Isospin breaking in tau decays and $(g-2)_\mu$

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in collaboration with
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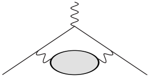
$(g-2)_\mu$ RECAP

$(g-2)_\mu$: discrepancy between exp vs theory ($\gtrsim 3\sigma$)

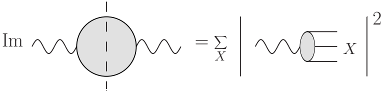
hadronic contributions dominate the error



$$a_\mu = \frac{\alpha}{\pi} \int \frac{ds}{s} K(s, m_\mu) \frac{\mathrm{Im}\Pi(s)}{\pi}$$ [Brodsky, de Rafael '68]

analyticity $\hat{\Pi}(s) = \Pi(s) - \Pi(0) = \frac{s}{\pi} \int_{4m_\pi^2}^{\infty} dx \frac{\mathrm{Im}\Pi(x)}{x(x-s-i\varepsilon)}$

unitarity

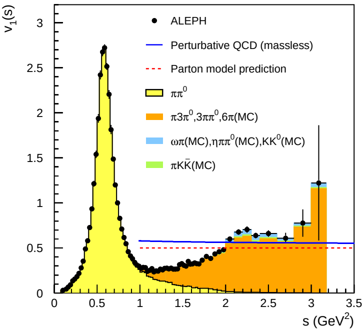


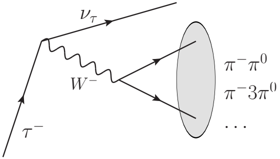
$$\frac{4\pi^2\alpha}{s} \frac{\mathrm{Im}\Pi(s)}{\pi} = \sigma_{e^+e^- \to \gamma^\star \to \mathrm{had}}$$

At present $O(30)$ channels: $\pi^0\gamma, \pi^+\pi^-, 3\pi, 4\pi, K^+K^-, \cdots$

$\pi\pi$ channel is $\sim 70\%$ of signal and $\sim 70\%$ of error

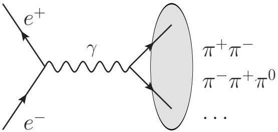
Motivations for τ





$V - A$ current

Final states $I = 1$ charged



EM current

Final states $I = 0, 1$ neutral

τ data can improve $a_\mu[\pi\pi]$

$\rightarrow 72\%$ of total Hadronic LO

or $a_\mu^{ee} \neq a^\tau \rightarrow$ NP [Cirigliano et al '18]

Isospin Corrections

Restriction to $e^+e^- \to \pi^+\pi^-$ and $\tau^- \to \pi^-\pi^0\,\nu_\tau$

$$v_0(s) = \frac{s}{4\pi\alpha^2}\sigma_{\pi^+\pi^-(\gamma)}(s)$$

$$v_-(s) = \frac{m_\tau^2}{6|V_{ud}|^2}\frac{\mathcal{B}_{\pi\pi^0}}{\mathcal{B}_e}\frac{1}{N_{\pi\pi^0}}\frac{dN_{\pi\pi^0}}{ds}\left(1-\frac{s}{m_\tau^2}\right)^{-1}\left(1+\frac{2s}{m_\tau^2}\right)^{-1}\frac{1}{S_{\rm EW}}$$

Isospin correction $v_0 = R_{\rm IB} v_-$ $\quad R_{\rm IB} = \frac{\rm FSR}{G_{\rm EM}}\frac{\beta_0^3|F_\pi^0|^2}{\beta_-^3|F_\pi^-|^2}$ [Alemani et al. '98]

0. $S_{\rm EW}$ electro-weak radiative correct. [Marciano, Sirlin '88][Braaten, Li '90]

1. Final State Radiation of $\pi^+\pi^-$ system [Schwinger '89][Drees, Hikasa '90]

2. $G_{\rm EM}$ (long distance) radiative corrections in τ decays
Chiral Resonance Theory [Cirigliano et al. '01, '02]
Meson Dominance [Flores-Talpa et al. '06, '07]

