Determining f_B and f_{B_s} on the Lattice

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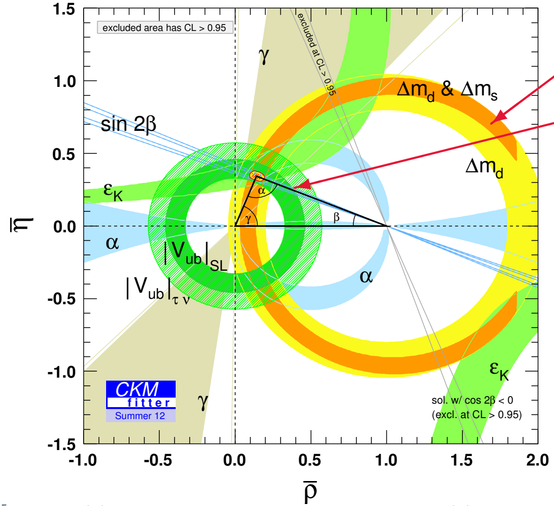
Phenomenological Importance



B^0–$\overline{B^0}$ mixing

$B \to \pi\ell\nu$ form factor

Possible deviations from SM

[Lunghi and Soni 2010/11]

- $\sin(2\beta)$ is 3.3σ lower
- $\mathrm{BR}(B \to \pi\ell\nu)$ is 2.8σ lower
- Likely sources $B_d(s)$-mixing and $\sin(2\beta)$

[Laiho,Lunghi,Van de Water 2012]

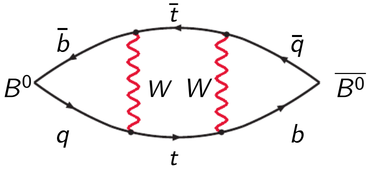
- New physics in $B \to \tau\nu$ decay and/or B_d mixing preferred
- Tension may hint to physics at a few-GeV mass scale

[http://ckmfitter.in2p3.fr, http://utfit.roma1.infn.it, http://www.latticeaverages.org]

B^0–$\overline{B^0}$ Mixing

- Allows us to determine the CKM matrix elements
- Dominant contribution in SM: box diagram with top quarks

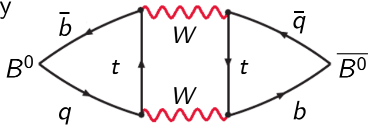
$$\left.\begin{array}{r} |V_{td}^* V_{tb}| \text{ for } B_d-\text{mixing} \\ |V_{ts}^* V_{tb}| \text{ for } B_s-\text{mixing} \end{array}\right\} \Delta M_q = \frac{G_F^2 m_W^2}{6\pi^2} \eta_B S_0 M_{B_q} f_{B_q}^2 B_{B_q} |V_{tq}^* V_{tb}|^2$$



- Nonperturbative contribution: $f_q^2 B_{Bq}$
- Define the $SU(3)$ breaking ratio $\xi^2 = f_{B_s}^2 B_{B_s} / f_{B_d}^2 B_{B_d}$

- CKM matrix elements are extracted by

$$\frac{\Delta M_s}{\Delta M_d} = \frac{M_{B_s}}{M_{B_d}} \xi^2 \frac{|V_{ts}|^2}{|V_{td}|^2}$$



- Experimental error of ΔM_q is better than a percent; lattice uncertainty for ξ is about 3%

$B \to \pi l \nu$ form factor

- Allows to determine the CKM matrix element V_{ub} from the experimental branching ratio

$$\frac{d\Gamma(B \to \pi l \nu)}{dq^2} = \frac{G_F^2 |V_{ub}|^2}{192\pi^3 M_B^3} \left[(M_B^2 + M_\pi^2 - q^2)^2 - 4 M_B^2 M_\pi^2 \right]^{3/2} |f_+(q^2)|^2$$

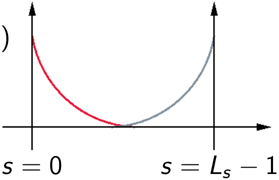
- Tension between exclusive determination and inclusive determinations of V_{ub} is greater than 3σ

