Synthesizing Lattice Results for 12 Flavor QCD

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Motivation

- Banks and Zaks discussed the appearance of an infrared stable fixed point in vector-like gauge theories (NPB 196 (1982) 189).
- A fundamental question about many-flavor QCD - does this IR fixed point exist and for what number of flavors?
- Extended technicolor models need slow evolution of couplings (walking) to separate scales at which fermion masses are generated.

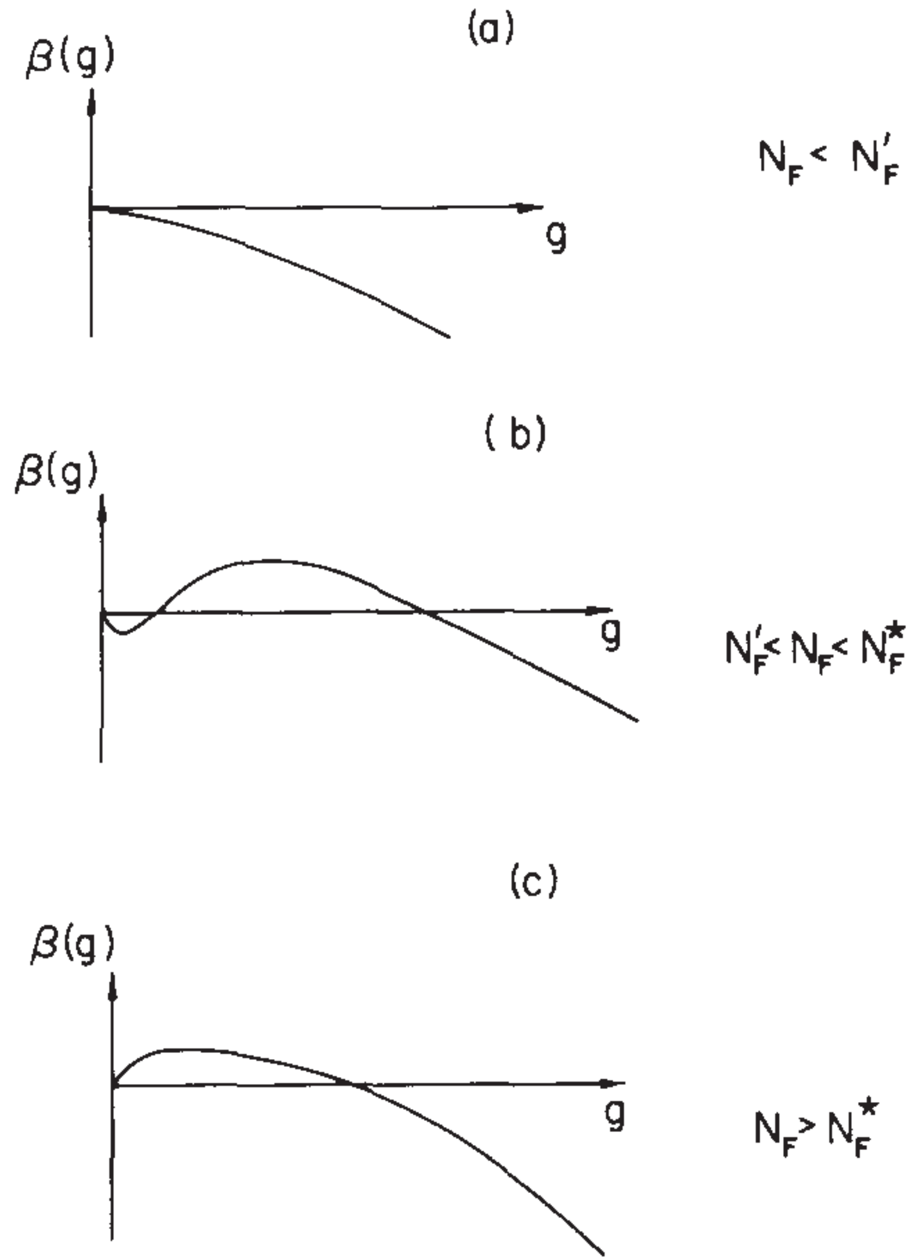
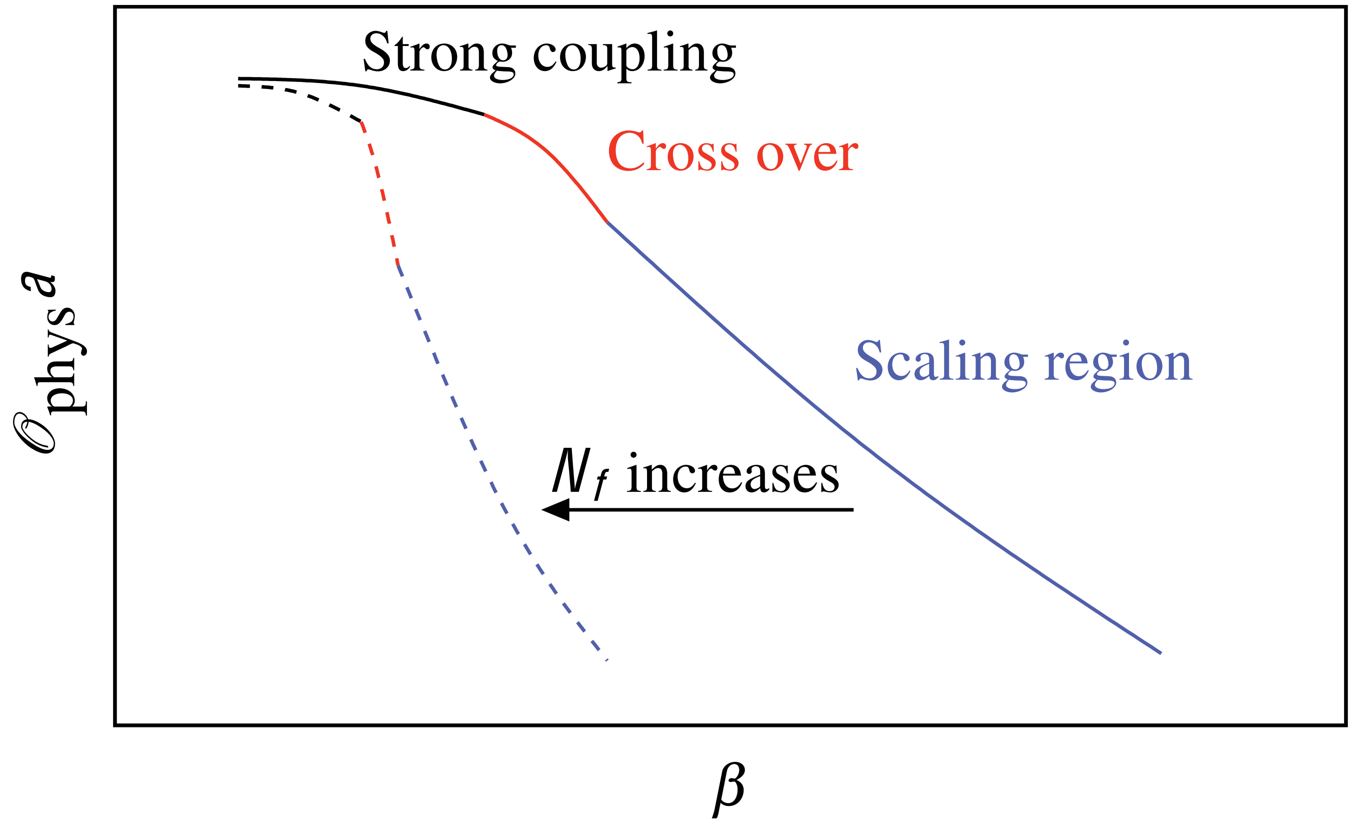


Fig. 1. 2 zero β function.

