The $B \to D^* \ell \nu$ semileptonic decay at non-zero recoil and its implications for $|V_{cb}|$ and $R(D^*)$

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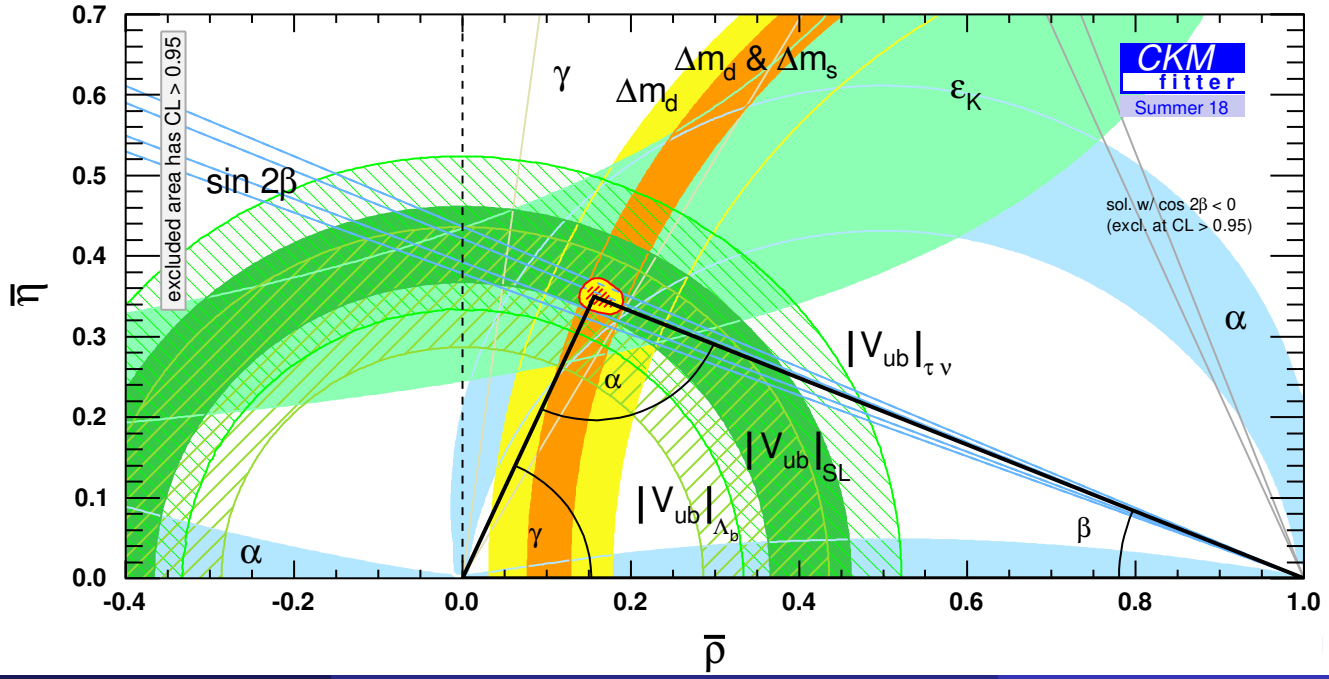
The V_{cb} matrix element: Tensions

$$\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

$\lvert V_{cb}\rvert\,(\cdot 10^{-3})$	PDG 2016	PDG 2018
Exclusive	39.2 ± 0.7	41.9 ± 2.0
Inclusive	42.2 ± 0.8	42.2 ± 0.8

- Matrix must be unitary (preserve the norm)
- BUT **current tensions (2019) stand at** $\approx 2\sigma - 3\sigma$



The V_{cb} matrix element: Measurement from exclusive processes

$$\underbrace{\frac{d\Gamma}{dw}\left(\bar{B}\to D^*\ell\bar{\nu}_\ell\right)}_{\text{Experiment}} = \underbrace{\frac{G_F^2 m_B^5}{48\pi^2}(w^2-1)^{\frac{1}{2}}P(w)\left|\eta_{ew}\right|^2}_{\text{Known factors}}\underbrace{\left|\mathcal{F}(w)\right|^2}_{\text{Theory}}\left|V_{cb}\right|^2$$

- The amplitude $\mathcal{F}$ must be calculated in the theory
  - Extremely difficult task, QCD is non-perturbative
- Can use effective theories (HQET) to say something about $\mathcal{F}$
  - Separate light (non-perturbative) and heavy degrees of freedom as $m_Q \to \infty$
  - $\lim_{m_Q\to\infty}\mathcal{F}(w) = \xi(w)$, which is the Isgur-Wise function
  - **We don't know what $\xi(w)$ looks like, but we know $\xi(1) = 1$**
  - At large (but finite) mass $\mathcal{F}(w)$ receives corrections $O\left(\alpha_s, \frac{\Lambda_{QCD}}{m_Q}\right)$
- Reduction in the phase space $(w^2-1)^{\frac{1}{2}}$ limits experimental results at $w \approx 1$
  - Need to extrapolate $\left|V_{cb}\right|^2\left|\eta_{ew}\mathcal{F}(w)\right|^2$ to $w = 1$
  - This extrapolation is done using well established parametrizations

The V_{cb} matrix element: The parametrization issue

All the parametrizations perform an expansion in the z parameter

$$z = \frac{\sqrt{w+1} - \sqrt{2N}}{\sqrt{w+1} + \sqrt{2N}}$$

- Boyd-Grinstein-Lebed (BGL) *Phys. Rev. Lett. 74 (1995) 4603-4606* *Phys.Rev. D56 (1997) 6895-6911* *Nucl.Phys. B461 (1996) 493-511*

$$f_X(w) = \frac{1}{B_{f_X}(z)\phi_{f_X}(z)} \sum_{n=0}^{\infty} a_n z^n$$

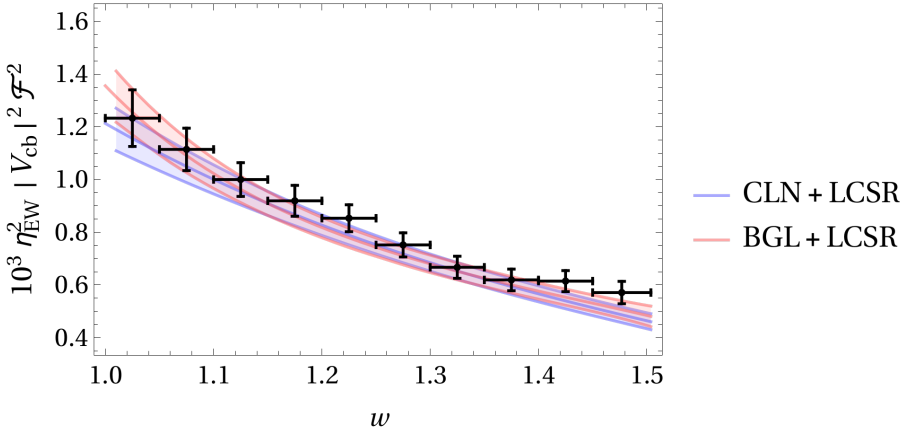
  - B_{f_X} Blaschke factors, includes contributions from the poles
  - ϕ_{f_X} is called *outer function* and must be computed for each form factor
  - Weak unitarity constraints $\sum_n |a_n|^2 \leq 1$

