$B \rightarrow \pi$, $B_s \rightarrow K$, and $B \rightarrow K$ decays from four-flavor lattice QCD

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Overview

1. Introduction
   a. Motivations
   b. Form factor definitions

2. Analysis of $B \to \pi$, $B_s \to K$, $B \to K$
   a. Actions and parameters
   b. Form factor fits

3. Outlook
Status of $|V_{ub}|$

Update of plot in arXiv:1711.08085
Tension in $B \rightarrow K \mu^+ \mu^-$

Older, less precise experiments omitted; cf. PRD:93.034005 [arXiv:1510.02349]
Tension in $B \rightarrow K \mu^+ \mu^-$

Older, less precise experiments omitted; cf. PRD:93.034005 [arXiv:1510.02349]
$B_s \rightarrow \pi(K)\ell\nu$: charged currents

\[
\frac{d\Gamma}{dq^2} = (\text{known factors}) \times |V_{ub}|^2 \times \{|f_+(q^2)|^2, |f_0(q^2)|^2\}
\]
$B \rightarrow \pi (K) \ell^+ \ell^-$: flavor-changing neutral currents

\[
\frac{d\Gamma}{dq^2} = (\text{known factors}) \times |V_{tb} V_{t\ell}^*|^2 \times \left\{ |f_+(q^2)|^2, |f_0(q^2)|^2, |f_T(q^2)|^2 \right\}
\]
Form factors I

These transitions can be mediated by vector, scalar, and tensor currents.

Taking Lorentz and discrete symmetries into account:

\[
\langle L(k)|V^\mu|B(p)\rangle = f_+(q^2) \left( p^\mu + k^\mu - \frac{M_B^2 - M_L^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_L^2}{q^2} q^\mu
\]

\[
\langle L(k)|S|B(p)\rangle = f_0(q^2) \frac{M_B^2 - M_L^2}{m_b - m_q} 
\]

\[
\langle L(k)|T^{\mu\nu}|B(p)\rangle = f_T(q^2) \frac{2}{M_B + M_L} (p^\mu k^\nu - p^\nu k^\mu)
\]

PCVC ensures that the vector and scalar currents lead to the same \( f_0 \).
Form factors I

These transitions can be mediated by vector, scalar, and tensor currents.

Taking Lorentz and discrete symmetries into account:

$$\langle L(k) | V^\mu | B(p) \rangle = f_+(q^2) \left( p^\mu + k^\mu - \frac{M_B^2 - M_L^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_B^2 - M_L^2}{q^2} q^\mu$$

$$= \sqrt{2M_B} \left[ f_\perp (E_L) k^\mu_\perp + f_\parallel (E_L) p^\mu / M_B \right],$$

$$\langle L(k) | S | B(p) \rangle = f_0(q^2) \frac{M_B^2 - M_L^2}{m_b - m_q}$$

$$\langle L(k) | T^{\mu\nu} | B(p) \rangle = f_T(q^2) \frac{2}{M_B + M_L} \left( p^\mu k^\nu - p^\nu k^\mu \right)$$

PCVC ensures that the vector and scalar currents lead to the same $f_0$. 
Form factors II

It is straightforward to extract the matrix elements

\[ f_\perp (E_L) = \frac{\langle P|\mathcal{V}_i|B \rangle}{\sqrt{2M_B}} \frac{1}{k^i} \]

\[ f_\parallel (E_L) = \frac{\langle P|\mathcal{V}_0|B \rangle}{\sqrt{2M_B}} \]

\[ f_T (E_L) = \frac{M_B + M_L}{\sqrt{2M_B}} \frac{\langle P|\mathcal{T}^0_i|B \rangle}{\sqrt{2M_B}} \frac{1}{k^i} \]

from three-point correlation functions.

Then \( f_+ \) and \( f_0 \) are linear combinations of \( f_\perp \) and \( f_\parallel \).
MILC (2+1+1)-flavor HISQ ensembles I

Sizes reflect number of configs analyzed in this talk.
MILC (2+1+1)-flavor HISQ ensembles II

Sea actions:
- Lüscher-Weisz gluons
- HISQ $q_l, s, c$

Valence actions:
- HISQ $q_l, s$
- Clover $b$ with Fermilab interpretation

Discretization effects
- $O(\alpha_s^2 a^2)$
- $O(\alpha_s a^2)$
- $O(\alpha_s a, a^2) f((m_b a)^2)$

Parameters for physical-mass ensembles:

<table>
<thead>
<tr>
<th>$a$ ($\approx$ fm)</th>
<th>$N_s^3 \times N_t$</th>
<th>$am'_l$</th>
<th>$am'_s$</th>
<th>$am'_c$</th>
<th>$\kappa'_b$</th>
<th>$N_{cfg} \times N_{src}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.15</td>
<td>$32^3 \times 48$</td>
<td>0.002426</td>
<td>0.06730</td>
<td>0.8447</td>
<td>0.07732</td>
<td>3630 \times 8</td>
</tr>
<tr>
<td>0.12</td>
<td>$48^3 \times 64$</td>
<td>0.001907</td>
<td>0.05252</td>
<td>0.6382</td>
<td>0.08574</td>
<td>986 \times 8</td>
</tr>
<tr>
<td>0.088</td>
<td>$64^3 \times 96$</td>
<td>0.0012</td>
<td>0.0363</td>
<td>0.432</td>
<td>0.09569</td>
<td>1535 \times 8</td>
</tr>
<tr>
<td>0.057</td>
<td>$96^3 \times 192$</td>
<td>0.0008</td>
<td>0.022</td>
<td>0.260</td>
<td>0.10604</td>
<td>1027 \times 8</td>
</tr>
</tbody>
</table>
Energies of pseudoscalar mesons are extracted from two-point correlators:

\[ C_2(t; \mathbf{k}) = \sum_{\mathbf{x}} e^{i \mathbf{k} \cdot \mathbf{x}} \left\langle \mathcal{O}_P(0, 0) \mathcal{O}_P^\dagger(t, \mathbf{x}) \right\rangle \]

\[ \Rightarrow \sum_m (-1)^{m(t+1)} \left| \left\langle 0 \left| \mathcal{O}_P \right| P^{(m)} \right\rangle \right|^2 \frac{2}{2E_L^{(m)}} \cdot e^{-E_L^{(m)} t} \]

Form factors are extracted from three-point correlators:

\[ C_3^{\mu(\nu)}(t, T; \mathbf{k}) = \sum_{\mathbf{x}, \mathbf{y}} e^{i \mathbf{k} \cdot \mathbf{y}} \left\langle \mathcal{O}_P(0, 0) J^{\mu(\nu)}(t, \mathbf{y}) \mathcal{O}_B^\dagger(T, \mathbf{x}) \right\rangle \]
Correlation-function fits

Form factors are obtained from direct fits to $C_3$:

\[ f_\perp(E_L) = Z_\perp \frac{\hat{C}_3^i(k)}{k^i} \]
\[ f_\parallel(E_L) = Z_\parallel \hat{C}_3^4(k) \]
\[ f_T(E_L) = Z_T \frac{M_B + M_L}{\sqrt{2M_B}} \frac{\hat{C}_3^{4i}(k)}{k^i} \]

We introduce a blinding procedure in our current renormalizations $J = Z_J J$, which use a mostly nonperturbative matching $Z_J = \rho_J \sqrt{Z_{V_{bb}} Z_{V_{qq}}}$.

Form factors are extrapolated to the chiral-continuum limit using heavy-meson rooted staggered chiral perturbation theory (HMrS$\chi$PT). [PRD:73.014515, PRD:76.014002]

Finally, they are extended to the full kinematic range using $z$-expansion methods.
Correlation-function fits for $B \to K$ at $a \approx 0.057$ fm

Form factors from direct fits are consistent with those from fits to the ratios $\tilde{R}$. 
Form factors for $B \to \pi$

- $w_0 \sqrt{f_\perp / \rho_\perp}$
- $w_0 \sqrt{f_\parallel / \rho_\parallel}$

- $a \approx 0.15$ fm, $m'_t / m'_s = \text{phys}$
- $a \approx 0.12$ fm, $m'_t / m'_s = 0.2$
- $a \approx 0.12$ fm, $m'_t / m'_s = 0.1$
- $a \approx 0.12$ fm, $m'_t / m'_s = \text{phys}$
- $a \approx 0.088$ fm, $m'_t / m'_s = 0.1$
- $a \approx 0.088$ fm, $m'_t / m'_s = \text{phys}$
- $a \approx 0.057$ fm, $m'_t / m'_s = \text{phys}$
Form factors for $B \rightarrow K$

\begin{align*}
\frac{1}{2} w_0 f_\perp / \rho_\perp & \approx 0.15 \text{ fm}, \quad m'_l/m'_s = \text{phys} \\
\frac{1}{2} w_0 f_\parallel / \rho_\parallel & \approx 0.12 \text{ fm}, \quad m'_l/m'_s = 0.2 \\
\frac{1}{2} w_0 f_\perp / \rho_\perp & \approx 0.12 \text{ fm}, \quad m'_l/m'_s = 0.1 \\
\frac{1}{2} w_0 f_\parallel / \rho_\parallel & \approx 0.12 \text{ fm}, \quad m'_l/m'_s = \text{phys} \\
\frac{1}{2} w_0 f_\perp / \rho_\perp & \approx 0.088 \text{ fm}, \quad m'_l/m'_s = 0.1 \\
\frac{1}{2} w_0 f_\parallel / \rho_\parallel & \approx 0.088 \text{ fm}, \quad m'_l/m'_s = \text{phys} \\
\frac{1}{2} w_0 f_\perp / \rho_\perp & \approx 0.057 \text{ fm}, \quad m'_l/m'_s = \text{phys}
\end{align*}
Form factors for $B_s \rightarrow K$