3. Phase Space ($\beta_{0,-}$) due to $(m_{\pi^\pm} - m_{\pi^0})$

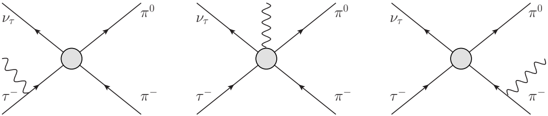
Long distance QED - I

At low energies relevant degrees of freedom are mesons

- Chiral Perturbation Theory [Cirigliano et al. '01, '02]
- Meson dominance model [Flores-Talpa et al. '06, '07]

Corrections casted in one function $v_-(s) \to v_-(s) G_{\mathrm{EM}}(s)$

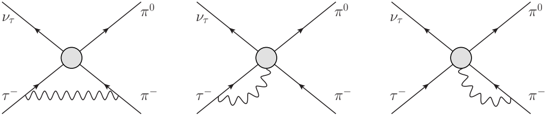
Real photon corrections



Real + virtual

$\to$ IR divergences cancel

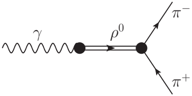
Virtual photon corrections



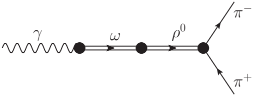
Pion form factors

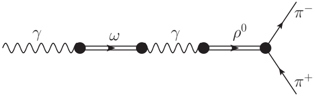
[Gounaris, Sakurai '68]
[Kühn, Santamaria '90]

$$F_\pi^0(s) \propto \frac{m_\rho^2}{D_\rho(s)}$$

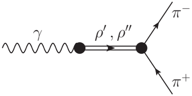


$$\times \left[1 + \color{red}{\delta_{\rho\omega}} \frac{s}{D_\omega(s)}\right]$$

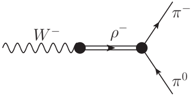




$$+ \frac{m_X^2}{D_X(s)} \qquad X = \rho' \,, \rho''$$



$$F_\pi^-(s) \propto \frac{m_{\rho^-}^2}{D_{\rho^-}(s)} + (\rho' \,, \rho'')$$



Sources of IB breaking in phenomenological models

$m_{\rho^0} \neq m_{\rho^\pm}$, $\Gamma_{\rho^0} \neq \Gamma_{\rho^\pm}$, $m_{\pi^0} \neq m_{\pi^\pm}$

$\rho - \omega$ mixing $\delta_{\rho\omega} \simeq O(m_\mathrm{u} - m_\mathrm{d}) + O(e^2)$

Status

$$a_\mu^{\rm HVP,LO}[\pi\pi, ee] = 503.51(3.5) \times 10^{-10} \text{ with } E \in [2m_\pi, 1.8 \text{ GeV}]$$

$$a_\mu^{\rm HVP,LO}[\pi\pi, \tau] = 531.3(3.3) \times 10^{-10}$$

$$a_\mu[\pi\pi, ee] - a_\mu[\pi\pi, \tau] = -12.0(2.6)$$ [Cirigliano et al.]

$$a_\mu[\pi\pi, ee] - a_\mu[\pi\pi, \tau] = -16.1(1.8)$$ [Davier et al. '09]

(≈ -12 due to $S_{\rm EW}$, rest R_{IB})

$$a_\mu[\tau] : \begin{cases} \text{model dependence} \\ e^+e^- \text{ data more precise} \end{cases} = \text{abandoned}$$

Additional $\rho\gamma$ mixing correction [Jegerlehner, Szafron '11]

partly accounted in $m_{\rho^0} - m_{\rho^-}$ in [Davier et al. '09]

$a_\mu[\pi\pi, ee] = 385.2(1.5)$ with $E \in [0.582 - 0.975]$ GeV

$a_\mu[\pi\pi, \tau] = 386.0(2.4)$ after R_{IB}

Contribution to a_μ

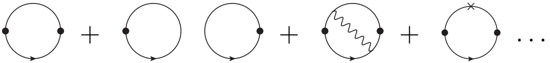
Time-momentum representation [Bernecker, Meyer, '11]

$$G^\gamma(t) = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^\gamma(x) j_k^\gamma(0)\rangle \quad \rightarrow \quad a_\mu = 4\alpha^2 \sum_t w_t G^\gamma(t)$$

