Assumed fluid symmetries (0+1D system):

- Boost-invariant in one dimension
- Uniform in the other dimensions

Navier-Stokes equation of motion for the **energy density ϵ**:

$$\partial_\tau \epsilon = -\frac{(\epsilon + P)}{\tau}\left[1 - \frac{4}{3\tau T}\frac{\eta}{s} - \frac{1}{\tau T}\frac{\zeta}{s}\right]$$

c_s^2: speed of sound
T: Temperature
s: Entropy
τ: "Time" $\sqrt{t^2 - z^2}$

Navier-Stokes equation of motion for the **temperature T**:

$$\frac{1}{T}\partial_\tau T = -\frac{c_s^2(T)}{\tau}\left[1 - \frac{V(T)}{\tau T}\right] \qquad V(T) \equiv \left(\frac{4}{3}\frac{\eta}{s}(T) + \frac{\zeta}{s}(T)\right)$$

- Nature of fluid enters through the equation of state (speed of sound c_s) and the viscosities η/s and ζ/s
- **Remember:**
  0+1D, shear and bulk viscosities indistinguishable because of symmetries

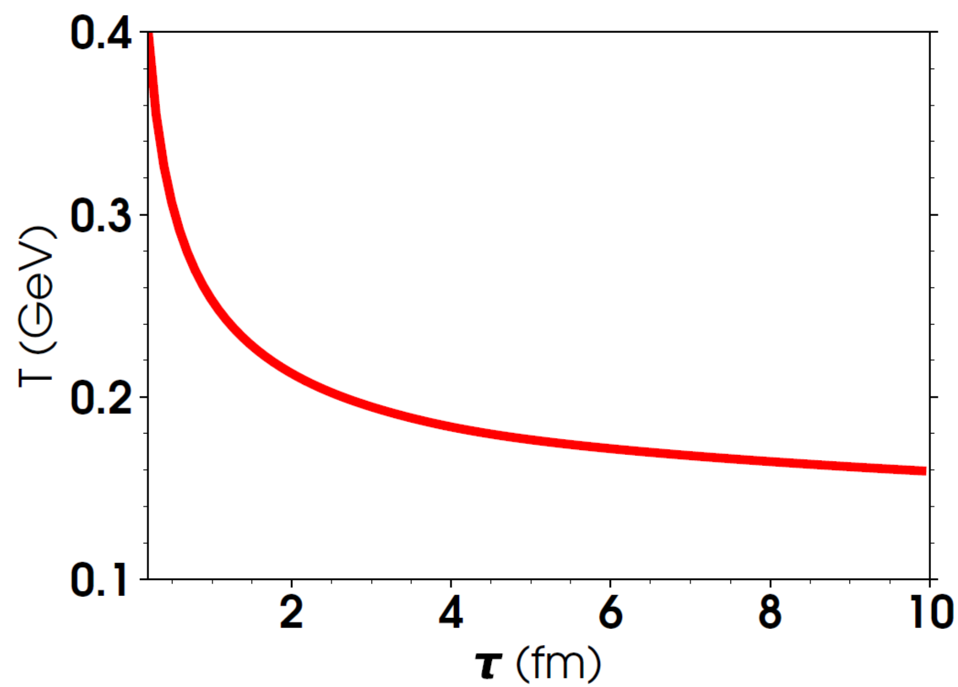
0+1D Navier-Stokes Bjorken hydrodynamics

Navier-Stokes equation of motion for the **temperature T**:

$$\frac{1}{T}\partial_\tau T = -\frac{c_s^2(T)}{\tau}\left[1 - \frac{V(T)}{\tau T}\right]$$

$$V(T) \equiv \left(\frac{4}{3}\frac{\eta}{s}(T) + \frac{\zeta}{s}(T)\right)$$

c_s^2: speed of sound
T: Temperature
s: Entropy
τ: "Time" $\sqrt{t^2 - z^2}$



Bjorken Navier-Stokes effective viscosity

Equation of motion for the temperature profile T:

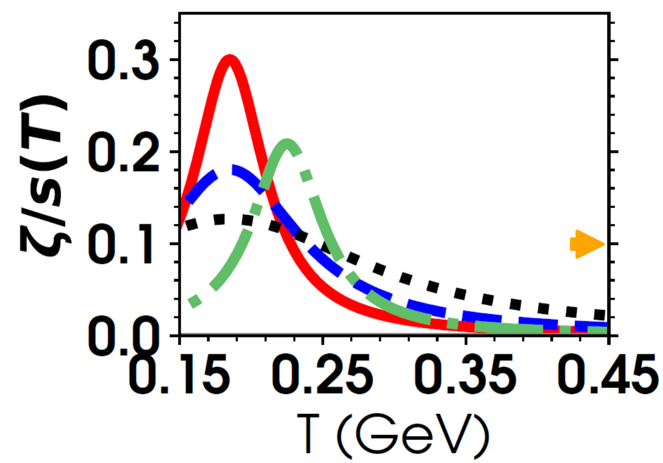
$$\frac{1}{T}\partial_\tau T = -\frac{c_s^2(T)}{\tau}\left[1 - \frac{V(T)}{\tau T}\right] \qquad V(T) \equiv \left(\frac{4}{3}\frac{\eta}{s}(T) + \frac{\zeta}{s}(T)\right)$$

Define quantity to be conserved:

$$\ln\left(\frac{T(\tau)}{T(\tau_0)}\right) = -\int_{\tau_0}^{\tau} d\tau' \frac{c_s^2(T(\tau'))}{\tau'} + \boxed{\int_{\tau_0}^{\tau} d\tau' \frac{c_s^2(T(\tau'))}{\tau'} \frac{V(T(\tau'))}{\tau' T(\tau')}}$$

Define effective viscosity:

$$\int_{\tau_0}^{\tau} d\tau' \frac{c_s^2}{\tau'} \frac{V(T(\tau'))}{\tau' T(\tau')} = \boxed{V_{\mathrm{eff}}} \int_{\tau_0}^{\tau} d\tau' \frac{c_s^2}{\tau'} \frac{1}{\tau' T(\tau')}$$



Bjorken N-S eff. viscosity: conformal case

Equation of motion for the temperature profile T:

$$\frac{1}{T}\partial_\tau T = -\frac{c_s^2(T)}{\tau}\left[1-\frac{V(T)}{\tau T}\right] \qquad V(T) \equiv \left(\frac{4}{3}\frac{\eta}{s}(T) + \frac{\zeta}{s}(T)\right)$$

Effective viscosity:

$$V_{\text{eff}} = \frac{\int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} \frac{V(T(\tau'))}{\tau' T(\tau')}}{\int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} \frac{1}{\tau' T(\tau')}}$$

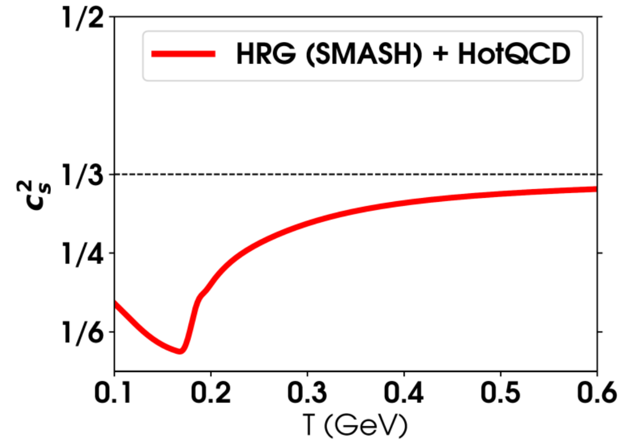
Assuming viscosity is a perturbation:

$$\boxed{V_{\text{eff}} \approx \frac{\int_{T_I(\tau)}^{T_0} dT'(T')^{\left(c_s^{-2}-2\right)} V(T')}{\int_{T_I(\tau)}^{T_0} dT'(T')^{\left(c_s^{-2}-2\right)}}}$$

Bjorken Navier-Stokes eff. viscosity: QCD

Equation of motion for the temperature profile T:

$$\frac{1}{T}\partial_\tau T = -\frac{c_s^2(T)}{\tau}\left[1 - \frac{V(T)}{\tau T}\right] \qquad V(T) \equiv \left(\frac{4}{3}\frac{\eta}{s}(T) + \frac{\zeta}{s}(T)\right)$$

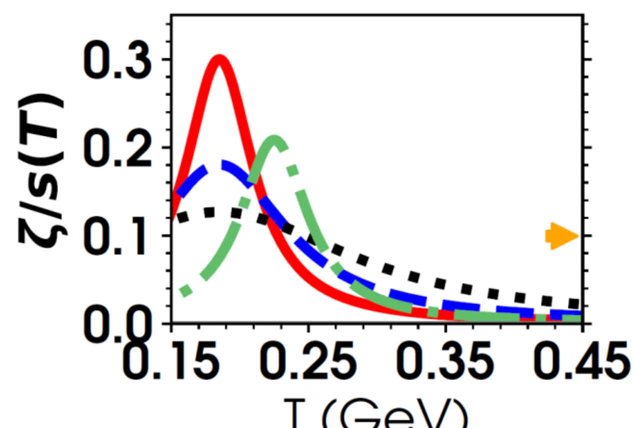


Effective viscosity:

$$V_{\rm eff} = \frac{\int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} \frac{V(T(\tau'))}{\tau' T(\tau')}}{\int_{\tau_0}^{\tau} \frac{d\tau'}{\tau'} \frac{1}{\tau' T(\tau')}}$$

Assuming viscosity is a perturbation:

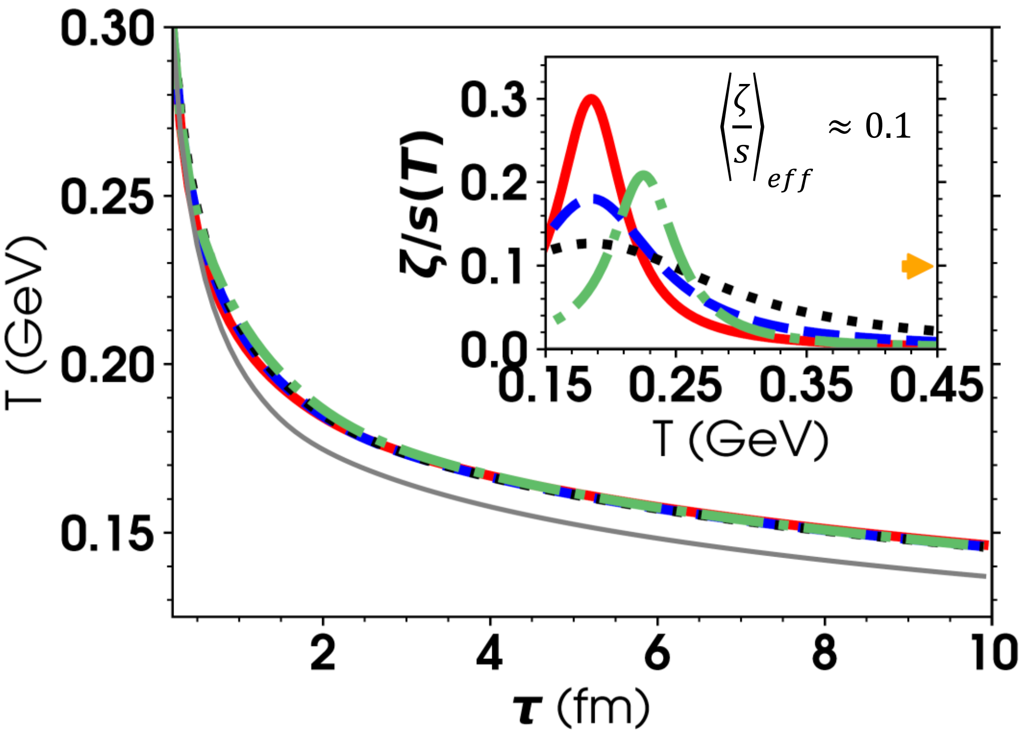
$$\boxed{V_{\rm eff} \approx \frac{\int_{T(\tau)}^{T_0} dT' \left(\frac{T}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2} V(T')}{\int_{T(\tau)}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2}}}$$



Bjorken QCD Navier-Stokes: $\zeta/s(T)$ degeneracy

Effective viscosity:

$$V_{\text{eff}} \approx \frac{\int_{T(\tau)}^{T_0} dT' \left(\frac{T}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2} V(T')}{\int_{T(\tau)}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2}}$$

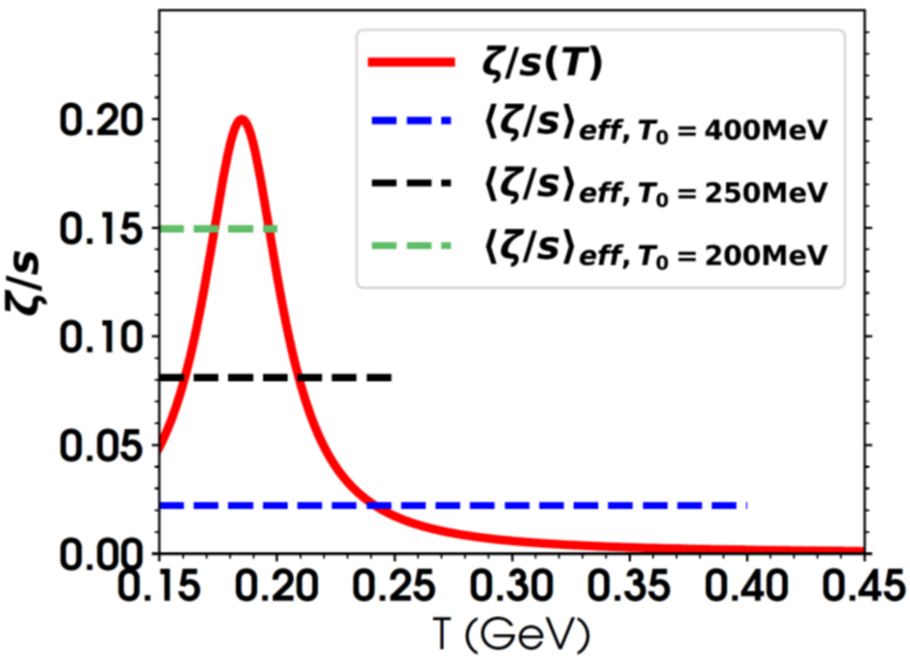


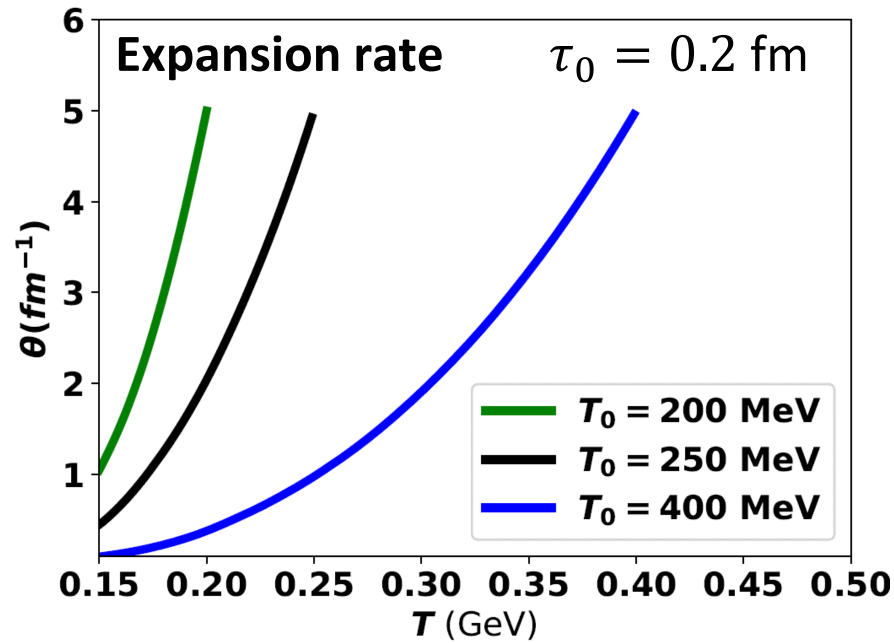
Degeneracy of ζ/s(T) in Bjorken Navier-Stokes hydrodynamics

Expansion rate and bulk visc. in Bjorken fluid

(Assume fixed initial time τ_0)

$$\theta^{\mathrm{Bjorken}}(T) = \frac{1}{\tau} \approx \frac{1}{\tau_0}\left(\frac{T}{T_0}\right)^{\frac{\bar{c}_s^{-2}}{\left(1-\frac{c_s^{-2}(\sqrt{TT_0})-\bar{c}_s^{-2}}{c_s^{-2}(T)}\right)}}$$

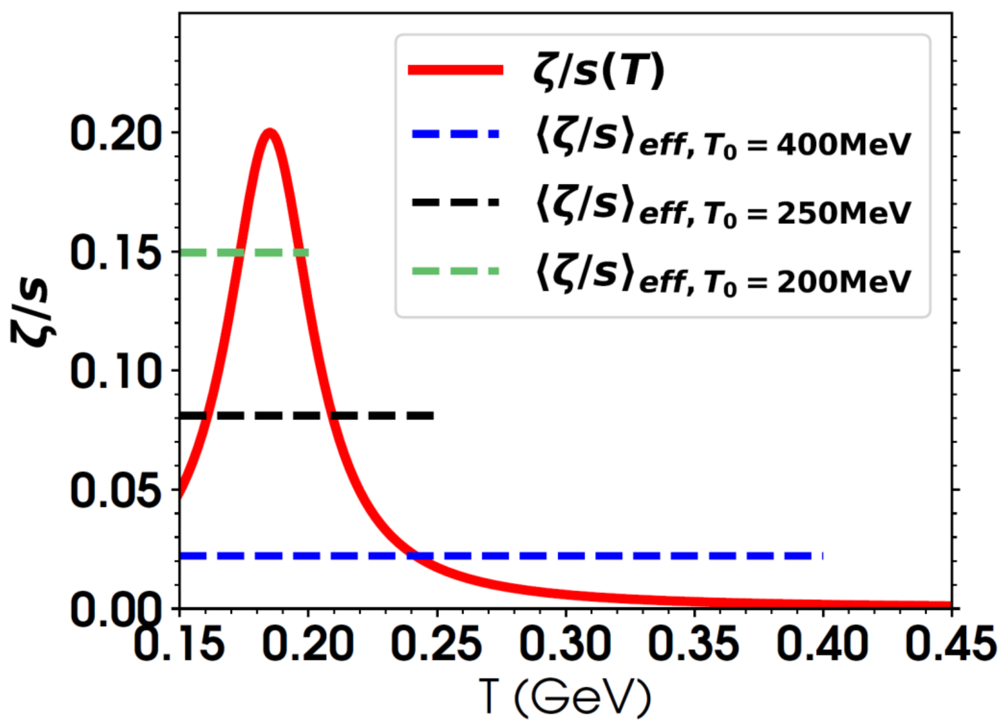




$$\frac{1}{T}u^\mu \partial_\mu T = -c_s^2(T)\left[\theta - \frac{1}{T}\boxed{\frac{\zeta}{s}\theta^2} - \frac{1}{T}\frac{\eta}{s}2\sigma_{\mu\nu}\sigma^{\mu\nu}\right]$$

Definition of effective viscosity in Bjorken fluid

$$\left\langle \frac{\zeta}{s} \right\rangle_{eff} \approx \frac{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2} \frac{\zeta}{s}(T')}{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2}}$$

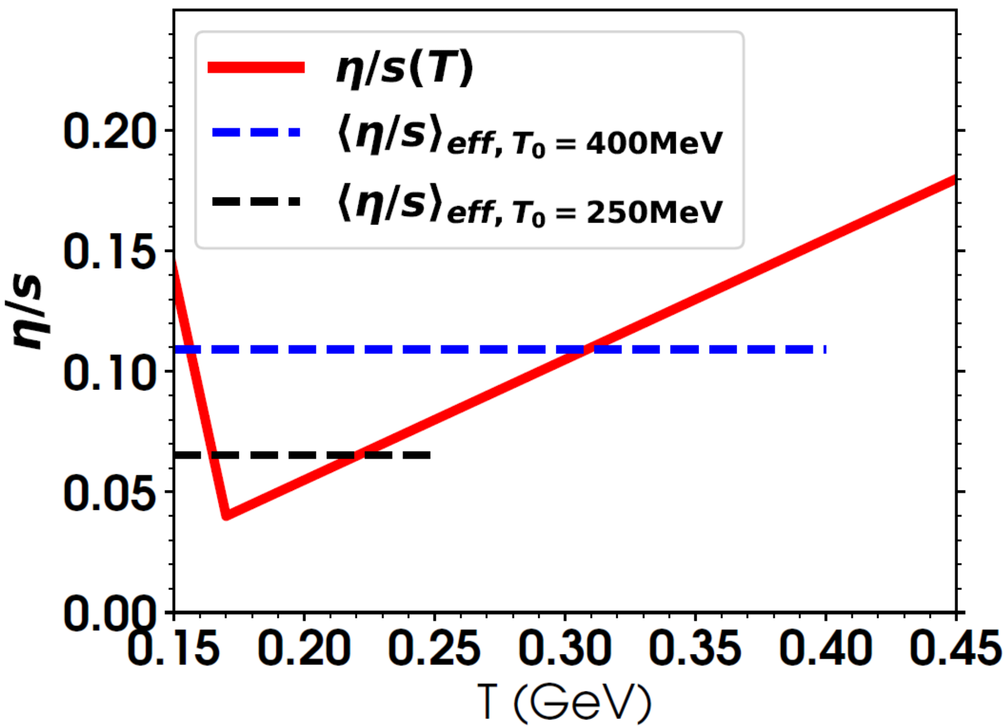


- Strong dependence on initial time, temperature ($\tau_0,\ T_0$)
- Generally weak dependence on "final" temperature
- If initial temperature is above ζ/s(T) "support":

$$\frac{\langle \zeta/s \rangle_{\mathrm{eff,\ a}}}{\langle \zeta/s \rangle_{\mathrm{eff,\ b}}} \approx \frac{\left(T_0^b\right)^2 - \left(T_f\right)^2}{\left(T_0^a\right)^2 - \left(T_f\right)^2}$$

