125 GeV Higgs Boson and the Type-II Seesaw Model

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PSBD, D. K. Ghosh, N. Okada, and I. Saha, JHEP **1303**, 150 (2013) [arxiv:1301.3453]

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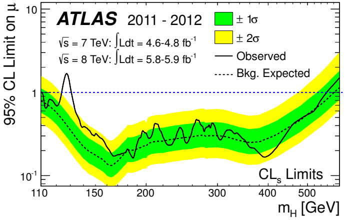
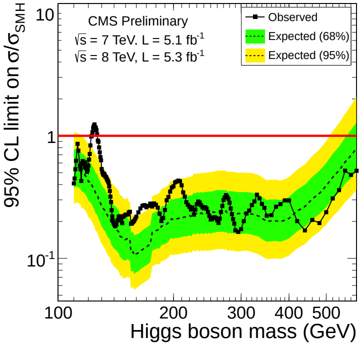
Outline

- Motivation
- Review of the Minimal Type-II Seesaw Model
- RG Equations
- Vacuum Stability and Perturbativity
- Predictions for $h \to \gamma\gamma$ and $h \to Z\gamma$
- Conclusion

Discovery of (a/the) Higgs Boson at the LHC





- Best-fit value for m_h:

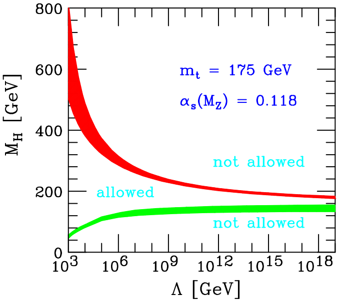
$$125.5 \pm 0.2(\text{stat})^{+0.5}_{-0.6}(\text{syst})\ \text{GeV (ATLAS)}$$

$$125.7 \pm 0.2(\text{stat}) \pm 0.3(\text{syst})\ \text{GeV (CMS)}$$

- What does it mean for the fate of the SM?

Theoretical Bounds on SM Higgs

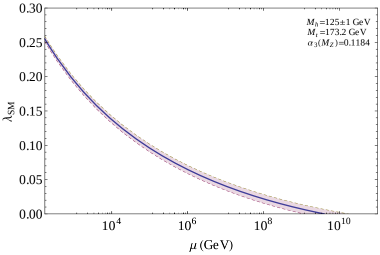
- $V_{\text{eff}}(\Phi) = -m_\Phi^2(\Phi^\dagger\Phi) + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2$.
- Require $\lambda > 0$ (stability bound) and finite (perturbativity bound).



[Hambye, Riesselmann '96]

- NNLO calculation for stability bound: [Degrassi *et al.* '12]
  M_h (GeV) $> 129.4 + 1.4\left(\frac{M_t\ (\text{GeV}) - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\text{th}}$.

RG running of SM Higgs Quartic Coupling



- Need to introduce new physics below $10^9 - 10^{10}$ GeV to make the electroweak vacuum stable.
- Not yet universally accepted (due to various theoretical/experimental uncertainties).

Neutrino Mass

- A conclusive evidence for beyond SM physics.
- Neutrino oscillation data require at least two nonzero neutrino masses.
- Neutrinos massless in the SM because of no RH neutrino (no Dirac mass) and a global $B - L$ symmetry (no Majorana mass).
- Add ν_R: for Dirac mass term $y_\nu \overline{L} \Phi \nu_R$ alone, require $y_\nu \lesssim 10^{-12}$.

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- A more natural way is to break $B-L$ by extra fields.
- Can be parametrized within the SM by Weinberg's dimension-5 operator.
- Tree-level realization: seesaw mechanism.
- Can the same physics make the elctroweak vacuum stable up to the Planck scale?
- Could be directly probed if the seesaw scale is $\mathcal{O}$(TeV).

The Type-II Seesaw Model

[Magg, Wetterich '80; Cheng, Li '80; Lazarides, Shafi, Wetterich '80; Schechter, Valle '80; Mohapatra, Senjanovic '81]

- Add a scalar field $\Delta(\mathbf{1}, \mathbf{3}, 2)$ under $SU(3)_c \times SU(2)_L \times U(1)_Y$.

$$\Delta = \frac{\sigma^i}{\sqrt{2}} \Delta_i = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix},$$

with $\Delta_1 = (\delta^{++} + \delta^0)/\sqrt{2},\ \Delta_2 = i(\delta^{++} - \delta^0)/\sqrt{2},\ \Delta_3 = \delta^+$.

