# Towards the N3LO Higgs cross-section

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in collaboration with
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# OUTLINE

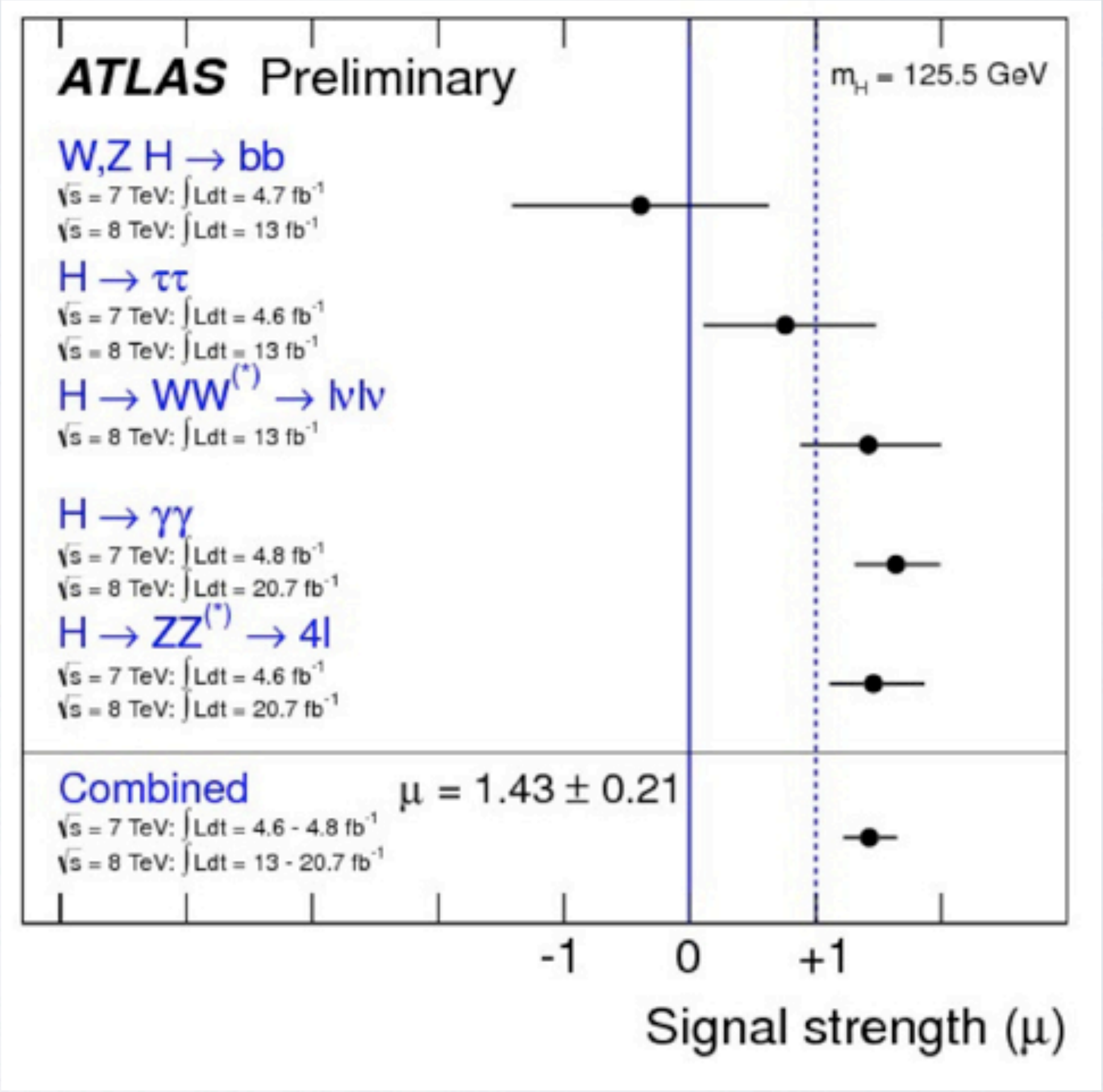
- Motivation and challenge
- A new method for expansions of phase-space integrals
- Conclusion and Outlook

# Why
# Next-to-next-to-next-to-LO?

- ATLAS and CMS have discovered a Higgs boson
- Mass of the Higgs boson was the last unknown parameter of the Standard Model: Higgs cross-section can be derived unambiguously from it.
- What is it? Does it agree with experiment?

# Experimental Prospects

- Higgs total cross-section already measured with an uncertainty of about 30%
- Uncertainty will be reduced below the current theoretical uncertainty with more data after the upgrade
- A more precise theory prediction is necessary in order to test the Standard Model and to reveal potential deviations due to new physics effects.




$\Rightarrow \mu = \mathbf{1.43 \pm 0.16\ (stat) \pm 0.14\ (sys)}$

# Why N3LO?

Theory prediction relies on perturbative QCD

- Two measures of theory uncertainty: series progression and scale variation

- Perturbative series converges slower

- NNLO scale variation uncertainty will be superseded by experiment

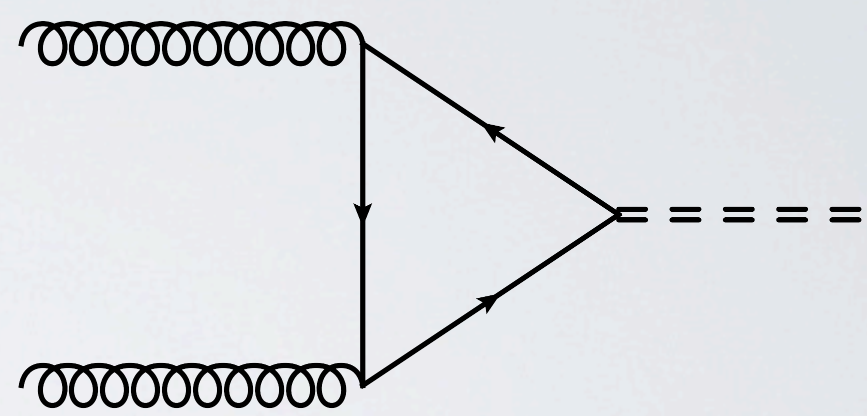
| | $\sigma^{8TeV}$ [pb] | $\delta\sigma$ [%] |
|---|---|---|
| LO | 9.6 | $\pm \sim 25$ |
| NLO | 16.7 | $\pm \sim 20$ |
| NNLO | 19.6 | $\pm \sim 9$ |
| N3LO | ? | $\pm \sim 4$ |