Effective shear viscosity in Bjorken fluid

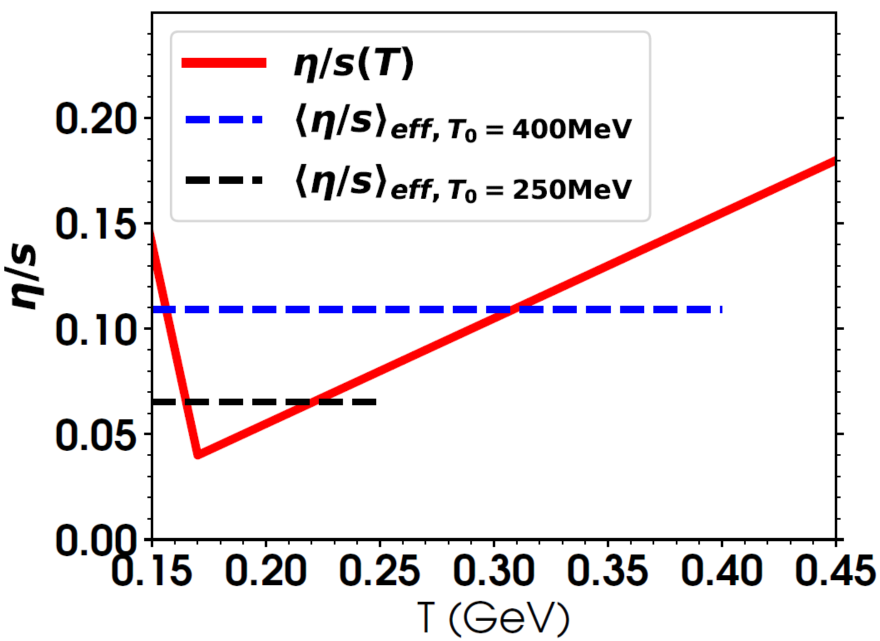
$$\left\langle \frac{\eta}{s} \right\rangle_{eff} \approx \frac{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2} \frac{\eta}{s}(T')}{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2}}$$

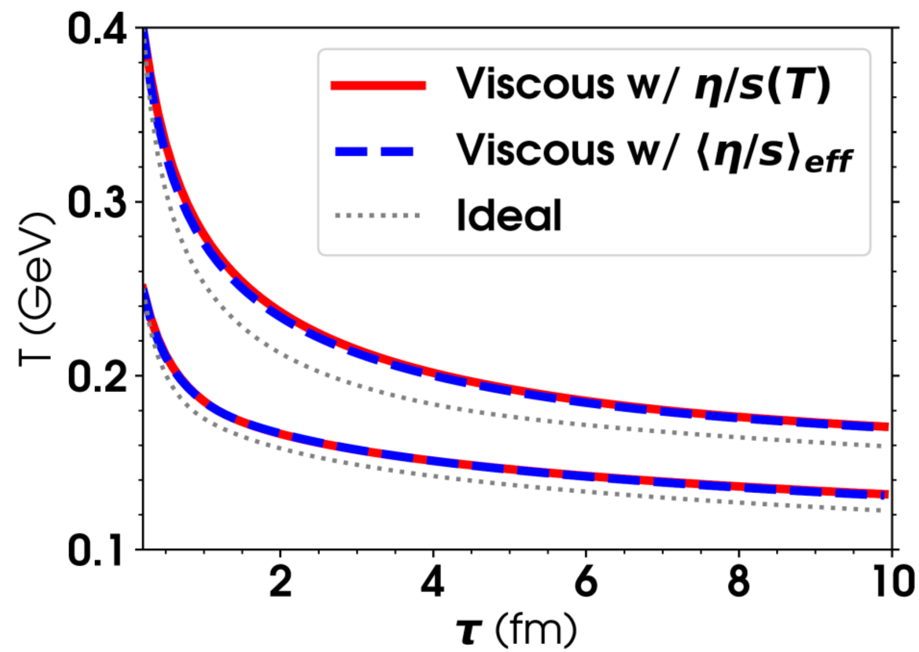


- Strong dependence on initial time, temperature ($\tau_0,\ T_0$)
- Weaker dependence on “final” temperature

Definition of effective viscosity in Bjorken fluid

$$\left\langle\frac{\eta}{s}\right\rangle_{eff} \approx \frac{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2} \frac{\eta}{s}(T')}{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2}}$$





From 1st to 2nd order hydrodynamics

- Previous results were for 1st order hydrodynamics (Navier-Stokes)
- Second-order Bjorken hydrodynamics?

$$\frac{d\ln T(\tau)}{d\tau} = -\frac{c_s^2(T(\tau))}{\tau}\left[1+\hat{\Pi}(\tau)\right]$$

$$\partial_\tau \hat{\Pi} = -\frac{(\hat{\Pi}-\hat{\Pi}_{NS})}{\tau_\Pi} + \frac{\hat{\Pi}}{\tau}\left[\frac{1}{3}+c_s^2\right] + \frac{\hat{\Pi}^2}{\tau}(1+c_s^2)$$

$$\hat{\Pi} = \Pi/(sT)$$

c_s^2: speed of sound
T: Temperature
Π: Bulk pressure
s: Entropy
τ: "Time" $\sqrt{t^2 - z^2}$
τ_Π: Relaxation time

- Evolution of bulk pressure depends on initial bulk pressure $\Pi(\tau_0)$ and relaxation time τ_Π

From 1^{st} to 2^{nd} order hydrodynamics

- Second-order Bjorken hydrodynamics?

$$\frac{d\ln T(\tau)}{d\tau} = -\frac{c_s^2(T(\tau))}{\tau}\left[1+\hat{\Pi}(\tau)\right] \qquad \hat{\Pi} = \Pi/(sT)$$

$$\partial_\tau \hat{\Pi} = -\frac{(\hat{\Pi}-\hat{\Pi}_{NS})}{\tau_\Pi} + \frac{\hat{\Pi}}{\tau}\left[\frac{1}{3}+c_s^2\right] + \frac{\hat{\Pi}^2}{\tau}(1+c_s^2)$$

c_s^2: speed of sound
T: Temperature
Π: Bulk pressure
s: Entropy
τ: "Time" $\sqrt{t^2 - z^2}$
τ_Π: Relaxation time

- Define "effective viscosity" or "effective viscous effect":

$$\int_{\tau_0}^{\tau} d\tau' \frac{c_s^2(T(\tau'))}{\tau'}\hat{\Pi}(\tau') = \langle \tau T \hat{\Pi}\rangle_{\text{eff}} \int_{\tau_0}^{\tau} d\tau' \frac{c_s^2(T(\tau'))}{\tau'} \frac{1}{\tau' T(\tau')}$$

$$\boxed{\langle \tau T \hat{\Pi}\rangle_{\text{eff}} \approx \left[\hat{\Pi}(\tau_0) - \hat{\Pi}_{NS}(\tau_0)\right] Y(T,T_0) + \langle \zeta/s\rangle_{\text{eff}}^{\text{NS}}}$$

$$Y(T,T_0) \approx -\frac{\int_T^{T_0} \frac{dT'}{T'} \exp\left[-\frac{\tau_0}{\tau_\Pi}\left[\left(\frac{T_0}{T'}\right)^{c_s^2(\sqrt{T_0 T'})}\right]\right]\left(\frac{T_0}{T'}\right)^{(1+\bar{c}_s^2)c_s^{-2}(\sqrt{T_0T'})}}{\frac{1}{\tau_0 T_0}\int_T^{T_0}\frac{dT'}{T'}\left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0T'}\right)-1}}$$

Effective viscosity in 2nd order hydrodynamics

- Does second-order Bjorken hydrodynamics break degeneracies found in first-order hydrodynamics?

$$\boxed{\langle \tau T \hat{\Pi} \rangle_{\rm eff} \approx \left[\hat{\Pi}(\tau_0) - \hat{\Pi}_{NS}(\tau_0) \right] Y(T, T_0) + \langle \zeta/s \rangle_{\rm eff}^{\rm NS}}$$

- Additional dependence on $\zeta/s(T_0)$ through $\widehat{\Pi}_{NS}(\tau_0) = -[\zeta/s(T_0)]/(\tau_0 T_0)$

- Scenarios:
  1) $|\widehat{\Pi}(\tau_0)| \gg |\widehat{\Pi}_{NS}(\tau_0)|$: effect of $\widehat{\Pi}_{NS}(\tau_0)$ negligible, still degeneracies
  2) $\left|\widehat{\Pi}(\tau_0)\right| \sim |\widehat{\Pi}_{NS}(\tau_0)|$: first term small, still degeneracies
     [i.e. if system starts close to Navier-Stoke state, 2nd order hydro ~ 1st order]

