Proton decay matrix elements on lattice

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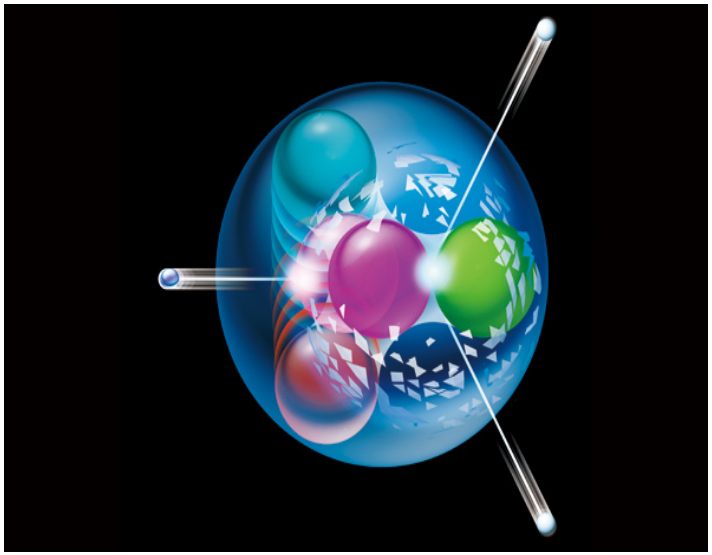
Introduction

Proton Decay



Process $p \longrightarrow \Pi + \ell$

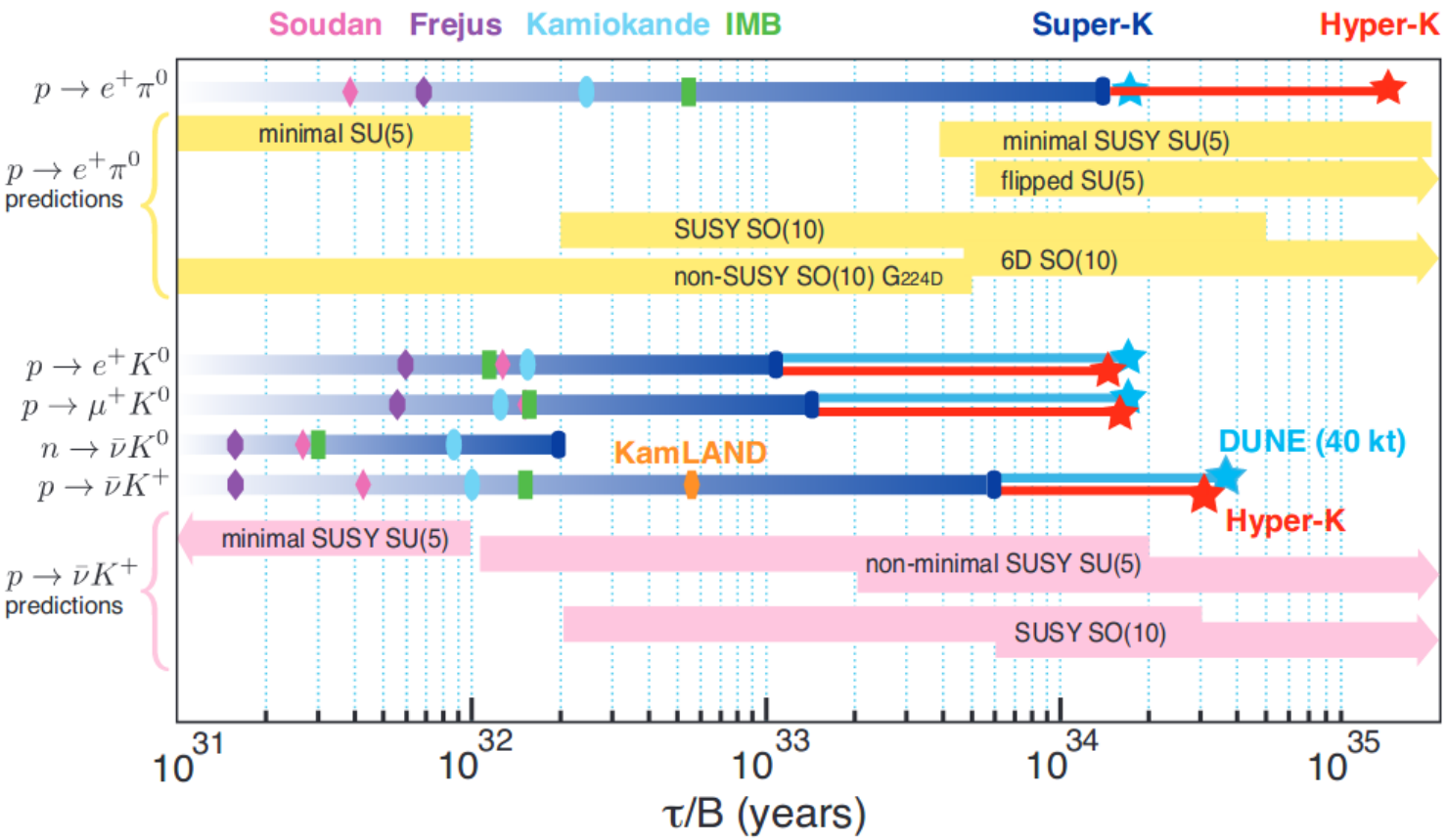
Baryon Number Violation

Haven't been observed.

Lifetime $> 10^{34}$ years

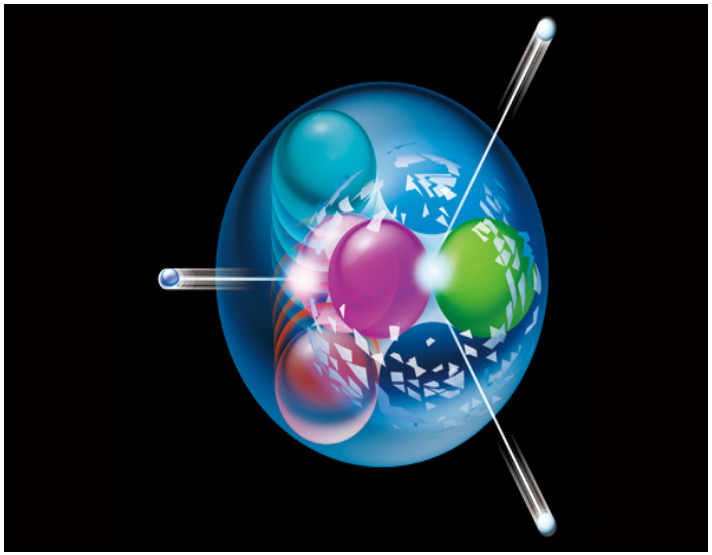
Does proton even decay?

