Recent Advances in Formal Theory

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The Standard Model Works!

- Is the EW scale fine tuned?
- Are there other particles below at few TeV ?

- Dark matter?

- Lots of problems for phenomenology.

Success of QFT

- Nature is well described by QFT.
- It is weakly coupled at the 100's of Gev's and reasonably understood.

- What is left to do?

Some problems of ``formal'' QFT

- Develop better computational techniques.
- Understand QFT in general. (Motivation: New fundamental physics and condensed matter theory. Condensed matter can give us lessons → recall the Higgs!)
- Strong coupling
- Gauge theories and strings.
- Understand CFT's (Fixed points) and the flows connecting them.
- Relation between field theory and quantum gravity (AdS/CFT)
- QFT in various dimensions.

Beyond QFT

- Quantum gravity and string theory.
- Quantum aspects of black holes.
- Cosmology, fundamentals of inflation, measure problems, initial singularity, quantum cosmology.

All of these topics have multiple connections with each other.

Supersymmetric theories

- Still ?
- They are simpler QFT's, and more solvable (like the mouse is a model for a human in biology).

- We can do exact computations.

- $\mathcal{N}$=4 SYM.
- $\mathcal{N}$=2 SUSY theories
- $\mathcal{N}$=1 SUSY theories
- $\mathcal{N}$=0 …

Great advances in calculational techniques

- Advances in computing quantities that depend on the coupling.

- 1- Integrability (in planar $\mathcal{N}$=4 SYM)

  Review : **arXiv:1012.3982**

  Minahan, Zarembo, Beisert, Staudacher, Gromov, Kazakov, Vieira, Basso, ….

- 2- Special observables in SUSY theories: Sphere partition functions, circular Wilson loops, etc. Full path integral → finite dimensional integral (localization)

  Nekrasov,
  Pestun, Kapustin, Jacob, Willet, …

- These become tools to explore these theories, testing dualities, etc.

$\mathcal{N}$=4 SYM

- $\mathcal{N}$=4 SYM = Maximally supersymmetric version of U(N) QCD. Gluons + fermions + scalars (all in the adjoint).
- Planar limit $N \rightarrow \infty$, $g^2 N$ = fixed .

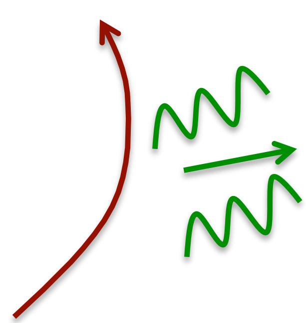
Example:

Power radiated by a moving quark in $\mathcal{N} = 4$ SYM

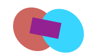
$$P = 2\pi B a^2$$

$$B = \frac{1}{4\pi^2}\frac{\sqrt{\lambda} I_2(\sqrt{\lambda})}{I_1(\sqrt{\lambda})} \qquad \lambda = g^2 N$$

Bessel functions



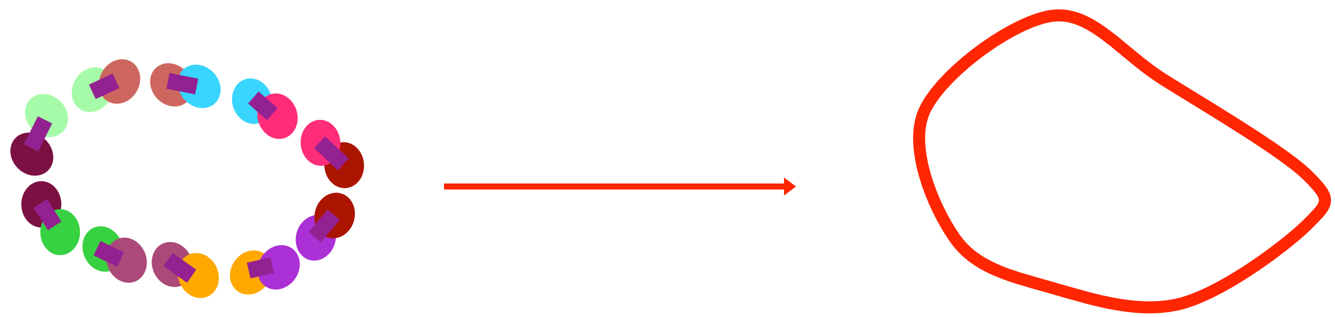
Large N gauge theories and strings

Gluon: color and anti-color 

Take N colors instead of 3, SU(N)