- Caprini-Lellouch-Neubert (CLN) *Nucl. Phys. B530 (1998) 153-181*

$$\mathcal{F}(w) \propto 1 - \rho^2 z + c z^2 - d z^3, \quad \text{with } c = f_c(\rho),\, d = f_d(\rho)$$

  - Relies strongly on HQET, spin symmetry and (old) inputs
  - Tightly constrains $\mathcal{F}(w)$: four independent parameters, one relevant at $w = 1$

The V_{cb} matrix element: The parametrization issue



From *Phys. Lett. B769 (2017) 441-445* using Belle data from arXiv:1702.01521 and the Fermilab/MILC'14 value at zero recoil

- CLN seems to underestimate the slope at low recoil
- The BGL value of $|V_{cb}|$ is compatible with the inclusive one

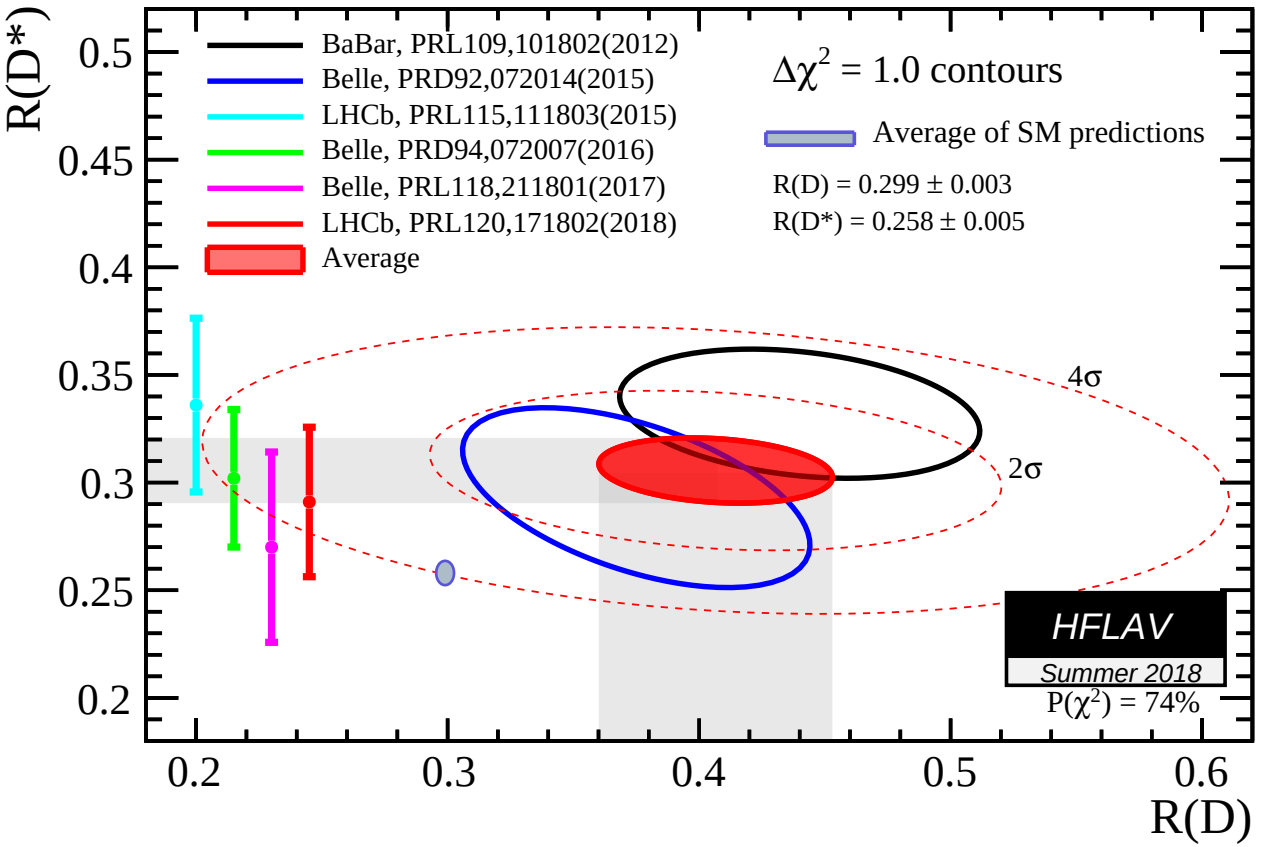
$$|V_{cb}| = 41.7 \pm 2.0(\times 10^{-3})$$

- Latest Belle dataset and Babar analysis seem to contradict this picture
  - From Babar's paper arXiv:1903.10002 **BGL is compatible with CLN and far from the inclusive value**
  - Belle's paper arXiv:1809.03290v3 finds **similar results in its last revision**
- The discrepancy inclusive-exclusive is not well understood
- Data at $w \gtrsim 1$ is **urgently needed** to settle the issue
- Experimental measurements perform badly at low recoil

We would benefit enormously from a high precision lattice calculation at $w \gtrsim 1$

The V_{cb} matrix element: Tensions in lepton universality

$$R\left(D^{(*)}\right) = \frac{\mathcal{B}\left(B \to D^{(*)}\tau\nu_\tau\right)}{\mathcal{B}\left(B \to D^{(*)}\ell\nu_\ell\right)}$$



- Current $\approx 3\sigma - 4\sigma$ tension with the SM

Calculating V_{cb} on the lattice: Formalism

- Form factors

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{V}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} = \frac{1}{2}\epsilon^{\nu *}\varepsilon^{\mu\nu}_{\rho\sigma}v_B^\rho v_{D^*}^\sigma \boldsymbol{h_V}(w)$$

$$\frac{\langle D^*(p_{D^*},\epsilon^\nu)|\,\mathcal{A}^\mu\,|\bar{B}(p_B)\rangle}{2\sqrt{m_B\,m_{D^*}}} =$$

$$\frac{i}{2}\epsilon^{\nu *}\left[g^{\mu\nu}\,(1+w)\,\boldsymbol{h_{A_1}}(w) - v_B^\nu\,(v_B^\mu \boldsymbol{h_{A_2}}(w) + v_{D^*}^\mu \boldsymbol{h_{A_3}}(w))\right]$$