\[ w_0^{1/2} f_\perp / \rho_\perp \]

\[ w_0^{1/2} f_\parallel / \rho_\parallel \]

\[ a \approx 0.15 \text{ fm}, \ m'/m_s = \text{phys} \]
\[ a \approx 0.12 \text{ fm}, \ m'/m_s = 0.2 \]
\[ a \approx 0.12 \text{ fm}, \ m'/m_s = 0.1 \]
\[ a \approx 0.088 \text{ fm}, \ m'/m_s = 0.1 \]
\[ a \approx 0.088 \text{ fm}, \ m'/m_s = \text{phys} \]
\[ a \approx 0.057 \text{ fm}, \ m'/m_s = \text{phys} \]
Form factors in the chiral continuum

\(SU(2)\) HMrS\(\chi\)PT description of form factors:

\[f_J = f_J^{(0)} \times \left( c_J^0 \left[ 1 + \delta f_J^{\text{logs}} \right] + \delta f_J^{\text{NLO}} + \delta f_J^{\text{N}^2\text{LO}} + \cdots \right) \times \left( 1 + \delta f_J^b \right)\]

\[f_J^{(0)} = \frac{g_\pi}{f_\pi (E_L + \Delta_{B^*})}\]

\[\delta f_J^{\text{NLO}} = c_J^l \chi_l + c_J^h \chi_h + c_J^E \chi_E + c_J^{E^2} \chi_{E^2} + c_J^{a^2} \chi_{a^2}\]

- \(\Delta_{B^*}\) involves a \(1^-\) or \(0^+\) \(M_{B^*}\)
- \(\delta f_J^{\text{logs}}\) denotes nonanalytic functions of \(m_l, a\)
- Perturbative part of \(Z_J\) is implemented with priors: \(\tilde{\rho}_J = 1 + \tilde{\rho}_J^{(1)} \alpha_s + \tilde{\rho}_J^{(2)} \alpha_s^2\)
- \(\delta f_J^b\) accounts for \(b\)-quark discretization effects
Form factors in the chiral continuum for $B_s \to K$

- $f_\perp$ and $f_\parallel$ are fit simultaneously.
- $N^2$LO $SU(2)$ HMrSχPT is used as the central fit.
Error budget for $B_s \rightarrow K$

- Errors at low $q^2$ are somewhat large due to statistics and discretization.
- $z$-expansion methods are then used to extrapolate the form factors to $q^2 = 0$. 

**PRELIMINARY**
Outlook

1. Calculate and blind $\rho_J$

2. Finalize error budget and $z$ expansions

3. Unblind analysis and confront experiments
   
   a. Determine $|V_{ub}|$ from charged-current decays
   
   b. Compare $B$ observables from neutral-current decays to test for new physics

4. Submit papers to arXiv in the fall
Overview of related projects

- Clover $b$ with Fermilab interpretation
  - Discretization effects of $O(\alpha_s a, a^2) f((m_b a)^2)$
  - $B \to D$, $B \to D^*$
    - MILC (2+1)-flavor asqtad ensembles
    - Analysis led by Alejandro Vaquero (see tomorrow’s talk)
  - $B_s \to K$, $B \to K$, $B \to \pi$
    - Analysis led by Z.G.

- HISQ $h$
  - Discretization effects of $O(\alpha_s a^2) f((m_h a)^2)$
  - $m_c \lesssim m_h \lesssim m_b$, where $am'_h \leq 0.9$
  - $H \to L$
    - Analysis led by William Jay
  - $H \to H'$
    - Analysis led by William Jay
Correlation functions for $H \rightarrow \pi$ at $a \approx 0.12 \text{ fm}$
Comparisons of noise-to-signal at $a \approx 0.12$ fm

$$\bar{C}^{V^4}_{H \rightarrow K} \left( t; |\hat{k}|=0 \right)$$

Data from W. Jay. HISQ $h$: $m_h = 1.4 m_c$
Comparisons of noise-to-signal at $a \approx 0.12$ fm

Data from W. Jay. HISQ $h$: $m_h = 1.4 m_c$
Thank you!

Logo by James Simone
Conformal mapping $q^2 \mapsto |z| \leq 1$ exploits analytic structure in complex plane to extend chiral-continuum form factors to low $q^2$.

\[
 z(q^2) = \frac{\sqrt{t^+ - q^2} - \sqrt{t^+ - t_0}}{\sqrt{t^+ - q^2} + \sqrt{t^+ - t_0}}
\]

where $t_\pm = (M_B \pm M_L)^2$

\[
 f(q^2) = \frac{1}{1 - \frac{q^2}{M^2}} \sum_n a_n z^n(q^2)
\]

- $t_0^{\text{opt}}$ minimizes $|z|$ in physical region $\Rightarrow |z| \leq \{0.30, 0.15\}$ for $L = \{\pi, K\}$
- Smallness of $|z|$ controls truncation
- Unitarity guarantees convergence