# THE CHALLENGE OF AN N3LO CALCULATION

- Has never been done before for a hadron collider process.
- Many conceptual and practical problems have not yet been solved at NNLO.
- N3LO is a true challenge and requires the best of our techniques and ideas

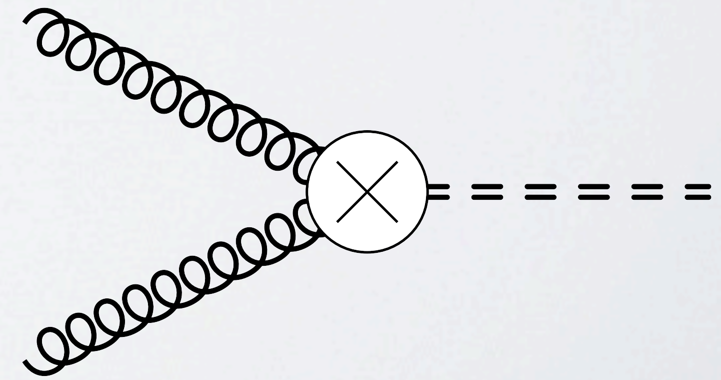
# Gluon Fusion Cross-Section

- Dominant production mechanism at hadron colliders: **Gluon Fusion**



- Loop induced process
- Higgs boson is lighter than the top quark
  $\longrightarrow$ Effective theory

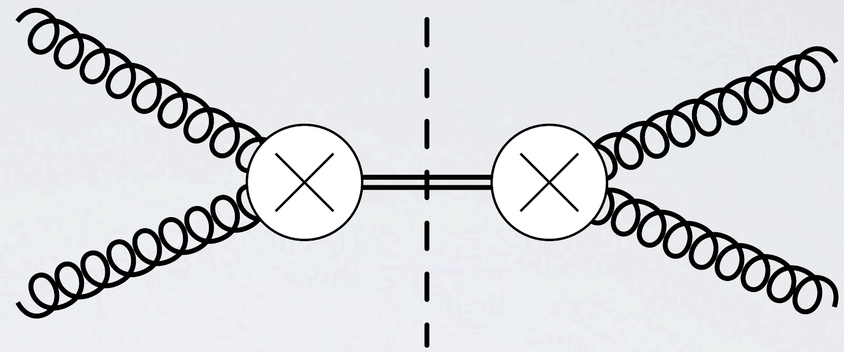
$$\mathcal{L} = \mathcal{L}_{QCD,5} - \frac{1}{4v} C_1 \, H \, G^a_{\mu\nu} \, G^{\mu\nu}_a$$