Experimental bound



Current proton decay bound in SK, (ABE et al., 2018)

Proton Decay



Process $p \longrightarrow \Pi + \ell$

Baryon Number Violation

Haven't been observed.

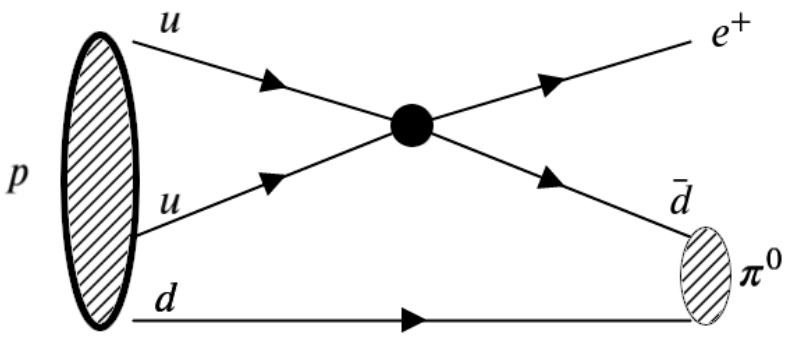
Lifetime $> 10^{34}$ years

Motivated by Baryon Asymmetry

Possible explanation by

GUT, SUSY-GUT

Effective operators



Four-fermion effective operators

$$p \longrightarrow \Pi + \ell$$

Hadronic states : Nonperturbative computation required.

Effective operator

GUT, SUSY-GUT

GUT

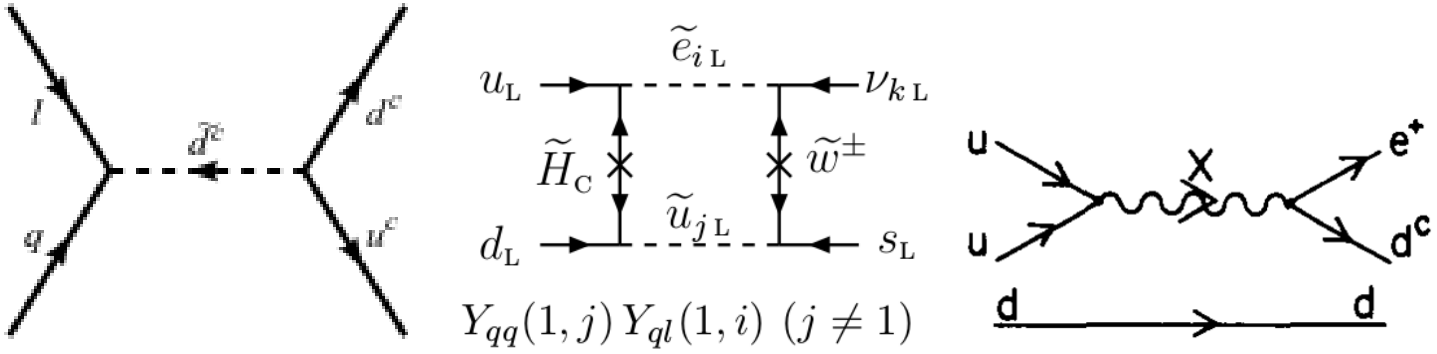
Symmetry group to be $G \supset SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$

- Coupling unification
- Baryon asymmetry

SUSY-GUT

- Superpartners to particles
- Better unification at higher scale

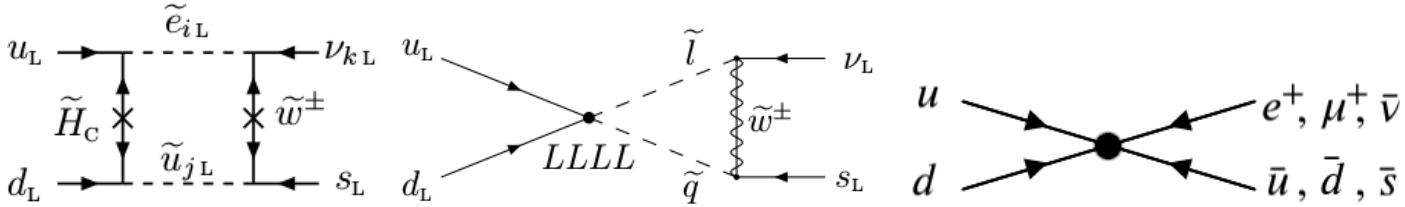
GUT,SUSY-GUT



(a) d=4 operator (b) d=5 operator (c) d=6 operator

Possible BV operators in (SUSY-)GUT

GUT,SUSY-GUT



(d) $\sim \Lambda_{GUT}$ (e) $\sim \Lambda_{SUSY}$ (f) $\sim \Lambda_{EW}$

Proton decay operator at different scales

Model parameters come into Wilson coefficients

(a) $Y_{qq}, Y_{ql}, Y_{ud}, Y_{ue}$

(b) M_{H_C}

(c) $m_{\tilde{l}}, m_{\tilde{q}}$, triangle loop integrals, ...