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- The relevant Lagrangian is

$$\begin{aligned} \mathcal{L} &= \mathcal{L}^{\rm SM}_{\rm kinetic} + \mathcal{L}^{\rm SM}_{Y} - \mathcal{V}(\Phi, \Delta) \\ &\quad + \mathrm{Tr}\left[(D_\mu \Delta)^\dagger (D^\mu \Delta)\right] - \left[\frac{1}{\sqrt{2}} (Y_\Delta)_{ij} L_i^T C i\sigma_2 \Delta L_j + \mathrm{H.c.}\right] \end{aligned}$$

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$$\begin{aligned} \mathcal{V}(\Phi, \Delta) &= -m_\Phi^2(\Phi^\dagger\Phi) + \frac{\lambda}{2}(\Phi^\dagger\Phi)^2 + M_\Delta^2 {\rm Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[{\rm Tr}(\Delta^\dagger\Delta)\right]^2 \\ &\quad + \frac{\lambda_2}{2}\left\{\left[{\rm Tr}(\Delta^\dagger\Delta)\right]^2 - {\rm Tr}\left[(\Delta^\dagger\Delta)^2\right]\right\} + \lambda_4(\Phi^\dagger\Phi){\rm Tr}(\Delta^\dagger\Delta) \\ &\quad + \lambda_5\Phi^\dagger[\Delta^\dagger, \Delta]\Phi + \left(\frac{\Lambda_6}{\sqrt{2}}\Phi^{\rm T} i\sigma_2\Delta^\dagger\Phi + {\rm H.c.}\right). \end{aligned}$$

Neutrino Masses and Mixing

- $\langle \phi^0 \rangle = v/\sqrt{2}$ and $\langle \delta^0 \rangle = v_\Delta/\sqrt{2}$.
- Majorana mass matrix for the neutrinos: $(M_\nu)_{ij} = v_\Delta (Y_\Delta)_{ij}$.
- ρ-parameter constraint requires $v_\Delta/v < 0.02$.
- EWSB conditions:

$$\begin{aligned} m_\Phi^2 &= \frac{1}{2}\lambda v^2 - \Lambda_6 v_\Delta + \frac{\lambda_4 - \lambda_5}{2} v_\Delta^2, \\ M_\Delta^2 &= \frac{1}{2}\frac{\Lambda_6 v^2}{v_\Delta} - \frac{1}{2}(\lambda_4 - \lambda_5)v^2 - \frac{1}{2}\lambda_1 v_\Delta^2. \end{aligned}$$

- For $M_\Delta \gg v$, we get a seesaw-like neutrino mass matrix:

$$M_\nu \simeq \frac{\lambda_6 v^2}{2M_\Delta} Y_\Delta$$

with the dimensionless parameter $\lambda_6 \equiv \Lambda_6/M_\Delta$.

Yukawa Coupling

- Fix the Yukawa structure from neutrino oscillation data:

$$Y_\Delta = \frac{M_\nu}{v_\Delta} = \frac{1}{v_\Delta} U^\mathsf{T} M_\nu^{\mathrm{diag}} U$$

where $M_\nu^{\mathrm{diag}} = \mathrm{diag}(m_1, m_2, m_3)$ and U is the PMNS mixing matrix:

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \times \mathrm{diag}(e^{i\alpha_1/2}, e^{i\alpha_2/2}, 1)$$

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- For illustration, take normal hierarchy with $m_1 = 0$.

$$Y_\Delta = \frac{10^{-2}\ {\rm eV}}{v_\Delta} \times \begin{pmatrix} 0.31 - 0.12i & -0.09 + 0.32i & -0.72 + 0.37i \\ -0.09 + 0.32i & 2.53 + 0.04i & 2.19 + 0.01i \\ -0.72 + 0.37i & 2.19 + 0.01i & 3.07 - 0.03i \end{pmatrix}$$

- Note: v_Δ cannot be arbitrarily small due to perturbativity!