# Perturbative Corrections

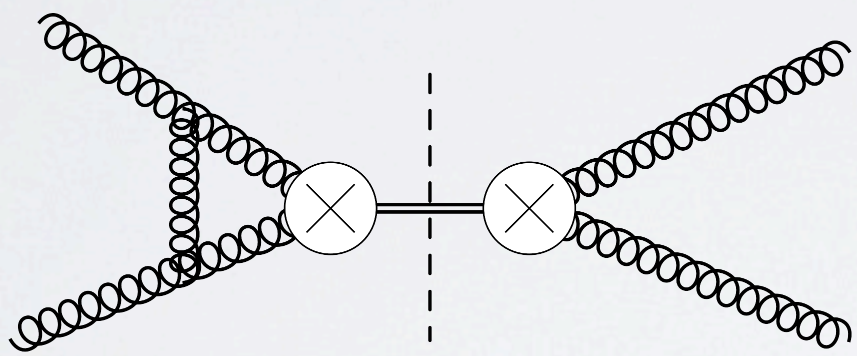
~ 1979

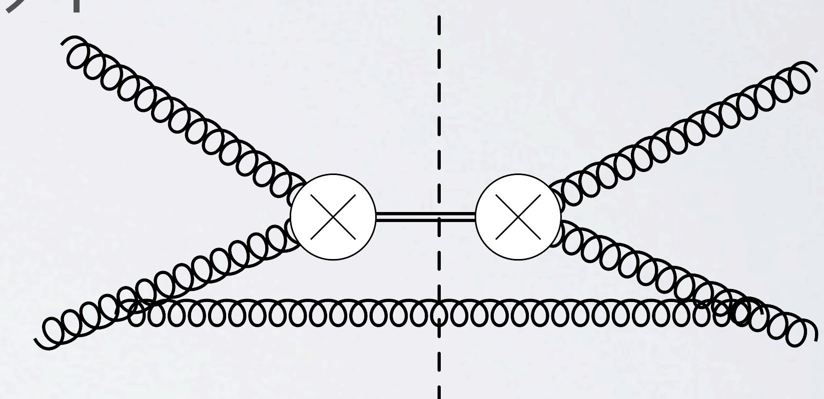
- LO



1991

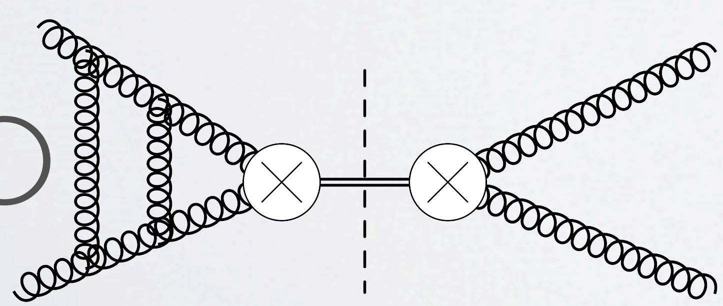
- NLO

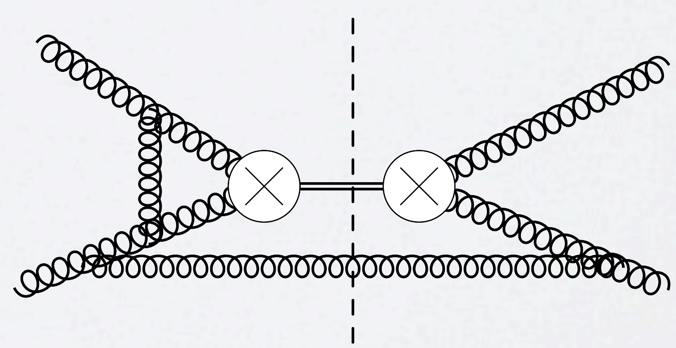


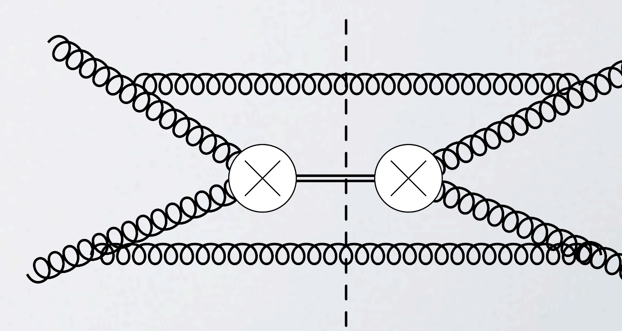


2002

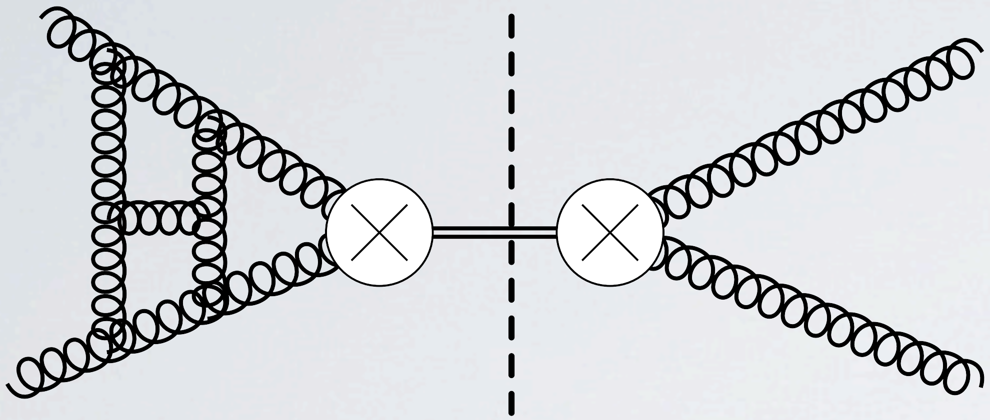
- NNLO







# N3LO CORRECTION

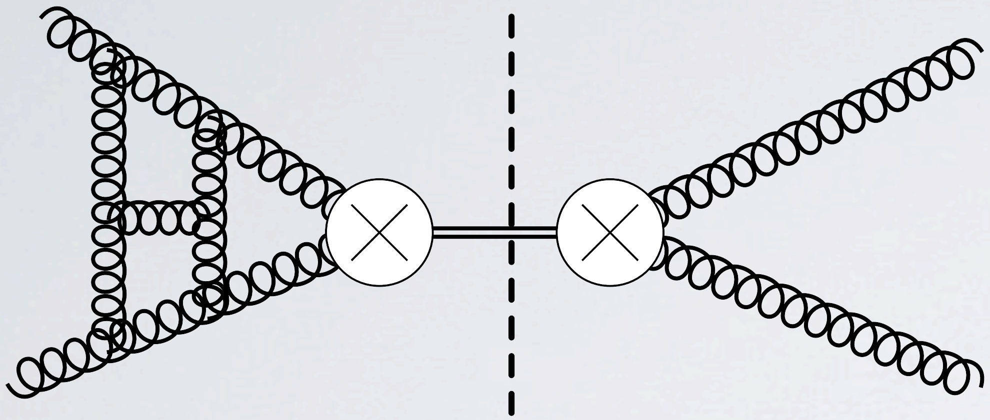


triple virtual

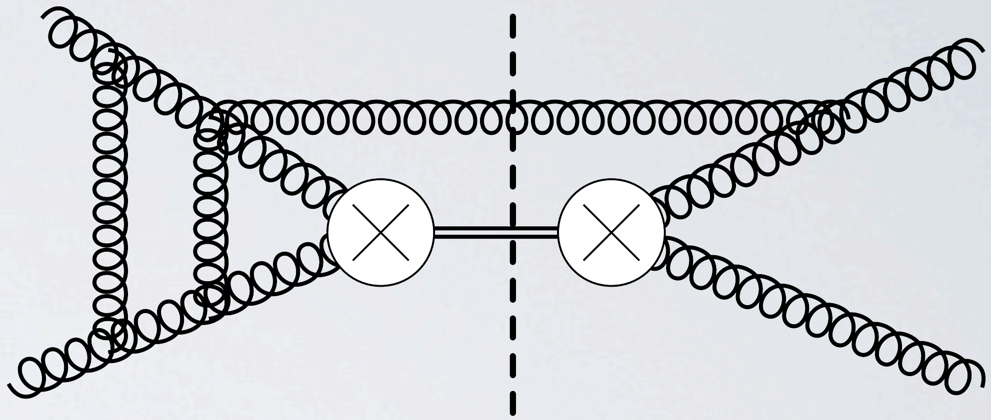
purely virtual contributions
is known: 3-loop QCD form -factor

[Baikov, Chetyrkin, Smirnov, Smirnov, Steinhauser; Gehrmann, Glover, Huber, Ikizlerli, Studerus]

