$B \to \pi$, $B_s \to K$, and $B \to K$ decays from four-flavor lattice QCD

Zech Gelzer

zgelzer@illinois.edu

University of Illinois at Urbana–Champaign

Fermilab Lattice and MILC Collaborations

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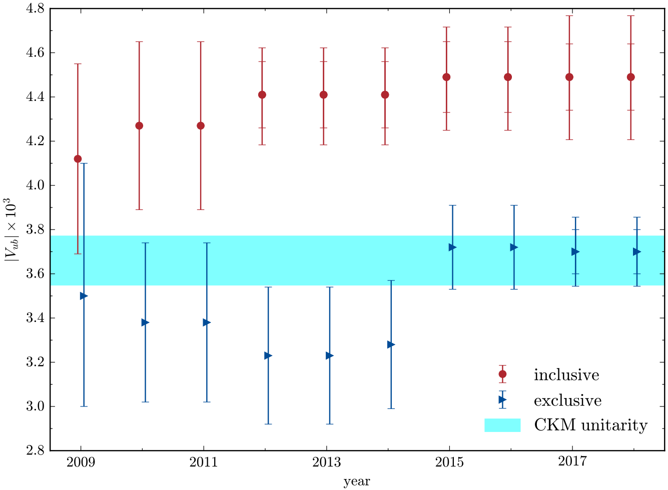
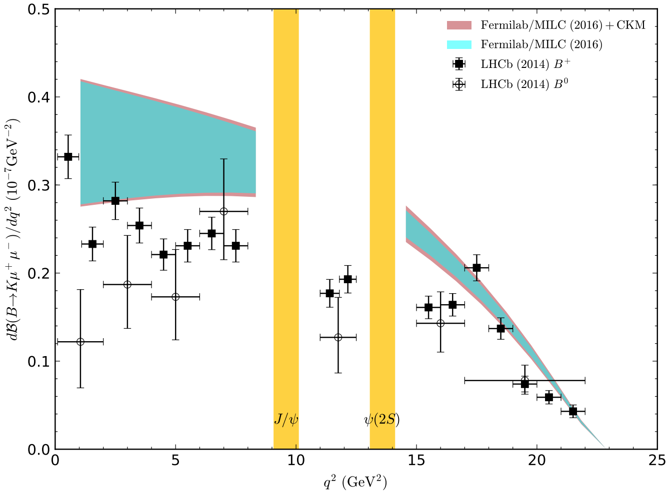
Overview

Status of $|V_{ub}|$



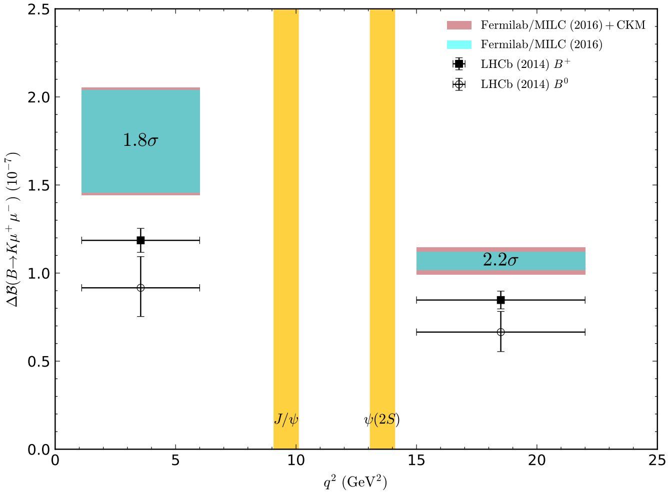
Update of plot in arXiv:1711.08085

Tension in $B \to K\mu^+\mu^-$



Older, less precise experiments omitted; cf. PRD:93.034005 [arXiv:1510.02349]

Tension in $B \to K\mu^+\mu^-$



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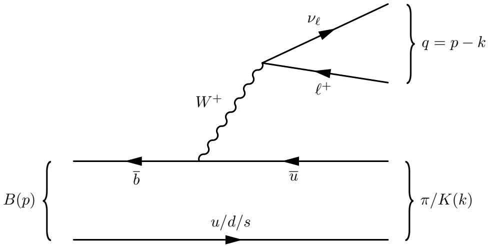
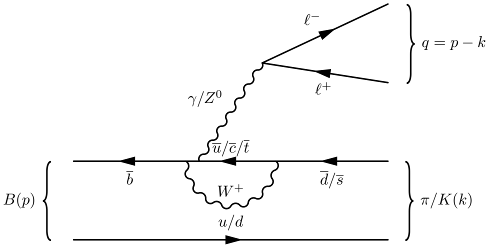
$B_{(s)} \to \pi(K)\ell\nu$: charged currents



$$\frac{d\Gamma}{dq^2} = (\text{known factors}) \times |V_{ub}|^2 \times \left\{|f_+(q^2)|^2, |f_0(q^2)|^2\right\}$$