Scalar Masses

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \phi + i\chi) \end{pmatrix}, \qquad \Delta = \begin{pmatrix} \frac{\delta^+}{\sqrt{2}} & \delta^{++} \\ \frac{1}{\sqrt{2}}(v_\Delta + \delta + i\eta) & -\frac{\delta^+}{\sqrt{2}} \end{pmatrix}$$

Upon minimization of $\mathcal{V}(\Phi, \Delta)$ gives physical mass eigenstates $H^{\pm\pm}, H^{\pm}, h, H^0, A^0$ and massless Goldstone bosons $G^{\pm}, G^0$ (eaten up to give masses to $W^{\pm}, Z$).

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$$\begin{aligned}
m^2_{H^{\pm\pm}} &= M^2_\Delta + \frac{1}{2}(\lambda_4 + \lambda_5)v^2 + \frac{1}{2}(\lambda_1 + \lambda_2)v^2_\Delta, \\
m^2_{H^\pm} &= \left(M^2_\Delta + \frac{1}{2}\lambda_4 v^2 + \frac{1}{2}\lambda_1 v^2_\Delta\right)\left(1 + \frac{2v^2_\Delta}{v^2}\right), \\
m^2_{A^0} &= \left(M^2_\Delta + \frac{1}{2}(\lambda_4 - \lambda_5)v^2 + \frac{1}{2}\lambda_1 v^2_\Delta\right)\left(1 + \frac{4v^2_\Delta}{v^2}\right), \\
m^2_h &= \frac{1}{2}\left(A + C - \sqrt{(A-C)^2 + 4B^2}\right), \\
m^2_{H^0} &= \frac{1}{2}\left(A + C + \sqrt{(A-C)^2 + 4B^2}\right),
\end{aligned}$$

$$\text{with } A = \lambda v^2, \qquad B = -\frac{2v_\Delta}{v}\left(M^2_\Delta + \frac{1}{2}\lambda_1 v^2_\Delta\right), \quad C = M^2_\Delta + \frac{1}{2}(\lambda_4 - \lambda_5)v^2 + \frac{3}{2}\lambda_1 v^2_\Delta.$$

Note that among the two *CP*-even neutral Higgs bosons, $m_{H^0} > m_h$ always.

Scalar Mixing

$$\begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} = \begin{pmatrix} \cos\beta' & \sin\beta' \\ -\sin\beta' & \cos\beta' \end{pmatrix} \begin{pmatrix} \phi^{\pm} \\ \delta^{\pm} \end{pmatrix},$$

$$\begin{pmatrix} h \\ H^{0} \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi \\ \delta \end{pmatrix},$$

$$\begin{pmatrix} G^{0} \\ A^{0} \end{pmatrix} = \begin{pmatrix} \cos\beta & \sin\beta \\ -\sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \chi \\ \eta \end{pmatrix},$$

where the mixing angles are given by

$$\tan\beta' = \frac{\sqrt{2}v_\Delta}{v},$$

$$\tan\beta = \frac{2v_\Delta}{v} \equiv \sqrt{2}\tan\beta',$$

$$\tan 2\alpha = \frac{2B}{A-C} = \frac{4v_\Delta}{v}\frac{M_\Delta^2 + \frac{1}{2}\lambda_1 v_\Delta^2}{M_\Delta^2 + \frac{1}{2}(\lambda_4 - \lambda_5 - 2\lambda)v^2 + \frac{3}{2}\lambda_1 v_\Delta^2}$$

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In the limit $v_\Delta \ll v$,

- h is dominantly $SU(2)_L$ doublet (unless h and H^0 are mass-degenerate), with $m_h^2 = \lambda v^2$ (as in the SM) independent of M_Δ.
- Other scalars are dominantly triplet, with M_Δ-dependent mass.

Stability and Unitarity Conditions

[Arhrib, Benbrik, Chabab, Moultaka, Rahili '11]

- Electroweak Vacuum Stability:

$$\lambda \geq 0, \quad \lambda_1 \geq 0, \quad 2\lambda_1 + \lambda_2 \geq 0,$$

$$\lambda_4 + \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \quad \lambda_4 + \lambda_5 + \sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} \geq 0,$$

$$\lambda_4 - \lambda_5 + \sqrt{\lambda\lambda_1} \geq 0, \quad \lambda_4 - \lambda_5 + \sqrt{\lambda\left(\lambda_1 + \frac{\lambda_2}{2}\right)} \geq 0.$$

