3-body Radiative Flavored Meson Decays

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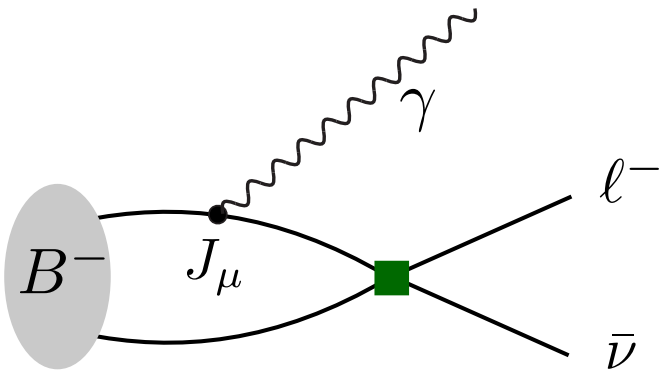
$B^- \to \ell^- \bar{\nu} \gamma$



- Hard photon removes helicity suppression $(m_\ell/m_B)^2$
- For large $E_\gamma^{(0)}$, simplest decay that probes the inverse moment of the B meson

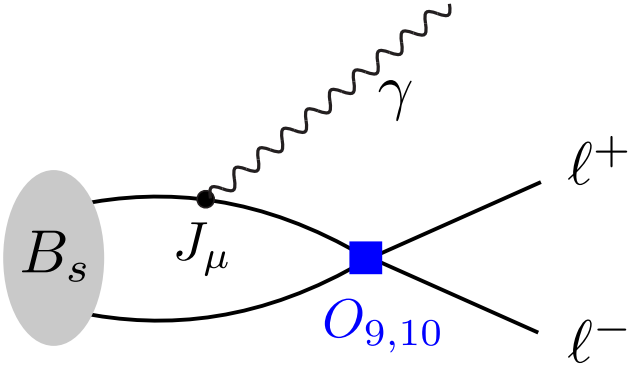
$$\frac{1}{\lambda_B} = \int_0^\infty \mathrm{d}\omega \, \frac{\Phi_{B+}(\omega)}{\omega}$$

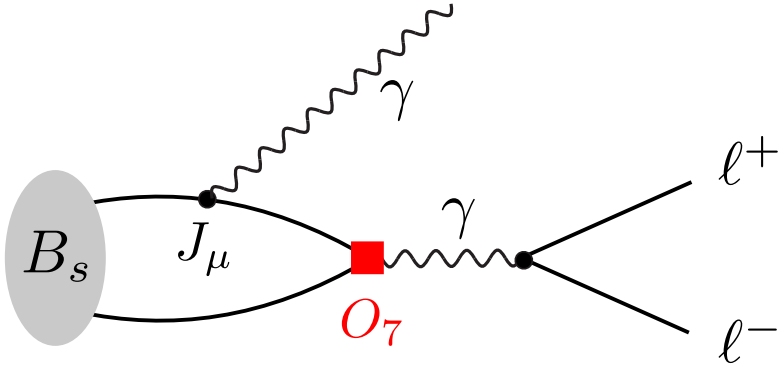
- λ_B important input in QCD factorization approach to exclusive B decays, currently not well known
  [See e.g., Beneke, Braun, Ji, Wei, arXiv:1804.04962/JHEP 2018;

  Beneke, Buchalla, Neubert, Sachrajda, arXiv:hep-ph/9905312/PRL 1999]
- Belle: $\mathcal{B}(B^+ \to \ell^+ \nu \gamma) < 3.0 \times 10^{-6}$ $(E_\gamma^{(0)} > 1$ GeV) SM: $\mathcal{O}(10^{-6})$

  [arXiv:1810.12976/PRD 2018]

$B_s^0 \to \ell^+\ell^-\gamma$ and $B_s \to \ell^+\ell^-\gamma$





- Hard photon removes helicity suppression $(m_\ell/m_B)^2$
- This process sensitive to all operators in the $b \to s\ell^+\ell^-$ weak effective Hamiltonian including O_9
- Might be possible to measure $B_s^0 \to \ell^+\ell^-\gamma$ at LHCb

  [F. Dettori, D. Guadagnoli, M. Reboud, arXiv:hep-ph/1610.00629]