Effective operators

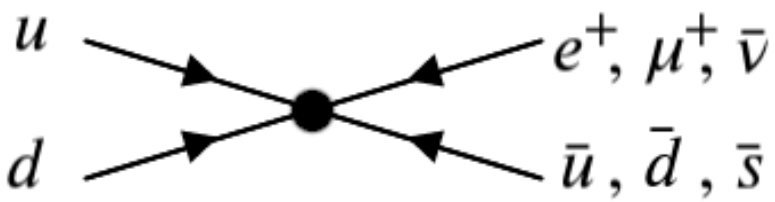


Figure 1: Four-fermion effective operators

Effective operator : $\mathcal{O}_{\Gamma\Gamma'} = (qq)_\Gamma (q\ell)_{\Gamma'}$,

$(XY)_\Gamma = (X^T \mathcal{C} P_\Gamma Y)$ $\qquad$ $\mathcal{C} :=$ (Charge Conjugation Matrix)

$\langle \Pi \bar{\ell} | p \rangle_{GUT} \sim C^{\Gamma\Gamma} \langle \Pi \bar{\ell} | \mathcal{O}_{\Gamma\Gamma'} | p \rangle_{SM} = C^{\Gamma\Gamma} \bar{v}_\ell \langle \Pi | (qq)_\Gamma P_{\Gamma'} q | p \rangle$,

where $C^{\Gamma\Gamma'}$ is a wilson coefficient, Π is a meson, and p is a proton.

Decay rate

The decay rate Γ is calculated from the hadronic matrix element,

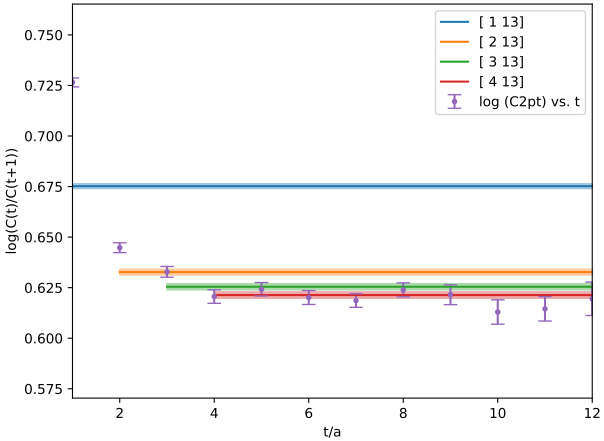
$$\begin{aligned}
&\langle \Pi(p')|O^{\Gamma\Gamma'}(q)|N(p,s)\rangle \\
&= \bar{v}_\ell P_{\Gamma'}\left[W_0^{\Gamma\Gamma'}(q^2) - \frac{i\not{q}}{m_N}W_1^{\Gamma\Gamma'}(q^2)\right]u_N(p,s) \qquad (1)\\
&= \bar{v}_\ell P_{\Gamma'}W_0^{\Gamma\Gamma'}(q^2)u_N(p,s) + O(m_l/m_N)\bar{v}_\ell u_N(p,s)
\end{aligned}$$

where Π a meson, N a nucleon, and $W_{0,1}$ decay form factor(AOKI et al., 2000). Then the decay rate is

$$\Gamma\left(p \to \Pi + \bar{\ell}\right) = \frac{(m_p^2 - m_\Pi^2)^2}{32\pi m_p^3}\left|\sum_l C_l W_0^l\left(p \to \Pi + \bar{\ell}\right)\right|^2. \qquad (2)$$

Matrix elements on lattice

Hadronic states



$$p_K = [0, 1, 1]p_{\text{min}}$$

Correlation function

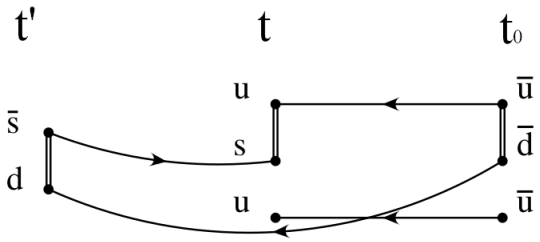
$$C_K^{2pt}(t, \vec{p})$$
$$= \sum_{\vec{x}} e^{i\vec{p}\cdot\vec{x}} \langle 0 | J_K(t, \vec{x}) J_K^\dagger(0, \vec{0}) | 0 \rangle$$
$$= \text{asymptotic states} + \ldots$$

Excited states at early times

Ground state at late times

Three-point function

(Meson)-(Decay Operator)-(Proton)



$$C^{3pt}(t,t')$$

$$= \sum_{\vec{x},\vec{x}'} e^{i(\vec{p}'\cdot\vec{x}'-\vec{q}\cdot\vec{x})} \langle 0|J_\Pi(x')\mathcal{O}(x)\bar{J}_N(x_0)|0\rangle$$

$$= \langle \Pi(\vec{p}')|\mathcal{O}|N(\vec{p})\rangle$$

$$\times \frac{C_\Pi^{2pt}(t'-t,\vec{p}')}{\sqrt{Z_\Pi}} \frac{\mathrm{Tr}[PC_p^{2pt}(t,\vec{p})]}{\sqrt{Z_p}}$$

Lattice settings

Lattice paramters

lattice size	$24^3 \times 64 \times 24$
gauge action	Iwasaki-DSDR
fermion	DWF
β	1.633
lattice cutoff	$a^{-1} = 1$GeV
$m_l a$	0.00107
$m_h a$	0.0850
$m_\pi a$	0.1387
$m_K a$	0.5051
m_{res}	0.00228
$m_\pi L$	3.3
Deflated CG	2000+1000
AMA	32+1
N_{cfg}	102

lattice size	$32^3 \times 64 \times 32$
gauge action	Iwasaki-DSDR
fermion	DWF
β	1.75
lattice cutoff	$a^{-1} = 1.37$GeV
$m_l a$	0.0001
$m_h a$	0.0450
$m_\pi a$	0.1046
$m_K a$	0.3602
m_{res}	
$m_\pi L$	3.3
Deflated CG	2000+250
AMA	32+1
N_{cfg}	20(and counting)