t' Hooft '74

Large N limit



g^2N = fixed, N → infinity

String coupling ~ 1/N

Closed strings → glueballs

Integrability in the gauge/string theory

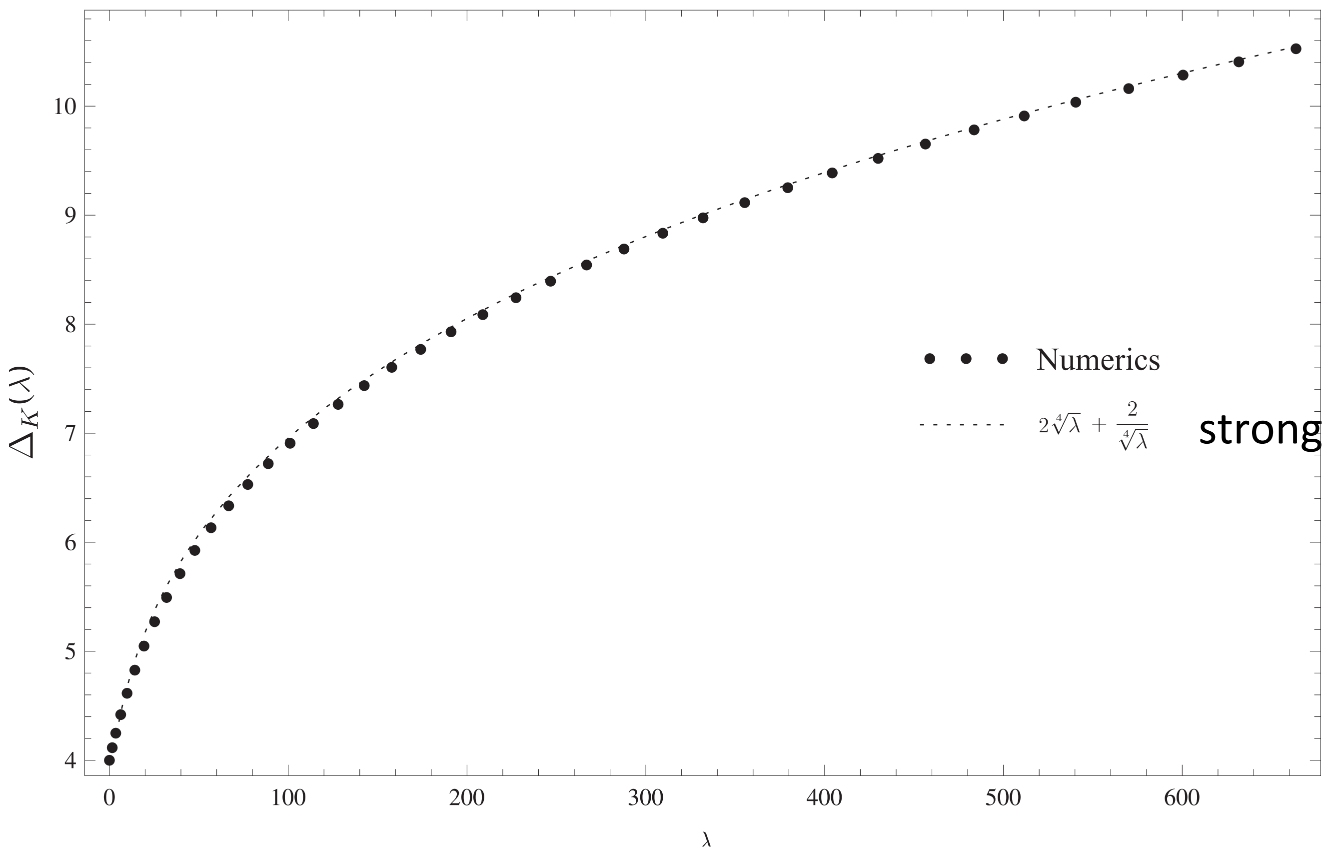
- Chains of gluons → strings → 2d theory on the string is integrable (i.e. solvable) for $\mathcal{N}$=4 SYM.
- (leads to interesting connections to previously studied condensed matter theories).
- Computation of anomalous dimensions.