# N3LO CORRECTION



triple virtual



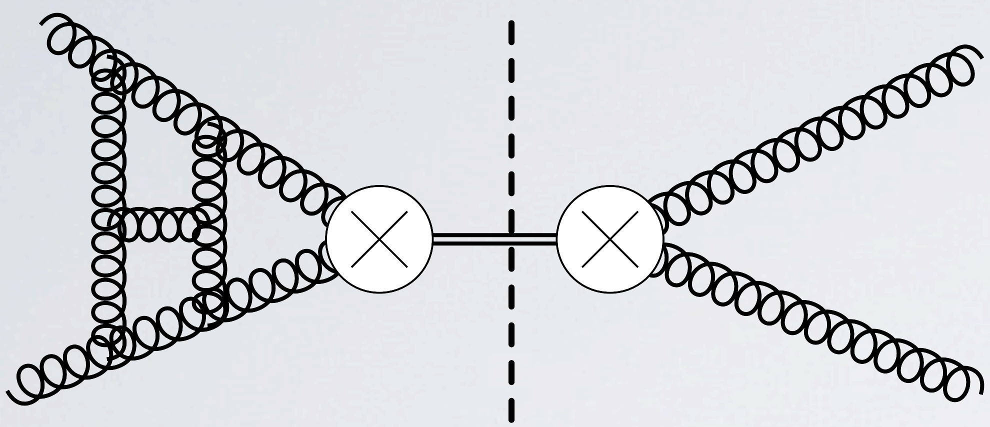
double-virtual real

- 2-loop QCD form factors known

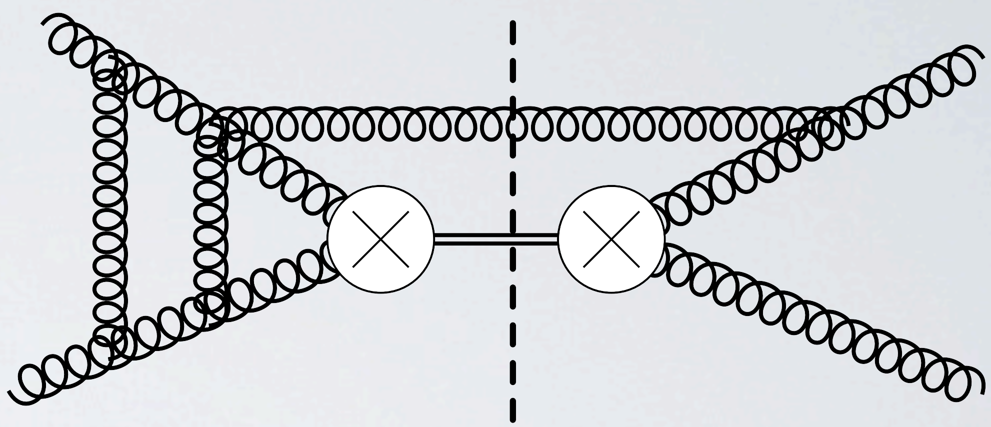
[Gonsalves; Kramer, Lampe; Gehrmann, Huber, Maître]