Depends on relaxation time τ_Π, on value of $\zeta/s(T_0)$, …

Degeneracy between viscosity and initial conditions

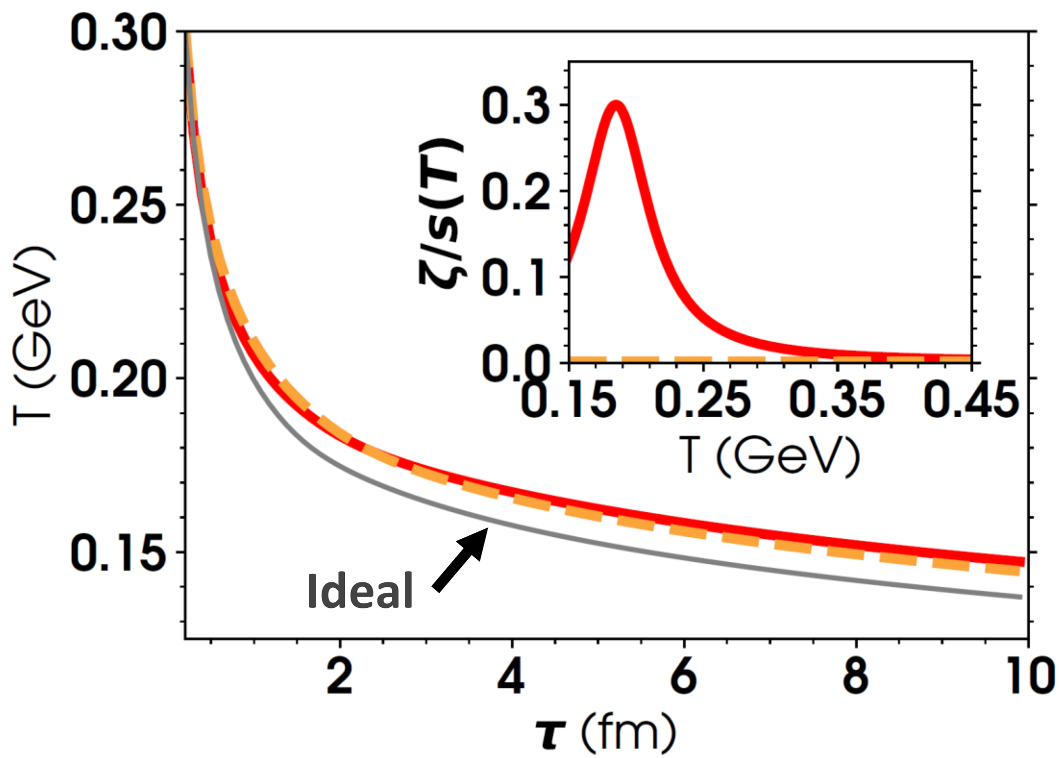
- Second-order Bjorken hydrodynamics: $\dfrac{d\ln T(\tau)}{d\tau} = -\dfrac{c_s^2(T(\tau))}{\tau}\left[1+\hat{\Pi}(\tau)\right]$

$$\boxed{\langle \tau T \hat{\Pi}\rangle_{\text{eff}} \approx \left[\hat{\Pi}(\tau_0) - \hat{\Pi}_{NS}(\tau_0)\right] Y(T,T_0) + \langle \zeta/s\rangle_{\text{eff}}^{\text{NS}}}$$

- Red solid line:
  $\Pi(\tau_0) = \Pi_{NS}(\tau_0)$
  $\zeta/s(\mathrm{T})$

- Yellow dashed line:
  $\Pi(\tau_0) = \dfrac{\langle \zeta/s\rangle_{eff}}{Y(T,T_0)}$
  $\zeta/s(\mathrm{T})$=0

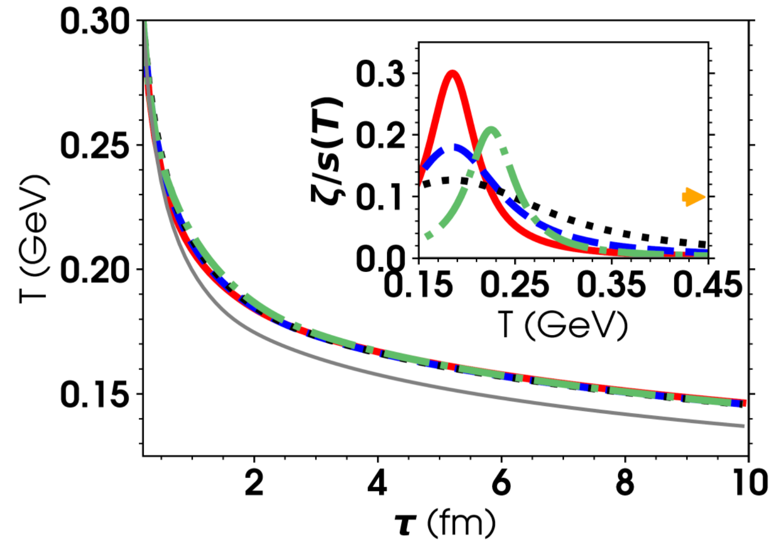
- Grey line: ideal solution (for reference)



Summary of Bjorken case

First-order Bjorken hydrodynamics:

$$\left\langle \frac{\zeta}{s} \right\rangle_{eff} \approx \frac{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2} \frac{\zeta}{s}(T')}{\int_{T_f}^{T_0} dT' \left(\frac{T'}{T_0}\right)^{c_s^{-2}\left(\sqrt{T_0 T'}\right)-2}}$$



- Relation between effective viscosity and temperature-dependent one
- Effective viscosity can be used to find families of ζ/s(T) with similar temperature profile

Second-order Bjorken hydrodynamics:

$$\boxed{\langle \tau T \hat{\Pi} \rangle_{\text{eff}} \approx \left[\hat{\Pi}(\tau_0) - \hat{\Pi}_{NS}(\tau_0)\right] Y(T, T_0) + \langle \zeta/s \rangle_{\text{eff}}^{\text{NS}}}$$

- Can (in theory) break degeneracies present in the Navier-Stokes case
- May not be a significant effect in practice
- Can introduce different “degeneracies”

Beyond Bjorken hydrodynamics

- Previous results assumed no transverse expansion (0+1D Bjorken hydrodynamics)
- Generalization beyond 0+1D?

Cylindrically symmetric Bjorken fluid (1+1D system):

- Boost-invariant in one dimension
- Cylindrical symmetry in transverse plane

[Better approximation for heavy ion collisions]

Hydrodynamic equations:

a) First order (Navier-Stokes)
~~b) Second order (Israel-Stewart)~~

Equation of states:

~~I. Conformal or constant speed of sound~~
II. QCD

Preliminary

Cylindrical Bjorken Navier-Stokes hydrodynamics

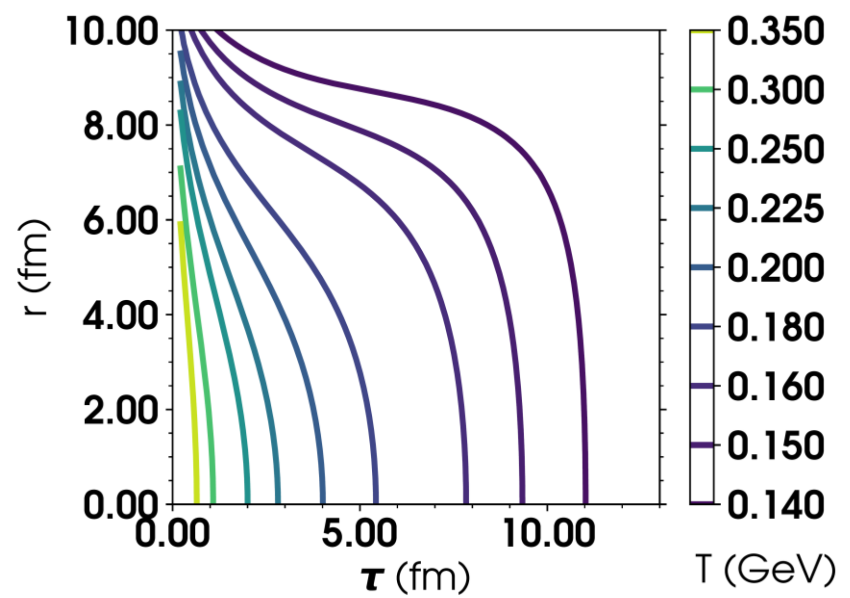
Cylindrically symmetric first-order Bjorken fluid (1+1D system):

- Boost-invariant in one dimension
- Cylindrical symmetry in transverse plane
- For simplicity: bulk viscosity only in what follows

$$u^\tau \partial_\tau \ln T + u^r \partial_r \ln T = -c_s^2(T) \left[\theta - \frac{1}{T} \frac{\zeta}{s} \theta^2 \right]$$

$$\theta = \frac{u^\tau}{\tau} + \frac{u^r}{r} + \partial_\tau u^\tau + \partial_r u^r$$

[Additional equation of motion for the flow velocity u^μ]



1+1D Bjorken N-S hydro, reformulated

Cylindrically symmetric first-order Bjorken fluid (1+1D system):

$$u^\tau \partial_\tau \ln T + u^r \partial_r \ln T = -c_s^2(T)\left[\theta - \frac{1}{T}\frac{\zeta}{s}\theta^2\right]$$

$$\theta = \frac{u^\tau}{\tau} + \frac{u^r}{r} + \partial_\tau u^\tau + \partial_r u^r$$

[Additional equation of motion for the flow velocity u^μ]

- Rewrite as ordinary differential equation with method of characteristics

$$\frac{d\tau(\chi)}{d\chi} = \sqrt{1 + u^r\left(\tau(\chi), r(\chi)\right)^2}$$

$$\frac{dr(\chi)}{d\chi} = u^r\left(\tau(\chi), r(\chi)\right)$$

$$\frac{d\ln T(\chi)}{d\chi} = -c_s^2(T(\chi))\theta\left(\tau(\chi), r(\chi)\right)\left[1 - \frac{\theta}{T}\frac{\zeta}{s}(T)\right]$$

$$\boxed{\begin{aligned}\tau(\chi = 0) &= \tau_0 \\ r(\chi = 0) &= r_0\end{aligned}}$$

Effective visc. in 1+1D N-S hydro

Cylindrically symmetric first-order Bjorken fluid (1+1D system):

$$\frac{d\tau(\chi)}{d\chi} = \sqrt{1 + u^r\left(\tau(\chi), r(\chi)\right)^2}$$
$$\frac{dr(\chi)}{d\chi} = u^r\left(\tau(\chi), r(\chi)\right)$$
$$\frac{d\ln T(\chi)}{d\chi} = -c_s^2(T(\chi))\theta\left(\tau(\chi), r(\chi)\right)\left[1 - \frac{\theta}{T}\frac{\zeta}{s}(T)\right]$$

$$\boxed{\begin{aligned} \tau(\chi = 0) &= \tau_0 \\ r(\chi = 0) &= r_0 \end{aligned}}$$

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle \zeta/s \rangle_{\text{eff}}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right)\frac{\theta}{T}\frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right)\frac{\theta}{T}}}$$