Radiative leptonic decays of $D^{\pm}_{(s)}$, $K^{\pm}$ and $\pi^{\pm}$ mesons

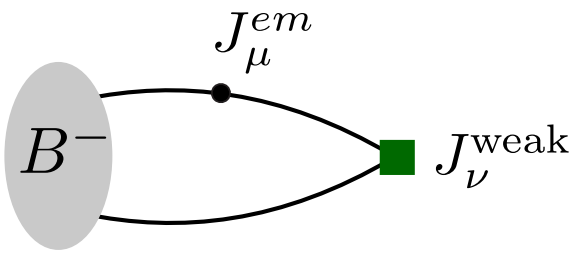
- $D_s^+ \to e^+\nu\gamma$: $\mathcal{B}(E_\gamma^{(0)} > 10 \text{ MeV }) < 1.3 \times 10^{-4}$ SM: $\mathcal{O}(10^{-4})$
- $D^+ \to e^+\nu\gamma$: $\mathcal{B}(E_\gamma^{(0)} > 10 \text{ MeV }) < 3.0 \times 10^{-5}$ SM: $\mathcal{O}(10^{-5})$
- $K^- \to e^-\bar{\nu}\gamma$, $K^- \to \mu^-\bar{\nu}\gamma$, $\pi^- \to e^-\bar{\nu}\gamma$, $\pi^- \to \mu^-\bar{\nu}\gamma$

  Partial branching fractions, photon-energy spectra, and angular distributions known from multiple experiments.

  Contributions from "inner bremsstrahlung", "structure-dependent", and interference terms are distinguished.

  [M. Bychkov, G. DAmbrosio (Particle Data Group), Form Factors for Radiative Pion and Kaon Decays, Section 68 of the Review of Particle Physics, 2018]

Hadronic Tensor and Form Factors



$$J_\mu^{em} = \sum_q e_q \bar{q}\gamma_\mu q, \qquad J_\nu^{weak} = \bar{u}\gamma_\nu(1-\gamma_5)b$$

$$\begin{aligned} T_{\mu\nu} &= -i\int \mathrm{d}^4x\; e^{ip_\gamma\cdot x}\,\langle 0|\,\mathbf{T}\big(J_\mu^{em}(x)J_\nu^{weak}(0)\big)\,|B^-(\vec{\mathbf{p}}_B)\rangle \\ &= \epsilon_{\mu\nu\tau\rho}p_\gamma^\tau v^\rho F_V + i\big[-g_{\mu\nu}(v\cdot p_\gamma) + v_\mu(p_\gamma)_\nu\big]F_A - i\frac{v_\mu v_\nu}{(v\cdot p_\gamma)}m_B f_B \\ &\quad + (p_\gamma)_\mu\text{-terms} \end{aligned}$$

$$E_\gamma^{(0)} = v\cdot p_\gamma,\; p_B = m_B v$$

Minkowski spectral decomposition of $T_{\mu\nu}$

Time ordering $t < 0$:

$$
\begin{aligned}
T^{<}_{\mu\nu} &= -i \int_{-\infty(1-i\epsilon)}^{0} \mathrm{d}t \int \mathrm{d}^3x \; e^{ip_\gamma \cdot x} \left\langle 0 \right| J^{weak}_{\nu}(0) J^{em}_{\mu}(x) \left| B^{-}(\vec{\mathbf{p}}_B) \right\rangle \\
&= -\sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma}} \frac{1}{E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B} - i\epsilon} \\
&\qquad \times \left\langle 0 \right| J^{weak}_{\nu}(0) \left| n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma) \right\rangle \left\langle n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma) \right| J^{em}_{\mu}(0) \left| B(\vec{\mathbf{p}}_B) \right\rangle
\end{aligned}
$$

(In infinite volume, the sum over n includes an integral over the continuous spectrum of multi-particle states.)

Minkowski spectral decomposition of $T_{\mu\nu}$

Time ordering $t > 0$:

$$
\begin{aligned}
T^{>}_{\mu\nu} &= -i \int_0^{\infty(1-i\epsilon)} \mathrm{d}t \int \mathrm{d}^3x \; e^{ip_\gamma \cdot x} \langle 0 | J^{em}_\mu(x) J^{weak}_\nu(0) \left| B^-(\vec{\mathbf{p}}_B) \right\rangle \\
&= \sum_m \frac{1}{2E_{m,\vec{\mathbf{p}}_\gamma}} \frac{1}{E_\gamma - E_{m,\vec{\mathbf{p}}_\gamma} - i\epsilon} \\
&\qquad \times \langle 0 | J^{em}_\mu(0) \left| m(\vec{\mathbf{p}}_\gamma) \right\rangle \langle m(\vec{\mathbf{p}}_\gamma) | J^{weak}_\nu(0) \left| B(\vec{\mathbf{p}}_B) \right\rangle
\end{aligned}
$$

(In infinite volume, the sum over n includes an integral over the continuous spectrum of multi-particle states.)