- Computation of scattering amplitudes in $\mathcal{N}$=4 SYM.

Simplest lowest dimensional operator. (Twist two operator at weak coupling)

$$O = Tr[\phi^I \phi^I]$$

Anomalous dimension



strong coupling

arXiv:0906.4240

Gromov, Kazakov, Vieira.

The gauge/gravity duality

- Duality between strongly coupled field theories and gravity.
- Gravity in AdS of other spacetimes with a boundary.
- Field theories are strongly coupled when the gravity approximation is correct.
- Example: $\mathcal{N}=4$ SYM and $AdS_5 \times S^5$

Gauge/gravity duality and strongly coupled systems.

- Using the duality to perform computations in strongly coupled systems.
- Black holes and thermal systems

Son, Kovtun, Starinets

- Quark gluon plasma $\eta/s = 1/4\pi$
- High T_c superconductors
- Cold atoms.

- Fermi surfaces and black holes with an AdS_2 region.

Gubser, Hartnol, Herzog, Horowitz, Faulkner, Liu, Mc. Greevy, Vegh, Zaanen, Schalm, Sachdev,…

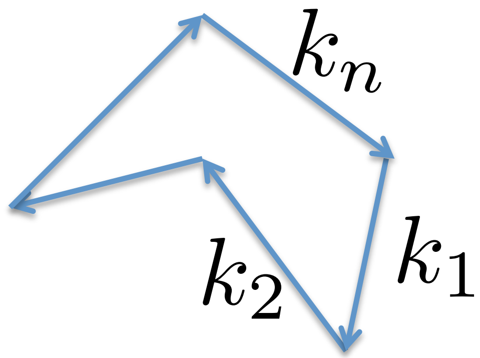
Amplitudes

Witten, Britto, Cachazo, Feng,
Arkani Hamed, et al. ...

Bern, Dixon, Kosower, ...

- BCFW recursion relations → fast way to compute tree amplitudes (and loop integrands).

- Amplitude/ Wilson loop connection.
- Simpler computation of amplitude.
- Use a lot of fun mathematical techniques, twistors, algebraic geometry, etc.



Momenta define a polygon.

Can define a Wilson loop with this contour

6 point, two loop amplitudes:

Bern, Dixon, Kosower, Roiban, Spradlin, Vergu, Volovich

Very complicated computer expression

↓ Amplitude/Wilson loop connection

Drummond, Henn, Korchemsky, Sokatchev

Analytic expression (6 pages)