- Phase-space integration missing

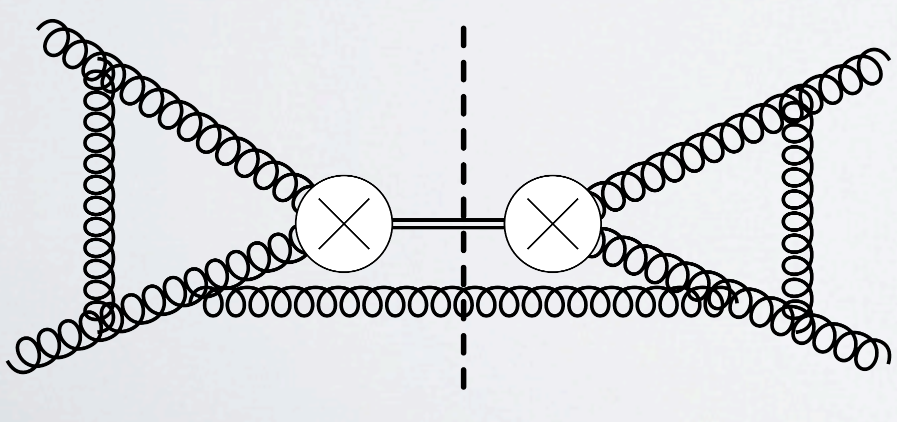
# N3LO CORRECTION



triple virtual

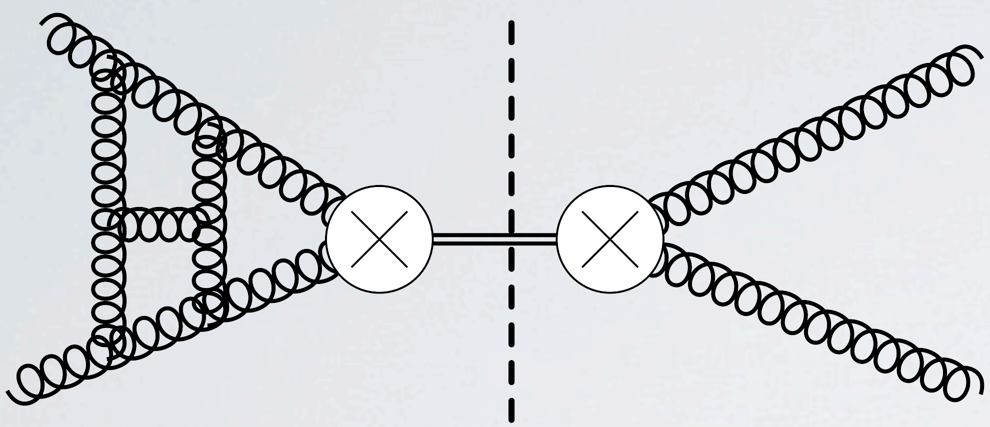


double-virtual real

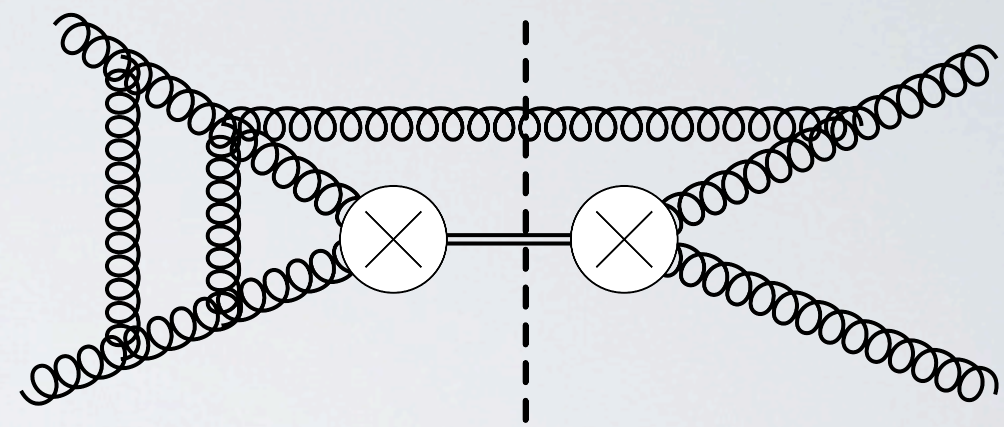


(real virtual)$^2$

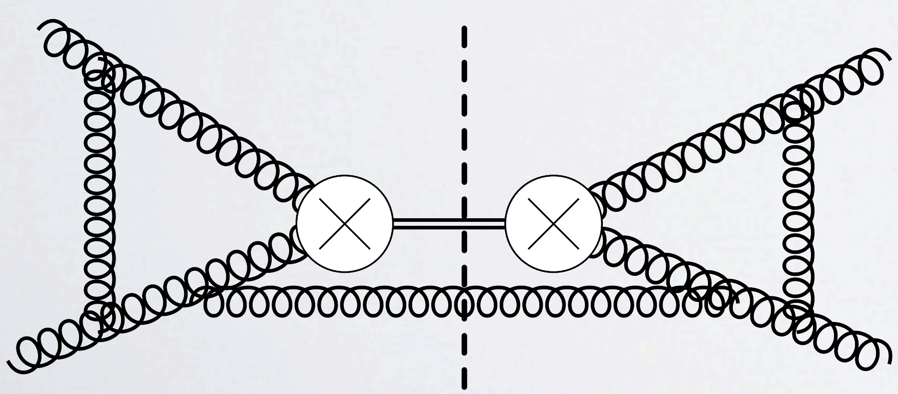
# N3LO CORRECTION



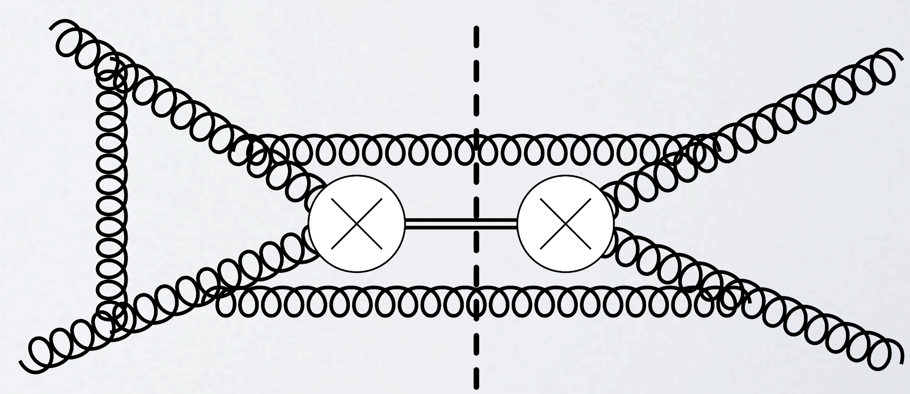
triple virtual



double-virtual real

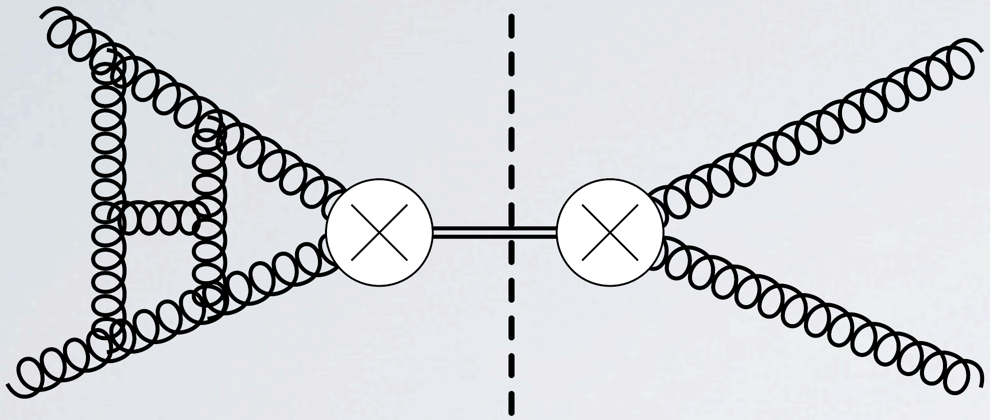


(real virtual)$^2$

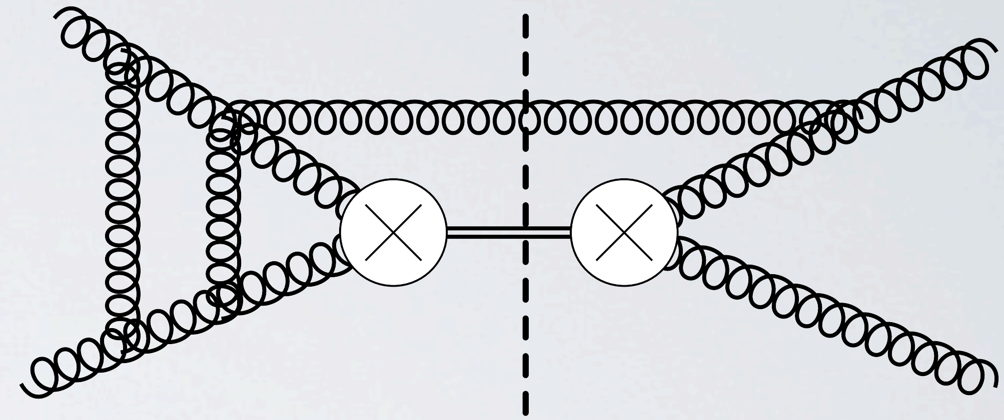


double-real virtual

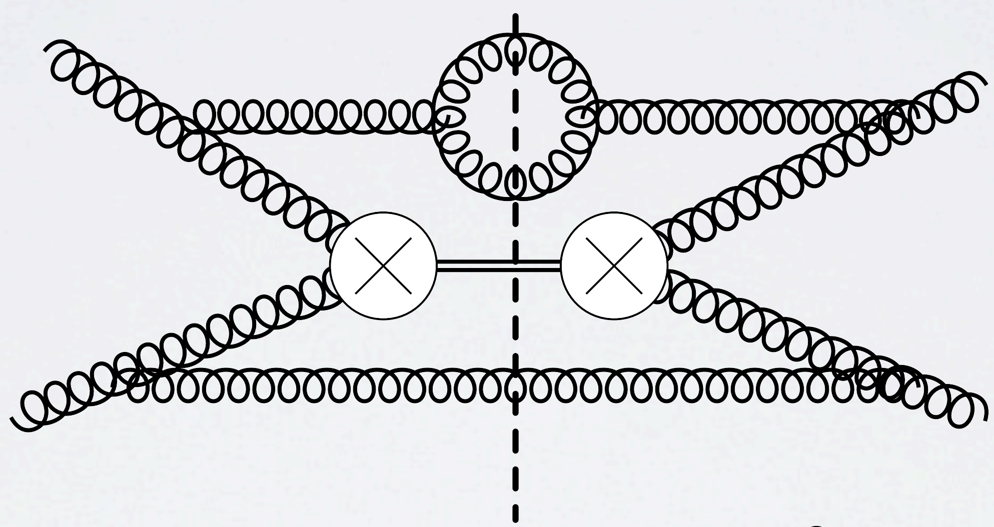
# N3LO CORRECTION



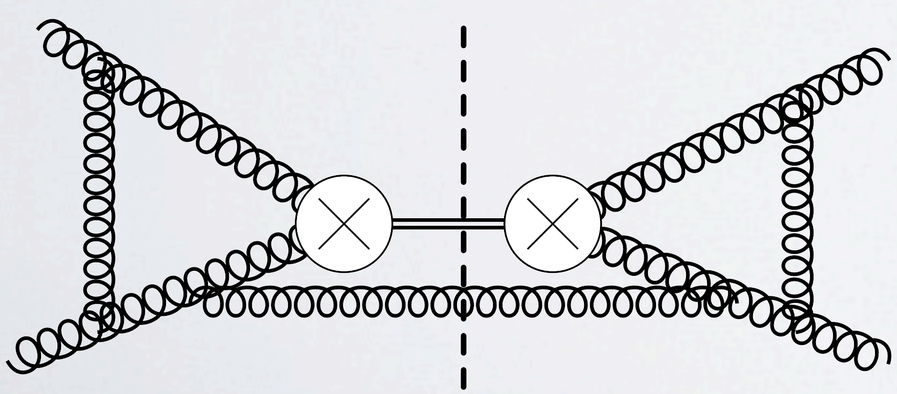