- $\mathcal{V}$ and $\mathcal{A}$ are the vector/axial currents in the continuum
- The h_X enter in the definition of $\mathcal{F}$
- We can calculate $h_{A_{1,2,3},V}$ directly from the lattice

Calculating V_{cb} on the lattice: Formalism

- Helicity amplitudes

$$H_{\pm} = \sqrt{m_B\, m_{D^*}}(w+1)\left(\boldsymbol{h_{A_1}}(w) \mp \sqrt{\frac{w-1}{w+1}}\boldsymbol{h_V}(w)\right)$$

$$H_0 = \sqrt{m_B\, m_{D^*}}(w{+}1)m_B\left[(w-r)\boldsymbol{h_{A_1}}(w) - (w-1)\left(r\,\boldsymbol{h_{A_2}}(w) + \boldsymbol{h_{A_3}}(w)\right)\right]/\sqrt{q^2}$$

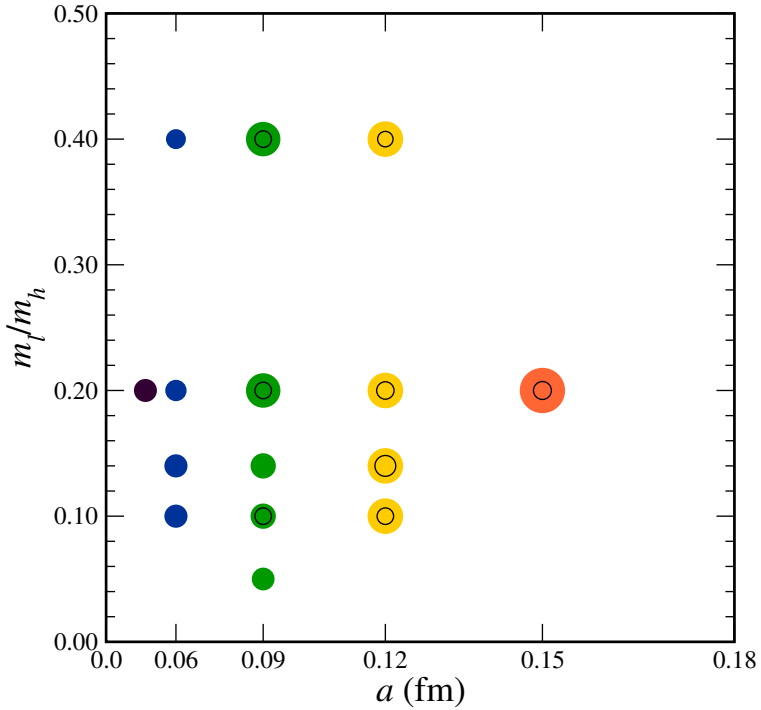
$$H_S = \sqrt{\frac{w^2-1}{r(1+r^2-2wr)}}\left[(1+w)\boldsymbol{h_{A_1}}(w) + (wr-1)\boldsymbol{h_{A_2}}(w) + (r-w)\boldsymbol{h_{A_3}}(w)\right]$$

- Form factor in terms of the helicity amplitudes

$$\chi(w)\,|\mathcal{F}|^2 = \frac{1-2wr+r^2}{12m_B\, m_{D^*}\,(1-r)^2}\left(H_0^2(w) + H_+^2(w) + H_-^2(w)\right)$$

Introduction: Available data and simulations

- Using 15 $N_f = 2 + 1$ MILC ensembles of sea asqtad quarks
- The heavy quarks are treated using the Fermilab action



Analysis: Probing different ratios

- In our previous talks we have shown some differences between experimental results of $|\mathcal{F}|^2$ at large recoil and our predictions
- The only missing puzzle in our calculation were the discretization errors, which have been preliminarly included in our chiral-continuum extrapolation
- We were expecting the discretization errors to account for this different behavior at large recoil

- Our strategy so far:
  - Fit the D^* two-points at zero and non-zero momentum
  - Use the fit results for the overlap factors and the energies to remove the extra factors arising in the ratios

Example: The double ratio

$$\frac{C^{3pt,A_j}_{B\to D^*}(p_\perp,t,T)\,C^{3pt,A_j}_{D^*\to B}(p_\perp,t,T)}{C^{3pt,V^4}_{D^*\to D^*}(0,t,T)\,C^{3pt,V^4}_{B\to B}(0,t,T)} =$$

$$\frac{M_{D^*}}{E_{D^*}(p_\perp)}\frac{Z^2_{D^*}(p_\perp)}{Z^2_{D^*}(0)}e^{-(E_{D^*}(p_\perp)-M_{D^*})T}\left(\frac{1+w}{2}h_{A_1}(w)\right)^2$$

Analysis: Probing different ratios

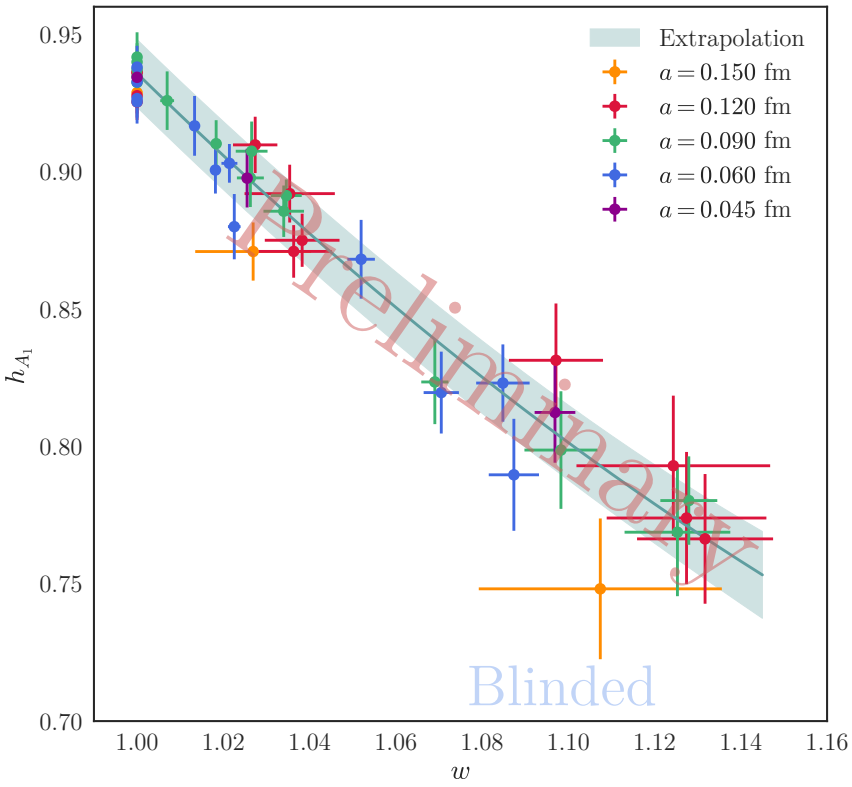
- We tried an alternative procedure that differs on the way the discretization errors are accounted for, specially at large recoil
- This procedure can act as a crosscheck of our results
- Remove the Z factors using a different ratio (not fit results)
- New ratio