T = 0 many-flavor QCD versus $\beta \sim 1/g^2$



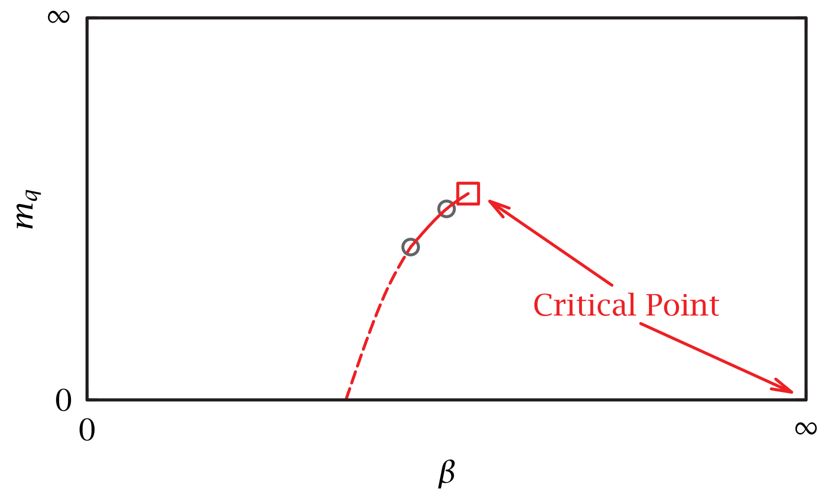
- Need $\beta > \beta_{\mathrm{crossover}}$ to be continuously connected to continuum physics
- Irrelevant (in continuum) operators can play a large role $\beta < \beta_{\mathrm{crossover}}$
- Strong to weak coupling may be discontinuous
- Steepness of crossover region may be hard to distinguish from walking.

12 Flavor QCD

- 6 groups with data for this subject, all using variants of staggered fermions
  - Y. Aoki, et. al.
  - Appelquist, Fleming, Neil
  - Cheng, Hasenfratz, Schaich
  - Deuzeman, Lombardo, Pallante
  - Fodor, Holland, Kuti, Nogradi, Schroeder
  - Jin, Mawhinney
- No consensus on $T = 0$ continuum phase, whether conformal or chirally broken
- We have explored system through basic low energy QCD observables and studies of the finite temperature phase transition using naive staggered fermions and the DBW2 gauge action
- Will give overview of our results, ask what we have learned, do some comparisons with Fodor, et. al. and suggest a next step in our work
- Almost all results shown here are from multiple volumes, so finite volume errors are minimal (few percent scale).

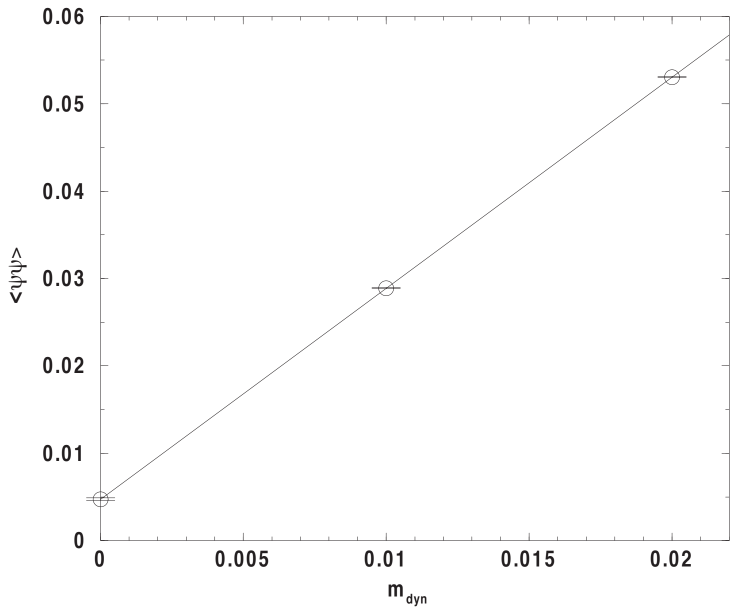
Lattice Phase Diagram for $N_f = 12$ QCD

- For naive staggered fermions and the DBW2 gauge action, we find the following phase diagram for zero temperature, 12 flavor QCD

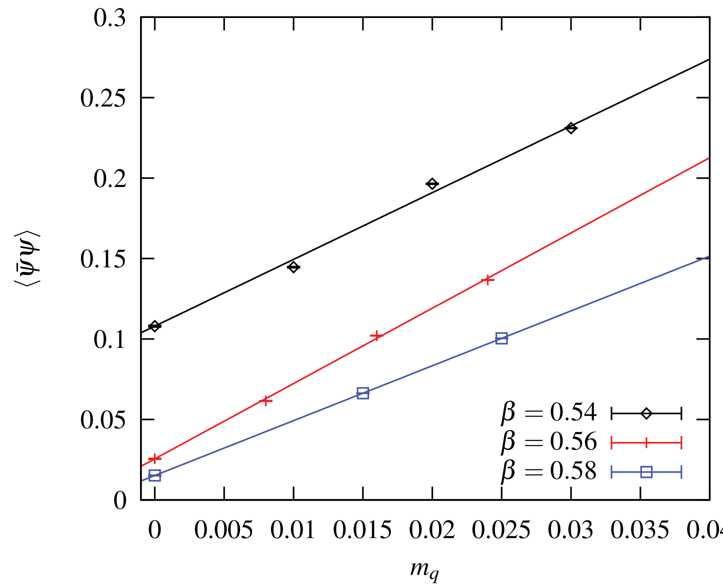


- A first order critical line ends at a second order critical point at finite β and m_q
- Critical point at finite β and m_q is lattice action dependent.
- What is happening across the first order line? At the endpoint?
- First look at our data in support of endpoint.
- Then try to understand our results along with those of some other groups.

Extrapolation of Chiral Condensate

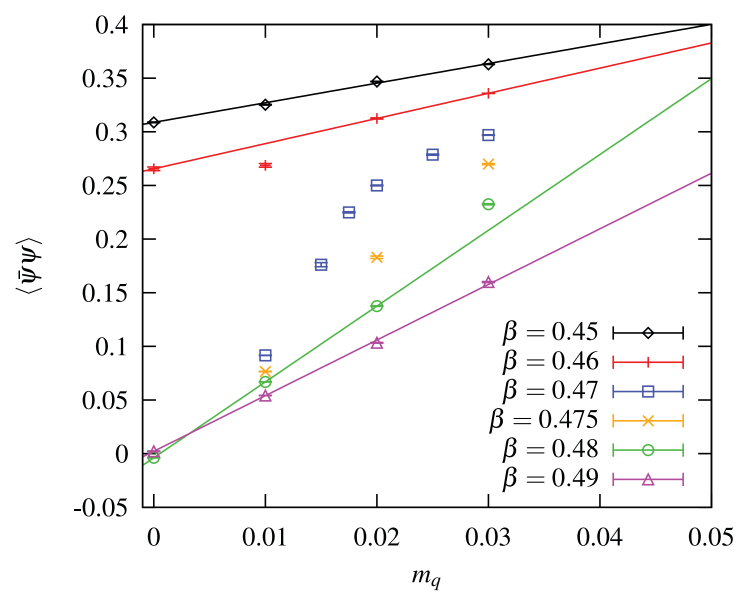


$N_f = 4$ (from long ago)