$B \to \pi(K)\ell^+\ell^-$: flavor-changing neutral currents



$$\frac{d\Gamma}{dq^2} = \text{(known factors)} \times |V_{tb}V_{tf}^*|^2 \times \left\{|f_+(q^2)|^2, |f_0(q^2)|^2, |f_T(q^2)|^2\right\}$$

Form factors I

These transitions can be mediated by vector, scalar, and tensor currents.

Taking Lorentz and discrete symmetries into account:

$$\langle L(k)|\mathcal{V}^\mu|B(p)\rangle = f_+(q^2)\left(p^\mu + k^\mu - \frac{M_B^2 - M_L^2}{q^2}q^\mu\right) + f_0(q^2)\frac{M_B^2 - M_L^2}{q^2}q^\mu$$

$$\langle L(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_L^2}{m_b - m_q}$$

$$\langle L(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_L}\left(p^\mu k^\nu - p^\nu k^\mu\right)$$

PCVC ensures that the vector and scalar currents lead to the same f_0.

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$$= \sqrt{2M_B}\left[f_\perp(E_L)k_\perp^\mu + f_\parallel(E_L)p^\mu/M_B\right],$$

$$\langle L(k)|\mathcal{S}|B(p)\rangle = f_0(q^2)\frac{M_B^2 - M_L^2}{m_b - m_q}$$

$$\langle L(k)|\mathcal{T}^{\mu\nu}|B(p)\rangle = f_T(q^2)\frac{2}{M_B + M_L}\left(p^\mu k^\nu - p^\nu k^\mu\right)$$

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Form factors II

It is straightforward to extract the matrix elements

$$f_\perp(E_L) = \frac{\langle P|\mathcal{V}^i|B\rangle}{\sqrt{2M_B}} \frac{1}{k^i}$$

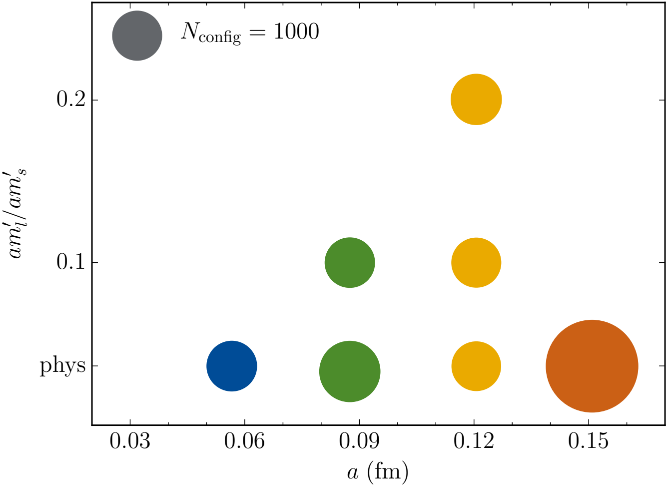
$$f_\parallel(E_L) = \frac{\langle P|\mathcal{V}^0|B\rangle}{\sqrt{2M_B}}$$

$$f_T(E_L) = \frac{M_B + M_L}{\sqrt{2M_B}} \frac{\langle P|\mathcal{T}^{0i}|B\rangle}{\sqrt{2M_B}} \frac{1}{k^i}$$

from three-point correlation functions.

Then f_+ and f_0 are linear combinations of $f_\perp$ and $f_\parallel$.

MILC (2+1+1)-flavor HISQ ensembles I



Sizes reflect number of configs analyzed in this talk.