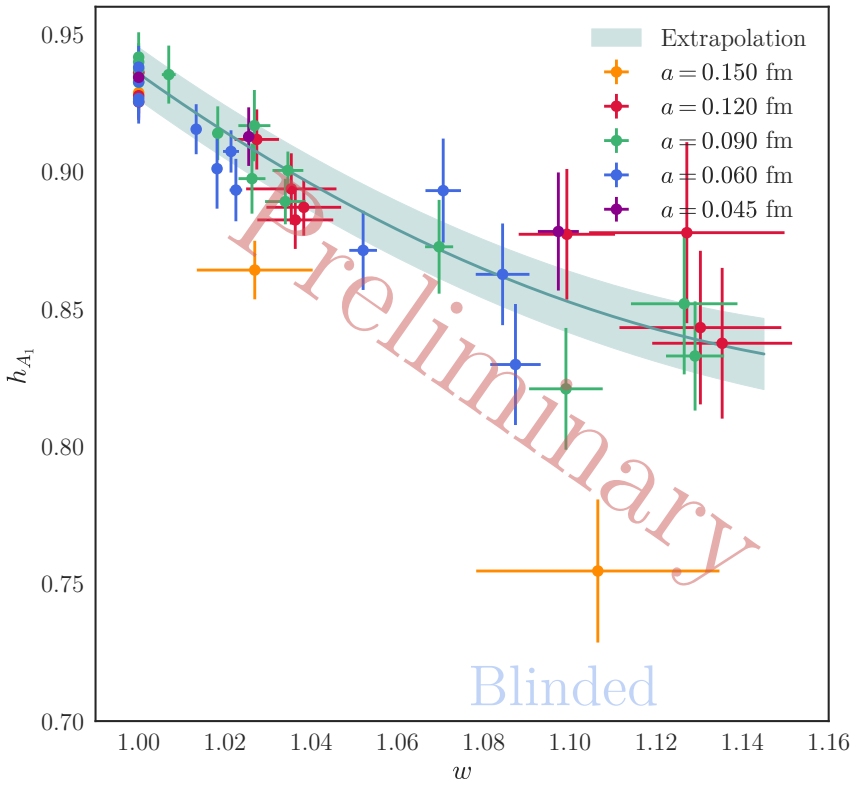
$$\frac{C^{3pt,A_1}_{B\to D^*}(p_\perp,t,T)}{C^{3pt,A_1}_{B\to D^*}(0,t,T)} \to \frac{C^{3pt,A_1}_{B\to D^*}(p_\perp,t,T)}{C^{3pt,A_1}_{B\to D^*}(0,t,T)} \times \sqrt{\frac{C^{2pt}_{D^*}(0,t)}{C^{2pt}_{D^*}(p_\perp,t)}}$$

- We still need to remove the energy factors
- The 2pts are averaged over neighbouring points

The main difference between the new and the old ratio is related to how the discretization (and statistical) errors affect the large momentum behavior