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- Unitarity:

$$\lambda \leq \frac{8}{3}\pi, \quad \lambda_1 - \lambda_2 \leq 8\pi, \quad 4\lambda_1 + \lambda_2 \leq 8\pi, \quad 2\lambda_1 + 3\lambda_2 \leq 16\pi,$$

$$|\lambda_5| \leq \frac{1}{2}\min\left[\sqrt{(\lambda \pm 8\pi)(\lambda_1 - \lambda_2 \pm 8\pi)}\right],$$

$$|\lambda_4| \leq \frac{1}{\sqrt{2}}\sqrt{\left(\lambda - \frac{8}{3}\pi\right)(4\lambda_1 + \lambda_2 - 8\pi)}.$$

RG Equations for $\mu < M_\Delta$

- For $\mu < M_\Delta$, $V_{\rm eff}(\Phi) = -m_\Phi^2(\Phi^\dagger\Phi) + \frac{1}{2}(\lambda - \lambda_6^2)(\Phi^\dagger\Phi)^2$ with $\lambda_6^2 \ll \lambda$.
- RGEs same as in the SM. At two-loop, for scalar quartic coupling:

$$\frac{d\lambda}{d\ln\mu} = \frac{\beta_\lambda^{(1)}}{16\pi^2} + \frac{\beta_\lambda^{(2)}}{(16\pi^2)^2},$$

with the β-functions [Machacek, Vaughn '83-85; Ford, Jack, Jones '92; Arason *et al* '92; Luo, Xiao '03]

$$\beta_\lambda^{(1)} = 12\lambda^2 - \left(\frac{9}{5}g_1^2 + 9g_2^2\right)\lambda + \frac{9}{4}\left(\frac{3}{25}g_1^4 + \frac{2}{5}g_1^2 g_2^2 + g_2^4\right) + 12y_t^2\lambda - 12y_t^4,$$

$$\begin{aligned}\beta_\lambda^{(2)} = & -78\lambda^3 + 18\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\lambda^2 - \left(\frac{73}{8}g_2^4 - \frac{117}{20}g_1^2 g_2^2 - \frac{1887}{200}g_1^4\right)\lambda - 3\lambda y_t^4 \\ & + \frac{305}{8}g_2^6 - \frac{289}{40}g_1^2 g_2^4 - \frac{1677}{200}g_1^4 g_2^2 - \frac{3411}{1000}g_1^6 - 64g_3^2 y_t^4 - \frac{16}{5}g_1^2 y_t^4 - \frac{9}{2}g_2^4 y_t^2 \\ & + 10\lambda\left(\frac{17}{20}g_1^2 + \frac{9}{4}g_2^2 + 8g_3^2\right)y_t^2 - \frac{3}{5}g_1^2\left(\frac{57}{10}g_1^2 - 21g_2^2\right)y_t^2 - 72\lambda^2 y_t^2 + 60y_t^6.\end{aligned}$$

- Use matching condition: $\lambda(\mu) = \frac{M_h^2}{v^2}\left[1 + \Delta_h(\mu)\right]$ to determine the boundary condition.

RG Equations for $\mu < M_\Delta$

- For the top Yukawa coupling,

$$\frac{dy_t}{d\ln\mu} = \left(\frac{\beta_t^{(1)}}{16\pi^2} + \frac{\beta_t^{(2)}}{(16\pi^2)^2} \right) y_t,$$

where [Machacek, Vaughn '83-85]

$$\begin{aligned}
\beta_t^{(1)} &= \frac{9}{2} y_t^2 - \left(\frac{17}{20} g_1^2 + \frac{9}{4} g_2^2 + 8 g_3^2 \right), \\
\beta_t^{(2)} &= -12 y_t^4 + \left(\frac{393}{80} g_1^2 + \frac{225}{16} g_2^2 + 36 g_3^2 \right) y_t^2 + \frac{1187}{600} g_1^4 - \frac{9}{20} g_1^2 g_2^2 + \frac{19}{15} g_1^2 g_3^2 \\
&\quad - \frac{23}{4} g_2^4 + 9 g_2^2 g_3^2 - 108 g_3^4 + \frac{3}{2} \lambda^2 - 6 \lambda y_t^2.
\end{aligned}$$

- Use matching condition: $y_t(\mu) = \frac{\sqrt{2} M_t}{v} [1 + \Delta_t(\mu)]$ to determine the boundary condition.