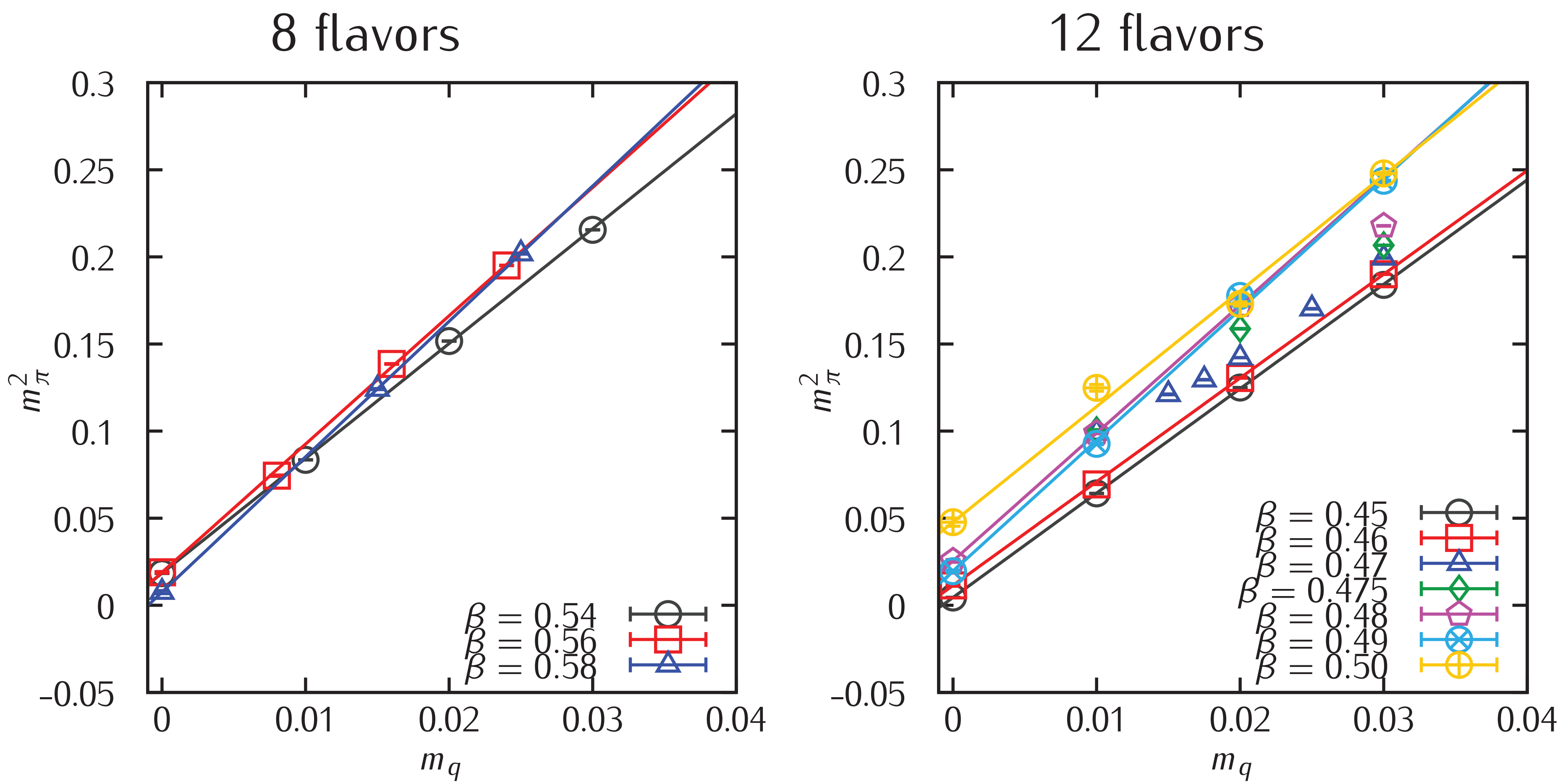
$N_f = 8$

- Unrenormalized, power divergent lattice quantity
- $N_f = 4$ and 8 extrapolate linearly to non-zero values.
- $N_f = 12$ at strong coupling shows χSB in massless limit
- $N_f = 12$ at weak coupling shows a rapid change in the system.



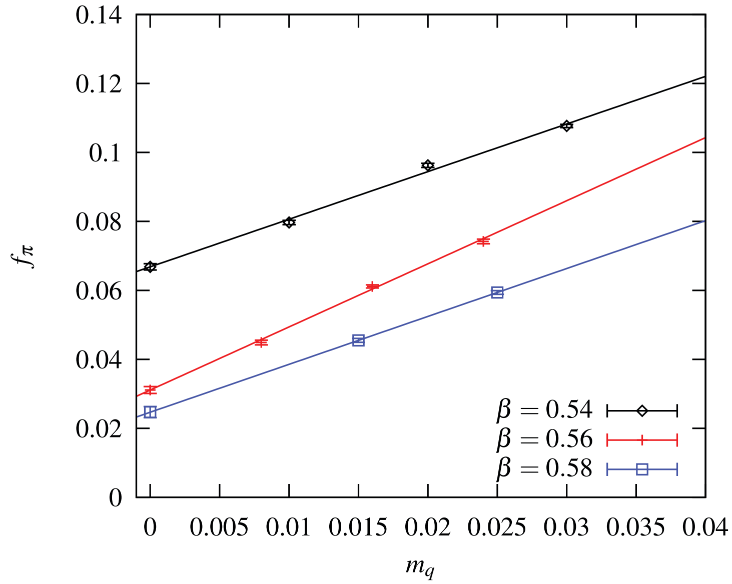
$N_f = 12$

m_π^2 vs. m_f for 8 and 12 flavors, T = 0

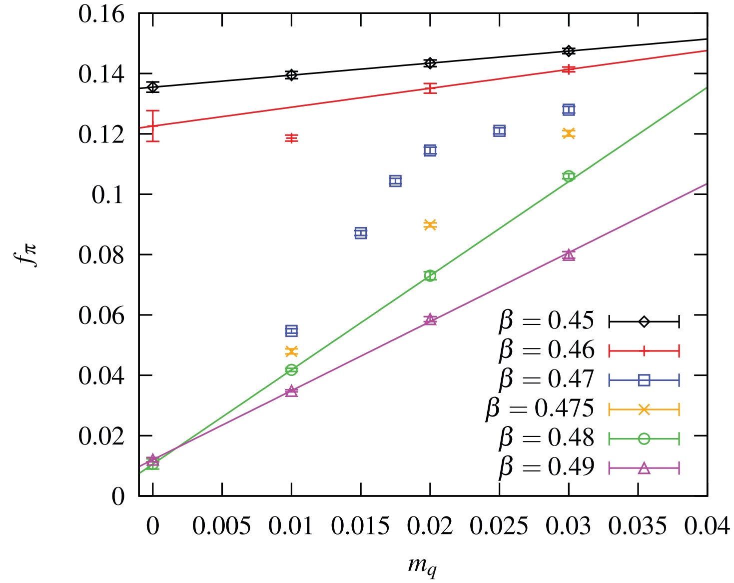


- m_π^2 roughly linear in m_q over large range - commonly seen in 2+1 flavor QCD
- Intercepts not precisely zero - finite volume effects, chiral logs, not χSB ...
- Notice slope of m_π^2 w.r.t m_q largely independent of coupling $\beta = 6/g^2$

Extrapolation of f_π



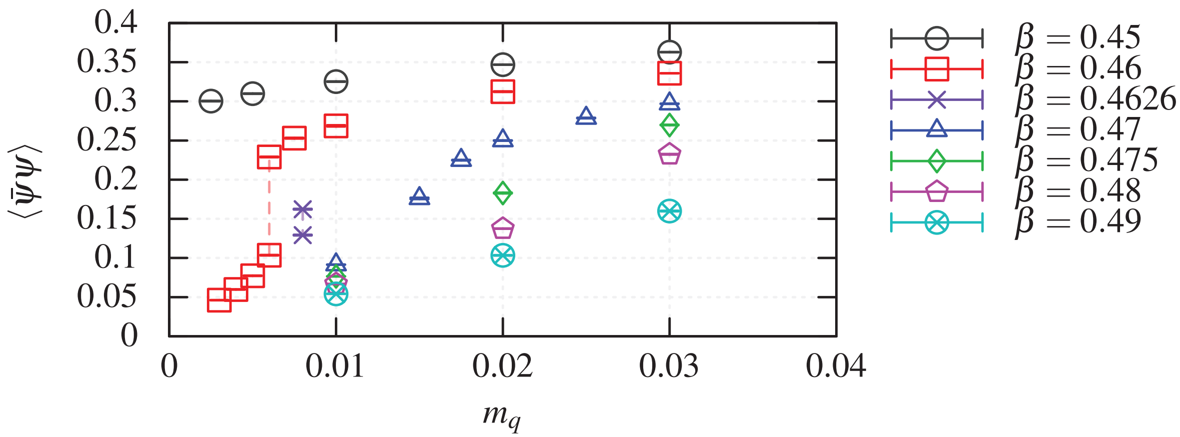
$N_f = 8$



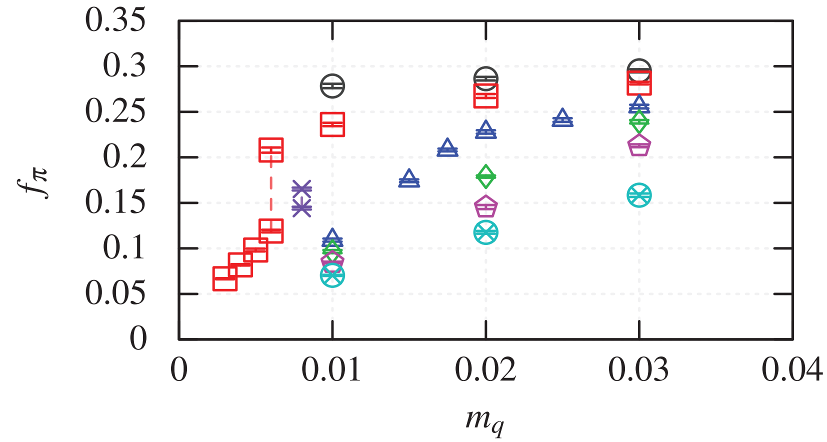
$N_f = 12$

- Extrapolated f_π for $N_f = 8$ shows 2× change across the region of rapid evolution.
- Extrapolated f_π for $N_f = 12$ shows ~10× change across the region of rapid evolution.
- Extrapolated, weak coupling f_π for $N_f = 12$ is non-zero.
- Clearly a lot is happening in this β region for 12 flavors. What is it and how do we control/avoid it having an effect on the continuum physics we are after?