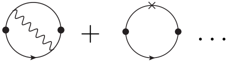
Isospin decomposition of u, d current

$$j_\mu^\gamma = \frac{i}{6}\left(\bar{u}\gamma_\mu u + \bar{d}\gamma_\mu d\right) + \frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right) = j_\mu^{(0)} + j_\mu^{(1)}$$

$$G_{00}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(0)}(0)\rangle =$$



$$G_{01}^\gamma \leftarrow \langle j_k^{(0)}(x) j_k^{(1)}(0)\rangle =$$



$$G_{11}^\gamma \leftarrow \langle j_k^{(1)}(x) j_k^{(1)}(0)\rangle =$$

Decompose $a_\mu = a_\mu^{(0,0)} + a_\mu^{(0,1)} + a_\mu^{(1,1)}$

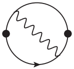
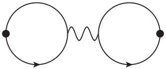
Neutral vs Charged

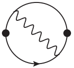
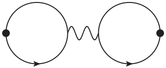
$$\frac{i}{2}\left(\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d\right), \begin{bmatrix} I = 1 \\ I_3 = 0 \end{bmatrix} \quad \rightarrow \quad j_\mu^{(1,-)} = \frac{i}{\sqrt{2}}\left(\bar{u}\gamma_\mu d\right), \begin{bmatrix} I = 1 \\ I_3 = -1 \end{bmatrix}$$

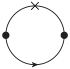
Isospin 1 charged correlator $G_{11}^W = \frac{1}{3}\sum_k \int d\vec{x}\, \langle j_k^{(1,+)}(x) j_k^{(1,-)}(0)\rangle$

[MB et al.' Latt18]

$$\delta G^{(1,1)} \equiv G_{11}^\gamma - G_{11}^W$$

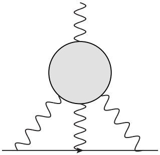
$$= Z_V^4 (4\pi\alpha) \frac{(Q_\mathrm{u} - Q_\mathrm{d})^4}{4} \Big[$$  $$+$$  $$\Big]$$

$$G_{01}^\gamma = Z_V^4 \frac{(Q_u^2 - Q_d^2)^2}{2} (4\pi\alpha) \Big[$$  $$+ 2\times$$  $$+$$  $$+ \ldots \Big]$$

$$+ Z_V^2 \frac{Q_u^2 - Q_d^2}{2} (m_u - m_d) \Big[2\times$$  $$+ \ldots \Big]$$

$\ldots$ = subleading diagrams currently not included

Synergy - I



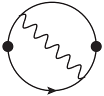
from QCD we need a 4-point function $f(x, y, z, t)$:

known kernel with details of photons and muon line

1 pair of point sources (x, y), sum over z, t exact at sink

stochastic sampling over (x, y) (based on $|x - y|$)

Successfull strategy: x10 error reduction [RBC '16]



from QCD we need a 4-point function $f(x, y, z, t)$:

$(g-2)_\mu$ kernel + photon propagator

Similar problem $\rightarrow$ re-use HLbL point sources!

The RBC & UKQCD collaborations

BNL and BNL/RBRC

Yasumichi Aoki (KEK)
Taku Izubuchi
Yong-Chull Jang
Chulwoo Jung
Meifeng Lin
Aaron Meyer
Hiroshi Ohki
Shigemi Ohta (KEK)
Amarjit Soni

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Luigi Del Debbio
Felix Erben
Vera Gülpers
Tadeusz Janowski
Julia Kettle
Michael Marshall
Fionn Ó hÓgáin
Antonin Portelli
Tobias Tsang
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Azusa Yamaguchi

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Ryan Hill
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James Richings
Chris Sachrajda