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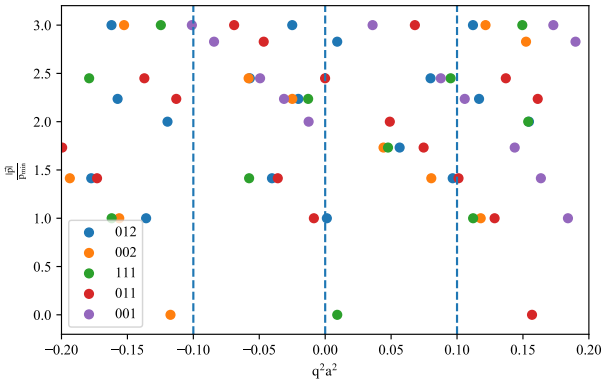
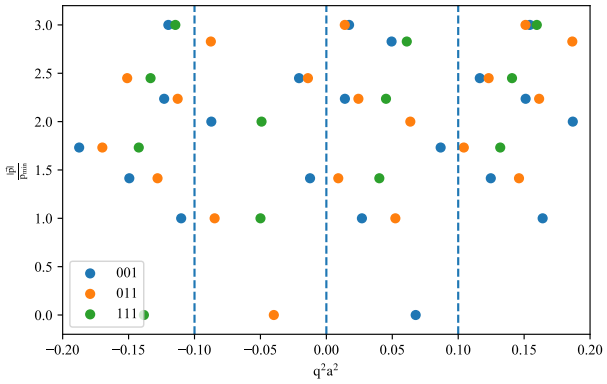
Kinematic Choice