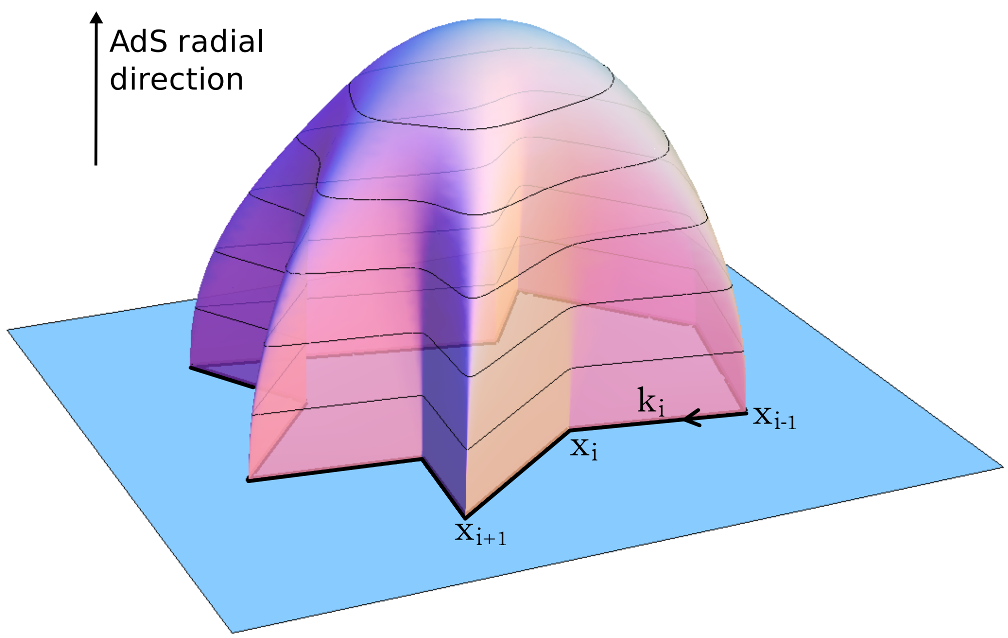
Del Duca, Dhur, Smirnov

↓ Symbology…

Goncharov, Spardlin, Vergu, Volovich

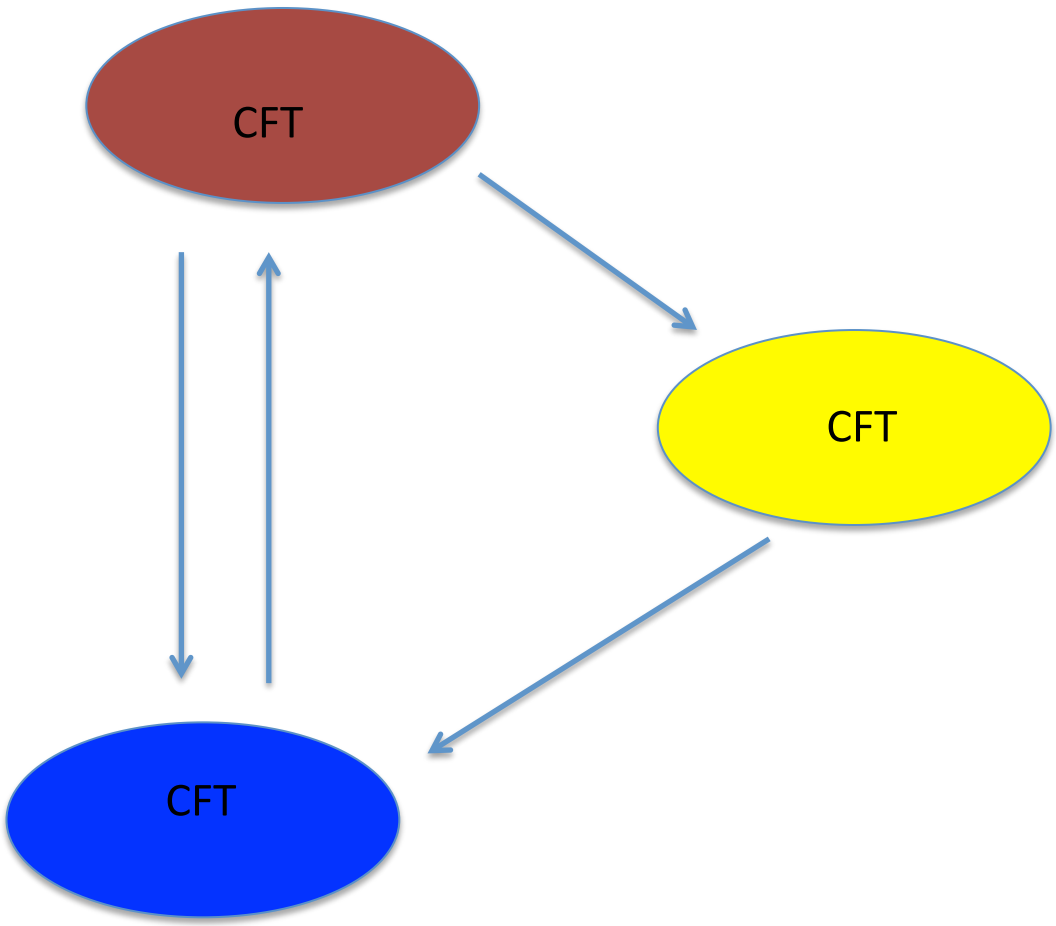
2 lines.