RG Equations for $\mu < M_\Delta$

- For the SM gauge couplings,

$$\frac{dg_i}{d\ln\mu} = -\frac{g_i^3}{16\pi^2}b_i - \frac{g_i^3}{(16\pi^2)^2}\sum_{j=1}^{3} b_{ij}g_j^2 - \frac{g_i^3 y_t^2}{(16\pi^2)^2}a_i,$$

where for $i = 1, 2, 3$, the β-function coefficients are given by [Machacek, Vaughn '83-85; Arason *et al* '92]

$$b_1 = -\frac{2}{3}N_F - \frac{1}{10}, \quad b_2 = \frac{22}{3} - \frac{2}{3}N_F - \frac{1}{6}, \quad b_3 = 11 - \frac{2}{3}N_F,$$

$$b_{ij} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \frac{136}{3} & 0 \\ 0 & 0 & 102 \end{pmatrix} - \frac{N_F}{2}\begin{pmatrix} \frac{19}{15} & \frac{1}{5} & \frac{11}{30} \\ \frac{3}{5} & \frac{49}{3} & \frac{3}{2} \\ \frac{44}{15} & 4 & \frac{76}{3} \end{pmatrix} - \begin{pmatrix} \frac{9}{50} & \frac{3}{10} & 0 \\ \frac{9}{10} & \frac{13}{6} & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

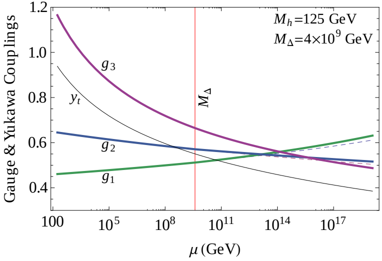
and $a_i = \left(\frac{17}{10}, \frac{3}{2}, 2\right)$. N_F is the effective number of flavors *below* the renormalization scale μ.

- Boundary conditions chosen as the $\overline{\mathrm{MS}}$ values at the Z-pole: $(\alpha_1, \alpha_2, \alpha_3)(M_Z) = (0.01681, 0.03354, 0.1184)$.

RG Equations for $\mu \geq M_\Delta$

- RGE for y_t remains unchanged since the coupling $ht\bar{t}$ is almost the same as in the SM.
- β-functions for electroweak gauge coupling RGEs different:

$$b_i^{\rm SM} = \left(-\frac{41}{10}, \frac{19}{6}, 7\right) \rightarrow b_i^{\rm type-II} = \left(-\frac{47}{10}, \frac{5}{2}, 7\right)$$

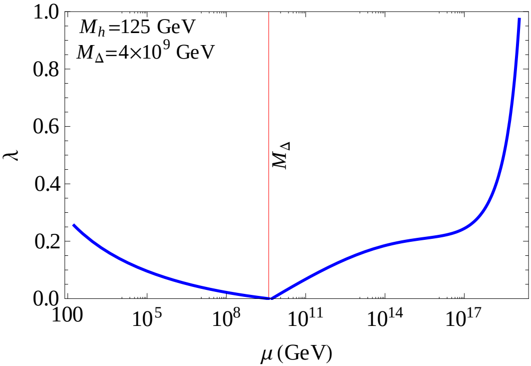


RG Equations for $\mu \geq M_\Delta$

- For scalar quartic coupling, $\beta_\lambda^{(1)} \to \beta_\lambda^{(1)} + 6\lambda_4^2 + 4\lambda_5^2$.
- RGEs for new scalar couplings [Schmidt '07; Chao, Zhang '07]