Energy-momentum conservation & $q^2 \sim 0$

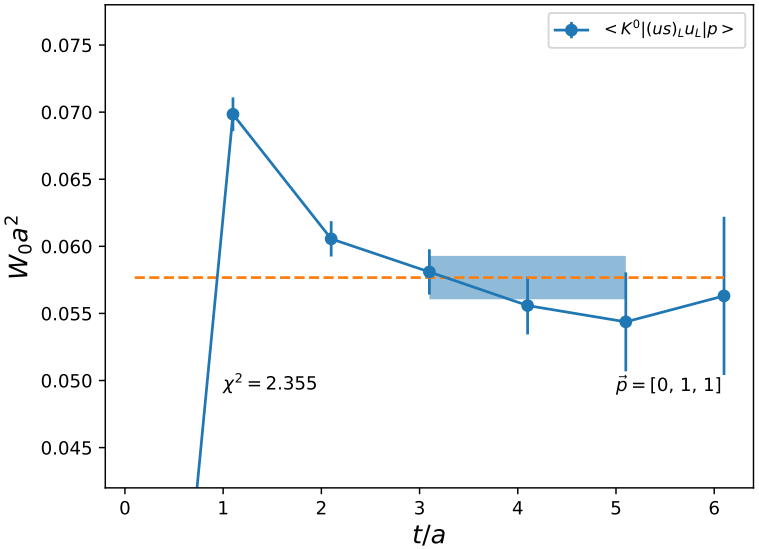


π



K

Matrix elements



decay form factor

$W_0^{LL}(p \to K^0 e^+)$ at $t_{sep} = 8$

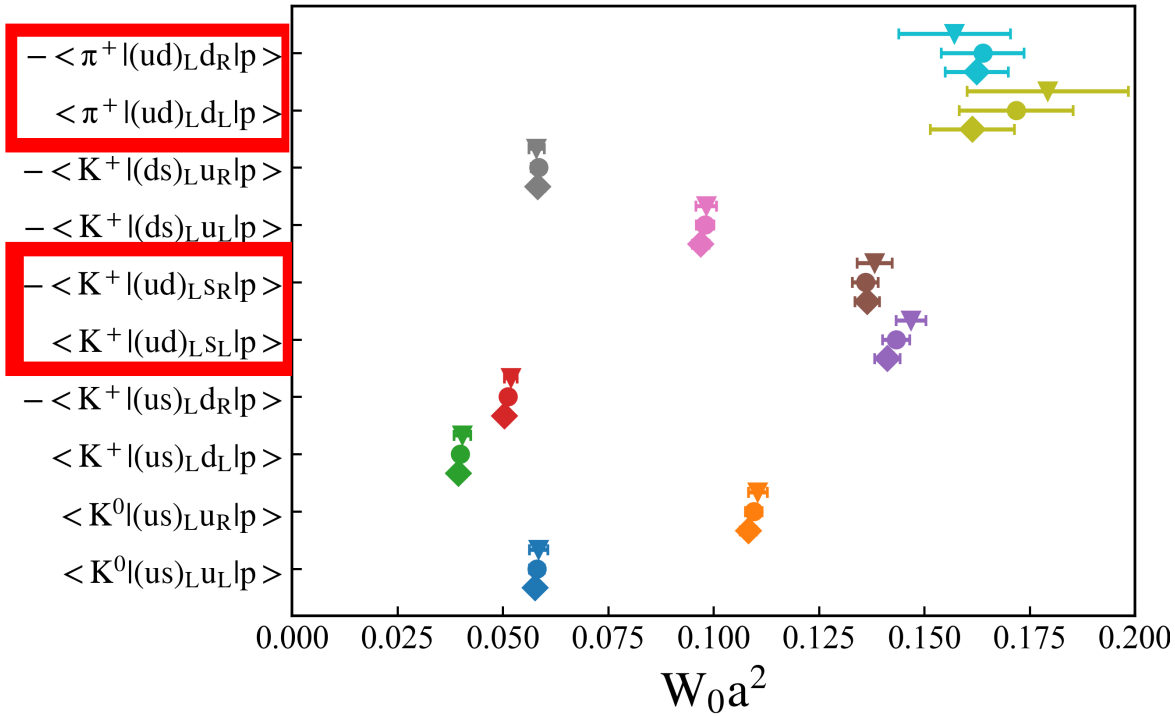
$p_p = [0, 0, 0]p_{\text{min}}$,

$p_K = [0, 1, 1]p_{\text{min}}$

plateau fit t=3–5

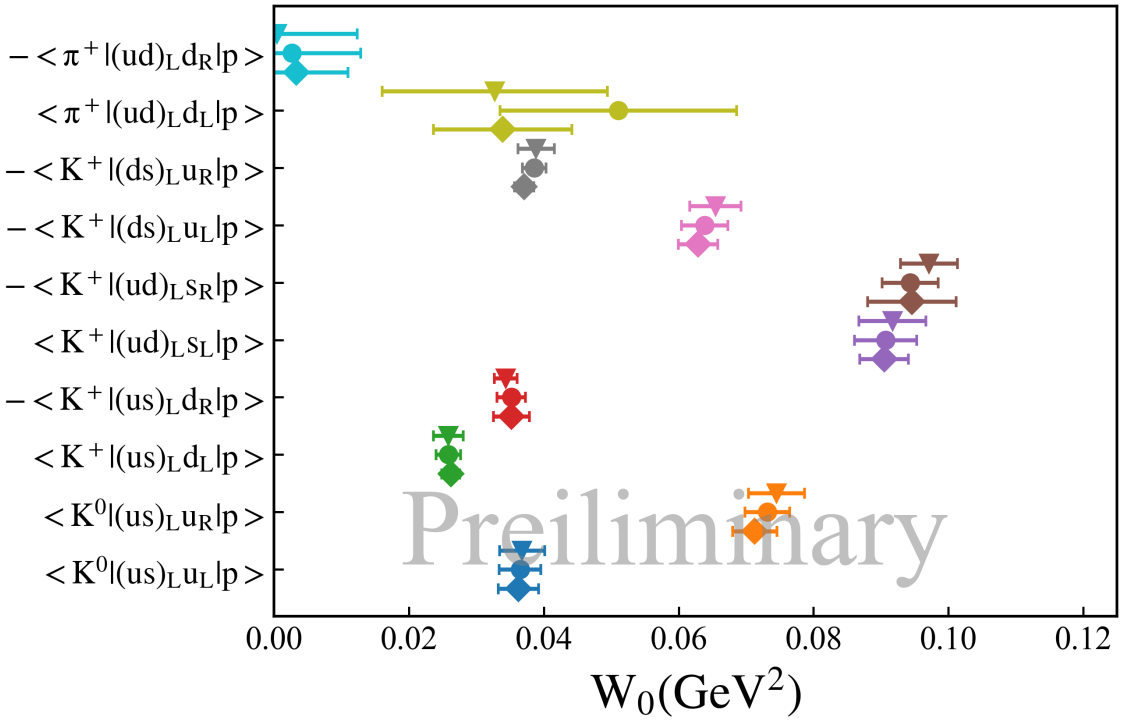
AMA 32+1, 102 configs

Matrix elements



Decay matrix elements w/ different src-sink separation {8,9,10}

Matrix elements



Decay matrix elements w/ different src-sink separation {8,9,10}

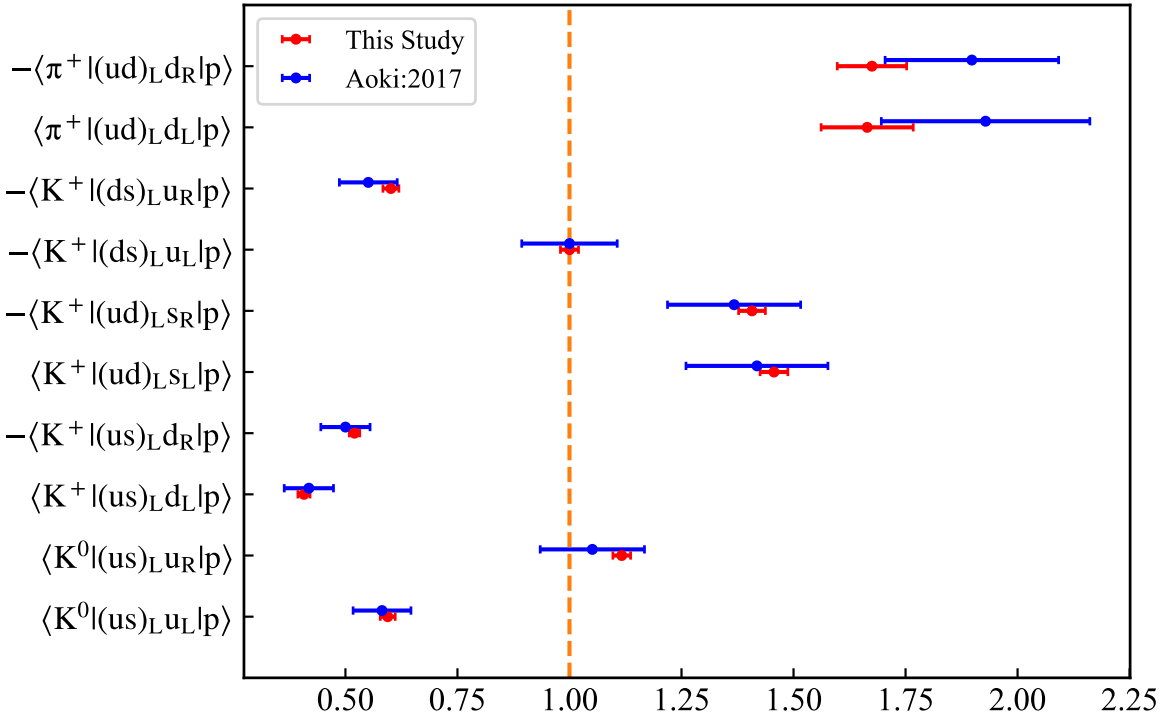
Comparison with earlier work

Bare value, but multiplicative renormalization only

$\longrightarrow$ ratio can be compared with renormalized values

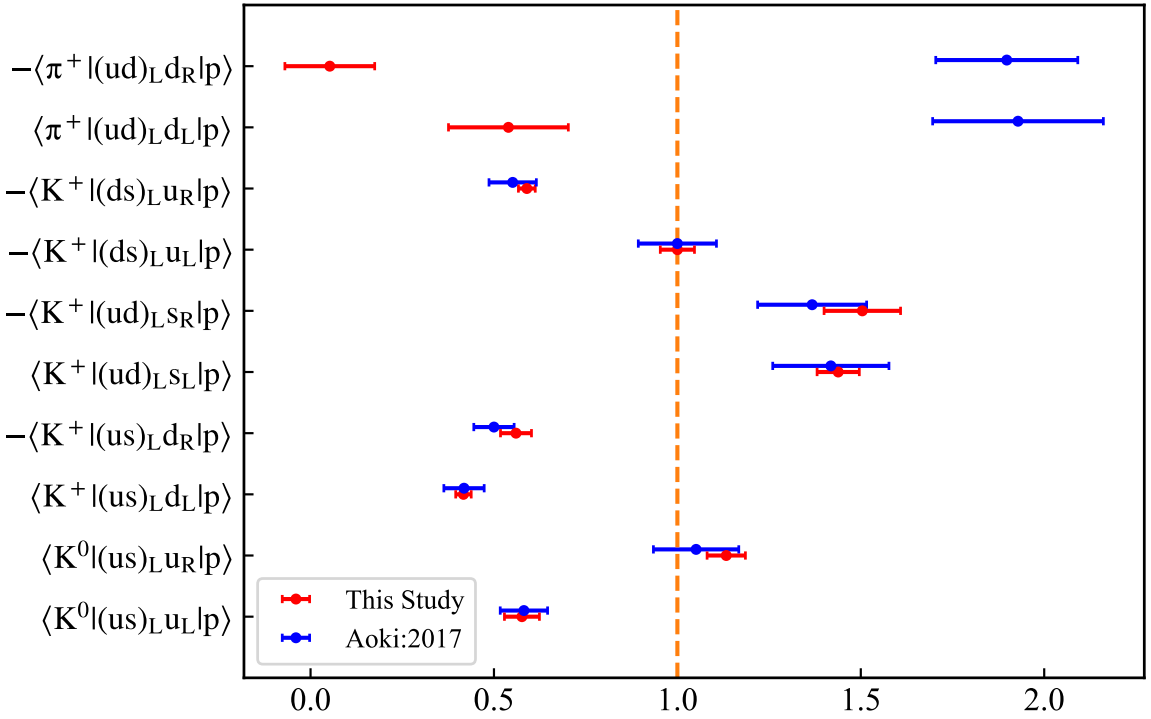
$$W_0^{norm} = \left| \frac{W_0^{\Gamma\Gamma'}(Channel)}{W_0^{\Gamma\Gamma'}(\langle K^+|(ds)_\Gamma u_{\Gamma'}|p\rangle)} \right| \tag{3}$$

Comparison with earlier work



Comparison with earlier study, (AOKI et al., 2017)

Comparison with earlier work



Comparison with earlier study, (AOKI et al., 2017)

Comparison to earlier work

	Stat. [%] (This study)	Stat.[%] (Aoki:2017)	Chiral extrapol.[%]	a^2 [%]	Δ_Z [%]
$\langle K^0 \vert (us)_L u_L \vert p \rangle$	2.80	3.5	3.1	5.0	8.1
$\langle K^0 \vert (us)_L u_R \vert p \rangle$	1.77	2.8	2.8	5.0	8.1
$\langle K^+ \vert (us)_L d_L \vert p \rangle$	3.32	4.4	7.5	5.0	8.1
$-\langle K^+ \vert (us)_L d_R \vert p \rangle$	2.24	3.7	3.5	5.0	8.1