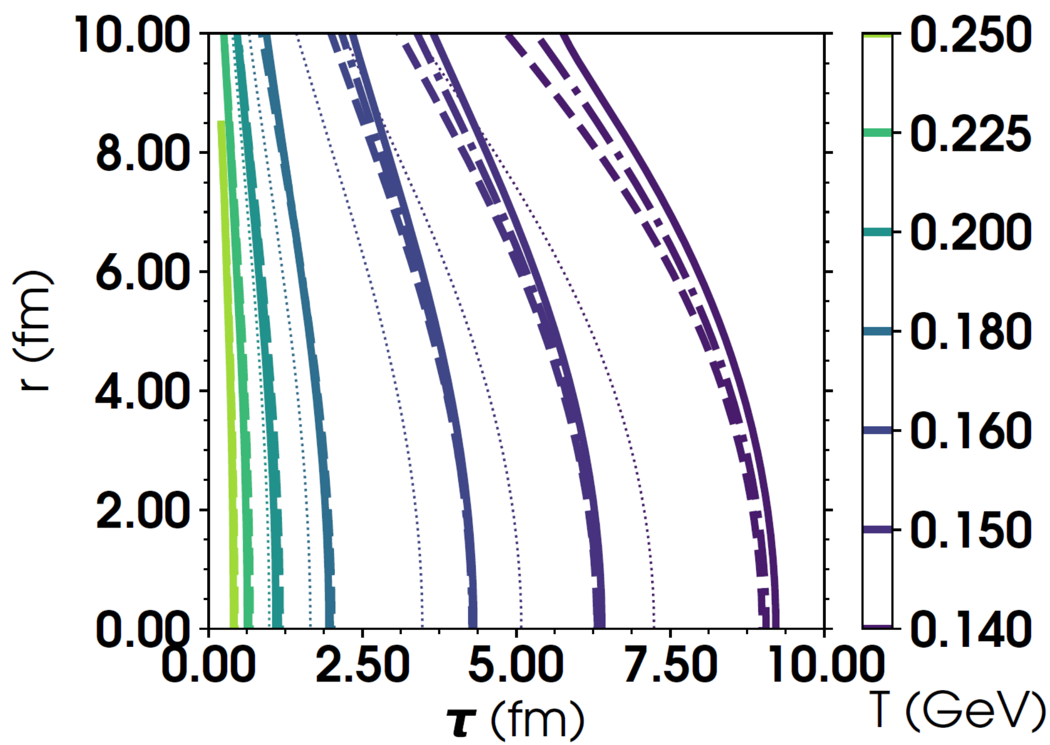
Effective viscosity now depends on the transverse (radial) position

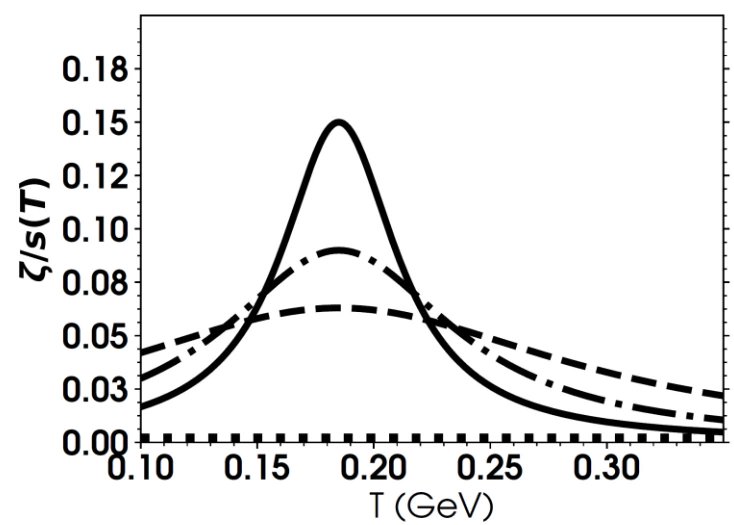
Effective visc. in 1+1D N-S hydro

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle\zeta/s\rangle_{\rm eff}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right)\frac{\theta}{T}\frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right)\frac{\theta}{T}}}$$

Example: forcing $\langle\zeta/s\rangle_{eff}$ to be the **same at r=0** does not lead to same temperature profile at r>0





$$T(\tau_0, r, \eta_s) = T_0 \exp(-r^2/\sigma^2)$$

$\tau_0 = 0.2\,fm,\ T_0 = 300$ MeV, $\sigma = 20$ fm, $u^r(\tau_0) = 0$

Effective visc. in 1+1D N-S hydro

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle \zeta/s\rangle_{\rm eff}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T}\frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T}}}$$

- In Bjorken case, an expression for $\theta(T)$ could be found (even for QCD equation of state)
- $\theta(T, r_0)$ not known for cylindrical hydrodynamics

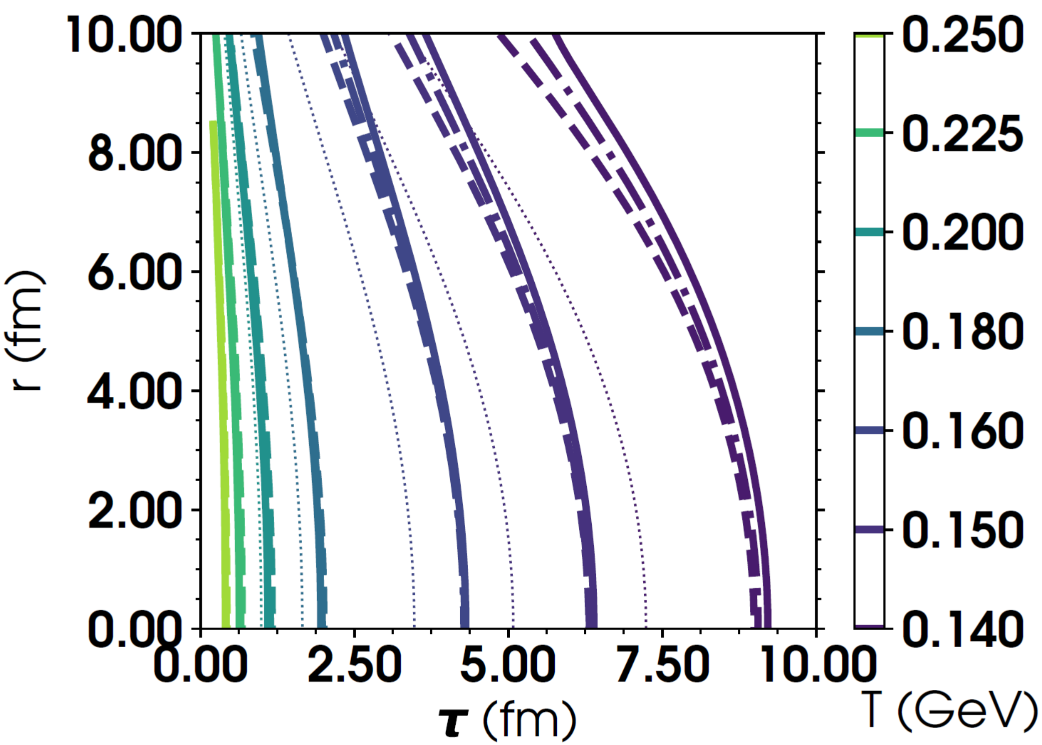
- Alternative approach:
  Solve ideal cylindrical hydrodynamics numerically, and use it to evaluate the effective viscosities

Effective visc. in 1+1D N-S hydro

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle \zeta/s \rangle_{\text{eff}}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T} \frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T}}}$$

Example: forcing $\langle \zeta/s \rangle_{eff}$ to be the same at r=0 does not lead to same temperature profile at r>0



- Need to find families of ζ/s(T) such that $\langle \zeta/s \rangle_{eff}(r_0)$ have similar for all r_0

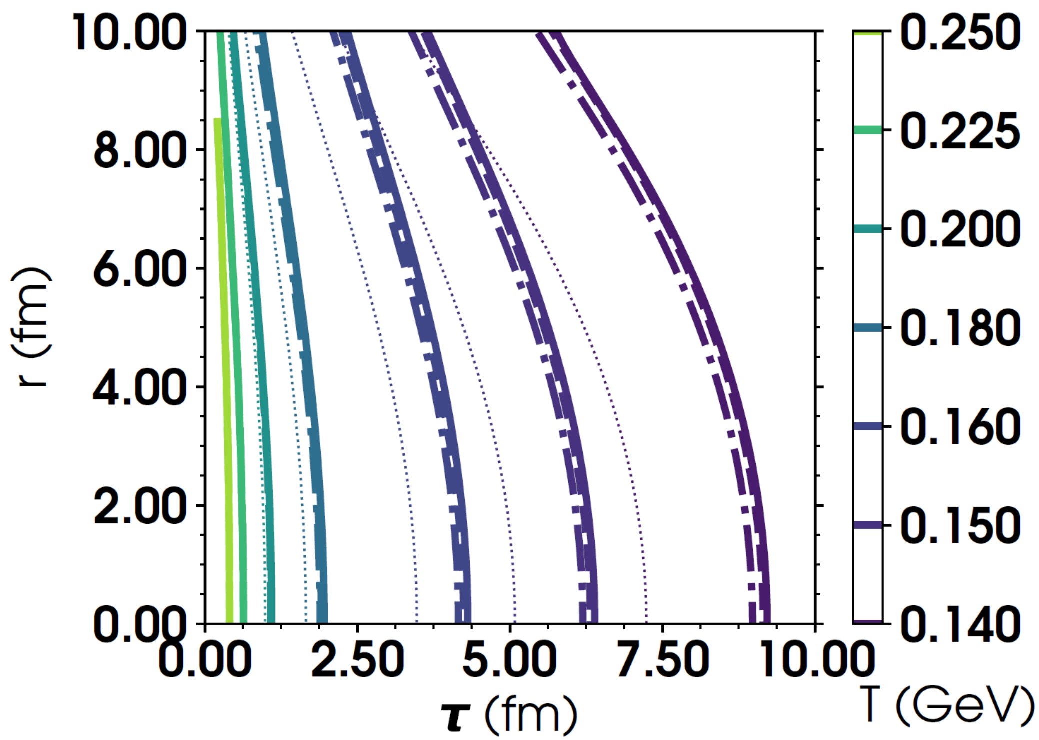
- In practice, for a smooth temperature profile: it is enough to optimize at finite number of r_0

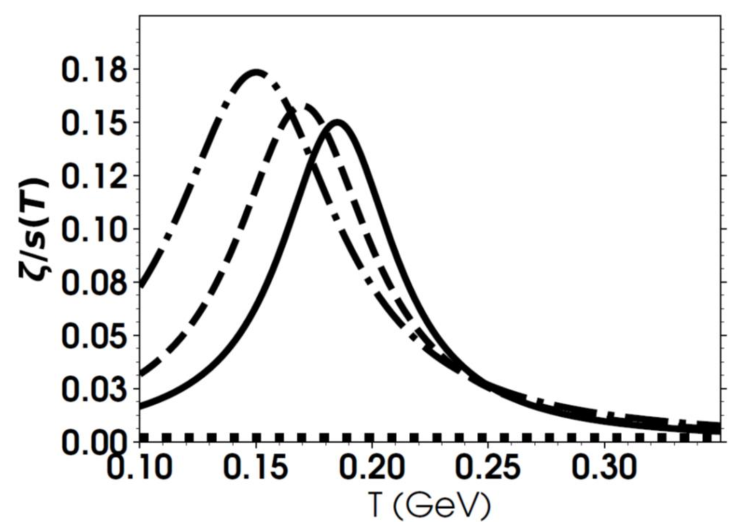
Effective visc. in 1+1D N-S hydro

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle \zeta/s \rangle_{\text{eff}}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T} \frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T}}}$$