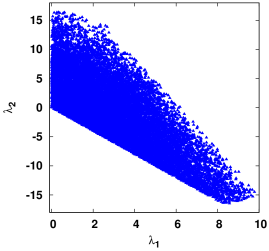
$$
\begin{aligned}
16\pi^2 \frac{d\lambda_1}{d\ln\mu} &= -\left(\frac{36}{5}g_1^2 + 24g_2^2\right)\lambda_1 + \frac{108}{25}g_1^4 + 18g_2^4 + \frac{72}{5}g_1^2 g_2^2 \\
&\quad +14\lambda_1^2 + 4\lambda_1\lambda_2 + 2\lambda_2^2 + 4\lambda_4^2 + 4\lambda_5^2 + 4\mathrm{Tr}\left[\mathbf{S}_\Delta\right]\lambda_1 - 8\mathrm{Tr}\left[\mathbf{S}_\Delta^2\right], \\
16\pi^2 \frac{d\lambda_2}{d\ln\mu} &= -\left(\frac{36}{5}g_1^2 + 24g_2^2\right)\lambda_2 + 12g_2^4 - \frac{144}{5}g_1^2 g_2^2 \\
&\quad +3\lambda_2^2 + 12\lambda_1\lambda_2 - 8\lambda_5^2 + 4\mathrm{Tr}\left[\mathbf{S}_\Delta\right]\lambda_2 + 8\mathrm{Tr}\left[\mathbf{S}_\Delta^2\right], \\
16\pi^2 \frac{d\lambda_4}{d\ln\mu} &= -\left(\frac{9}{2}g_1^2 + \frac{33}{2}g_2^2\right)\lambda_4 + \frac{27}{25}g_1^4 + 6g_2^4 \\
&\quad +\left(8\lambda_1 + 2\lambda_2 + 6\lambda + 4\lambda_4 + 6y_t^2 + 2\mathrm{Tr}\left[\mathbf{S}_\Delta\right]\right)\lambda_4 + 8\lambda_5^2 - 4\mathrm{Tr}\left[\mathbf{S}_\Delta^2\right], \\
16\pi^2 \frac{d\lambda_5}{d\ln\mu} &= -\frac{9}{2}g_1^2\lambda_5 - \frac{33}{2}g_2^2\lambda_5 - \frac{18}{5}g_1^2 g_2^2 \\
&\quad +\left(2\lambda_1 - 2\lambda_2 + 2\lambda + 8\lambda_4 + 6y_t^2 + 2\mathrm{Tr}\left[\mathbf{S}_\Delta\right]\right)\lambda_5 + 4\mathrm{Tr}\left[\mathbf{S}_\Delta^2\right]; \\
16\pi^2 \frac{d\mathbf{S}_\Delta}{d\ln\mu} &= 6\,\mathbf{S}_\Delta^2 - 3\left(\frac{3}{5}g_1^2 + 3g_2^2\right)\mathbf{S}_\Delta + 2\mathrm{Tr}[\mathbf{S}_\Delta]\mathbf{S}_\Delta. \quad (\text{where } \mathbf{S}_\Delta = Y_\Delta^\dagger Y_\Delta).
\end{aligned}
$$

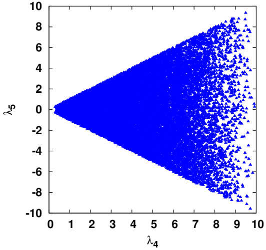
RG Running of the Scalar Quartic Coupling



Allowed Parameter Space

- High Seesaw Scale: $M_\Delta = \mathcal{O}(10^9)$ GeV





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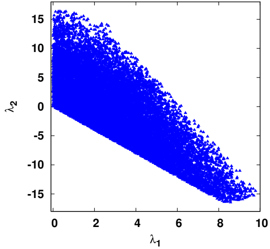
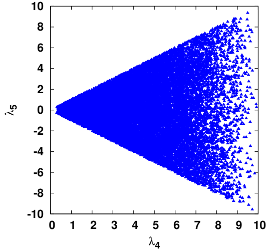
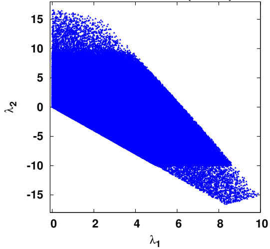
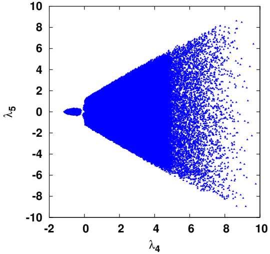




- Low Seesaw Scale: $M_\Delta = \mathcal{O}(100)$ GeV





Experimental Limits

- Direct search limits: $M_{H^{\pm\pm}} > 300$ - 400 GeV (LHC).
- Assumes dominant $\ell^{\pm}\ell^{\pm}$ decay mode.
- Valid for $v_{\Delta} < 10^{-4}$ GeV (large Yukawa coupling).
- For $v_{\Delta} > 10^{-4}$ GeV (small Yukawa), BR to $\ell^{\pm}\ell^{\pm}$ drops significantly.
- Mass limit lowered to about 100 GeV. [Melfo, Nemevsek, Nesti, Senjanovic, Zhang '11]
- LEP limit on $M_{H^{\pm}} > 90$ GeV.