MILC (2+1+1)-flavor HISQ ensembles II

Sea actions: | Discretization effects
- Lüscher-Weisz gluons — $O(\alpha_s^2 a^2)$
- HISQ q_l, s, c — $O(\alpha_s a^2)$

Valence actions:
- HISQ q_l, s — $O(\alpha_s a^2)$
- Clover b with Fermilab interpretation — $O(\alpha_s a, a^2)\, f\left((m_b a)^2\right)$

Parameters for physical-mass ensembles:

a ($\approx$ fm)	$N_s^3 \times N_t$	am_l'	am_s'	am_c'	κ_b'	$N_{\text{cfg}} \times N_{\text{src}}$
0.15	$32^3 \times 48$	0.002426	0.06730	0.8447	0.07732	3630×8
0.12	$48^3 \times 64$	0.001907	0.05252	0.6382	0.08574	986×8
0.088	$64^3 \times 96$	0.0012	0.0363	0.432	0.09569	1535×8
0.057	$96^3 \times 192$	0.0008	0.022	0.260	0.10604	1027×8

Correlation functions

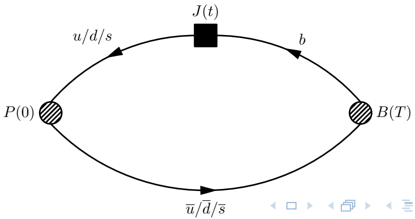
Energies of pseudoscalar mesons are extracted from two-point correlators:

$$C_2(t;\boldsymbol{k}) = \sum_{\boldsymbol{x}} e^{i\boldsymbol{k}\cdot\boldsymbol{x}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, \mathcal{O}_P^\dagger(t,\boldsymbol{x}) \right\rangle$$

$$\Rightarrow \sum_m (-1)^{m(t+1)} \frac{\left|\langle 0|\mathcal{O}_P|P^{(m)}\rangle\right|^2}{2E_L^{(m)}} e^{-E_L^{(m)}t}$$

Form factors are extracted from three-point correlators:

$$C_3^{\mu(\nu)}(t,T;\boldsymbol{k}) = \sum_{\boldsymbol{x},\boldsymbol{y}} e^{i\boldsymbol{k}\cdot\boldsymbol{y}} \left\langle \mathcal{O}_P(0,\mathbf{0})\, J^{\mu(\nu)}(t,\boldsymbol{y})\, \mathcal{O}_B^\dagger(T,\boldsymbol{x}) \right\rangle$$



Correlation-function fits

Form factors are obtained from direct fits to C_3:

$$f_\perp(E_L) = Z_\perp \frac{\widehat{C}_3^i(\boldsymbol{k})}{k^i}$$

$$f_\parallel(E_L) = Z_\parallel \widehat{C}_3^4(\boldsymbol{k})$$

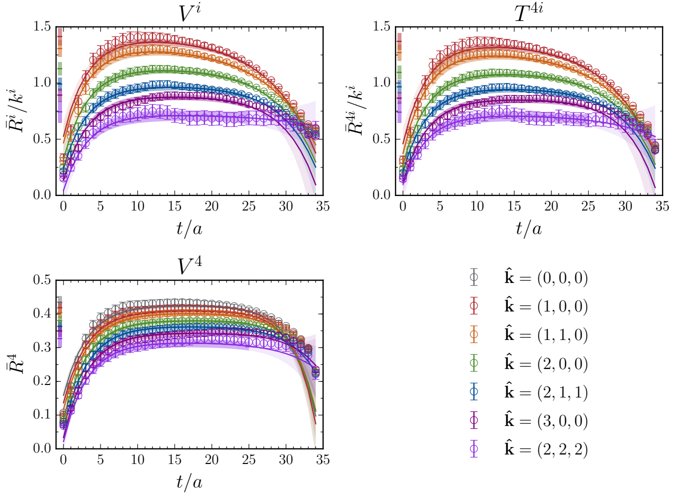
$$f_T(E_L) = Z_T \frac{M_B + M_L}{\sqrt{2M_B}} \frac{\widehat{C}_3^{4i}(\boldsymbol{k})}{k^i}$$

We introduce a blinding procedure in our current renormalizations $\mathcal{J} = Z_J J$, which use a mostly nonperturbative matching $Z_J = \rho_J \sqrt{Z_{V_{bb}^4} Z_{V_{qq}^4}}$.