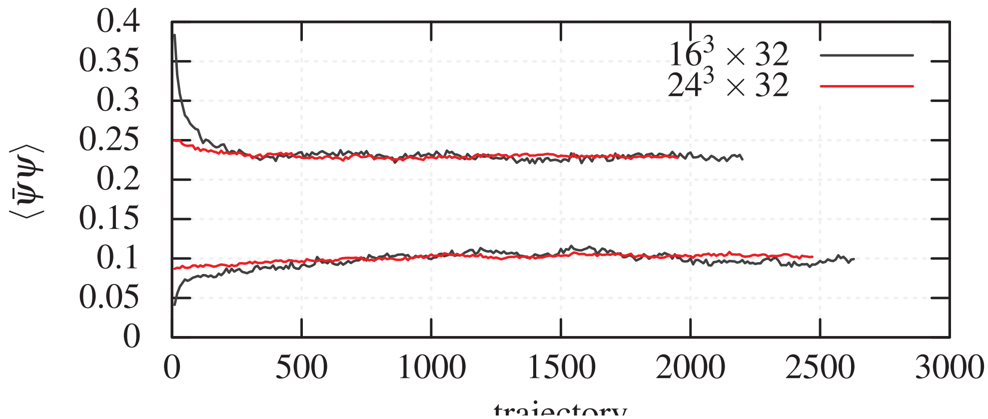
Locating 12 Flavor Bulk Transition

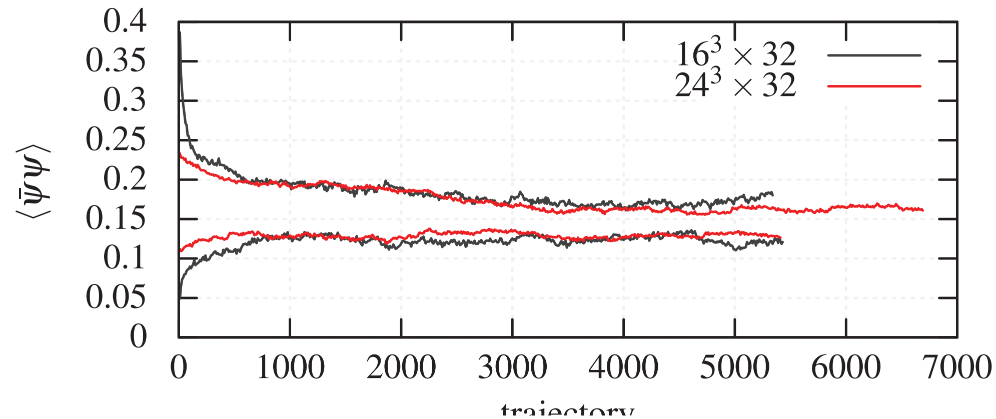


(a) $\langle\bar{\psi}\psi\rangle$



(b) f_π

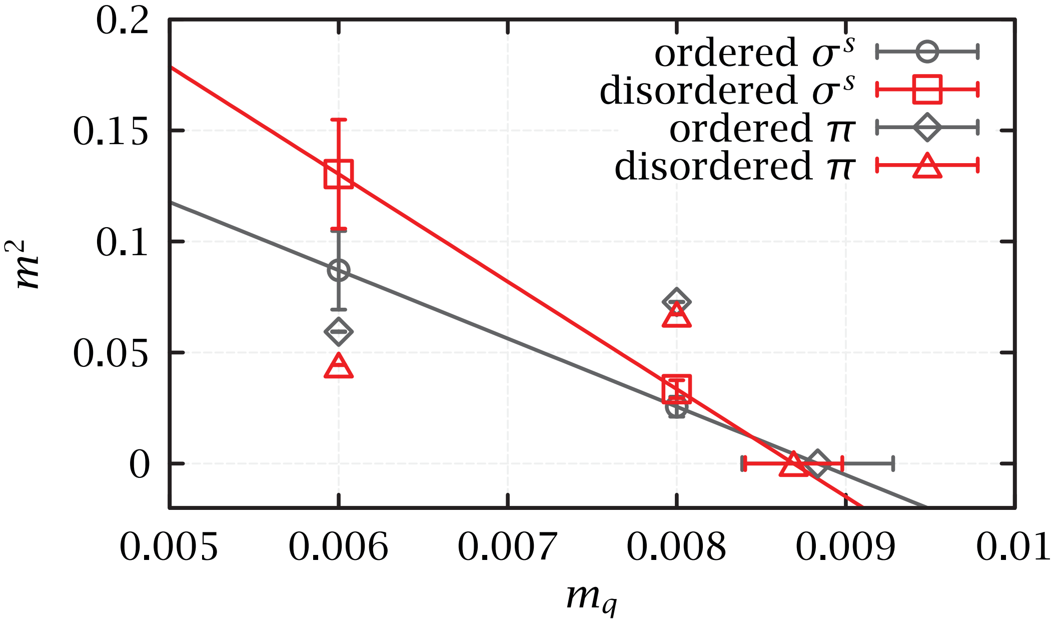




- Very clear signal for a bulk transition, with stability for thousands of MD time units
- Where does it end? Is there a second order critical point?

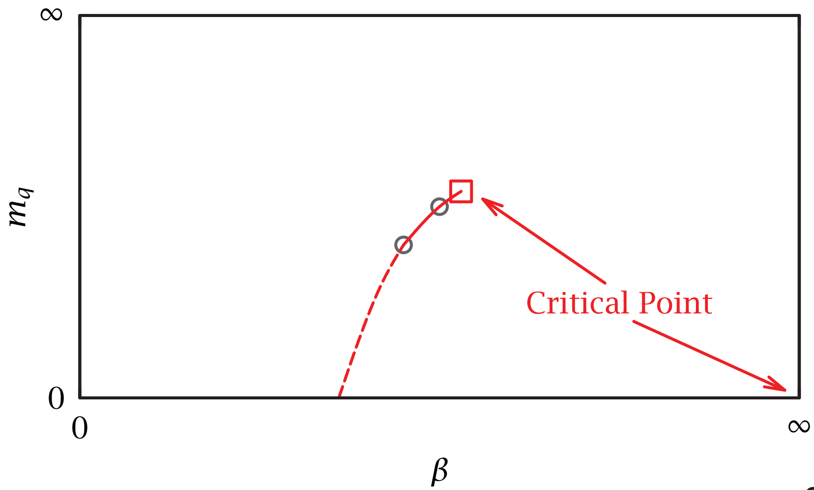
Scalar Singlet Meson

- Measure in both ordered (weak coupling side) and disordered (strong coupling) phases
- Linear extrapolation in both phases produces a consistent endpoint, where $m_\sigma = 0$.