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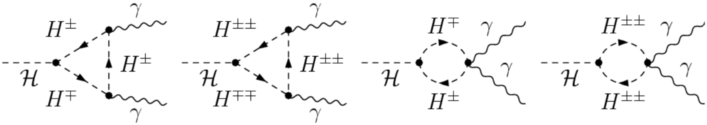
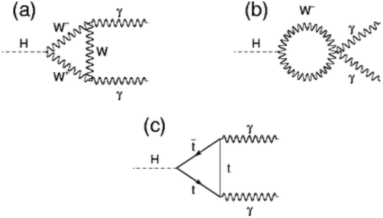
- Constraints from LFV require $v_\Delta M_{H^{\pm\pm}} \gtrsim 150\ \mathrm{eV\ GeV}$. [Melfo, Nemevsek, Nesti, Senjanovic, Zhang '11]
- Constraints from S, T, U parameters require $\Delta M \equiv |M_{H^{\pm\pm}} - M_{H^\pm}| \lesssim 40$ GeV. [Chun, Lee, Sharma '12]

New $h \to \gamma\gamma$ (and $Z\gamma$) Diagrams

- Higgs production rates as well as the tree-level decay rates are very similar to that in the SM.
- Additional contributions in loop-induced decay modes.



in addition to those in the SM:



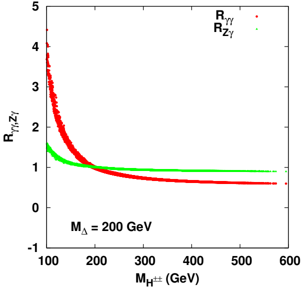
Predictions for $h \to \gamma\gamma$ and $Z\gamma$

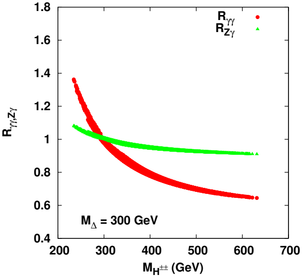
$$\begin{aligned}
\Gamma(h \to \gamma\gamma) &= \frac{\alpha^2 G_F M_h^3}{128\sqrt{2}\pi^3} \Big| \sum_f N_c Q_f^2 g_{hf\bar{f}} A_{1/2}^h(\tau_f) + g_{hW^+W^-} A_1^h(\tau_W) \\
&\quad + \tilde{g}_{hH^\pm H^\mp} A_0^h(\tau_{H^\pm}) + 4\tilde{g}_{hH^{\pm\pm}H^{\mp\mp}} A_0^h(\tau_{H^{\pm\pm}}) \Big|^2 . \\
\Gamma(h \to Z\gamma) &= \frac{\alpha G_F^2 M_W^2 M_h^3}{64\pi^4} \left(1 - \frac{M_Z^2}{M_h^2}\right)^3 \Big| \frac{1}{c_W} \sum_f N_c Q_f (2 I_3^f - 4 Q_f s_W^2) g_{hf\bar{f}} A_{1/2}^h(\tau_h^f, \tau_Z^f) \\
&\quad + c_W g_{hW^+W^-} A_1^h(\tau_h^W, \tau_Z^W) - 2 s_W g_{ZH^\pm H^\mp} \tilde{g}_{hH^\pm H^\mp} A_0^h(\tau_h^{H^\pm}, \tau_Z^{H^\pm}) \\
&\quad - 4 s_W g_{ZH^{\pm\pm}H^{\mp\mp}} \tilde{g}_{hH^{\pm\pm}H^{\mp\mp}} A_0^h(\tau_h^{H^{\pm\pm}}, \tau_Z^{H^{\pm\pm}}) \Big|^2 .
\end{aligned}$$