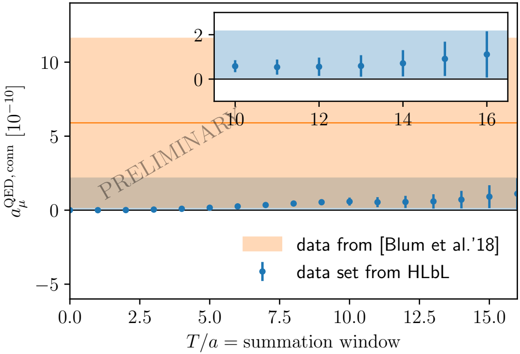
Stony Brook University

Jun-Sik Yoo
Sergey Syritsyn (RBRC)

Synergy - II

Stat. improvements from data of HLbL project [Phys.Rev.Lett. 118 (2017)]

contribution of diagrams V, S to a_μ



$O(2000)$ point-src per conf.

~ 3000 combinations

$O(10)$ configurations

$\times 4$ reduction in error

expected QED conn. error $\leq 3 \times 10^{-10}$ $\rightarrow$ matches target

Last slide, then Plots!

Restriction to 2π $\rightarrow$ neglect pure $I=0$ part $a_\mu^{(0,0)}[\pi^0\gamma, 3\pi\,, \ldots]$

$$\text{Lattice: } \Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \;\times\; \left[G_{01}^{\gamma}(t) + G_{11}^{\gamma}(t) - G_{11}^{W}(t)\right]$$

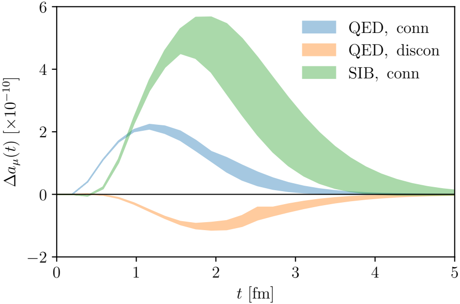
$$\text{Pheno: } \Delta a_\mu[\pi\pi,\tau] = \int_{4m_\pi^2}^{m_\tau^2} ds K(s) \;\left[\quad v_0(s) \quad - v_-(s)\right]$$

Conversion to Euclidean time for direct comparison

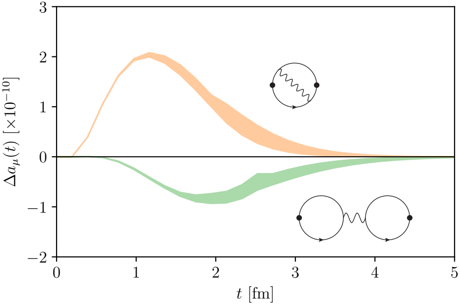
$$\Delta a_\mu[\pi\pi,\tau] = 4\alpha^2 \sum_t w_t \times \left\{ \frac{1}{12\pi^2} \int d\omega\; \omega^2 e^{-\omega t} \left[R_{\mathrm{IB}}(\omega^2) - 1\right] v_-(\omega^2) \right\}$$

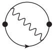
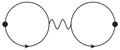
Lattice: Preliminary results - I

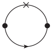
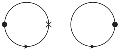
$\Delta a_\mu \to G_{01} + \delta G_{11}$:



Pure $I = 1$ only $O(\alpha)$ terms:



$V =$  $F =$  $S =$ 

$M =$  $O =$  relevant, negative, not-included

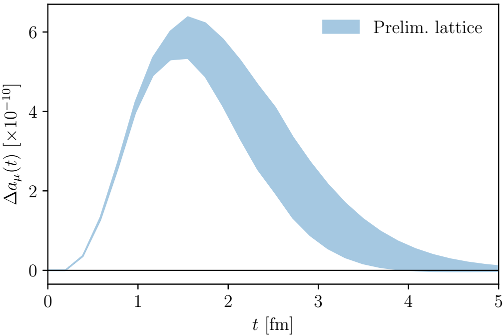
Lattice: Preliminary results - II

Study integrand in euclidean time $\rightarrow$ as important as integral

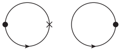
direct comparison
Lattice vs. EFT+Pheno

1. validate previous estimates of R_{IB}
2. study neutral/charged ρ and ω properties