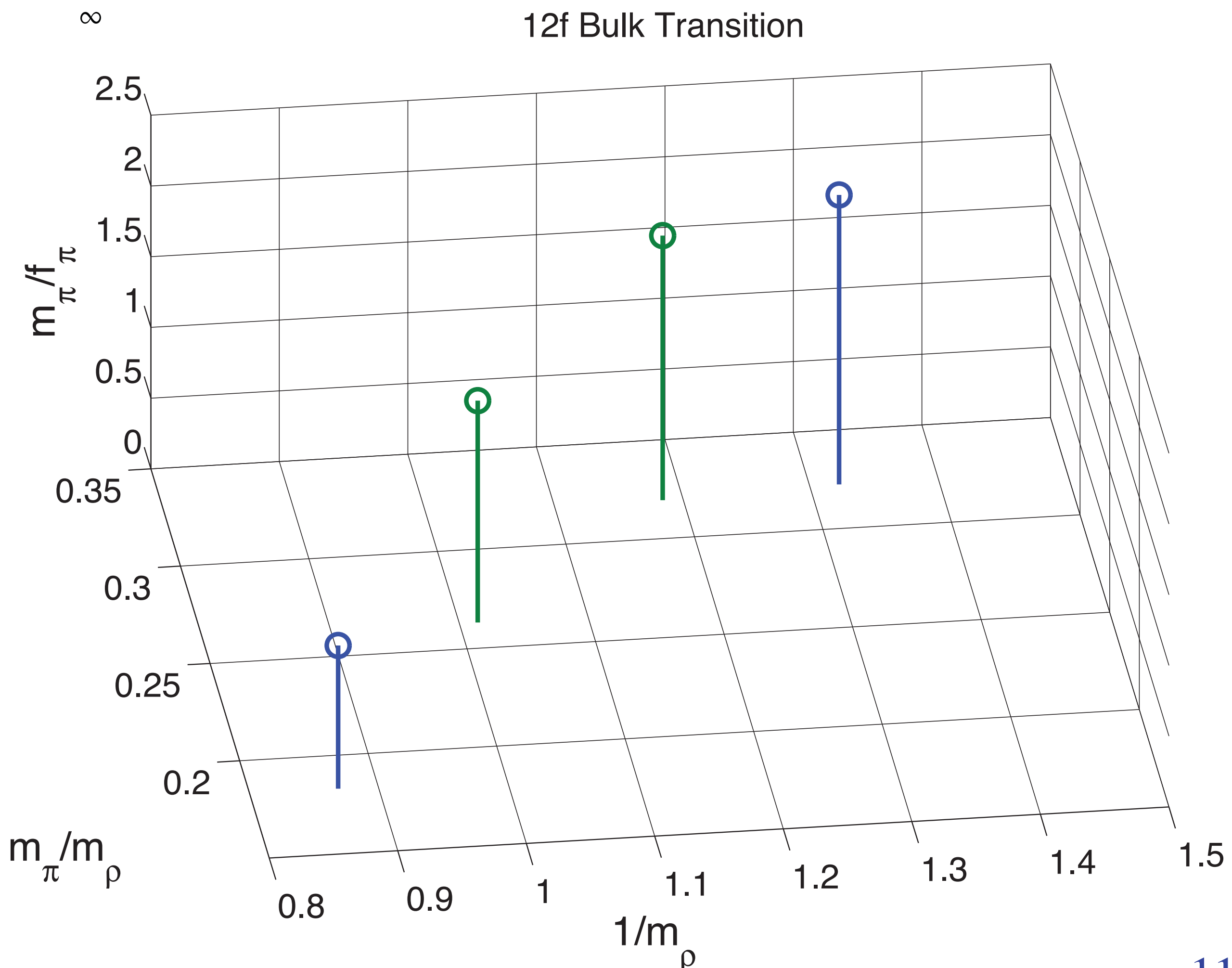
- With $m_\sigma < m_\pi$ in some region of the phase diagram, extrapolations (ChPT or others) require care.
- How far must one be from this second order critical point, to see continuum physics?
- How does the large scale change through the bulk transition effect our interpretation of simulation results?
- Details of this transition are almost surely lattice action dependent, but is there something universal to learn here?

12 Flavor QCD Phase Diagram for Staggered+DBW2

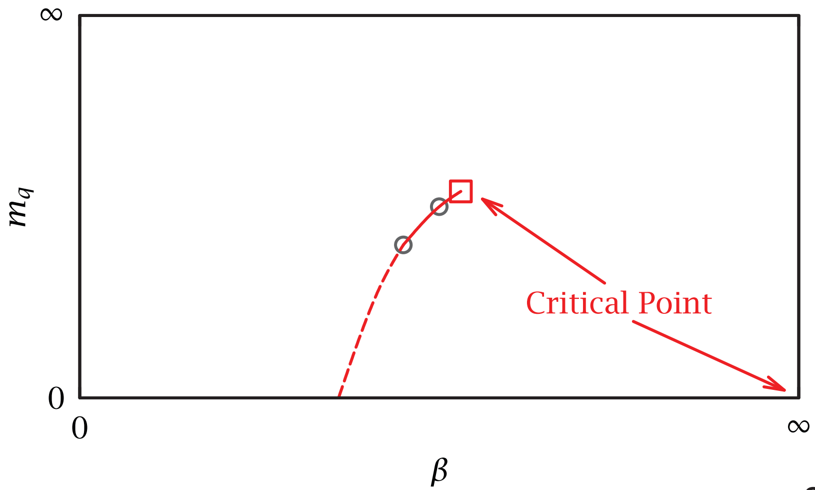


- There are (at least) two continuum limits possible
- Likely second order endpoint at finite β gives a free scalar field theory.

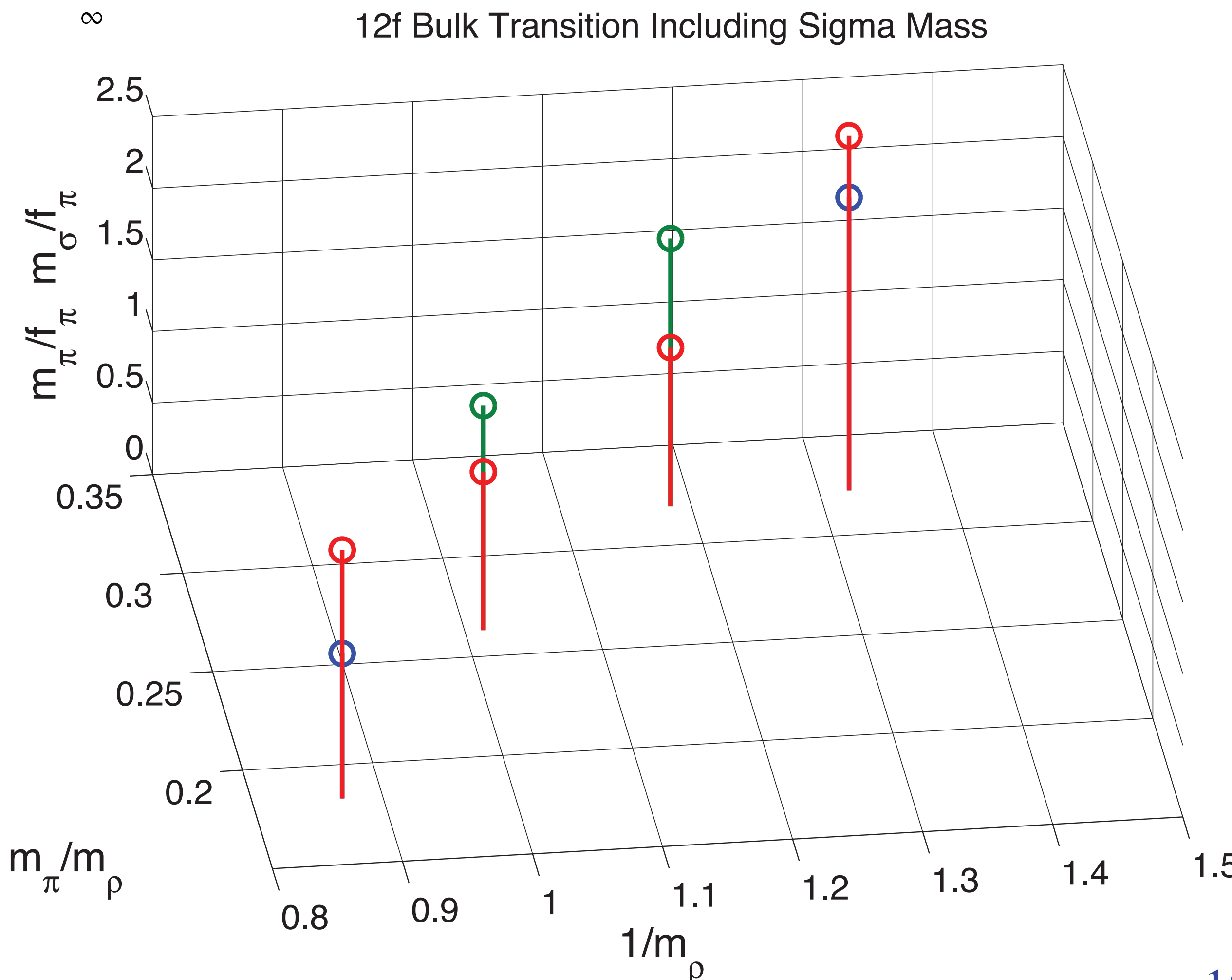
- Green points are strong/weak coupling phase at $\beta = 0.4626$, i.e. nearest to second order endpoint
- Blue points are farther from second order end-point, i.e. they have smaller bare quark mass-es
- In weak coupling phase, smaller bare quark mass gives larger m_π/f_π m_π/m_ρ