where

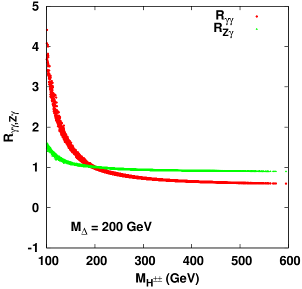
$$\begin{aligned}
g_{hf\bar{f}} &= \frac{\cos\alpha}{\cos\beta'} , \quad g_{hW^+W^-} = \cos\alpha + 2\sin\alpha \frac{v_\Delta}{v} \\
\tilde{g}_{hH^{++}H^{--}} &= \frac{M_W}{g M_{H^{\pm\pm}}^2} g_{hH^{++}H^{--}} , \quad \tilde{g}_{hH^+H^-} = \frac{M_W}{g M_{H^\pm}^2} g_{hH^+H^-} \\
g_{hH^{++}H^{--}} &\simeq (\lambda_4 + \lambda_5) v , \quad g_{hH^+H^-} \simeq \lambda_4 v \\
g_{ZH^+H^-} &= -\tan\theta_W , \quad g_{ZH^{++}H^{--}} = 2\cot 2\theta_W .
\end{aligned}$$

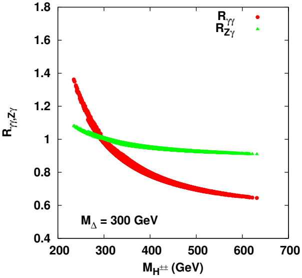
Predictions for $h \to \gamma\gamma$ and $Z\gamma$

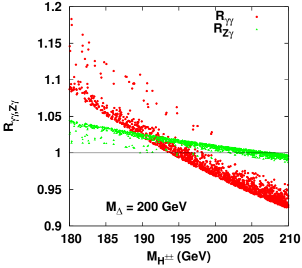


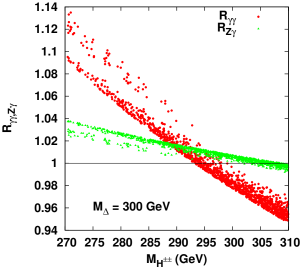


Predictions for $h \to \gamma\gamma$ and $Z\gamma$

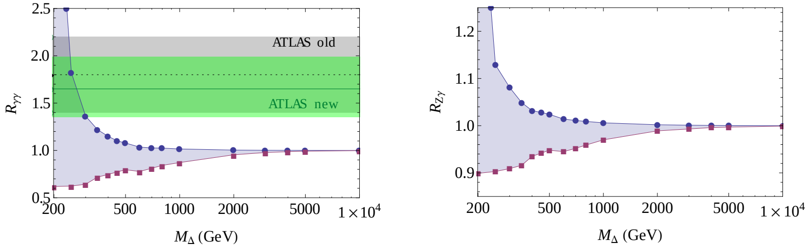




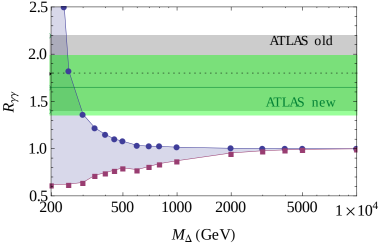


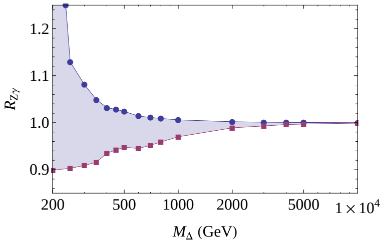


Upper Limit on the Seesaw Scale



Upper Limit on the Seesaw Scale





M_Δ [GeV]	$(R_{\gamma\gamma})_{\max(\min)}$	$(R_{\gamma\gamma})_{\max(\min)}$
200	4.41 (0.62)	1.59 (0.90)
300	1.36 (0.64)	1.08 (0.91)
400	1.15 (0.75)	1.03 (0.93)
500	1.08 (0.80)	1.02 (0.95)
1000	1.02 (0.87)	1.01 (0.97)

Conclusion

- For a 125 GeV Higgs, the SM vacuum is (most likely) not stable up to the Planck scale.
- This can be cured in a minimal type-II seesaw model for neutrino mass.
- There exists a large allowed parameter space, irrespective of the seesaw scale.
- The $h \to \gamma\gamma$ and $Z\gamma$ rates could be different from the SM predictions in this model.
- The two rates are correlated for most of the allowed parameter space.
- For a given enhancement in the $\gamma\gamma$ rate, there exists an upper limit on the seesaw scale.
- If more than 10% enhancement persists, the upper limit is about 450 GeV.
- Completely within reach of the LHC.