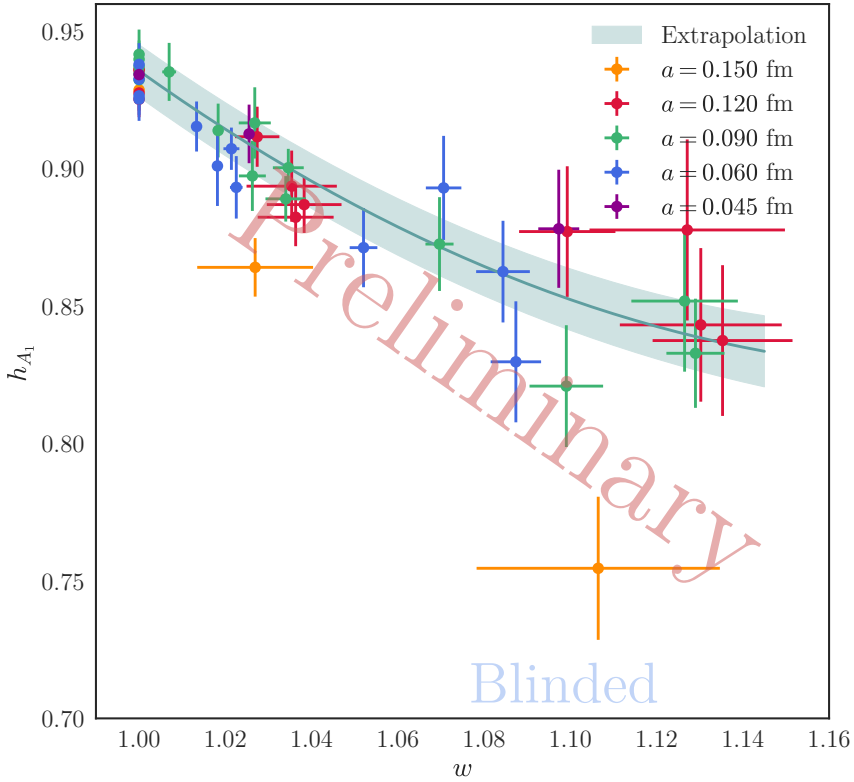
Results: Chiral-continuum fits

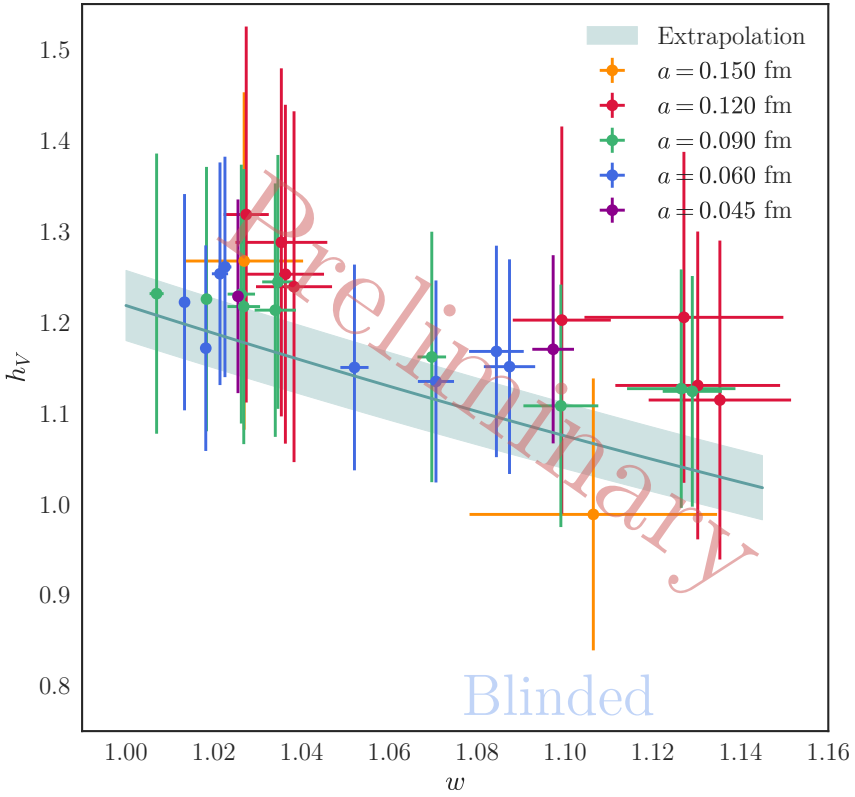




- **Left** Old fit, **Right** New fit. Preliminary blinded results.
- Both plots differ on the accounting of discretization effects, which seem to be large at large recoil

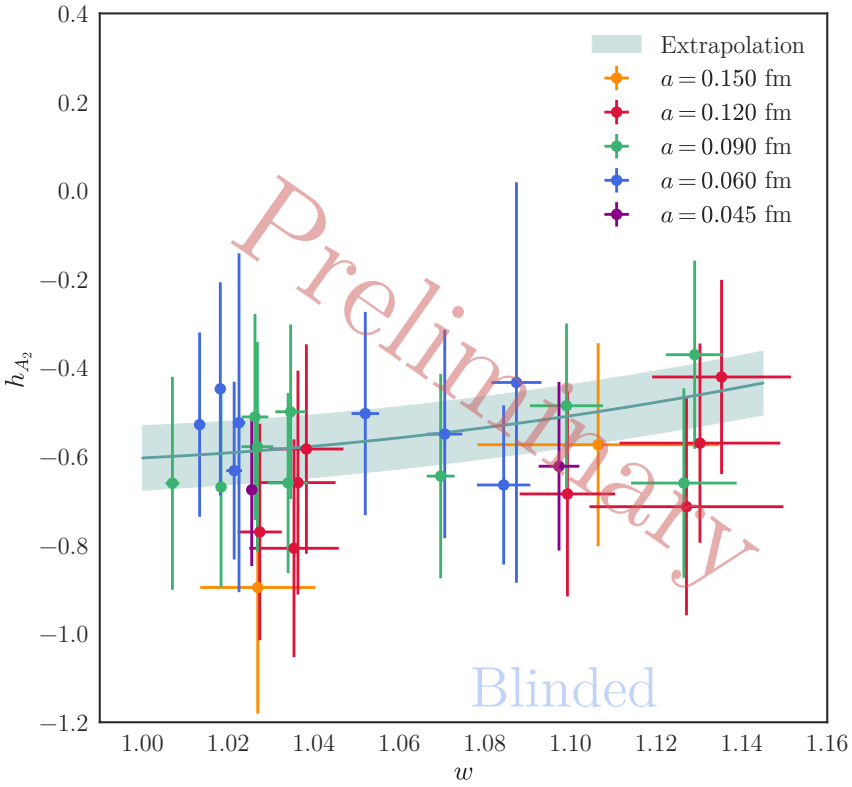
Results: Chiral-continuum fits

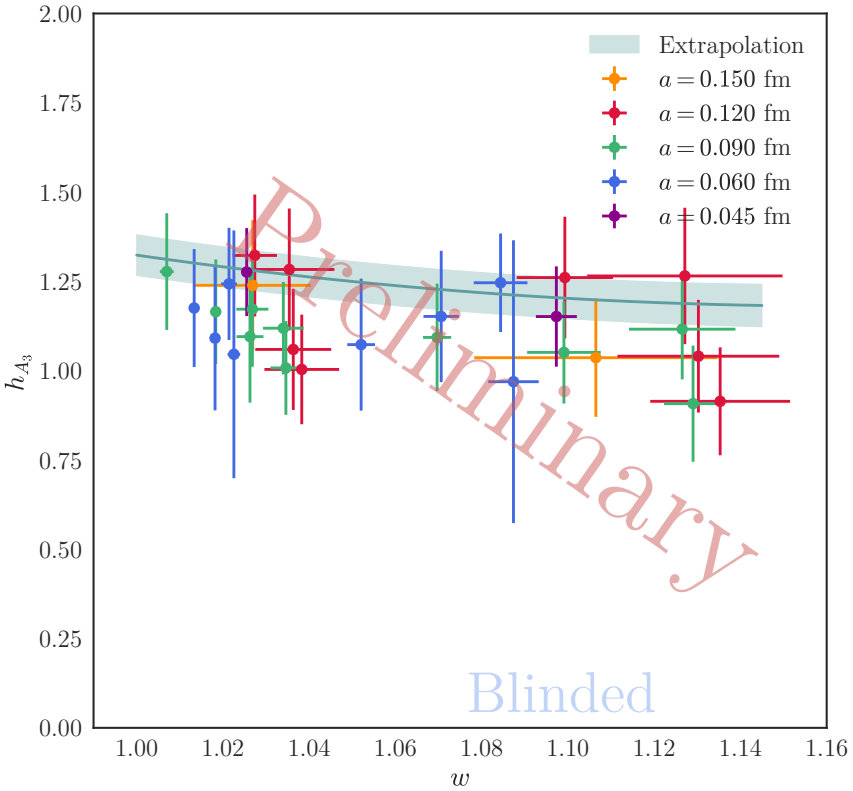




- Preliminary **blinded** results

Results: Chiral-continuum fits





- Preliminary **blinded** results

Analysis: Preliminary error budget

Our preliminary chiral-continuum extrapolation includes all the errors, and we show the most significant ones in the error budget