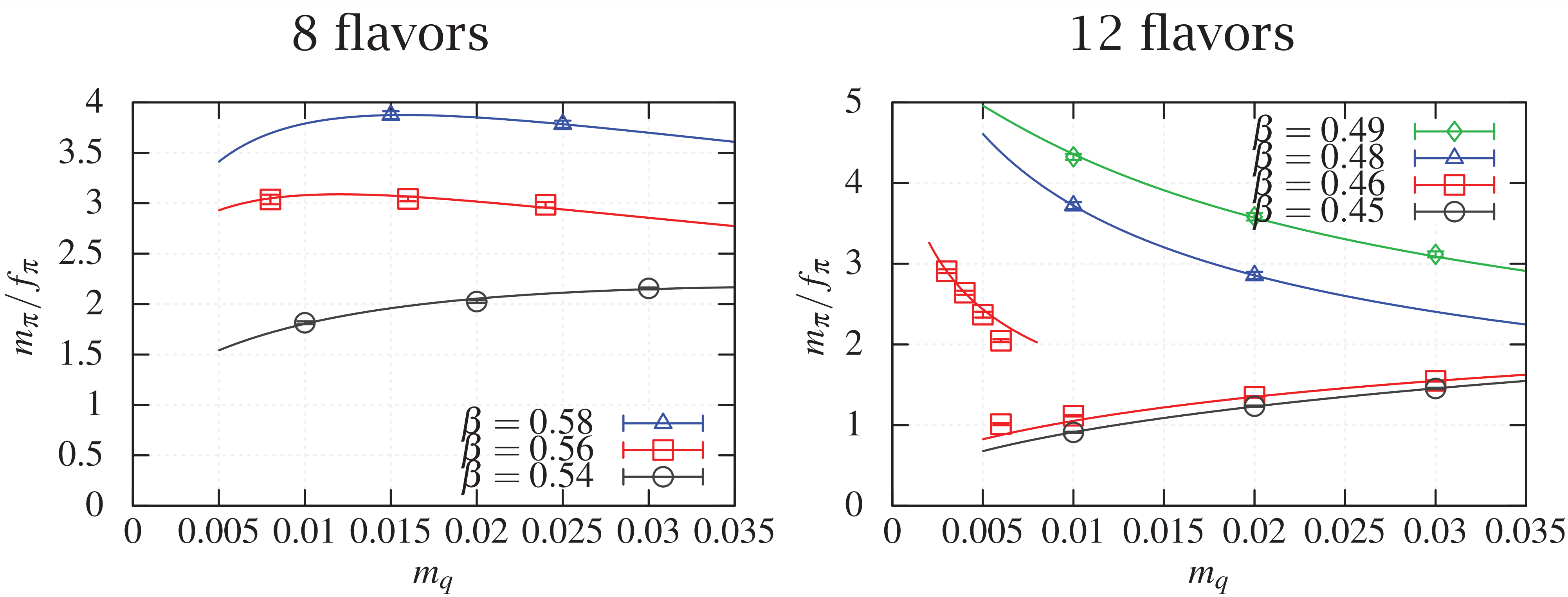
12 Flavor QCD Phase Diagram for Staggered+DBW2



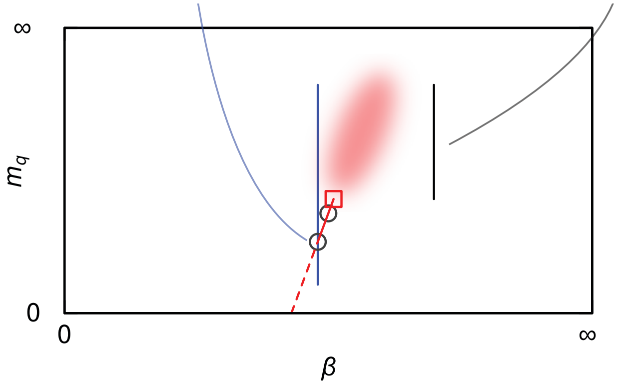
- Red points in 3d plot are m_σ/f_π



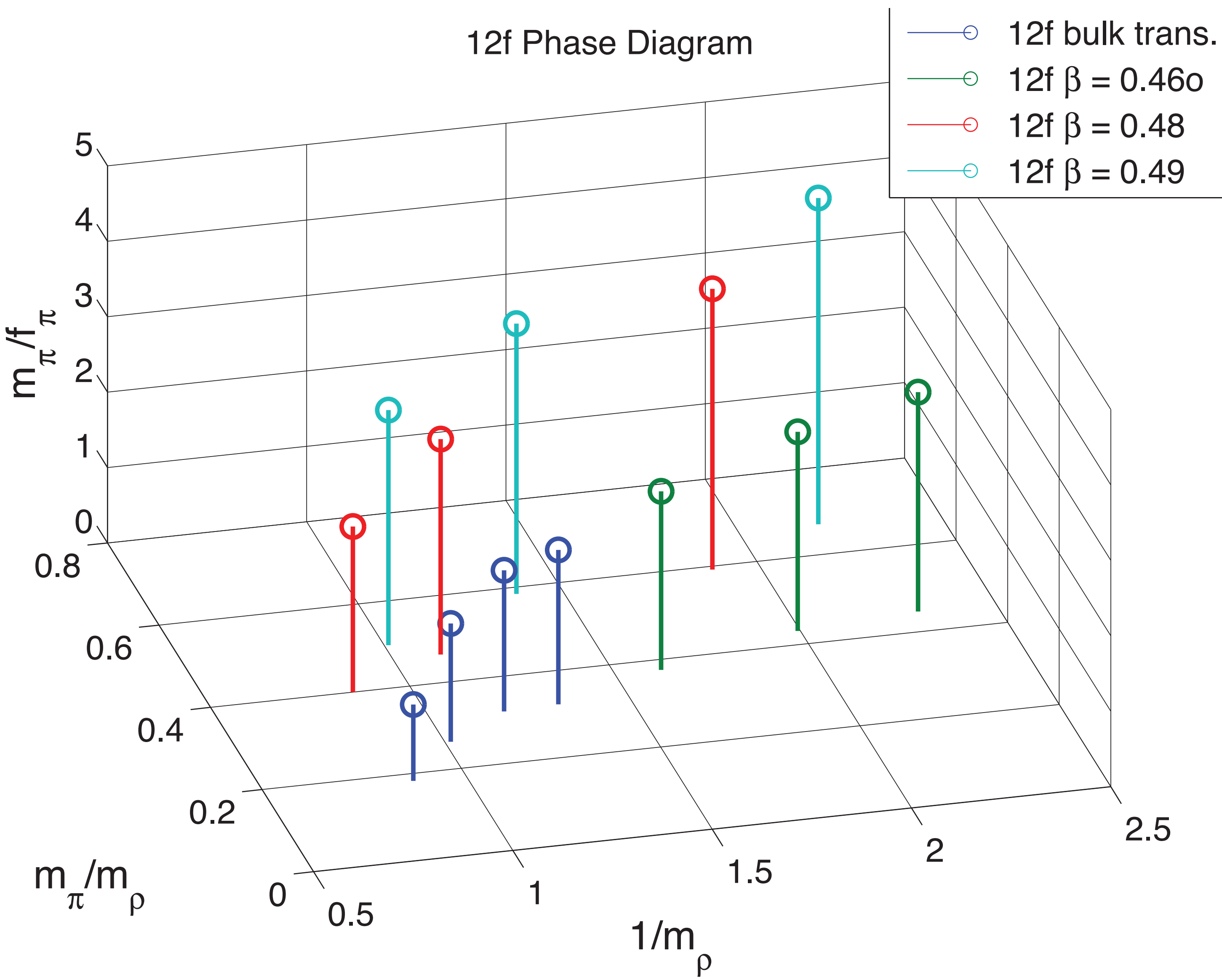
Bare Quark Mass Dependence of m_π/f_π



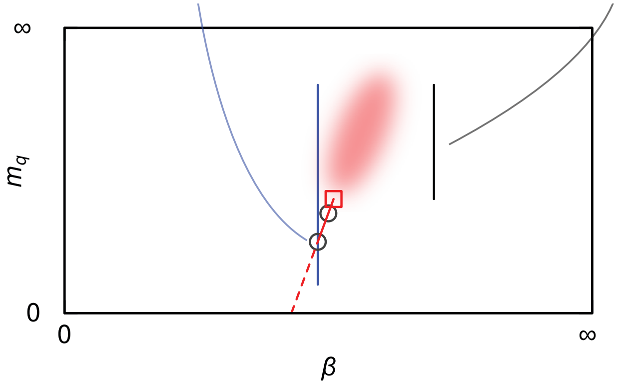
Change (m_q, β) for (m_π, m_ρ)