Our Project

- Use domain-wall light quarks and nonperturbatively tuned relativistic b-quarks to compute at few-percent precision
  - B^0–$\overline{B^0}$ mixing
  - Decay constants f_B and f_{B_s}
  - $B \to \pi\ell\nu$ form factor [T. Kawanai]
  - $g_{B^*B\pi}$ coupling constant [B. Samways]
- Tune RHQ parameters using bottom-strange states and high statistics
  - Improve upon exploratory studies and verify made assumptions
  - Validate tuning procedure by computing $b\bar{b}$ masses and splittings
- Derive lattice perturbation theory for matching lattice results to continuum 1-loop in tadpole-improved lattice perturbation [C. Lehner]
  - Improve matching using a mostly-nonperturbative scheme for f_B, f_{B_s} and $B \to \pi\ell\nu$

2+1 Flavor Domain-Wall Gauge Field Configurations

- Domain-wall fermions for the light quarks (u, d, s) [Kaplan 1992, Shamir 1993]
- Iwasaki gauge action [Iwasaki 1983]
- Configurations generated by RBC and UKQCD collaborations [C. Allton et al. 2008], [Y. Aoki et al. 2010]



L	a(fm)	m_l	m_s	m_π(MeV)	approx. # configs.	# time sources
24	≈ 0.11	0.005	0.040	331	1636	1
24	≈ 0.11	0.010	0.040	419	1419	1
32	≈ 0.08	0.004	0.030	307	628	2
32	≈ 0.08	0.006	0.030	366	889	2
32	≈ 0.08	0.008	0.030	418	544	2

Relativistic Heavy Quark Action for the *b*-Quarks

- Relativistic Heavy Quark action developed by Christ, Li, and Lin for the *b*-quarks in 2-point and 3-point correlation functions [Christ, Li, Lin 2007; Lin and Christ 2007]
- Builds upon Fermilab approach [El Khadra, Kronfeld, Mackenzie 1997] by tuning all parameters of the clover action non-perturbatively; close relation to the Tsukuba formulation [Aoki, Kuramashi, Tominaga 2003]
- Heavy quark mass is treated to all orders in $(m_b a)^n$
- Expand in powers of the spatial momentum through $O(\vec{p}a)$
  - Resulting errors will be of $O(\vec{p}^2 a^2)$
  - Allows computation of heavy-light quantities with discretization errors of the same size as in light-light quantities
- Applies for all values of the quark mass
- Has a smooth continuum limit

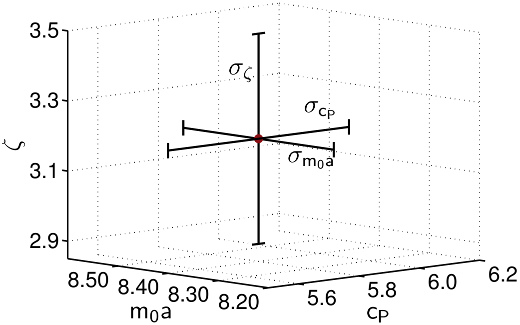
Nonperturbative Tuning of the RHQ Action Parameters

- Start from an educated guess for our three parameters $m_0 a$, c_P, and ζ
- Probe parameter space at seven points by measuring

  spin-averaged mass: $\overline{M} = (M_{B_s} + 3M_{B_s^*})/4$

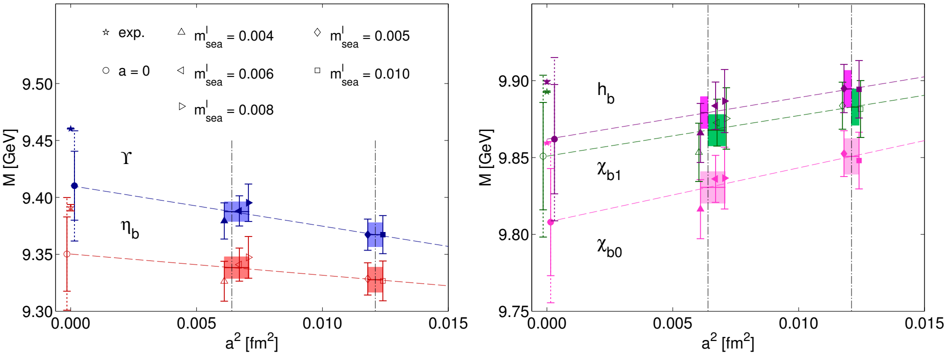
  hyperfine-splitting: $\Delta_M = M_{B_s^*} - M_{B_s}$