$\langle K^+ \vert (ud)_L s_L \vert p \rangle$	2.13	3.0	3.9	5.0	8.1
$-\langle K^+ \vert (ud)_L s_R \vert p \rangle$	2.12	3.2	1.6	5.0	8.1
$-\langle K^+ \vert (ds)_L u_L \vert p \rangle$	2.01	2.8	2.1	5.0	8.1
$-\langle K^+ \vert (ds)_L u_R \vert p \rangle$	2.96	3.6	2.7	5.0	8.1
$-\langle \pi^+ \vert (ud)_L d_R \vert p \rangle$	6.17	3.4	2.7	5.0	8.1
$\langle \pi^+ \vert (ud)_L d_R \vert p \rangle$	4.62	3.0	2.7	5.0	8.1

Left : Comparison of statistical errors. Right: Systematic errors in chiral extrapolation, $O(a^2)$, Δ_Z ((AOKI et al., 2017))

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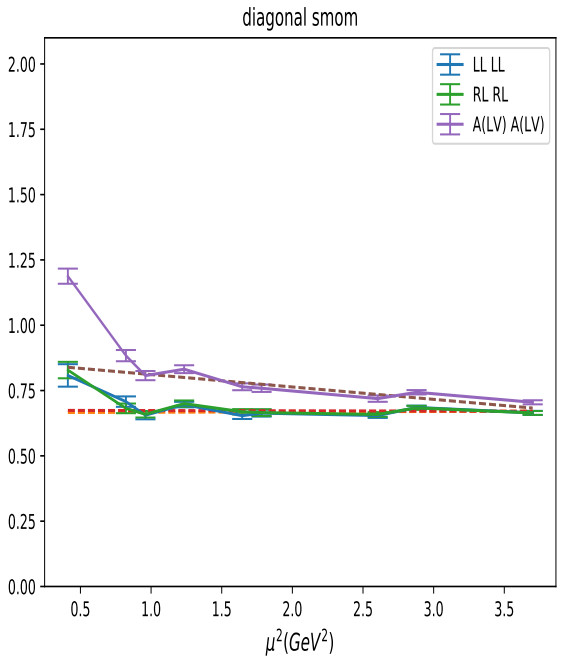
Renormalization



Three quark op. renormalization

32+1 AMA, 20 configs, Landau gauge

RI-SMOM scheme

Two-loop Matching (GRACEY, 2012)

fit region $(pa)^2 = 1 - 3.5\mathrm{GeV}^2$

$$U_{LL}^{\overline{MS}\leftarrow latt}(\mu = 2\mathrm{GeV}) = 0.67(2)$$

$$U_{RL}^{\overline{MS}\leftarrow latt}(\mu = 2\mathrm{GeV}) = 0.68(1)$$

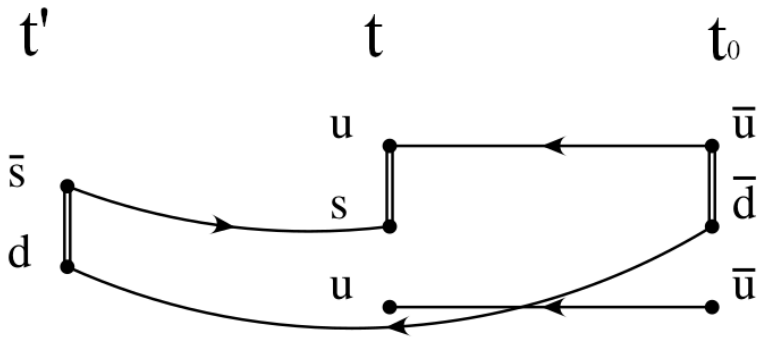
Future projects

Future projects

Proton decay matrix elements can be investigated futher to see:

- Induced Nucleon Decay from Dark matter
- Vector meson channels from proton decay
- Three body decay channel from proton decay

Vector meson channels



- Same computation with different Γ structures
- Different form factor decomposition
- Asymptotic vector meson channel should be there.