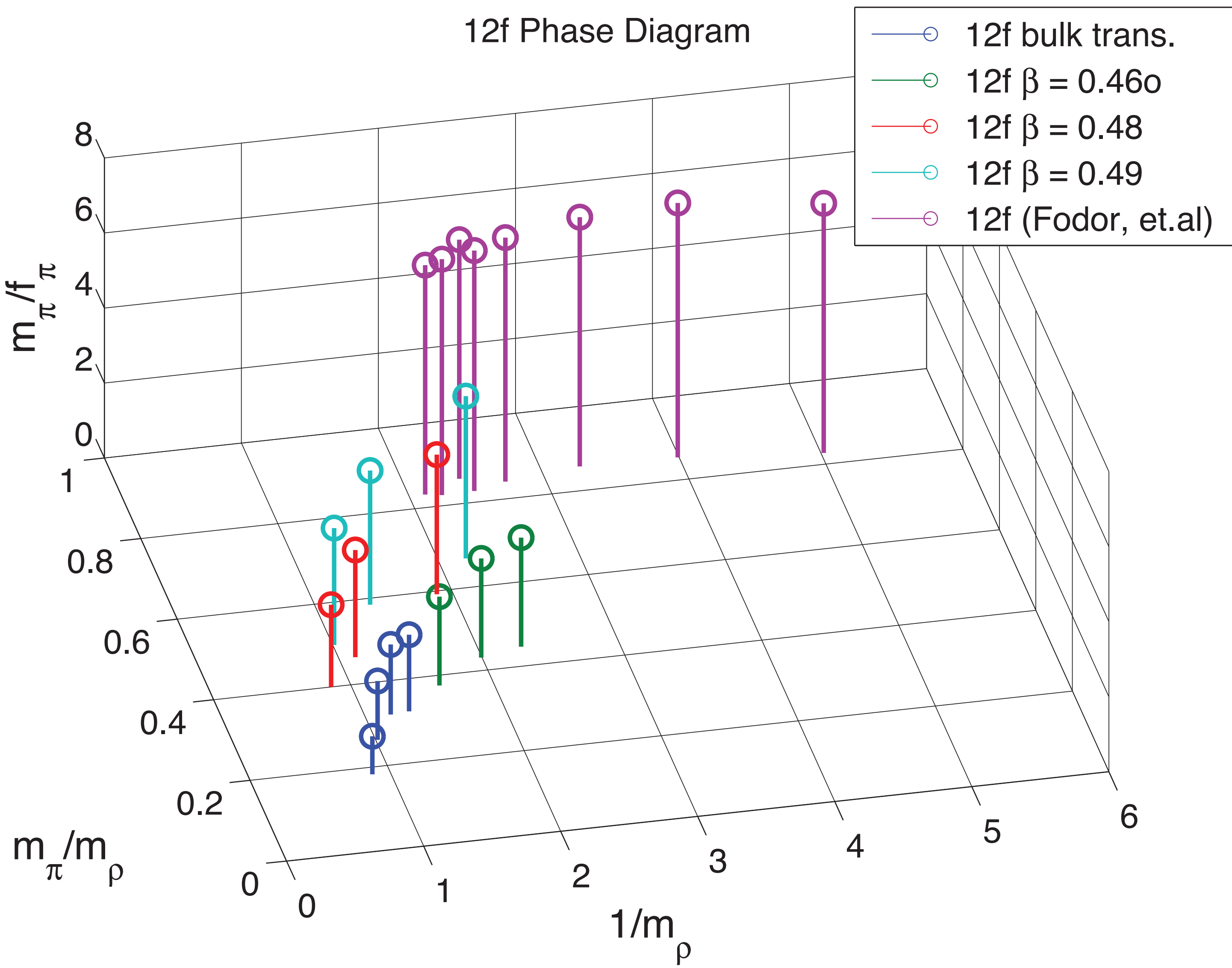
- Decreasing m_q at fixed β has not (yet?) caused m_π/m_ρ to decrease.
- There may be some slight curvature for m_π/m_ρ for smallest quark masses at $\beta = 0.4$



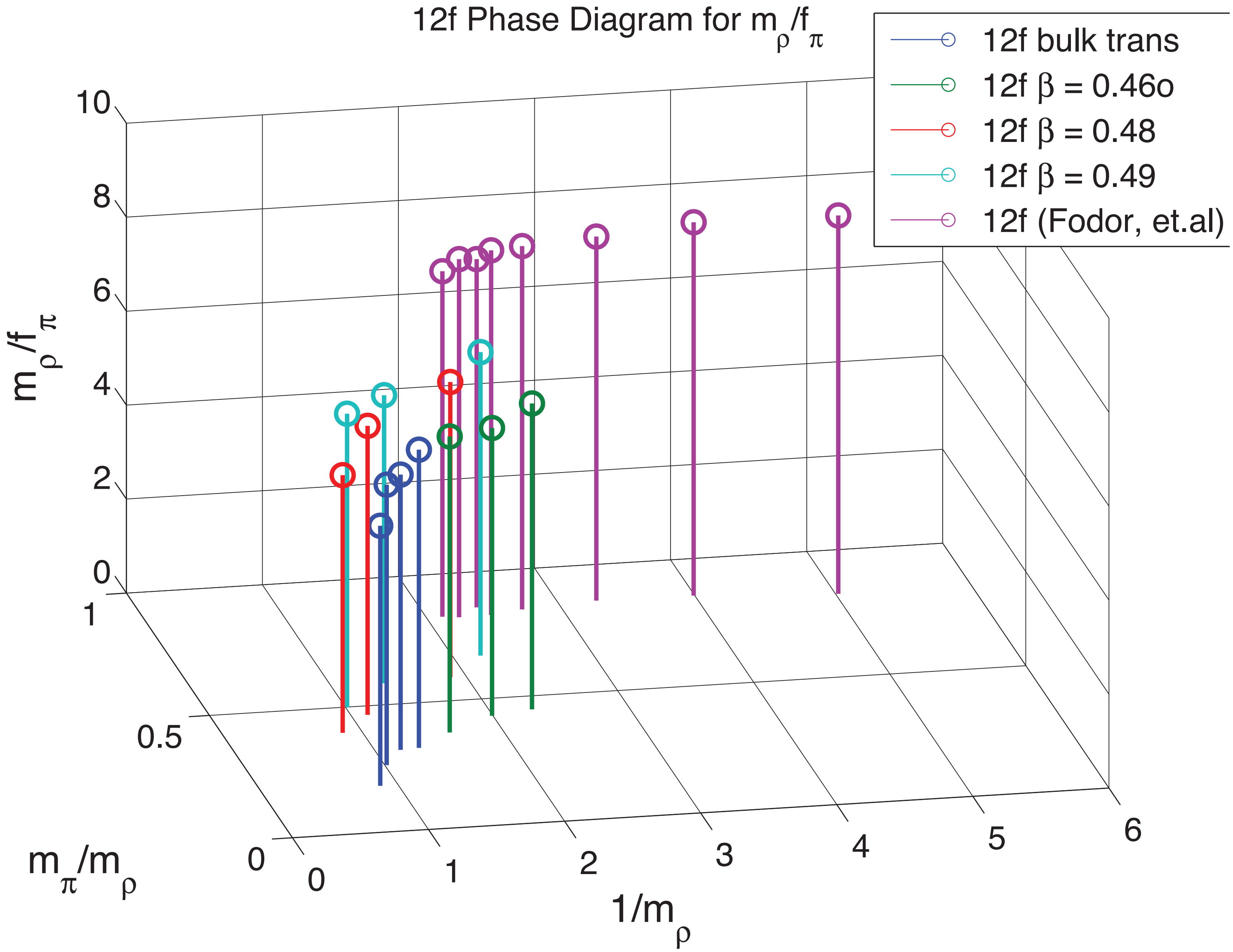
Universality?



- Include Fodor, et. al. β = 2.20 data.
- At much smaller lattice spacing, but pion is not light in physical units.
- See similar effect: decreasing bare quark mass at fixed β does not take m_π to 0.