triple virtual



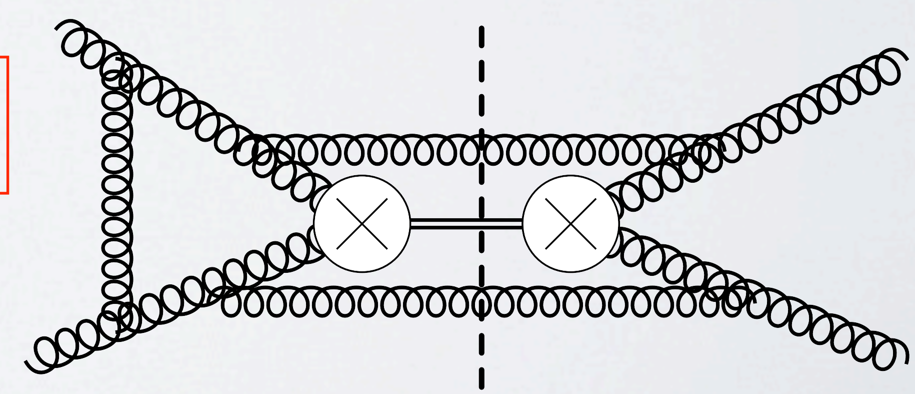
double-virtual real



**triple real**



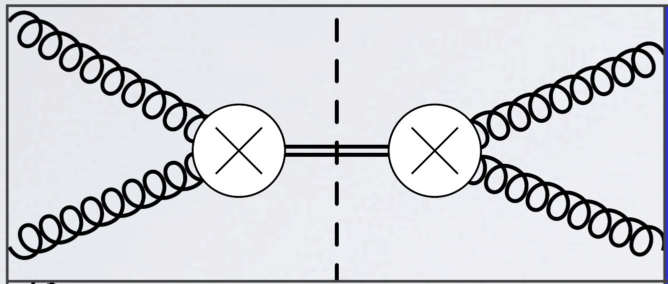
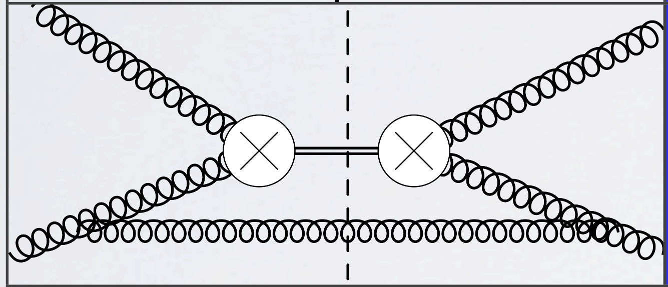
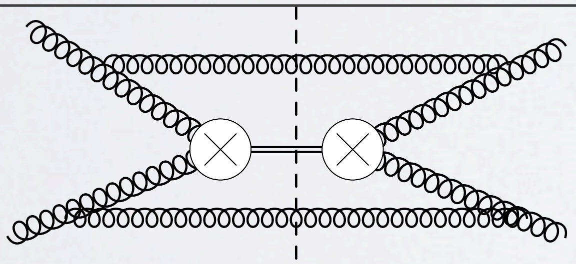
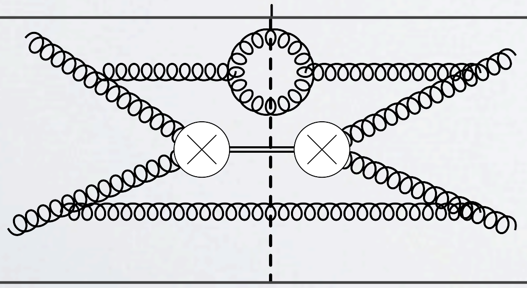
(real virtual)$^2$



double-real virtual

# Triple RRReal Contribution

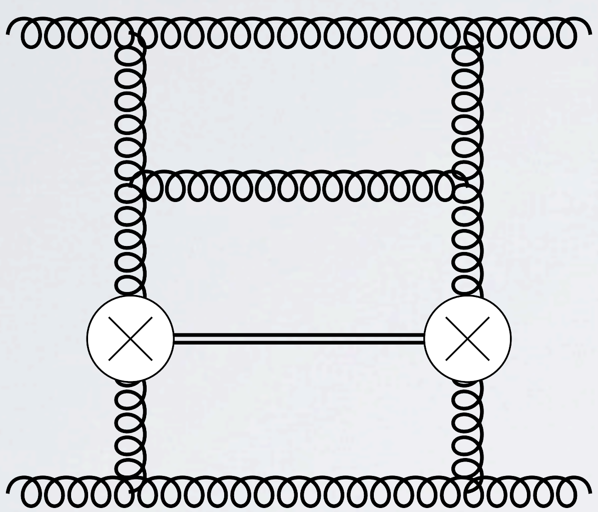
- We start with the contributions that was most complicated @ NNLO

| | | |
|---|---|---|
|  | 1 diagram | 1 Integral |
|  | 10 diagrams | 1 Integral |
|  | 351 diagrams | 18 Integrals |
|  | 26 565 diagrams | ~200 Integrals |