- Need to find families of $\zeta/s(T)$ such that $\langle \zeta/s \rangle_{eff}(r_0)$ have similar for "all" r_0





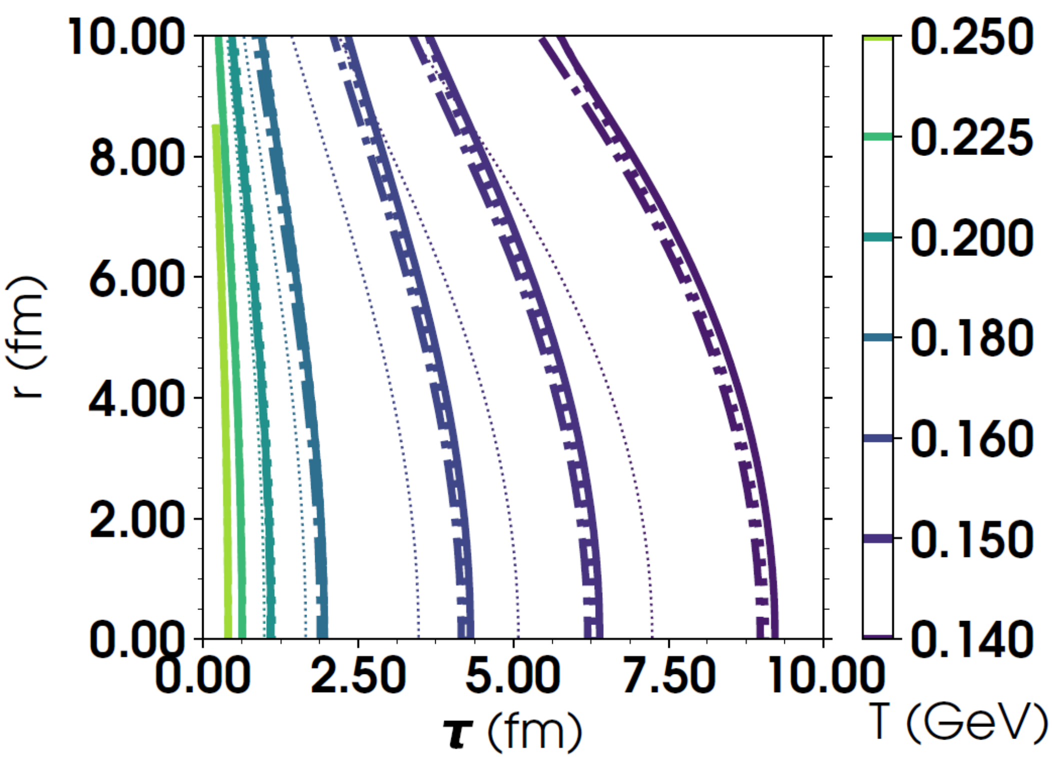
In practice, only two r_0 used here

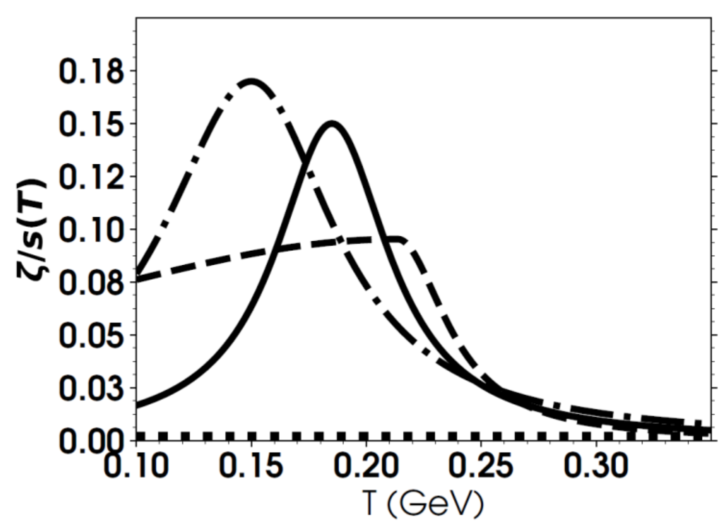
Effective visc. in 1+1D N-S hydro

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle\zeta/s\rangle_{\mathrm{eff}}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right)\frac{\theta}{T}\frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right)\frac{\theta}{T}}}$$

- Need to find families of $\zeta/s(T)$ such that $\langle\zeta/s\rangle_{eff}(r_0)$ have similar for "all" r_0





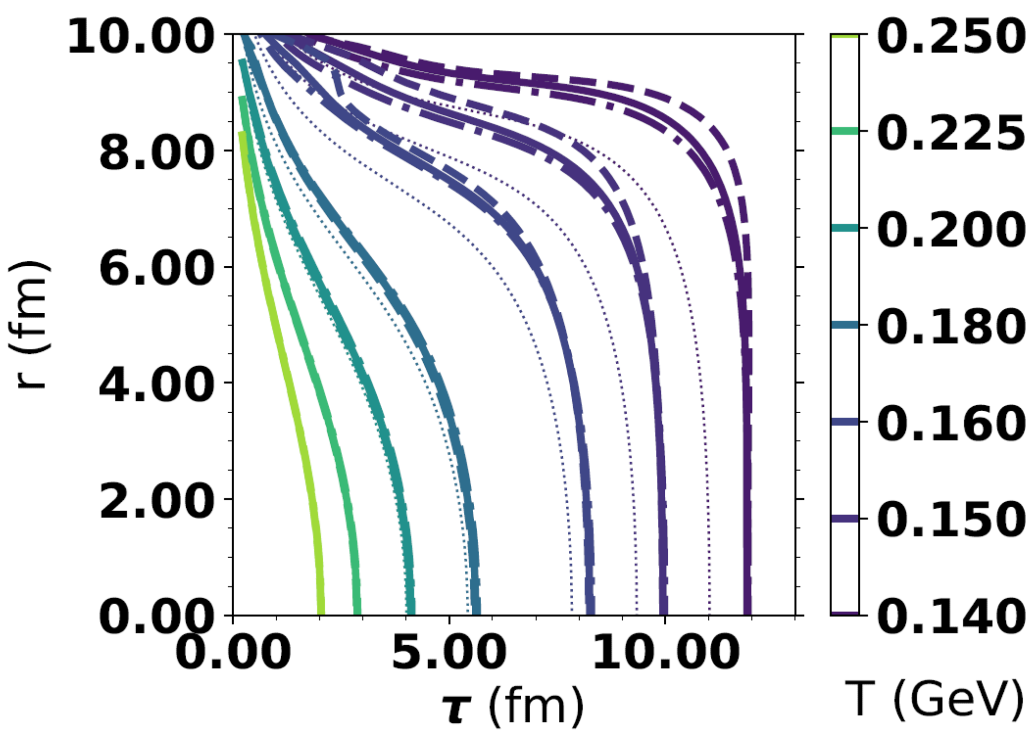
More flexible parametrization of $\zeta/s(T)$

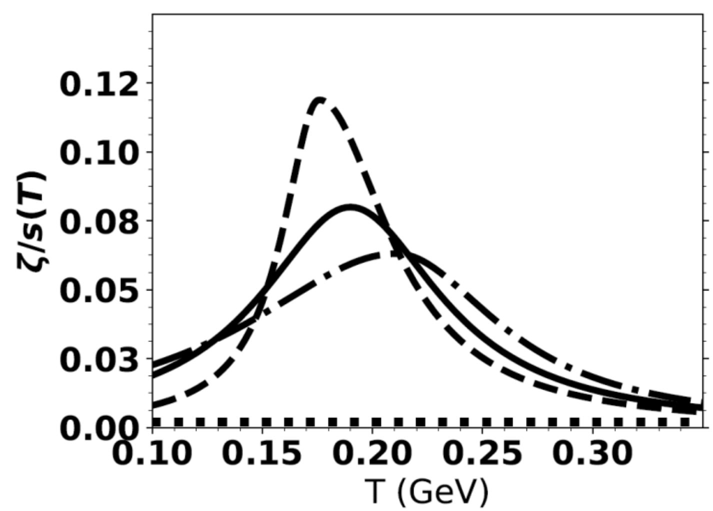
Effective visc. in 1+1D N-S hydro

- Along a characteristic, define effective viscosity the same way as in Bjorken case

$$\boxed{\langle \zeta/s \rangle_{\rm eff}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T} \frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi), r(\chi)\right) \frac{\theta}{T}}}$$

- Need to find families of $\zeta/s(T)$ such that $\langle \zeta/s \rangle_{eff}(r_0)$ have similar for "all" r_0

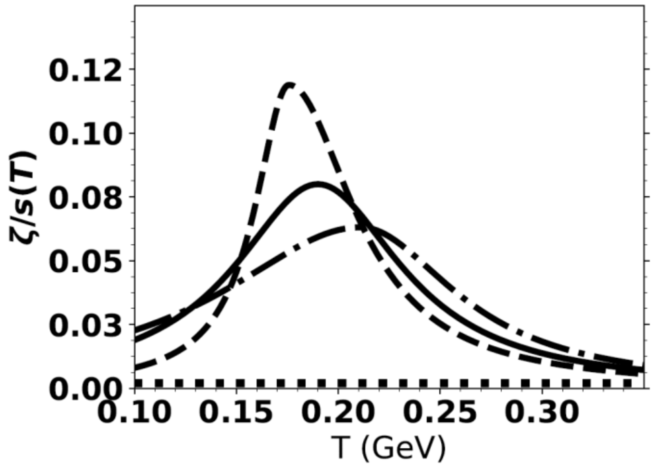




$$T(\tau_0, r, \eta_s) = T_0 \exp(-r^2/\sigma^2)$$

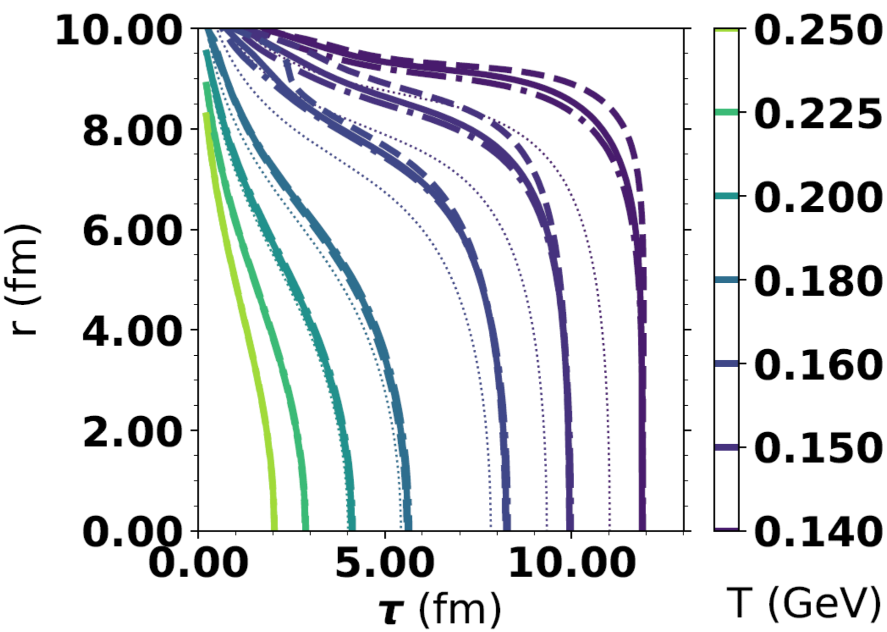
$\tau_0 = 0.2\, fm,\ T_0 = 500$ MeV, $\sigma = 10$ fm, $u^r(\tau_0) = 0$