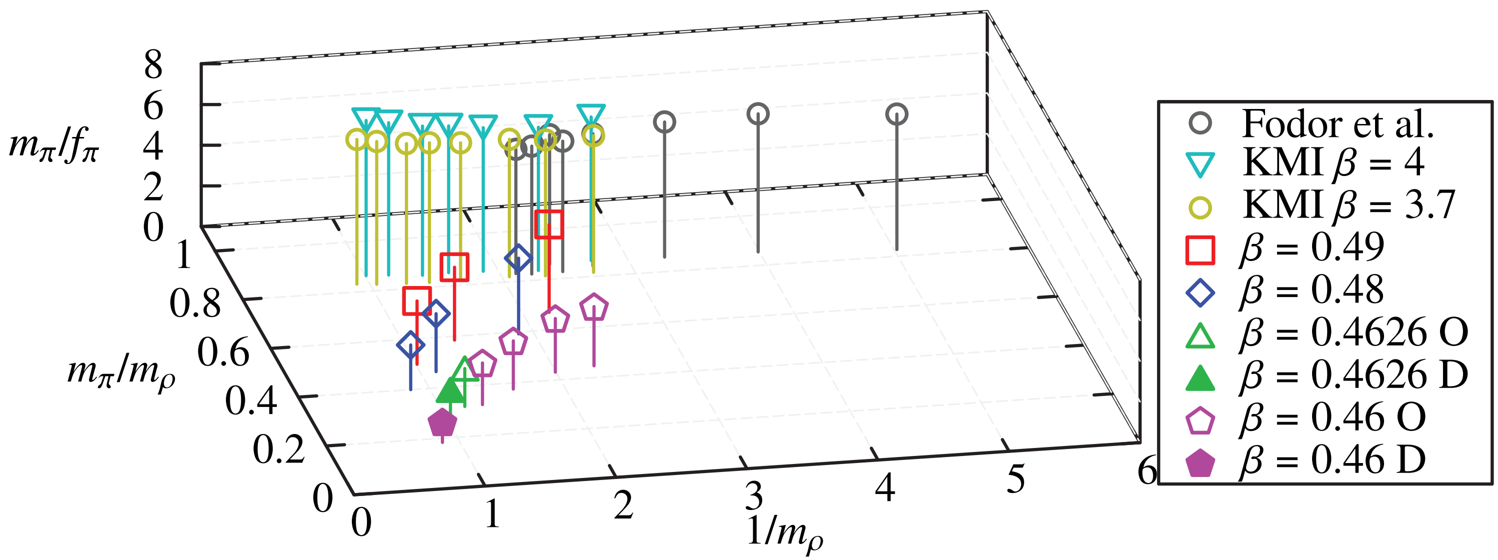


Check m_ρ/f_π

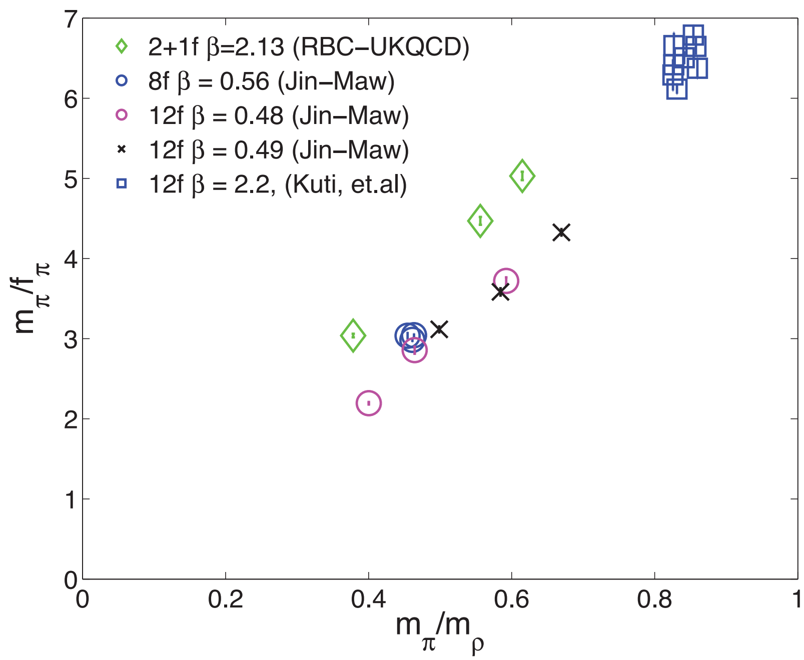


This is scaling well in $1/m_\rho$ and is larger when m_π/m_ρ increases

Add in Data of LatKMI Collaboration

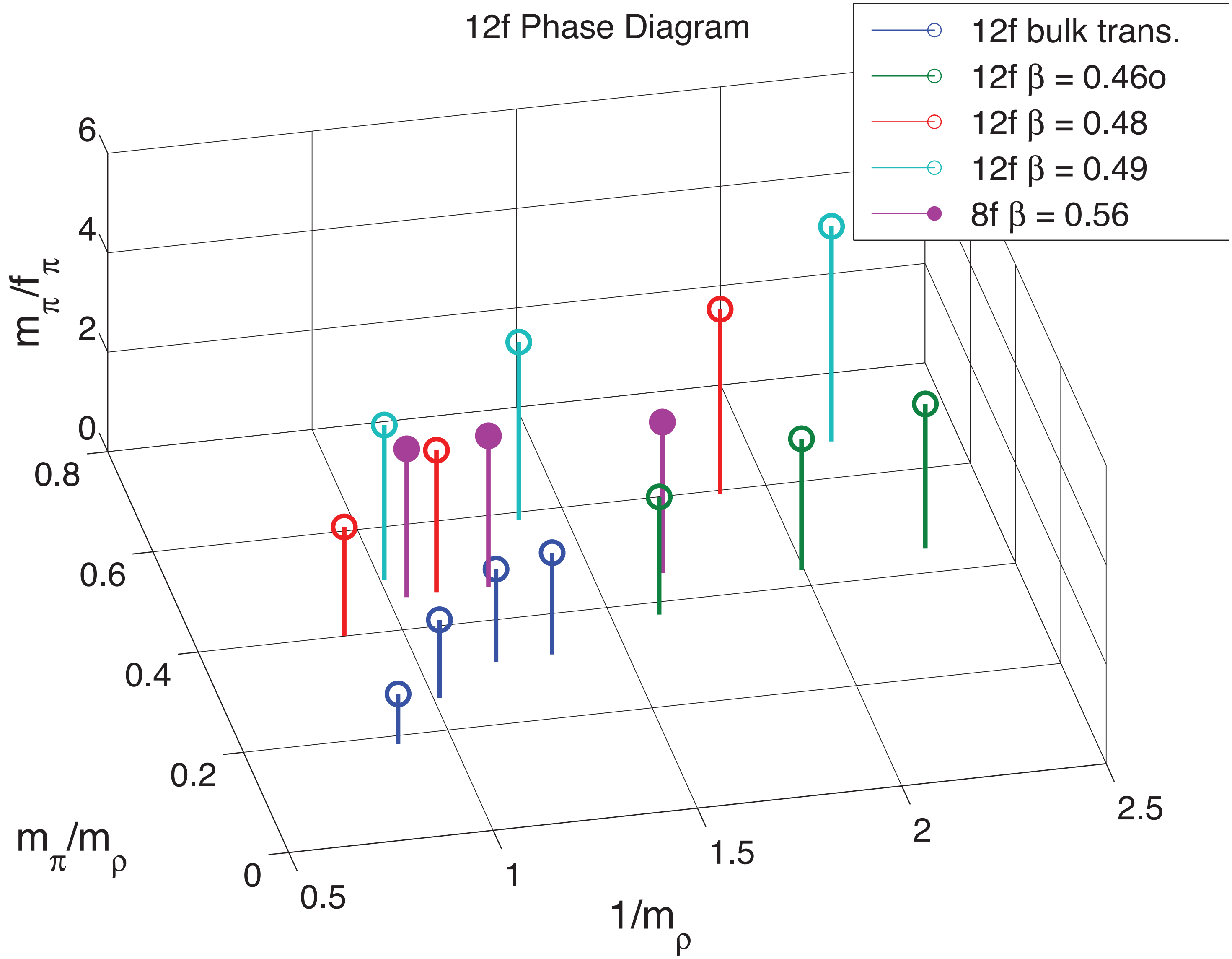


Misleading Simple Scaling Plot



- Observe a relatively consistent pattern between 2+1 flavor, 8f and 12f. (Note, unsure of normalization of Fodor, Kuti, et. al. f_π.)
- 2+1 flavor points have m_π/f_π decreasing with m_q
- 8 flavor points do not change even when m_q changed by 3×
- 12 flavor points have m_π/f_π increasing with m_q, as we have just seen.

Add 8f QCD to 12f (m_π,m_ϱ) Phase Diagram



- Also see decreasing m_q decreases m_ϱ without any change in m_π/f_π
- General consensus is the 8f is not conformal, so 8f data should move toward $m_\pi = 0$

Summary

- We have a clear understanding of the location of the lattice artifact, bulk transition for 12f QCD with naive staggered fermions and the DBW2 gauge action.
- There is a rapid scale change across the transition and we see confined, massive particle states for all simulations done to date.
- We have located the critical endpoint and see that the scalar meson mass vanishes there
- Moving to small bare quark mass, at fixed β, on the weak coupling side of this transition is decreasing the hadronic scales (m_ρ and f_π) so rapidly, that m_π/f_π is increasing
- Chiral perturbation theory analysis requires a small quark mass and hadronic scales that are not strongly dependent on quark masses.
- The quark mass at the critical endpoint of the lattice artifact transition gives us a hint of the size of quark masses, at that β, that can wildly distort continuum physics. It may not be surprising that much lighter quark masses, at that β, are required to see χSB physics (Goldstone mode, etc.), if it exists.
- The existing lattice data does not constrain the zero quark mass extrapolation.