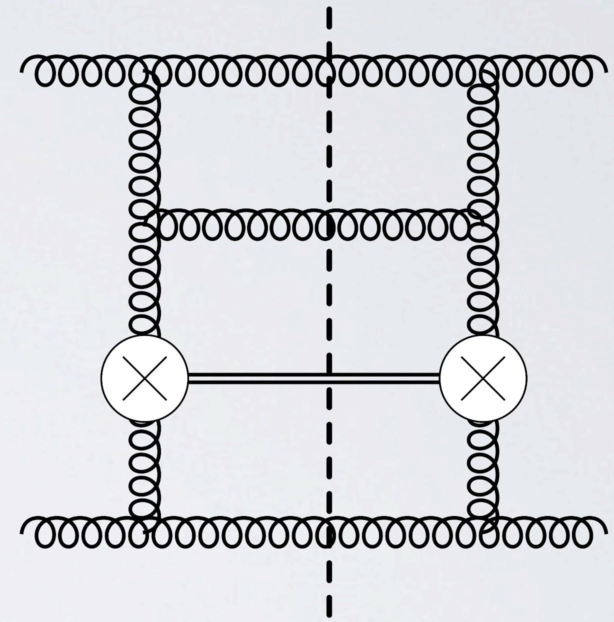
# Unitarity

- Optical theorem

$Im$



$$= \int d\Phi_4$$



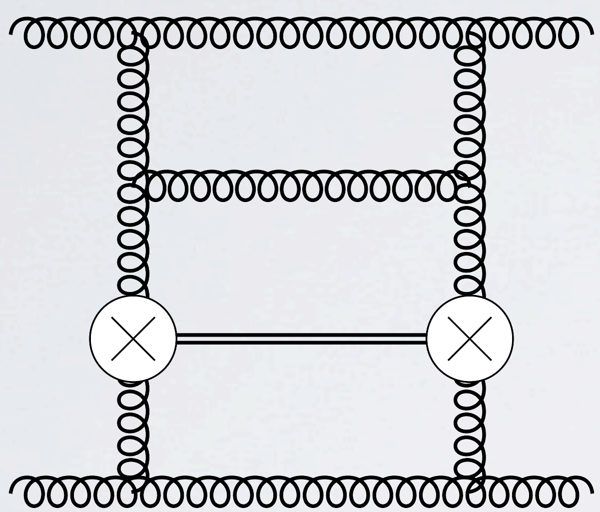
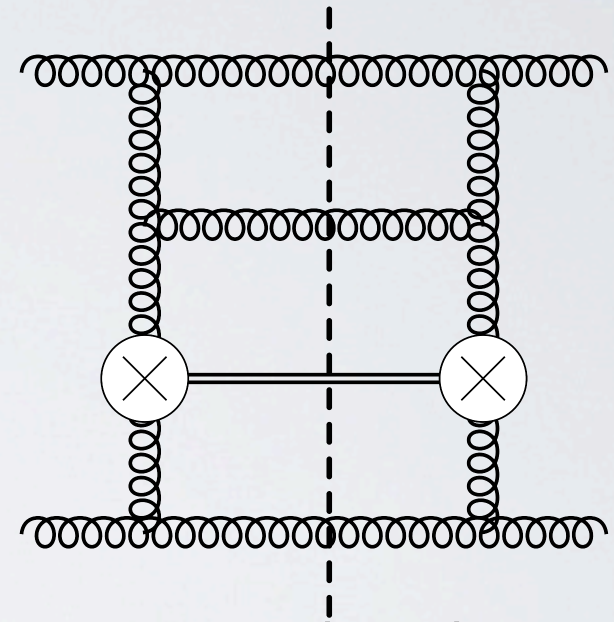
- Discontinuities of loop integrals are phase-space integrals

- Cutkosky's rule

$$\frac{1}{p^2 - m^2 + i\epsilon} \to \delta_+(p^2 - m^2) = \theta(p^0)\delta(p^2 - m^2)$$

# Reverse Unitarity

- Optical theorem

$Im$  $= \int d\Phi_4$ 

- Invert the relation: Regard phase-space cuts as unitarity cuts of loop integrals

[Anastasiou, Melnikov]

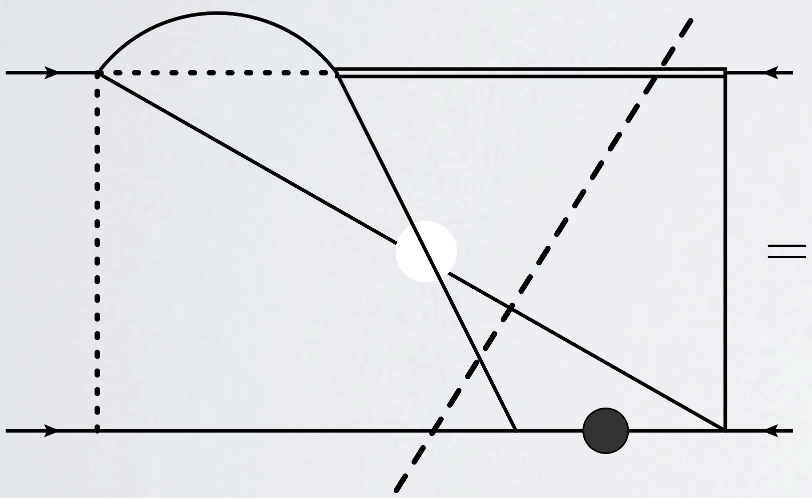
$$\delta_+(q^2) \to \left(\frac{1}{q^2}\right)_c = \frac{1}{2\pi i}\left(\frac{1}{q + i\epsilon} - \frac{1}{q^2 - i\epsilon}\right)$$

- Loop integral technology:
  Integration-by-parts, Master integrals, differential equations.