Effective visc. in 1+1D N-S hydro

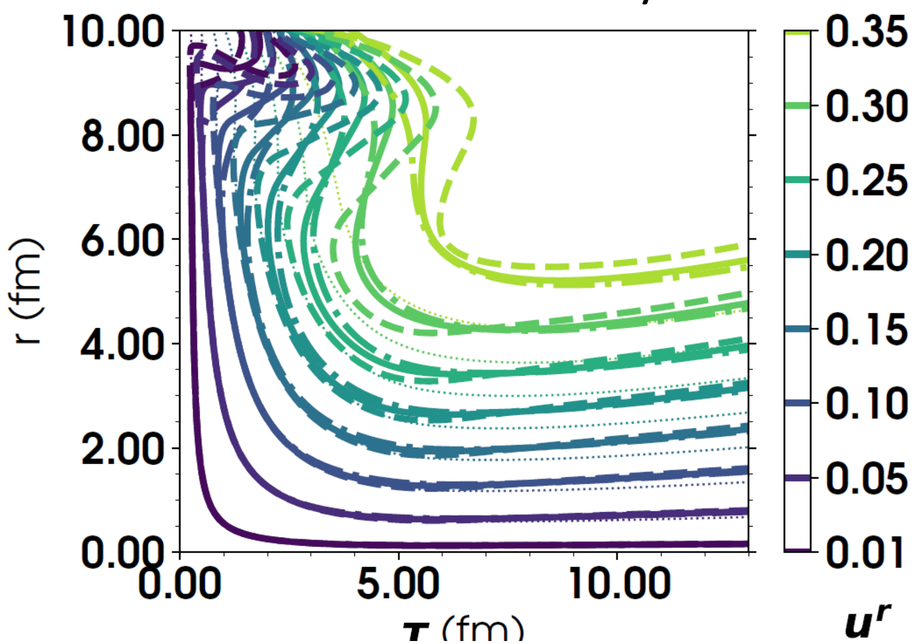


$$T(\tau_0, r, \eta_s) = T_0 \exp(-r^2/\sigma^2)$$

$\tau_0 = 0.2\ fm,\ T_0 = 500$ MeV,
$\sigma = 10$ fm, $\quad u^r(\tau_0) = 0$



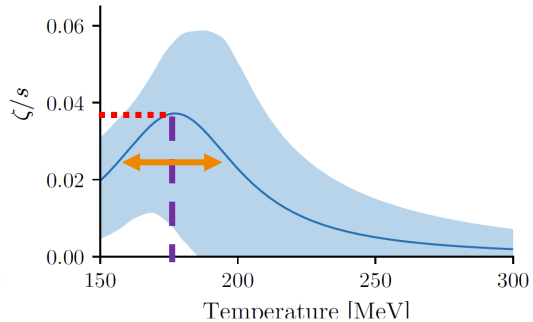
Radial flow velocity u^r



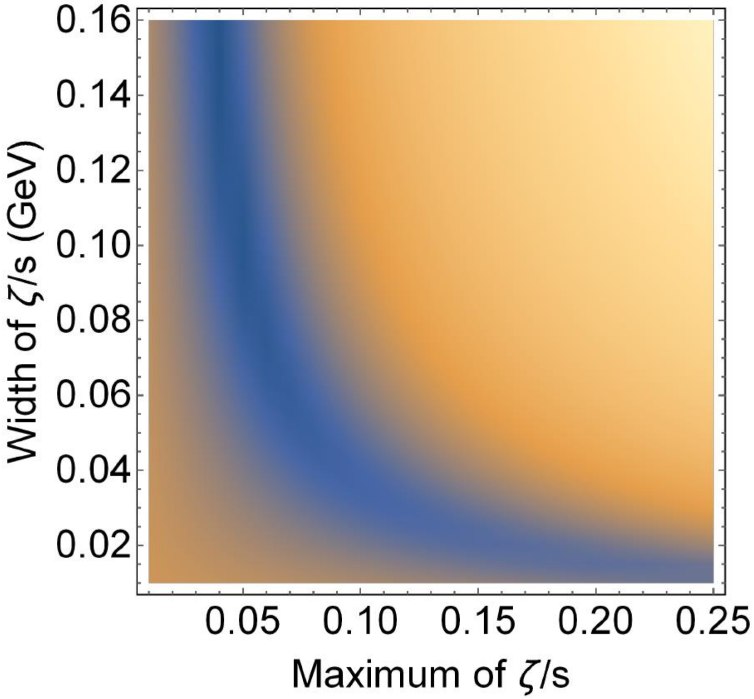
Correlations between width and max of $\zeta/s(T)$?

- Given a parametrization of ζ/s(T)

$$(\zeta/s)(T) = \frac{(\zeta/s)_{\max}}{1+\left(\frac{T-T_{peak}}{width}\right)^2}$$



- What are the correlations between the max and the width such that the effective viscosity between two parametrization is minimized?



$$\boxed{\langle\zeta/s\rangle_{\text{eff}}(r_0) \approx \frac{\int d\chi c_s^2(T)\theta\left(\tau(\chi),r(\chi)\right)\frac{\theta}{T}\frac{\zeta}{s}(T)}{\int d\chi c_s^2(T)\theta\left(\tau(\chi),r(\chi)\right)\frac{\theta}{T}}}$$

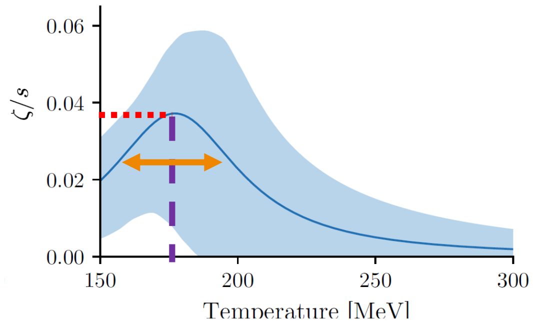
$$\ln\sum_{r_0}\left(\left\langle\frac{\zeta}{s}(T)\right\rangle_{eff}-\left\langle\frac{\zeta\prime}{s}(T)\right\rangle_{eff}\right)^2$$

[~ Measure of similarity temperature profile]

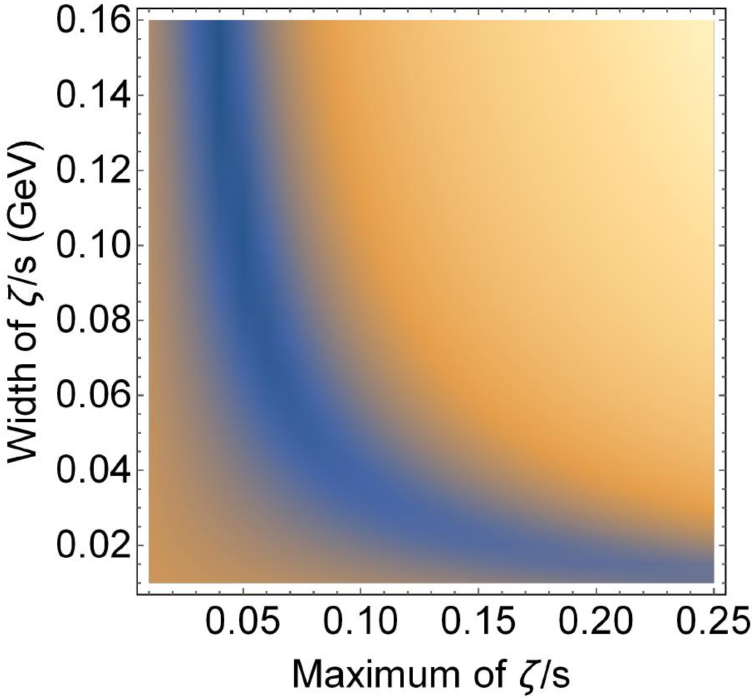
Correlations between width and max of ζ/s?

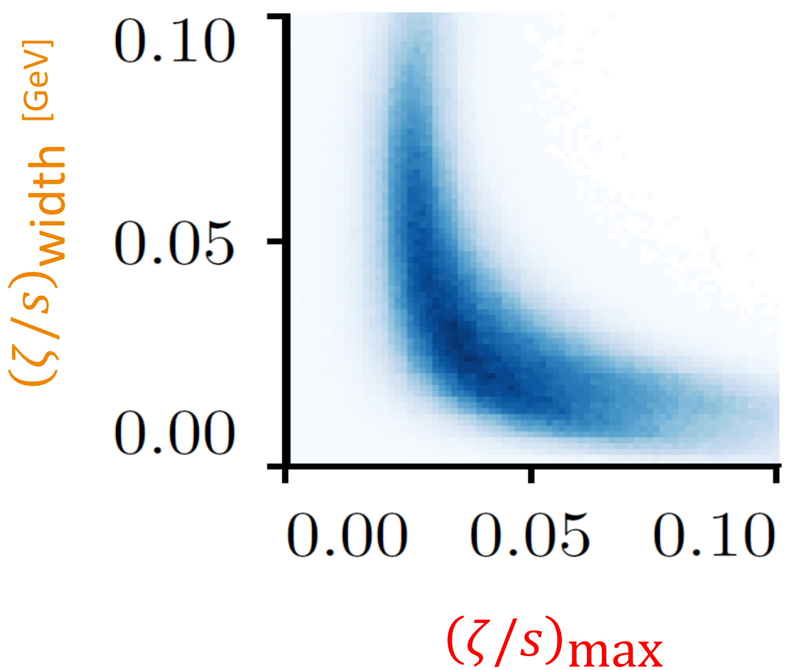
- Given a parametrization of ζ/s(T)

$$(\zeta/s)(T) = \frac{(\zeta/s)_{\text{max}}}{1 + \left(\frac{T - T_{peak}}{width}\right)^2}$$



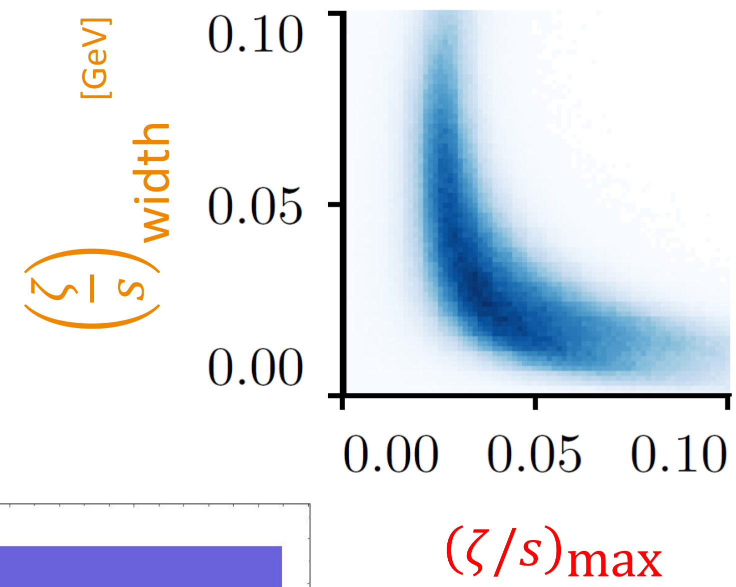
- What are the correlations between the max and the width such that the effective viscosity between two parametrization is minimized?