Source	h_V (%)	h_{A_1} (%)	h_{A_2} (%)	h_{A_3} (%)
Statistics	1.1	0.4	4.9	1.9
Isospin effects	0.0	0.0	0.6	0.3
χPT/cont. extrapolation	**1.9**	**0.7**	**6.3**	**2.9**
Matching	*1.5*	*0.4*	*0.1*	*1.5*
*Heavy quark discretization**	*2.5*	*1.2*	*9.0*	*6.0*

Errors at $w = 1.10$

***Preliminary estimate, analysis in progress**

- The inclusion of the discretization errors in the chiral-continuum extrapolation puts in evidence that **the discretization errors are the most important contribution to the final error**
- Our discretization errors **are not final** and must be crosschecked carefully
- **Bold** marks errors to be reduced/removed when using HISQ for light quarks
- *Italic* marks errors to be reduced/removed when using HISQ for heavy quarks

Analysis: z-Expansion

- The BGL expansion is performed on different (more convenient) form factors

Phys.Lett. **B**769, 441 (2017), Phys.Lett. **B**771, 359 (2017)

$$g = \frac{h_V(w)}{\sqrt{m_B\, m_{D^*}}} = \frac{1}{\phi_g(z) B_g(z)} \sum_j a_j z^j$$

$$f = \sqrt{m_B\, m_{D^*}}(1+w) h_{A_1}(w) = \frac{1}{\phi_f(z) B_f(z)} \sum_j b_j z^j$$

$$\mathcal{F}_1 = \sqrt{q^2} H_0 = \frac{1}{\phi_{\mathcal{F}_1}(z) B_{\mathcal{F}_1}(z)} \sum_j c_j z^j$$

$$\mathcal{F}_2 = \frac{\sqrt{q^2}}{m_{D^*}\sqrt{w^2-1}} H_S = \frac{1}{\phi_{\mathcal{F}_2}(z) B_{\mathcal{F}_2}(z)} \sum_j d_j z^j$$

- Constraint $\mathcal{F}_1(z=0) = (m_B - m_{D^*}) f(z=0)$
- Constraint $(1+w) m_B^2 (1-r) \mathcal{F}_1(z = z_{\rm Max}) = (1+r)\mathcal{F}_2(z = z_{\rm Max})$
- BGL (weak) unitarity constraints (all HISQ will use strong constraints)

$$\sum_j a_j^2 \leq 1, \qquad \sum_j b_j^2 + c_j^2 \leq 1, \qquad \sum_j d_j^2 \leq 1$$

Analysis: z expansion fit procedure

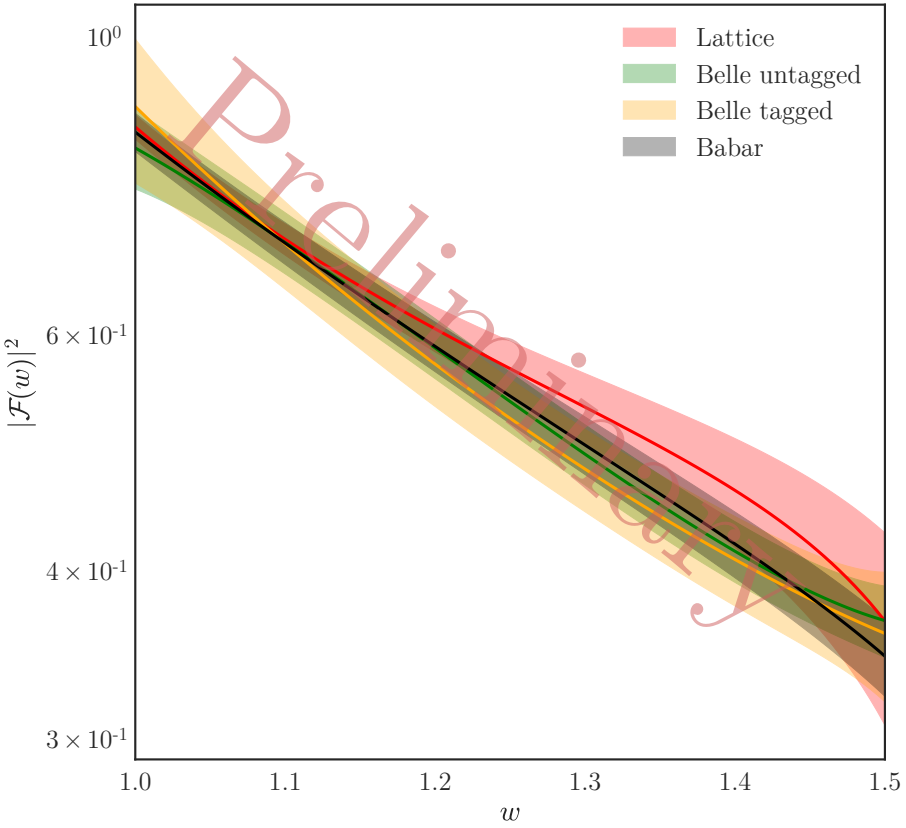
- Several different datasets
  - Our lattice data
  - BaBar BGL fit arXiv:1903.10002
  - Belle tagged dataset arXiv:1702.01521
  - Belle untagged dataset arXiv:1809.03290
- Several different fits
  - Lattice form factors only
  - Experimental data only (one fit per dataset)
  - Joint fit lattice + experimental data
- Each dataset is given in a different format, and requires a different amount of processing
- Different fitting strategy per dataset

Assume $V_{cb} = V_{cb}^{\mathrm{BaBar}}$ for the only Belle data fits to have a **common normalization** for the coefficients (just for the plots)