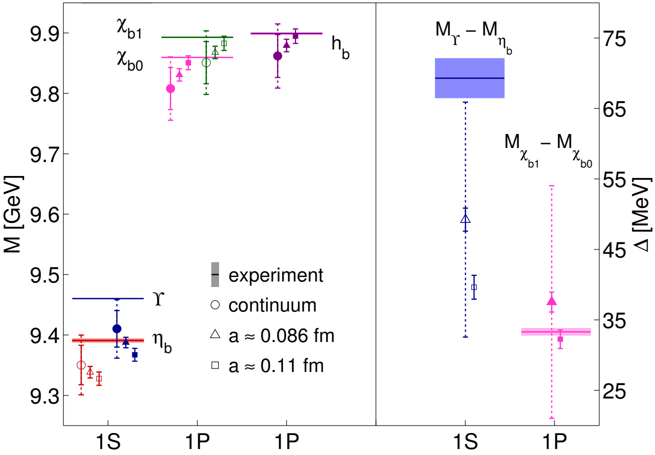
  ratio: $M_1/M_2 = M_{\text{rest}}/M_{\text{kinetic}}$
- Assume linearity to relate parameters and oberservables
- Use PDG values to match parameters to experimental results
- Test and verify parameters [Y. Aoki et al. 2012]



Predictions for the Heavy-Heavy States

- RHQ action describes heavy-light as well as heavy-heavy mesons
- Tuning the parameters in the B_s-system we can predict bottomonium states and mass splittings





$\Upsilon = 9410(30)(38)$ MeV

$\eta_b = 9350(33)(37)$ MeV

$h_b = 9862(36)(39)$ MeV

$\chi_{b1} = 9851(35)(39)$ MeV

$\chi_{b0} = 9808(35)(39)$ MeV

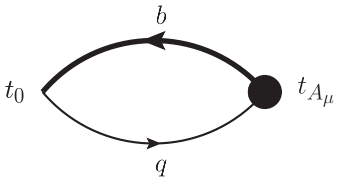
$M_\Upsilon - M_{\eta_b} = 49(02)(17)$

$M_{\chi_{b1}} - M_{\chi_{b0}} = 38(01)(16)$

[Y. Aoki et al. 2012]

B-meson Decay Constant Calculation

- Use point-source light quark and generate Gaussian smeared-source heavy quark
- Computation performed with seven parameter box and interpolated to the tuned RHQ parameters
- Axial current will be 1-loop $O(a)$ improved
- Result will use mostly nonperturbative renormalization
- Combined chiral and continuum extrapolation using heavy meson χPT



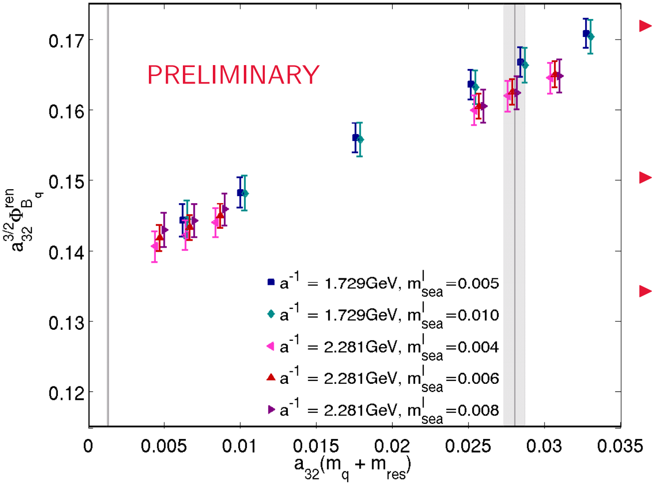
Mostly Nonperturbative Renormalization

For f_B, f_{B_s} and $B \to \pi$ we plan to compute mostly non-perturbative renormalization factors á la [El Khadra et al. 2001]