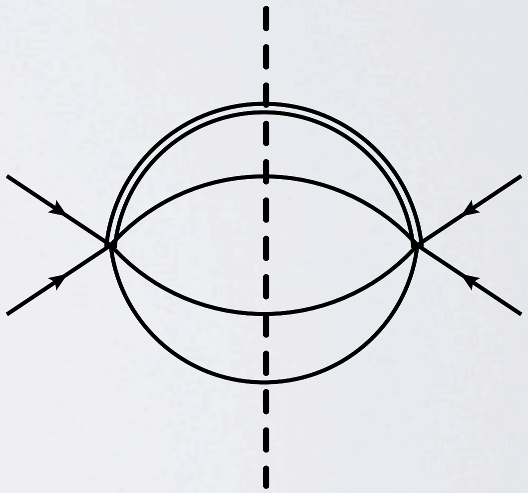
# Reduction to Master Integrals

- Loop technology:
  Systematic reduction of phase-space integrals
  by "Integration-by-parts" - identities

- Limited basis of Master integrals to express all integrals



$$= -\frac{(\epsilon-1)(2\epsilon-1)(3\epsilon-2)(3\epsilon-1)(6\epsilon-5)(6\epsilon-1)}{\epsilon^4(\epsilon+1)(2\epsilon-3)}$$

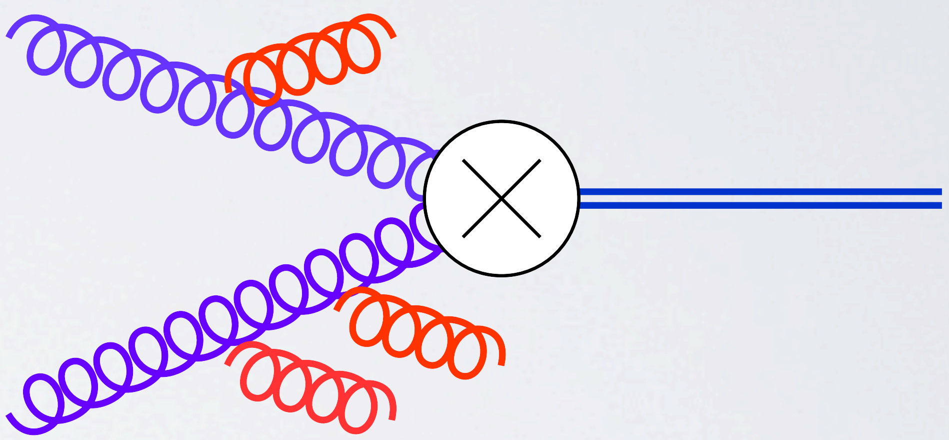


# Soft Expansion

- Reduction and calculation of integrals is still enormously difficult

⭐ **Perform an expansion of the cross-section**

- Threshold expansion:
  - Higgs almost on-shell
  - All final state partons are **soft**



- Small parameter

$$\bar{z} = (1 - z) = \left(1 - \frac{M_H^2}{\hat{s}}\right)$$

# A New Method

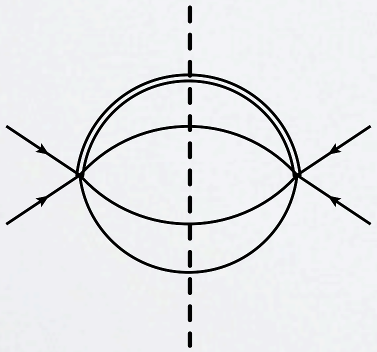
- New idea: **Cut-propagators can be differentiated and expanded!**

$$\left(\frac{1}{k^2+2\bar{z}(k\cdot q)}\right)_c = \frac{1}{2\pi i}\left(\frac{1}{k^2+2\bar{z}(k\cdot q)+i\epsilon}-\frac{1}{k^2+2\bar{z}(k\cdot q)-i\epsilon}\right)$$

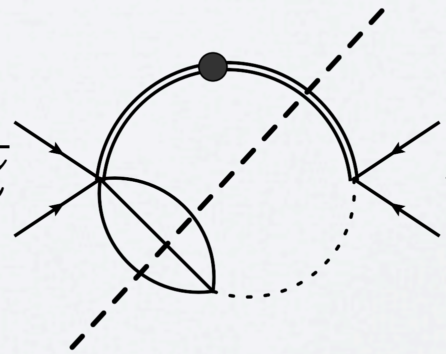
$$= \left(\frac{1}{k^2}\right)_c \sum_{i=0}^{\infty} \bar{z}^i \left(\frac{-(k\cdot q)}{k^2}\right)^i$$

  - Expansion at the integrand level
  - Resulting integrand has a diagrammatic interpretation

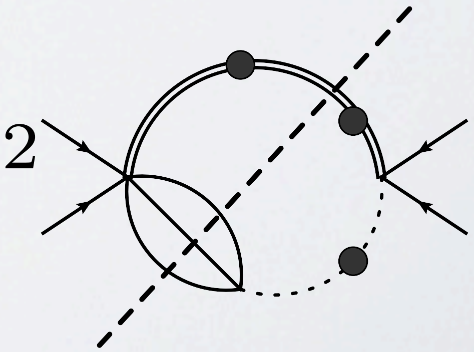
$\int d\Phi_4 = \bar{z}^{5-6\epsilon}$ [



$-\bar{z}$



$+\bar{z}^2$



$+\ldots$ ]

# Soft Expansion

- We obtain the **first two terms** in the soft expansion for all 2 to H + 3 parton processes

- We reduce to soft Master integrals

- **Only 10** soft Master integrals - all solved analytically

$$\hat{\sigma}(z) = \bar{z}^{3(D-4)-1}\left(\hat{\sigma}^{S} + \bar{z}\hat{\sigma}^{NS} + \sum_{k=2}^{\infty} \bar{z}^{k}\hat{\sigma}^{N^k S}\right)$$

**Only numbers**

# CONCLUSION

- First N3LO result for a LHC process
- We took a first essential step towards computing the N3LO Higgs boson production cross-section
- We devised a new method of creating and calculating a threshold expansion of real phase-space integrals
- We computed the soft and next-to-soft term in the threshold expansion
- We computed analytically 10 universal soft phase-space Master integrals

# OUTLOOK

Many possibilities to proceed!

- Compute higher orders in the soft expansion of the triple real emission - Easily accessible using our method
- Extend our method to perform a threshold expansion of loop contributions
- Obtain the full N3LO Higgs boson cross-section
- Apply our methods to other processes - such as Drell-Yan