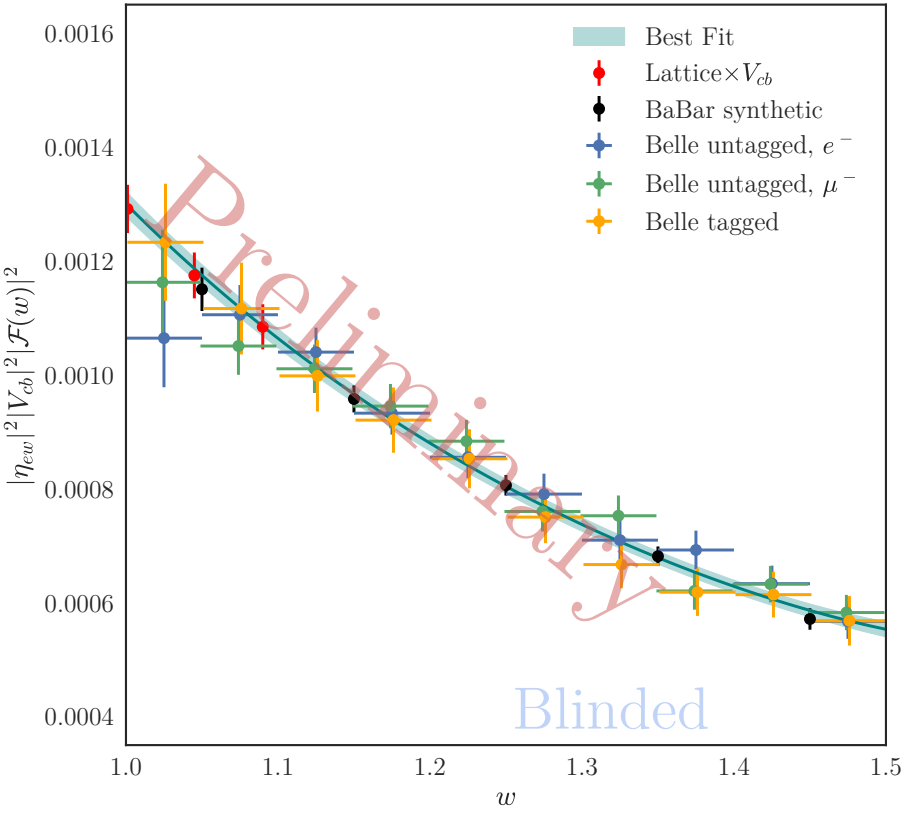
All the experimental and theoretical **correlations are included** in all fits