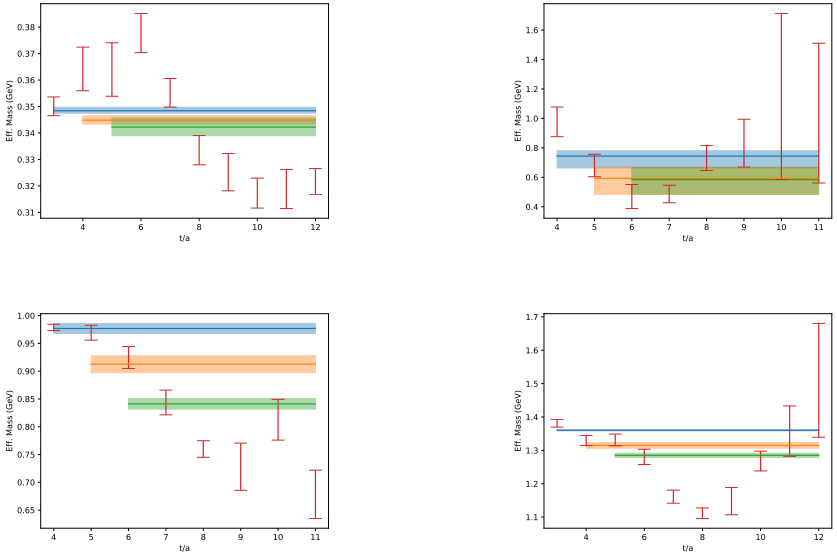
$$\langle K^{*i}(Q)\ell(p')|O_{d=6}|p(p,s)\rangle = \epsilon^i_\mu \bar{v}^c_\ell [F_1\gamma_5\gamma^\mu + F_2 i\gamma_5\sigma^{\mu\nu}Q_\nu + F_3\gamma_5 Q^\mu + F'_1\gamma^\mu + F'_2 i\sigma^{\mu\nu}Q_\nu + F'_3 Q^\mu]u_N$$

Vector meson channels

Lattice paramters(BOYLE et al., 2016)

lattice size	$24^3 \times 64 \times 16$
gauge action	Iwasaki
fermion	DWF
β	2.13
lattice cutoff	$a^{-1} = 1.78$GeV
$m_l a$	0.0005
$m_h a$	0.04
m_π	339.6MeV
$m_\pi L$	4.568

Vector meson channels



Two point fcn.

$m_\pi = 345(6)$MeV, p-val $= 0.08$

$m_\rho = 916(46)$MeV, p-val $= 0.00$

$m_\sigma = 595(40)$MeV, p-val $= 0.99$

$m_p = 1.315(16)$GeV, p-val$=0.924$

*No disconnected diagram is included.

Three body decay

A few reasons to compute three body decay :

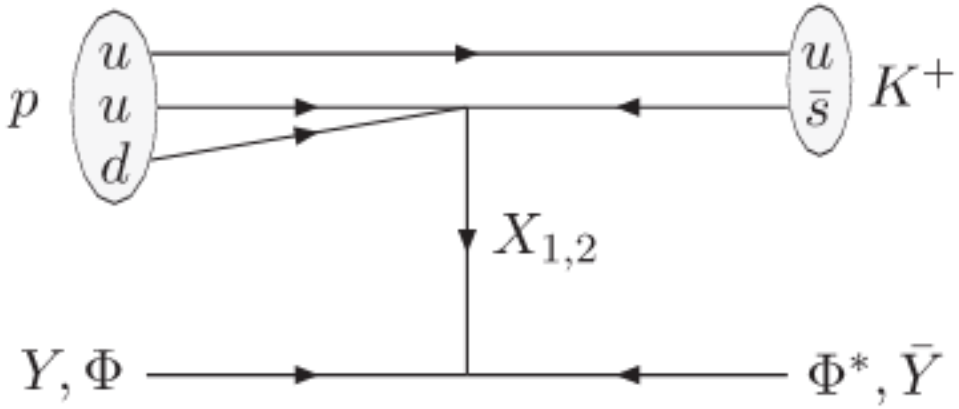
- Resonant to vector meson channels : $p \to K^* + \bar{\ell} \to (K\pi)\bar{\ell}$
- Decay rate ratio $(\Gamma(p \to \pi\pi e^+)/\Gamma(p \to \pi e^+))$ estimates to $\sim$ 24–150% (WISE; BLANKENBECLER; ABBOTT, 1981)
- Prime channel of next generation experiment
- Numerically cheapest among three body decay channels

Decay Mode	Water Cherenkov		Liquid Argon TPC	
	Efficiency	Background	Efficiency	Background
$p \to K^+\bar{\nu}$	19%	4	97%	1
$p \to K^0\mu^+$	10%	8	47%	< 2
$p \to K^+\mu^-\pi^+$			97%	1
$n \to K^+e^-$	10%	3	96%	< 2
$n \to e^+\pi^-$	19%	2	44%	0.8

DUNE proton decay efficiency

Future projects

Induced Nucleon Decay model (DAVOUDIASL et al., 2010)



- DM can annihilate the nucleon
- $\mathcal{L}_{eff} \sim (1/\Lambda^3) u_R d_R d_R Y_R \Phi +$ h.c.
- $\langle \Pi(p') | O^{\Gamma\Gamma'}(q) | N(p,s) \rangle = P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) + \frac{m_Y}{m_N} W_1^{\Gamma\Gamma'}(q^2) \right] u_N(p,s)$

Conclusion

Conclusion

- Proton decay matrix elements on the two lattice ensemble with chiral fermions at physical scale
- Three quark op. non perturbative renormalization
- New channels(vector meson, three body)
- Induced nucleon decay by DM

Reference

Reference

Reference

ABE, K. et al. Hyper-Kamiokande Design Report. 2018.