Euclidean three-point function

$$C_{\mu\nu}(t, t_B) = \int \mathrm{d}^3 y \int \mathrm{d}^3 x \; e^{-ip_\gamma \cdot x} e^{ip_B \cdot y} \left\langle J_\mu^{em}(t, \vec{\mathbf{x}}) J_\nu^{weak}(0, \vec{\mathbf{0}}) \phi_B^\dagger(\vec{\mathbf{y}}, t_B) \right\rangle$$

$$\phi_B^\dagger \sim \bar{b}\gamma_5 u$$

Time integral of three-point function

Time ordering $t < 0$: (for large negative t_B)

$$
\begin{aligned}
I^{<}_{\mu\nu}(t_B, T) &= \int_{-T}^{0} \mathrm{d}t \; e^{E_\gamma t} C_{\mu\nu}(t, t_B) \qquad \text{(* all times are now Euclidean)} \\
&= \langle B(\vec{\mathbf{p}}_B) | \, \phi_B^\dagger(0) \, | 0 \rangle \, \frac{1}{2E_{B,\vec{\mathbf{p}}_B}} e^{-E_B t_B} \\
&\times \sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma}} \frac{\langle 0 | \, J_\nu^{weak}(0) \, | n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma) \rangle \, \langle n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma) | \, J_\mu^{em}(0) \, | B(\vec{\mathbf{p}}_B) \rangle}{E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B}} \\
&\times \left[1 - e^{-(E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B}) T} \right]
\end{aligned}
$$

Require $E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B} > 0$ to get rid of unwanted exponential

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States $|n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)\rangle$ has same flavor quantum numbers as B meson

Time integral of three-point function

Time ordering $t < 0$: (for large negative t_B)

$$I^{<}_{\mu\nu}(t_B, T) = \int_{-T}^{0} \mathrm{d}t \; e^{E_\gamma t} C_{\mu\nu}(t, t_B)$$

(* all times are now Euclidean)

$$= \langle B(\vec{\mathbf{p}}_B) | \, \phi_B^\dagger(0) \, |0\rangle \, \frac{1}{2E_{B,\vec{\mathbf{p}}_B}} e^{-E_B t_B}$$

$$\times \sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma}} \frac{\langle 0| \, J_\nu^{weak}(0) \, |n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)\rangle \, \langle n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)| \, J_\mu^{em}(0) \, |B(\vec{\mathbf{p}}_B)\rangle}{E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B}}$$

$$\times \left[1 - e^{-(E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B})T} \right]$$

Require $E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B} > 0$ to get rid of unwanted exponential

States $|n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)\rangle$ has same flavor quantum numbers as B meson

$\rightarrow E_{n,\mathbf{p}_B - \mathbf{p}_\gamma} \geq E_{B,\mathbf{p}_B - \mathbf{p}_\gamma} = \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2}$

Time integral of three-point function

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Require $E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B} > 0$ to get rid of unwanted exponential

States $|n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma)\rangle$ has same flavor quantum numbers as B meson

$\rightarrow E_{n,\mathbf{p}_B - \mathbf{p}_\gamma} \geq E_{B,\mathbf{p}_B - \mathbf{p}_\gamma} = \sqrt{m_B^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2}$

For $\mathbf{p}_\gamma \neq 0$, $|\mathbf{p}_\gamma| + \sqrt{m_n^2 + (\mathbf{p}_B - \mathbf{p}_\gamma)^2} > \sqrt{m_B^2 + \mathbf{p}_B}$ is automatically satisfied

Time integral of three-point function

Time ordering: $t > 0$ (for large negative t_B)

$$I^{>}_{\mu\nu}(t_B, T) = \int_0^T \mathrm{d}t \; e^{E_\gamma t} C_{\mu\nu}(t, t_B)$$

(* all times are now Euclidean)

$$\begin{aligned} &= -\langle B(\vec{\mathbf{p}}_B)| \, \phi_B^\dagger(0) \, |0\rangle \, \frac{1}{2E_{B,\vec{\mathbf{p}}_B}} e^{-E_B t_B} \\ &\quad \times \sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_\gamma}} \langle 0| \, J_\mu^{em}(0) \, |n(\vec{\mathbf{p}}_\gamma)\rangle \, \langle n(\vec{\mathbf{p}}_\gamma)| \, J_\nu^{weak}(0) \, |B(\vec{\mathbf{p}}_B)\rangle \\ &\quad \times \frac{1}{E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma}} \left[1 - e^{(E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma})T} \right] \end{aligned}$$