$$\varrho^{bl} = \frac{Z_V^{bl}}{\sqrt{Z_V^{bb} Z_V^{ll}}}$$

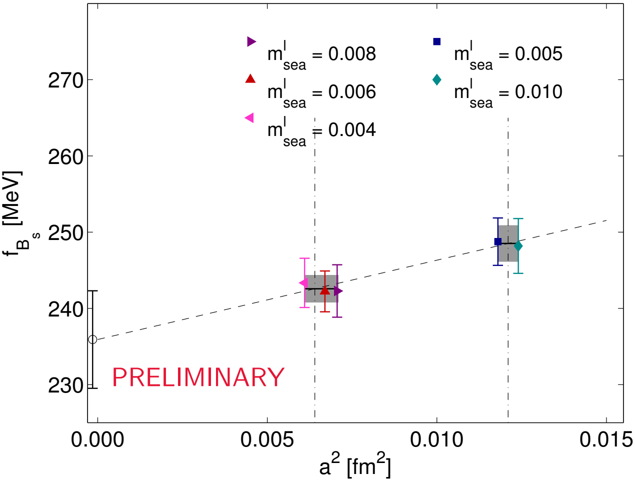
- Compute Z_V^{ll} and Z_V^{bb} non-perturbatively and only ϱ^{bl} perturbatively
- Enhanced convergence of perturbative serious of ϱ^{bl} w.r.t. Z_V^{bl} because tadpole diagrams cancel in the ratio
- Bulk of the renormalization is due to flavor conserving factor $\sqrt{Z_V^{ll} Z_V^{bb}} \sim 3$
- ϱ^{bl} is expected to be of $\mathcal{O}(1)$; receiving only small corrections
- For domain-wall fermions $Z_A = Z_V + \mathcal{O}(m_{\text{res}})$ i.e. we know Z_V^{ll} [Y. Aoki et al. 2011] and compute Z_v^{bb} ourselves

Preliminary Results for f_B and f_{B_s}



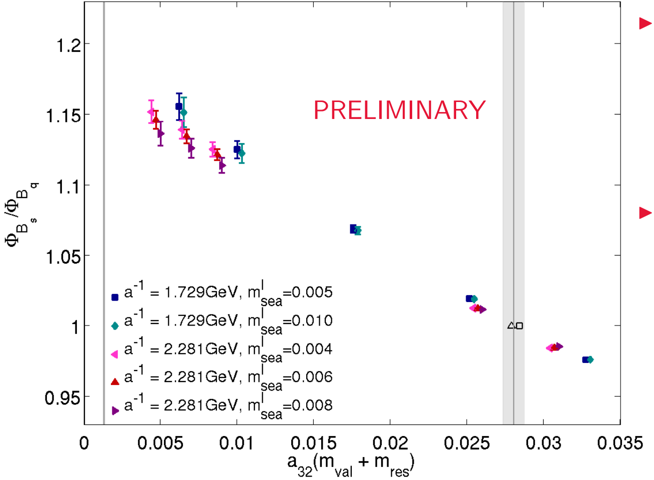
- On the lattice we compute Φ_{B_q}
  $$f_B = \Phi^{ren}_{B_q} \cdot a_{32}^{-3/2} / \sqrt{M_{B_q}}$$
- Working on combined chiral and continuum extrapolation
- Difficulties to fit our data using (N)NLO SU(2) or SU(3) HMχPT

Preliminary Results f_{B_s}



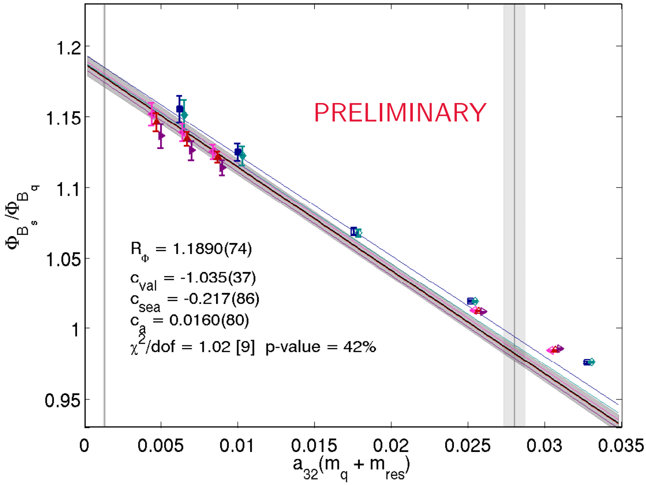
- Data for f_{B_s} are quite linear and show no sea-quark mass dependence
- Average data at same lattice spacing and assume a^2 scaling