Preliminary lattice (full) calculation: $G_{01}^{\gamma} + \delta G$



Not included:

1.  relevant
2. sub-leading $1/N_c$, $SU(N_f)$

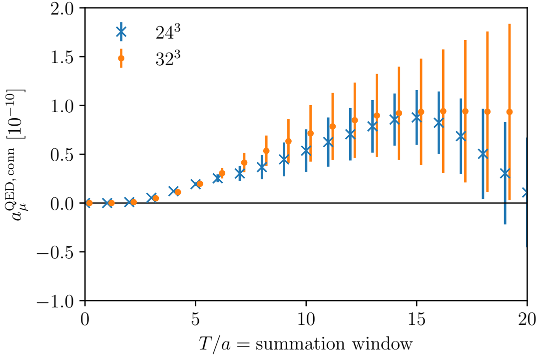
Under consideration:

3. finite-volume errors
4. discretization errors

Systematic errors

$$a_\mu^{\mathrm{QED,conn}} = V + 2S$$

FV study at coarse
$a^{-1} \sim 1$ GeV



Finite volume errors

empirical observation: diagrams may have largish FV errors

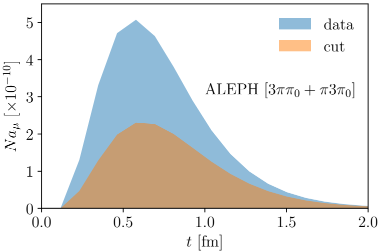
cancellation of FV effects in physical combinations

Systematic Errors - II

Lattice fully inclusive

1. cut $E < m_\tau$
2. higher multiplicity channels, 4π

effects above ~ 1.8 GeV suppressed by (muon) kernel



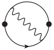
Smeared step function
$\Theta(s) = (1 + e^{2k(E-E_0)})^{-1}$
$\rightarrow$ control syst. effect of 4π

manipulate $G(t)$ (e.g. Backus-Gilbert)

preliminary: w/o cut $\Delta a_\mu[4\pi] \approx 2(1)$

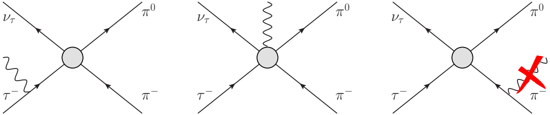
$E_0 = 2\ \text{GeV}, 1/(2k) = 0.25\ \text{GeV}$

Towards a comparison

Lattice contains $\pi^0\pi^-\gamma$ states $\rightarrow$ 

Re-evaluation of $G_{\mathrm{EM}} \rightarrow G^{\pi}_{\mathrm{EM}}$ [in collab. with Cirigliano]

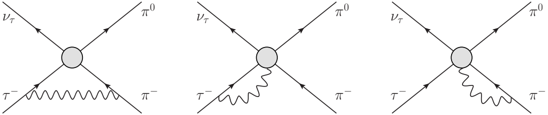
Real photon corrections



G^{π}_{EM} w/o $\pi^0\pi^-\gamma$ FSR

$\frac{v_-}{G^{\pi}_{\mathrm{EM}}}$ w $\pi^0\pi^-\gamma$ FSR

Virtual photon corrections



Outlook

Inclusive studies:

kernels to suppress high channels

suppression of short/long distances (cutoff effects/noise)

expand τ-decay program [with M. Gonzalez-Alonso]

e.g. $K\pi$ channel in vector-vector correlator

e.g. $SU(3)$ breaking $\pi\pi - K\pi$

Exclusive study: long term goal, proper isospin-breaking in $\pi\pi$ form factor

Conclusions

These are exciting times for $(g-2)_\mu$:

- 1% goal for lattice results to be expected soon
- QED+SIB crucial to reach target uncertainty