Form factors are extrapolated to the chiral-continuum limit using heavy-meson rooted staggered chiral perturbation theory (HMrSχPT). [PRD:73.014515, PRD:76.014002]

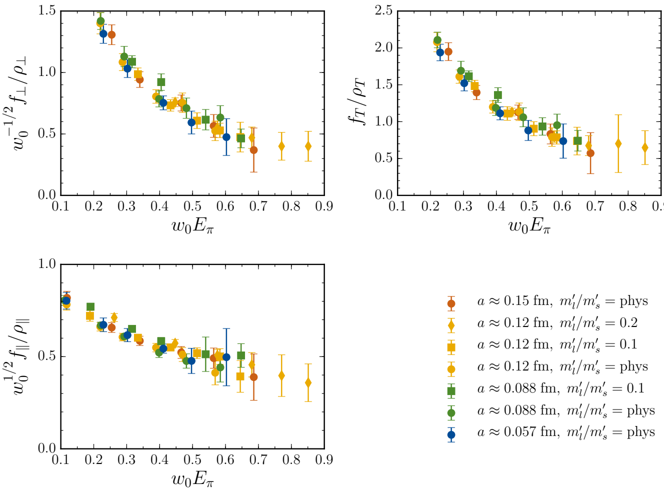
Finally, they are extended to the full kinematic range using z-expansion methods.

Correlation-function fits for $B \to K$ at $a \approx 0.057$ fm



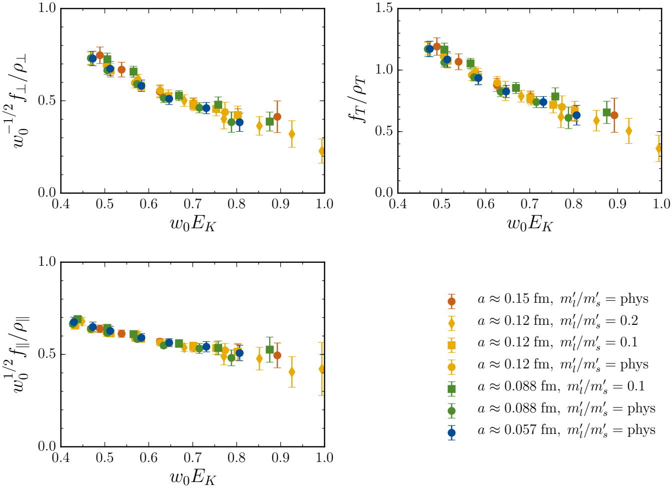
Form factors from direct fits are consistent with those from fits to the ratios $\bar{R}$.

Form factors for $B \to \pi$



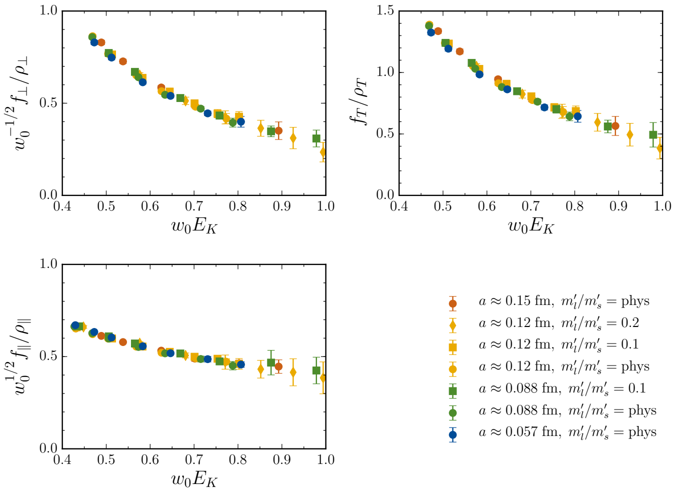
PRELIMINARY

Form factors for $B \to K$



PRELIMINARY

Form factors for $B_s \to K$



PRELIMINARY

Form factors in the chiral continuum

$SU(2)$ HMrSχPT description of form factors:

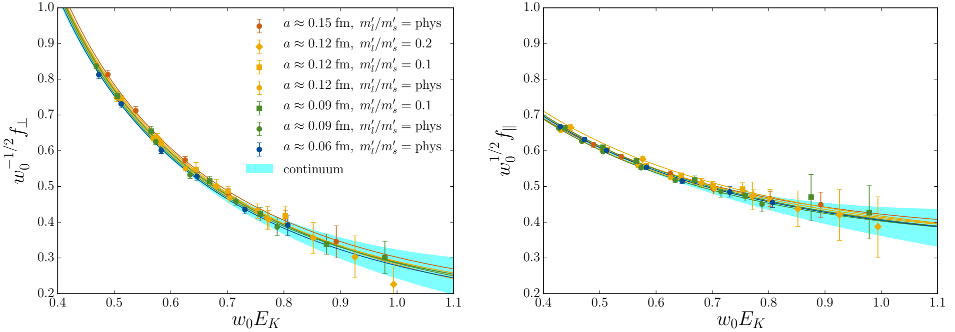
$$f_J = f_J^{(0)} \times \left(c_J^0 \left[1 + \delta f_J^{\rm logs} \right] + \delta f_J^{\rm NLO} + \delta f_J^{\rm N^2LO} + \cdots \right) \times \left(1 + \delta f_J^b \right)$$