AOKI, S. et al. Nucleon decay matrix elements from lattice QCD. *Phys. Rev.*, D62, p. 014506, 2000.

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Introduction

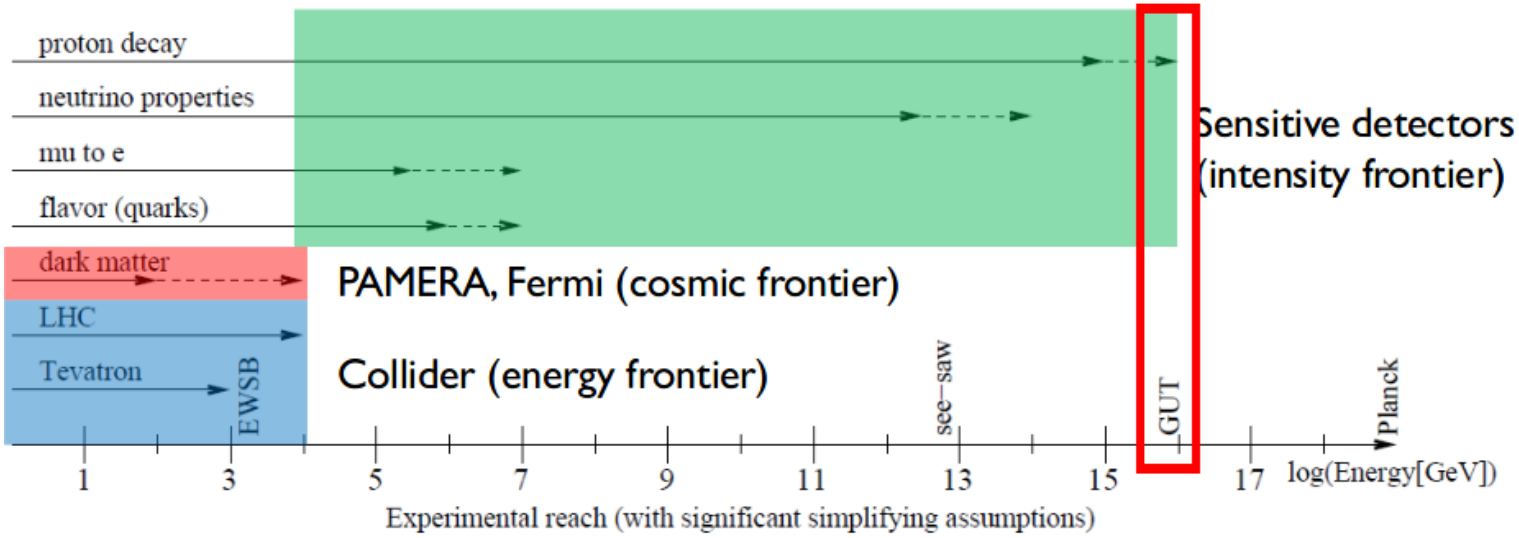

Figure 2: Energy scale of search, Zoltan Ligeti

Baryon asymmetry

Nonzero net baryon number

$\frac{n_B - \bar{n}_B}{n_\gamma} \sim 10^{-10}$

Sakharov's conditions

- At least one B violating process
- C- and CP-violation
- interactions outside of thermal equilibrium

Matrix elements

Define the ratio $R_3(t,t') = \frac{C^{3pt}(t,t')}{C_\Pi^{2pt}(t'-t,\vec{p}')\mathrm{Tr}[PC_p^{2pt}(t,\vec{p})]}\sqrt{Z_\Pi}\sqrt{Z_p}$.
As $t \to \infty$, $R_3(t,t') \to \langle \Pi(p')|O^{\Gamma\Gamma'}(q)|N(p,s)\rangle$, giving decay form factors $W_{0,1}(q^2)$

$$\mathrm{Tr}[R_3 P_L P_4] = W_0^{\Gamma L}(q^2) - \frac{iq_4}{m_N} W_1^{\Gamma L}(q^2).$$

$$\mathrm{Tr}[R_3 P_L i P_4 \gamma_j] = \frac{q_j}{m_N} W_1^{\Gamma L}(q^2)$$

Momentum transfer is chosen to be $q^2 \sim 0$: $\vec{p} = \vec{q} + \vec{p'}$

Other computations

		$\lvert\alpha\rvert$ [GeV^3]	$\lvert\beta\rvert$ [GeV^3]	
QCD model calculation	Donoghue and Goldwich [4]	0.003		Bag model
	Thomas and McKellar [7]	0.02		Bag model
	Meljanac et al. [5]	0.004		Bag model
	Ioffe [2]	0.009		Sum rule
	Krasnikov et al. [6]	0.003		Sum rule
	Ioffe and Smilga [8]	0.006		Sum rule
	Tomozawa [3]	0.006		Quark model
	Brodsky et al. [9]	0.03		
Lattice QCD $N_f = 0$	Hara et al. [10]	0.03		WF, $a = 0.11$ fm
	Bowler et al. [11]	0.013	0.010	WF, $a = 0.22$ fm
	Gavela et al. [12]	0.0056(8)	$\simeq \lvert\alpha\rvert$	WF, $a = 0.09$ fm
	JLQCD [13]	0.015(1)	0.014(1)	WF, $a = 0.09$ fm
	CP-PACS & JLQCD [14]	$0.0090(09)(^{+5}_{-19})$	$0.0096(09)(^{+6}_{-20})$	WF, continuum limit
	Aoki et al. [15]	0.0100(19)	0.0108(21)	DWF, $a = 0.15$ fm
Lattice QCD $N_f = 2$	Aoki et al. [15]	0.0118(21)	0.0118(21)	DWF, $a = 0.12$ fm
Lattice QCD $N_f = 2+1$	This work	0.0112(25)	0.0120(26)	DWF, $a = 0.12$ fm

Figure 3