Amplitudes at strong coupling → Area of minimal area surfaces in AdS



Alday, JM

Constraining renormalization group flows between Conformal Field Theories



c, f, a are quantities that decrease along renormalization group Flows in d=2,3,4 dimensions.

$$T_\mu^\mu = cR$$

$$T_\mu^\mu = aE = a(R^2_{\mu\nu\rho\sigma} + \cdots)$$

$$f = -\log Z_{S^3}$$

$$\log Z_{S^2} \sim c \log \Lambda$$

$$\log Z_{S^4} \sim a \log \Lambda$$

c, a and f theorems

- They decrease along the renormalization group flow.
- This puts some order in the space of QFT's.
- General statements, no susy required.

- c : in 2d → proof using stress tensor ward identities. Newer proof using entanglement entropy.

Zamolodchikov
Casini, Huerta

- a: in 4d → proof using anomaly matching argument for the conformal anomaly + unitarity. Komargodski, Schwimmer

- f: In 3d → argument using entanglement.

Casini, Huerta

Entanglement

$$|\Psi\rangle \sim |+\rangle_1|-\rangle_2 + |-\rangle_1|+\rangle_2$$

- The vacuum in QFT is highly entangled.

Bombelli, Koul, Lee, Sorkin
Shredniki

$$H = H_A \times H_B$$

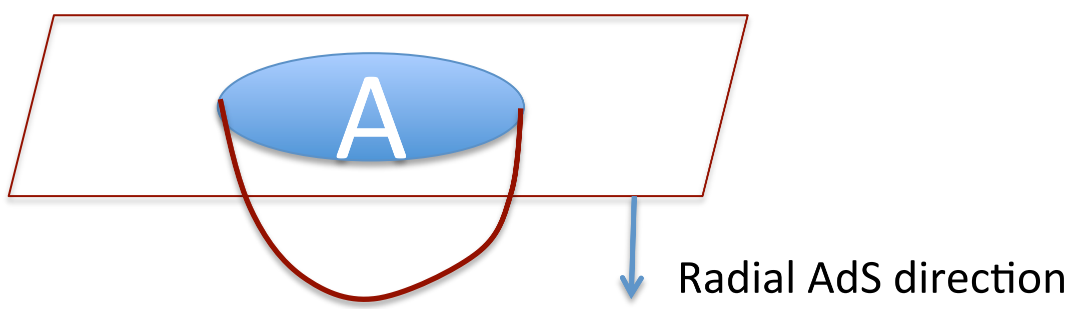
$$\rho_A = Tr_{H_B}\left[|0\rangle\langle 0|\right]$$

$$S_A = -Tr[\rho_A \log \rho_A]$$

In 2 +1 dimensions, for a circular region:

$$S = \frac{R}{\epsilon} + \text{Finite}$$

- Now: In 3d: Entanglement of a circular region = sphere partition function. Casini, Huerta, Myers
- Theories with AdS duals → Geometric computation in terms of a minimal surface in the bulk. Ryu, Takayanagi



Black holes

- Precision counting of the microstates of black holes. Connections with mathematics. Wall crossing → bound states coming in an out.

Sen, et al

- Black hole information problem.
- How the black hole interior is realized ?

Almeheri, Marolf, Polchinski, Sully

- Black holes have been a source of information!!
- And they will probably continue to be…

Conclusions

- Many interesting directions
- Long standing problems are being solved
- Great increase in calculational ability for special theories.
- Greater understanding of various dualities and relations between theories.
- Interesting interplay between gravity, quantum field theory, quantum information, etc.

Thank you