$$f_J^{(0)} = \frac{g_\pi}{f_\pi (E_L + \Delta_{B^*})}$$

$$\delta f_J^{\rm NLO} = c_J^l \chi_l + c_J^h \chi_h + c_J^E \chi_E + c_J^{E^2} \chi_E^2 + c_J^{a^2} \chi_{a^2}$$

- Δ_{B^*} involves a 1^- or 0^+ M_{B^*}
- $\delta f_J^{\rm logs}$ denotes nonanalytic functions of m_l, a
- Perturbative part of Z_J is implemented with priors: $\tilde{\rho}_J = 1 + \tilde{\rho}_J^{(1)} \alpha_s + \tilde{\rho}_J^{(2)} \alpha_s^2$
- δf_J^b accounts for b-quark discretization effects

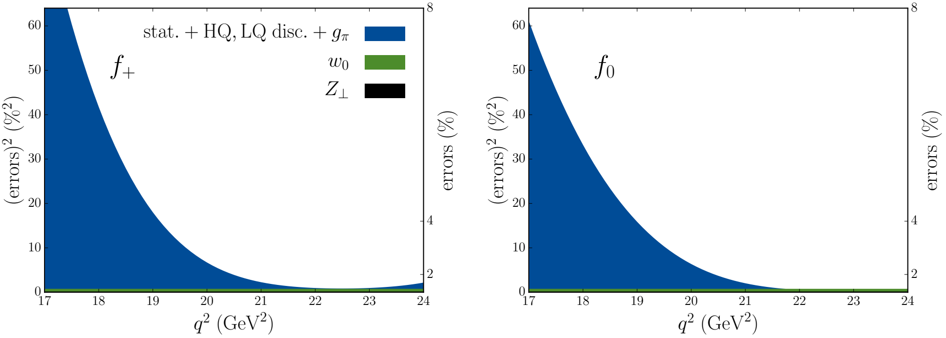
Form factors in the chiral continuum for $B_s \to K$



PRELIMINARY PRELIMINARY

- $f_\perp$ and $f_\parallel$ are fit simultaneously.
- $\mathrm{N^2LO}$ $SU(2)$ HMrSχPT is used as the central fit.

Error budget for $B_s \to K$



PRELIMINARY PRELIMINARY

- Errors at low q^2 are somewhat large due to statistics and discretization.
- z-expansion methods are then used to extrapolate the form factors to $q^2 = 0$.

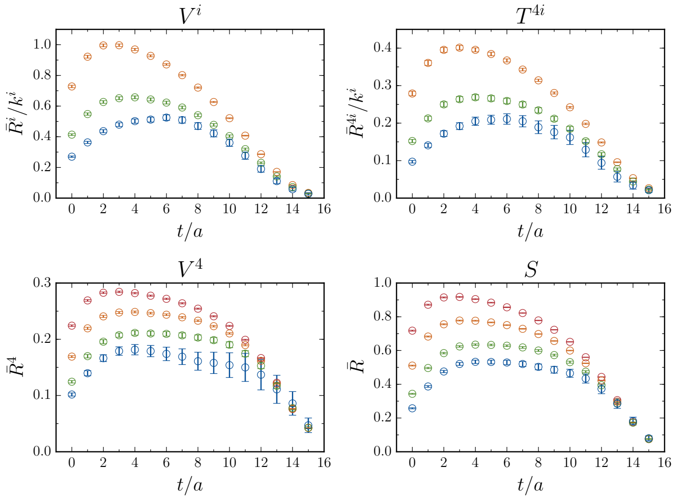
Outlook

1. Calculate and blind ρ_J
2. Finalize error budget and z expansions
3. Unblind analysis and confront experiments
    a. Determine $|V_{ub}|$ from charged-current decays
    b. Compare $\mathcal{B}$ observables from neutral-current decays to test for new physics
4. Submit papers to arXiv in the fall