Results: Pure-lattice prediction and joint fit

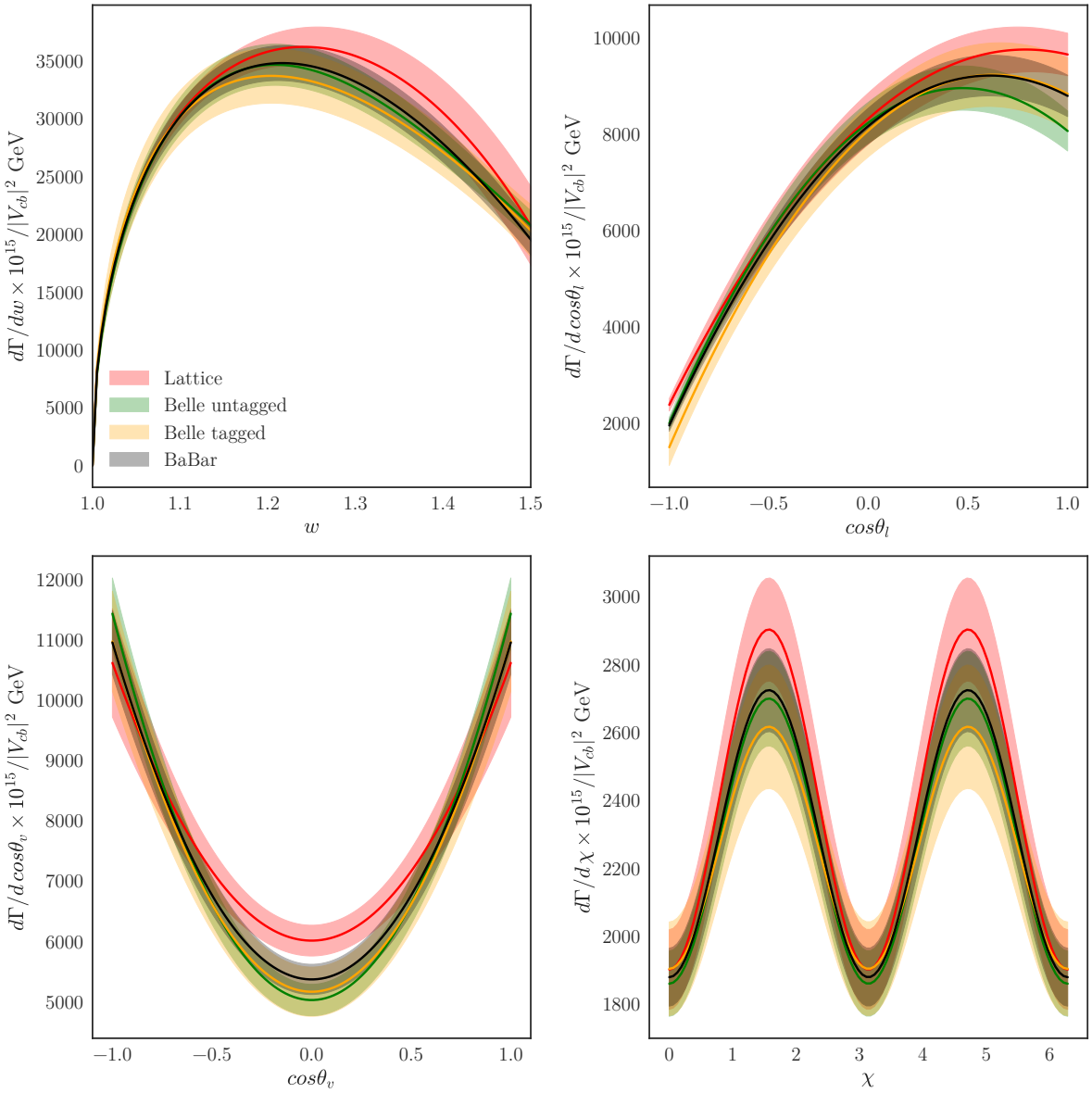
Separate fits



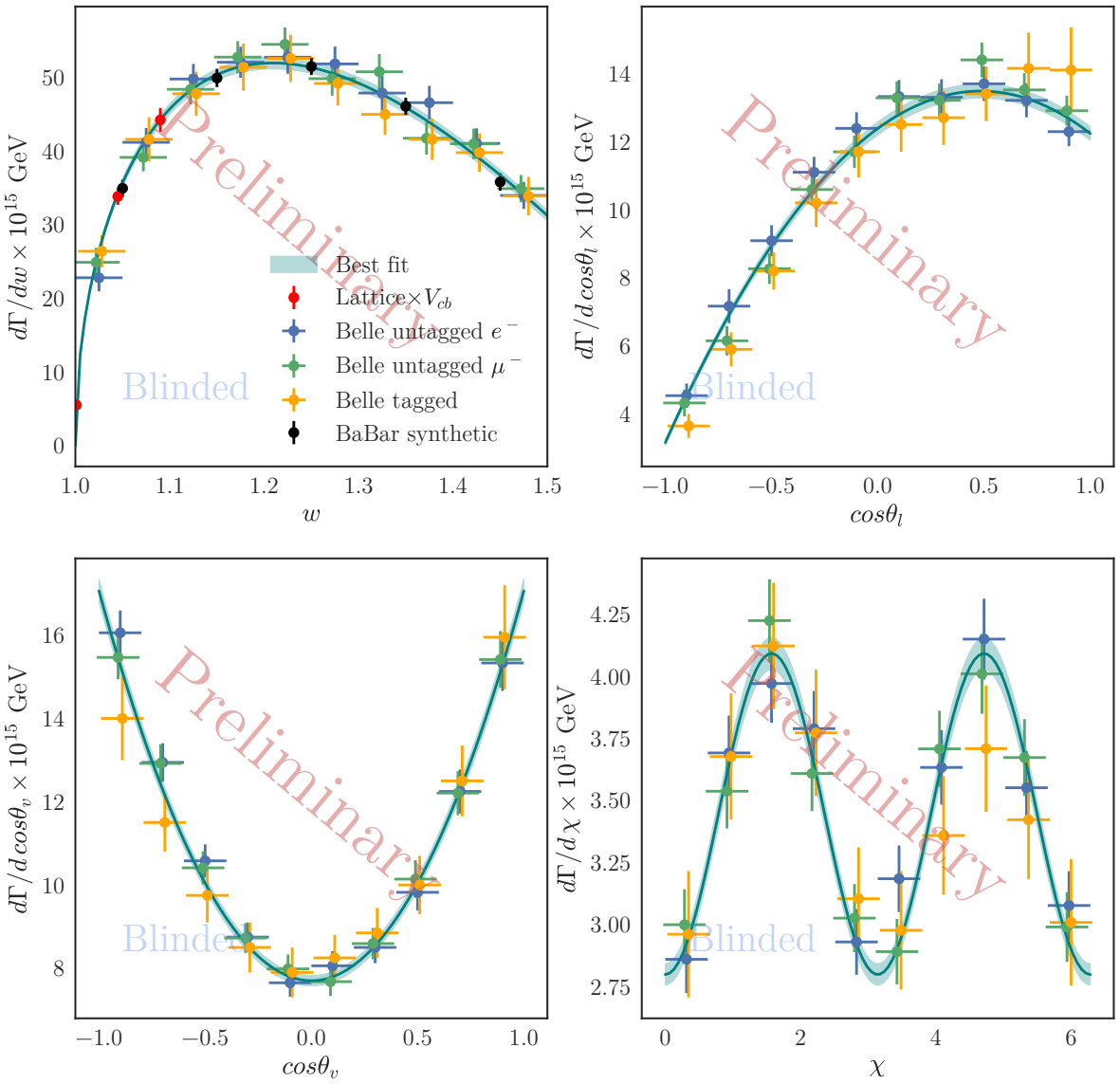
Separate fits



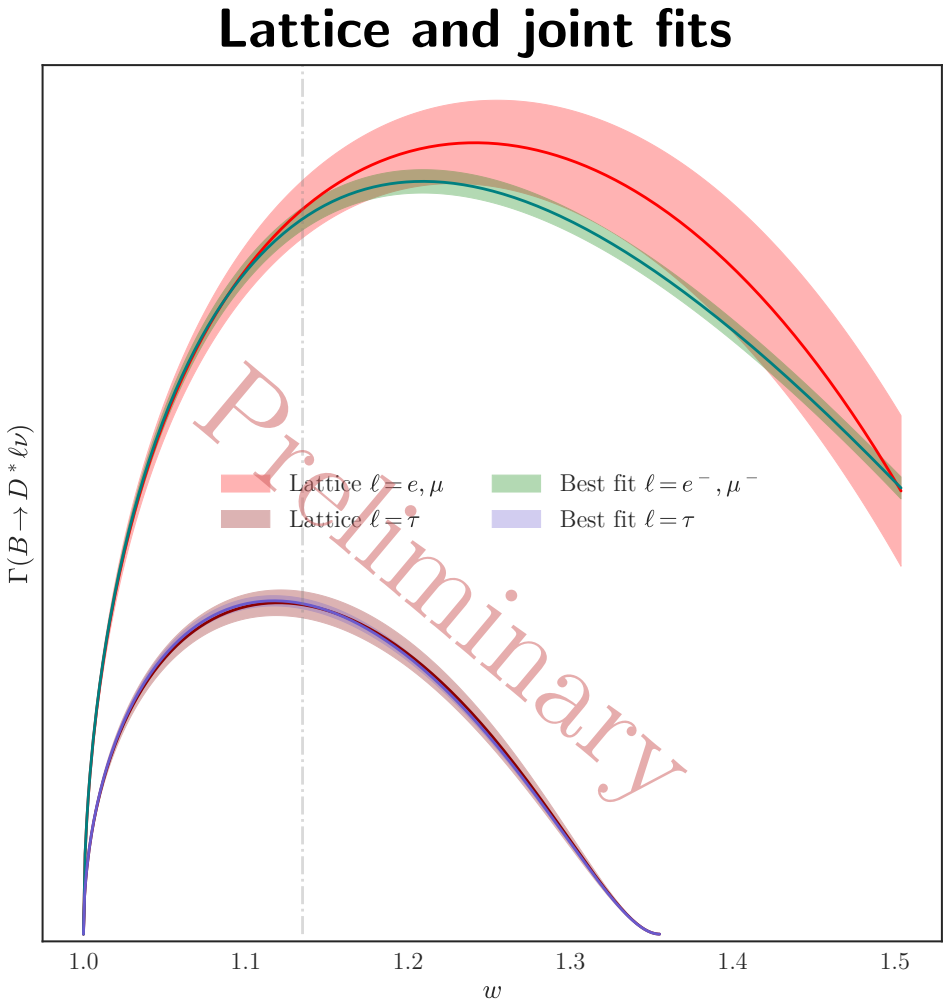
Results: Separate fits, angular bins



Results: Joint fit, angular bins with new ratio



Results: $R(D^*)$


Lattice and joint fits
Preliminary
Lattice $\ell = e, \mu$
Lattice $\ell = \tau$
Best fit $\ell = e^-, \mu^-$
Best fit $\ell = \tau$
$\Gamma(B \to D^* \ell \nu)$
1.0
1.1
1.2
1.3
1.4
1.5
w

Conclusions

- We are experiencing significant delays due to unexpected difficulties in the calculation
  - The new ratio shows that the **discretization errors** (which have been included very recently) **are large, and we need to carefully account for them to keep them under control**
  - This was expected, but the magnitude of the discretization effects is larger than what we initially thought
- The **large slope** for the decay amplitude showed in previous talks is under review
- As we say on every talk, **please, do not use our preliminary results in any calculation**
- **We need to understand better the systematic errors of our data**
- Well established roadmap to reduce errors in our calculation with newer lattice ensembles
- The net steps in our roadmap should largely reduce our systematic errors