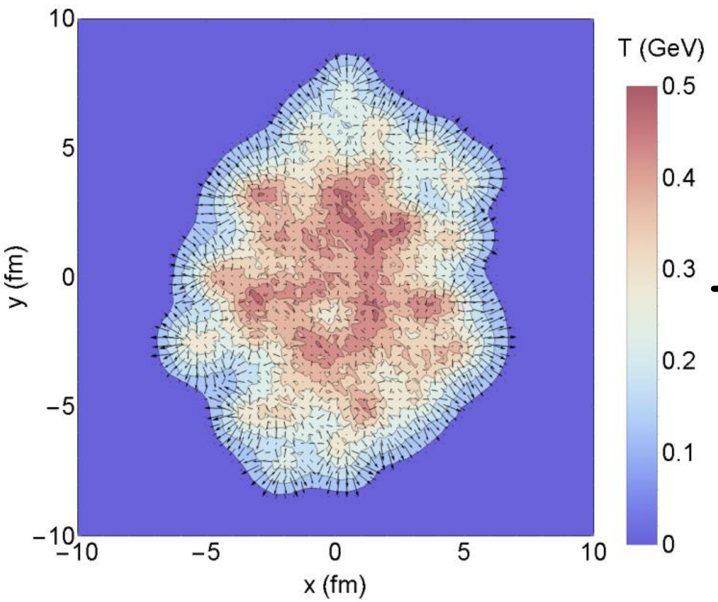


Origin of degeneracy?

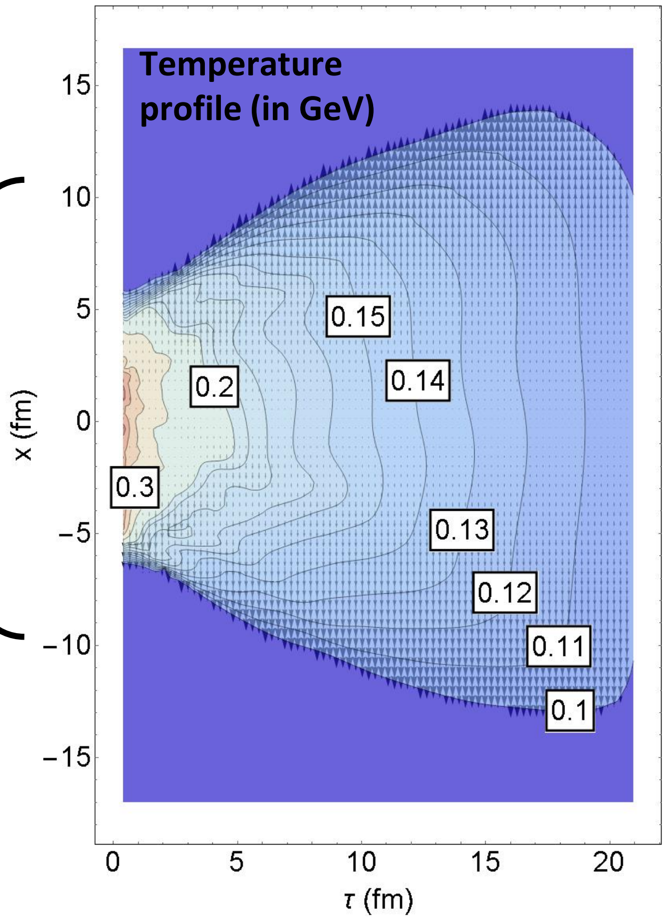
- Not enough measurements?
- Measurements not sensitive enough?



Symmetries?

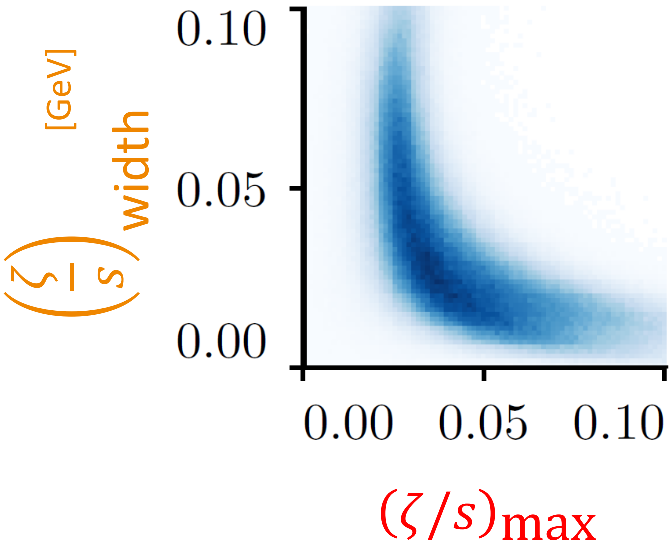


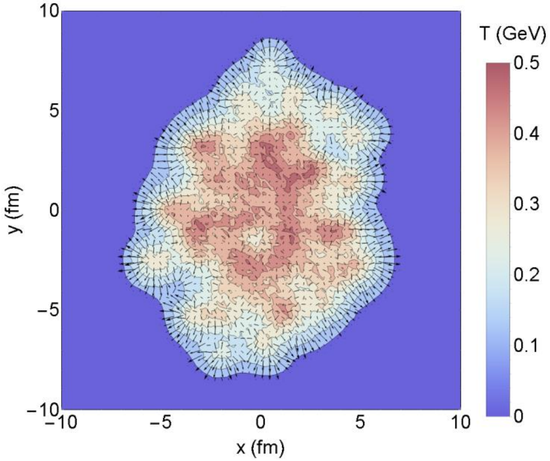
Initial temperature profile



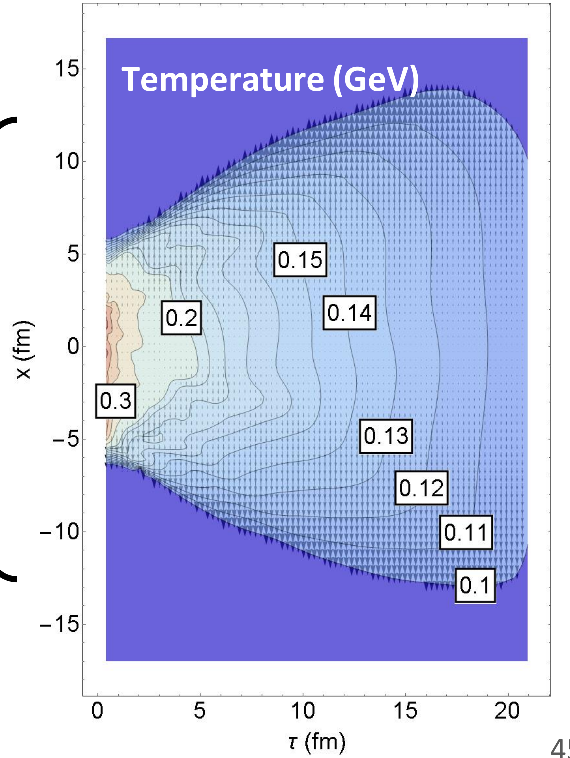
Origin of degeneracy?

- Unclear if analysis of 1+1D Navier-Stokes hydro applies to heavy ion collisions
- Second order transport such as relaxation time coefficient **may** play a significant role
- Heavy ion collisions are not cylindrically symmetric…
  yet late stage is not as asymmetric as could be expected



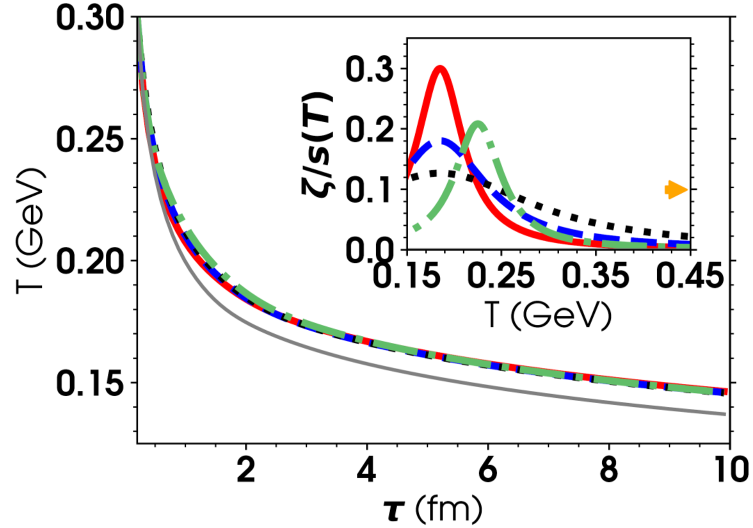


Initial temperature profile



Summary

- To what extent can $\eta/s(T)$ and $\zeta/s(T)$ be constrained from heavy ion collisions?
- Used 0+1D Bjorken hydrodynamics to study degeneracies in $\eta/s(T)$ and $\zeta/s(T)$
  - Role of the equation of state
  - First vs second order hydrodynamics
- Studied 1+1D (cylindrical) hydrodynamics for more realistic scenario



<u>Next steps:</u>

- Study 1+1D case further [e.g. more flexible parametrization of $\zeta/s(T)$; and $\frac{\eta}{s(T)}$, quantify "closeness/similarity" of different temperature/flow profiles]

Questions?