Preliminary Results f_{B_s}/f_B



- Use our close to strange quark propagators to compute the ratio
$$\frac{f_{B_s}}{f_B} = \frac{\Phi_{B_s}}{\Phi_{B_d}}\sqrt{\frac{M_{B_d}}{M_{B_s}}}$$
- χPT fits are still tricky but . . .

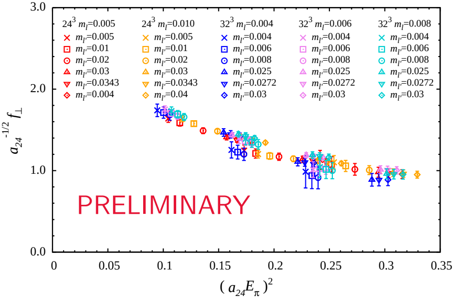
Preliminary Results f_{B_s}/f_B

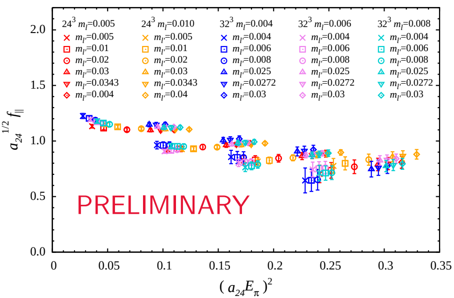


... we have a nice analytic fit for the chiral data

Status of $B \to \pi\ell\nu$ [T. Kawanai]

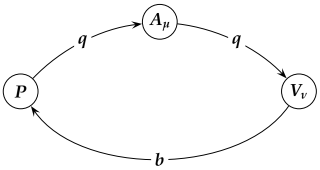
- The $B \to \pi\ell\nu$ hadronic weak matrix element is parameterized by
$$\langle\pi|\mathcal{V}^\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{m_B^2-m_\pi^2}{q^2}q^\mu\right) f_0(q^2)\frac{m_B^2-m_\pi^2}{q^2}q^\mu$$
- On the lattice we determine
$$\langle\pi|\mathcal{V}^\mu|B\rangle = \sqrt{2M_B}\left[v^\mu f_\parallel(E_\pi) + p_\perp^\mu f_\perp(E_\pi)\right]$$

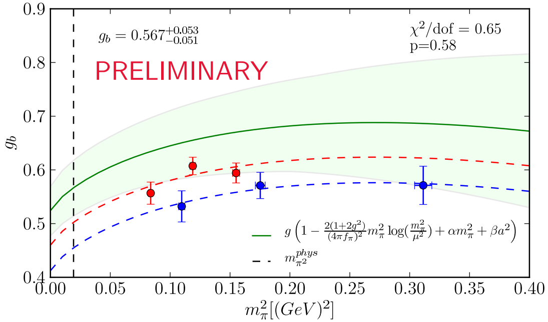




Preliminary Results $g_{B^*B\pi}$ coupling constant [B. Samways]

- χPT expressions for f_B, $B \to \pi\ell\nu$ form factors or B-meson mixing matrix elements require knowledge on $g_{B^*B\pi}$
- On the lattice compute $B^*B\pi$ three-point function





Conclusion

- We have completed tuning the parameters of the RHQ action for b-quarks, and find good agreement between our predictions for bottomonium masses and fine splittings with experiment.
- Given this success, we are now using this method for B-meson quantities such as decay constants, neutral B-meson mixing parameters and form factors, and expect to obtain errors competitive with other groups.
- We are finalizing the analysis of f_B, f_{B_s} and f_{B_s}/f_B ...