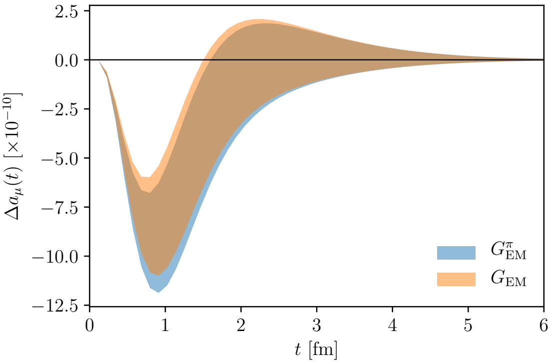
As a bi-product we get $\Delta a_\mu[\tau]$:

1. first lattice calculation of $\Delta a_\mu[\tau]$ almost complete
2. tests/checks previous calculations
   - comparing v_- with experiment requires G^{π}_{EM}
   - study G^{γ}_{01} alone $\rightarrow$ $\rho\omega$ mixing; $\delta G^{(1,1)}$ alone $\rightarrow$ ρ^0 vs ρ^-
3. possibly bound new physics

Thanks for your attention

Experimental results

$$\Delta a_\mu(t) = 4\alpha^2 \sum_t w_t \Big\{ \int ds\, h(s,t) \Big[v_0(s) - \frac{v_1(s)}{G_{\rm EM}(s)} \Big] \Big\}$$



v_0 BaBar, v_1 Aleph

preliminary

$\rho\gamma$ mixing - I

Gounaris-Sakurai based on VMD model w/o EM gauge invariance

- generation of a photon mass

$+$ based on phase shift (proper pion rescattering behavior)

widely used: e.g. PDG estimates of m_ρ, Γ_ρ

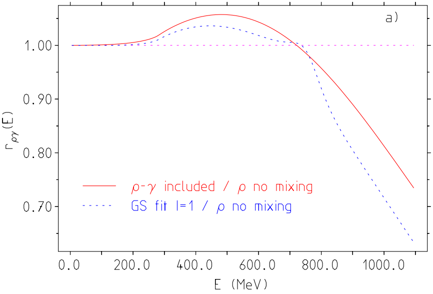
VMD model with gauge-invariance [Kroll, Lee, Zumino '67]

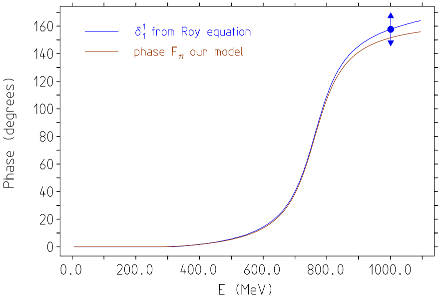
at 1-loop s-dependent mass matrix [Jegerlehner, Szafron '11]

limits of validity pion-loop? high enough energy must break down

$\rho\gamma$ MIXING - II

[Jegerlehner, Szafron '11]





30% correction at 1 GeV

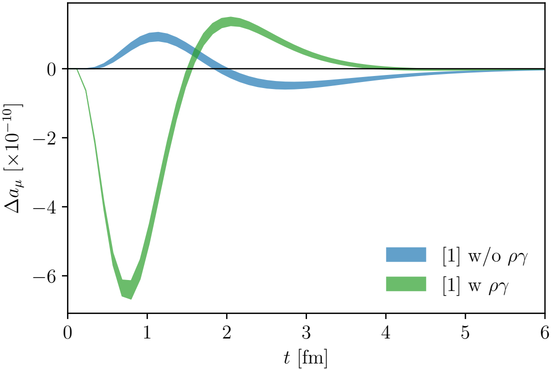
δ_1^1 in good agreement $E < 800$ MeV

perhaps restrict the $\rho\gamma$ below 800 MeV?

$\rho\gamma$ MIXING-III

$[1] =$ [Jegelehner, Szafron '17]

modified $\rho\gamma$ coupling
large negative Δa_μ



Modified $\rho\gamma$ suggests different behavior from lattice data

direct comparison with lattice not possible $\rightarrow$ hard cut at 1 GeV