Overview of related projects

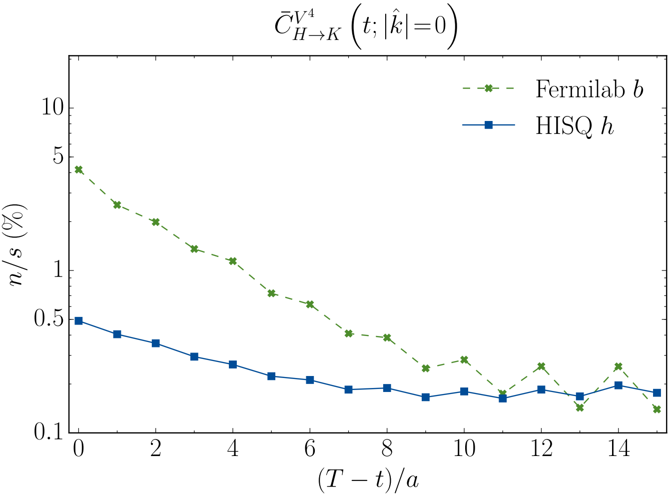
- **b** Clover b with Fermilab interpretation
  - Discretization effects of $O(\alpha_s a, a^2)\, f\big((m_b a)^2\big)$
  1. $B \to D$, $B \to D^*$
     - MILC (2+1)-flavor asqtad ensembles
     - Analysis led by Alejandro Vaquero (see tomorrow's talk)
  2. $B_s \to K$, $B \to K$, $B \to \pi$
     - Analysis led by Z.G.
- **h** HISQ h
  - Discretization effects of $O(\alpha_s a^2)\, f\big((m_h a)^2\big)$
  - $m_c \lesssim m_h \lesssim m_b$, where $am'_h \leq 0.9$
  3. $H \to L$
     - Analysis led by William Jay
  4. $H \to H'$
     - Analysis led by William Jay

Correlation functions for $H \to \pi$ at $a \approx 0.12$ fm



Data from W. Jay. PRELIMINARY

Comparisons of noise-to-signal at $a \approx 0.12$ fm



Data from W. Jay. HISQ h: $m_h = 1.4 m_c$

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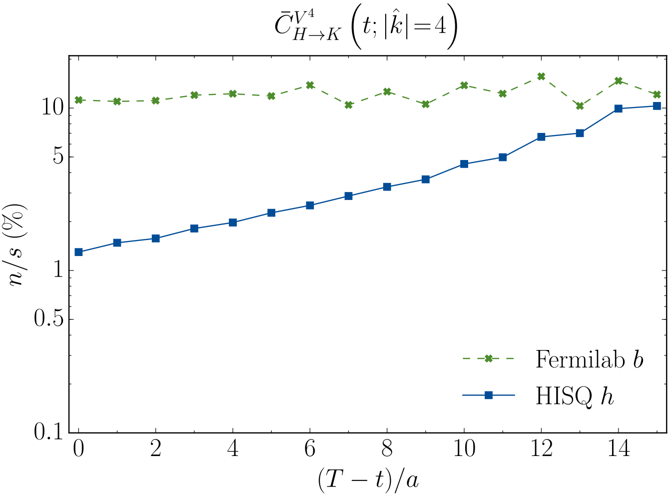


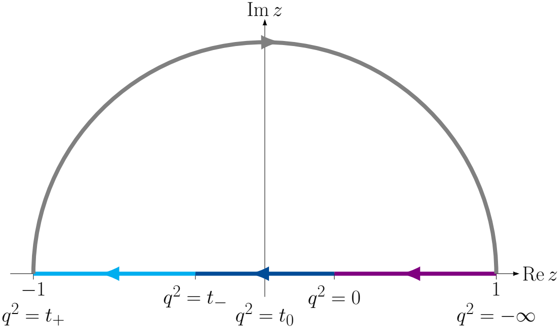
Data from W. Jay. HISQ h: $m_h = 1.4 m_c$

Thank you!

Logo by James Simone

z expansion

Conformal mapping $q^2 \mapsto |z| \leq 1$ exploits analytic structure in complex plane to extend chiral-continuum form factors to low q^2.



$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_\pm = (M_B \pm M_L)^2$

$$\Downarrow$$

$$f(q^2) = \frac{1}{1 - \frac{q^2}{M^2}} \sum_n a_n z^n(q^2)$$

- t_0^{opt} minimizes $|z|$ in physical region $\Rightarrow |z| \leq \{0.30, 0.15\}$ for $L = \{\pi, K\}$
- Smallness of $|z|$ controls truncation
- Unitarity guarantees convergence

return to analysis