Require $E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma} < 0$

Time integral of three-point function

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Require $E_\gamma - E_{n,\vec{\mathbf{p}}_\gamma} < 0$

Because the states $\left|n(\mathbf{p}_\gamma)\right\rangle$ have mass, $\sqrt{m_n^2 + \mathbf{p}_\gamma^2} > |\mathbf{p}_\gamma|$ is automatically satisfied

For $\mathbf{p}_\gamma \neq \mathbf{0}$,

$$T_{\mu\nu} = \lim_{T\to\infty} \lim_{t_B\to-\infty} \frac{-2E_{B,\vec{\mathbf{p}}_B} e^{-E_B t_B}}{\langle B(\vec{\mathbf{p}}_B)| \phi_B^\dagger(0) |0\rangle} I_{\mu\nu}(t_B, T)$$

Simulation parameters

So far we have considered:

$$K^- \to \gamma \ell^- \bar{\nu}, \qquad D^+ \to \gamma \ell^+ \nu, \qquad D_s^+ \to \gamma \ell^+ \nu$$

- $\mathbb{Z}_2$ random wall sources at weak current location
- Neglect disconnected diagrams
- Up/down/strange valence quarks: same domain-wall action as sea quarks
- Charm valence quarks: Möbius domain-wall with "stout" smearing
- "Mostly nonperturbative" renormalization
- All-mode averaging with 16 sloppy and 1 exact samples per config

see [Kane, Lehner, Meinel, Soni, arXiv:1907.00279] for more details

$D^+ \to \gamma \ell^+ \nu$ and $D_s^+ \to \gamma \ell^+ \nu$ runs

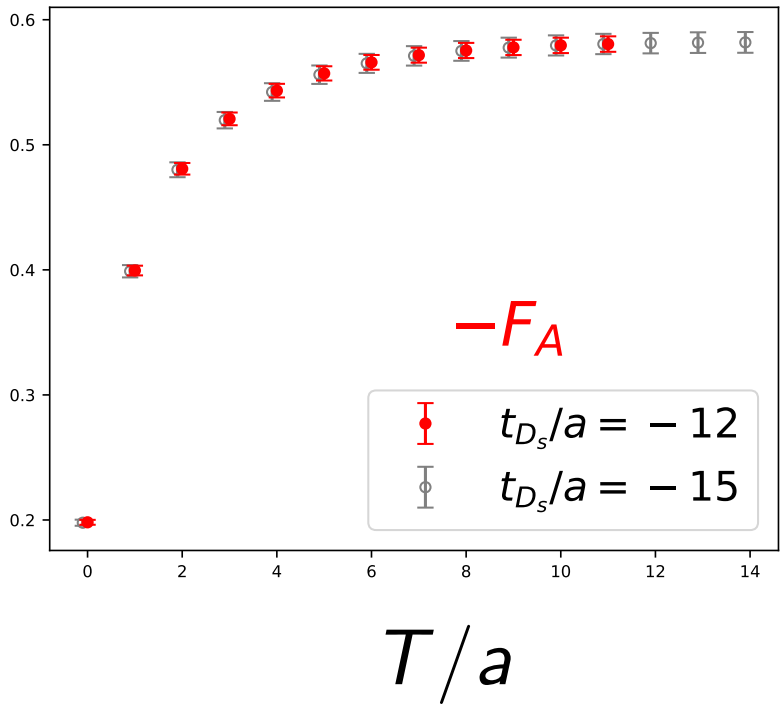
RBC/UKQCD Ensemble: $24^3 \times 64$, $m_\pi \approx 340$ (MeV), $a \approx 0.11$(fm)

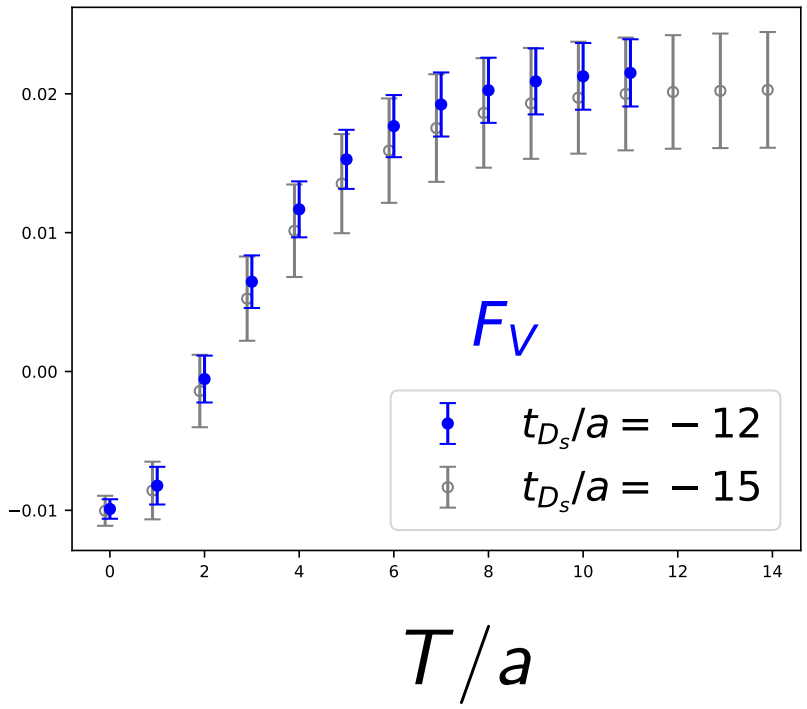
Meson rest frame:

Lattice	$t_{D_{(s)}}/a$	$\mathbf{p}_{D_{(s)}}(\frac{2\pi}{L})$	$\mathbf{p}_\gamma^2(\frac{2\pi}{L})^2$	# configs
$24^3 \times 64$	(-12, -15)	(0,0,0)	$\{1, 2, 3, 4, 5\}$	25

Show preliminary results for $D_s^+ \to \gamma \ell^+ \nu$

$D_s^+ \to \gamma \ell^+ \nu$: Form factors vs T, $p_\gamma = (0,0,1)\frac{2\pi}{L}$



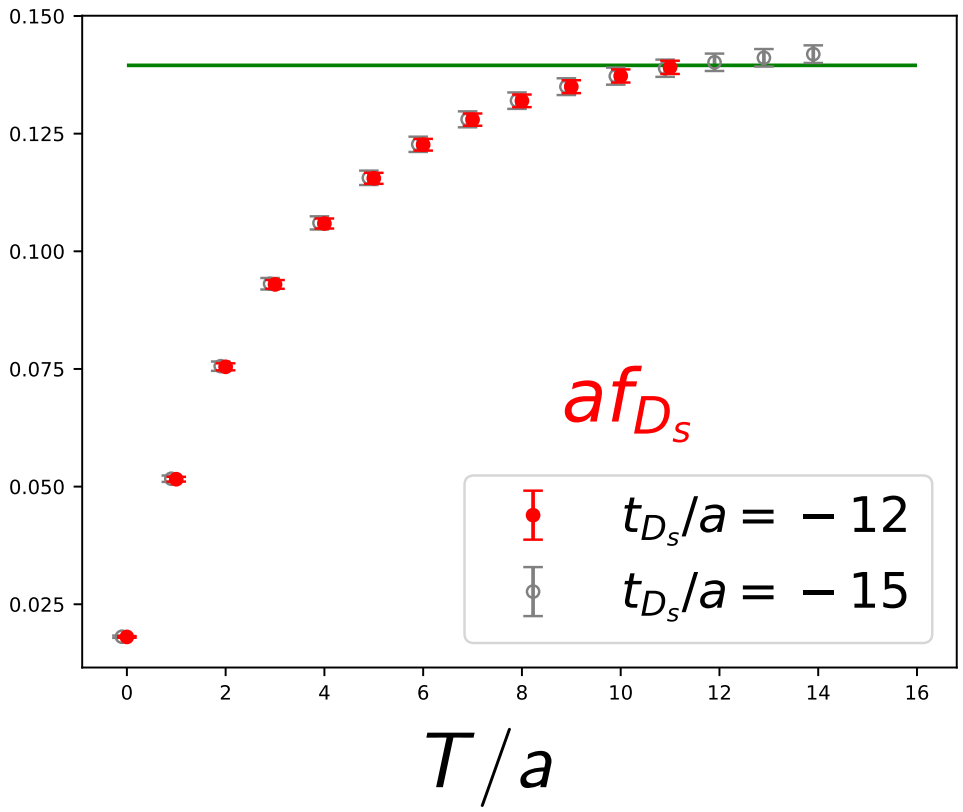


Hadronic tensor contains $D_{(s)}$ meson decay constant

$$T_{\mu\nu} = \epsilon_{\mu\nu\tau\rho} p_\gamma^\tau v^\rho F_V + i\big[- g_{\mu\nu}(p_\gamma \cdot v) + v_\mu (p_\gamma)_\nu \big] F_A - i \frac{v_\mu v_\nu}{p_\gamma \cdot v} m_{D_{(s)}} f_{D_{(s)}}$$
$$+ (p_\gamma)_\mu\text{-terms}$$

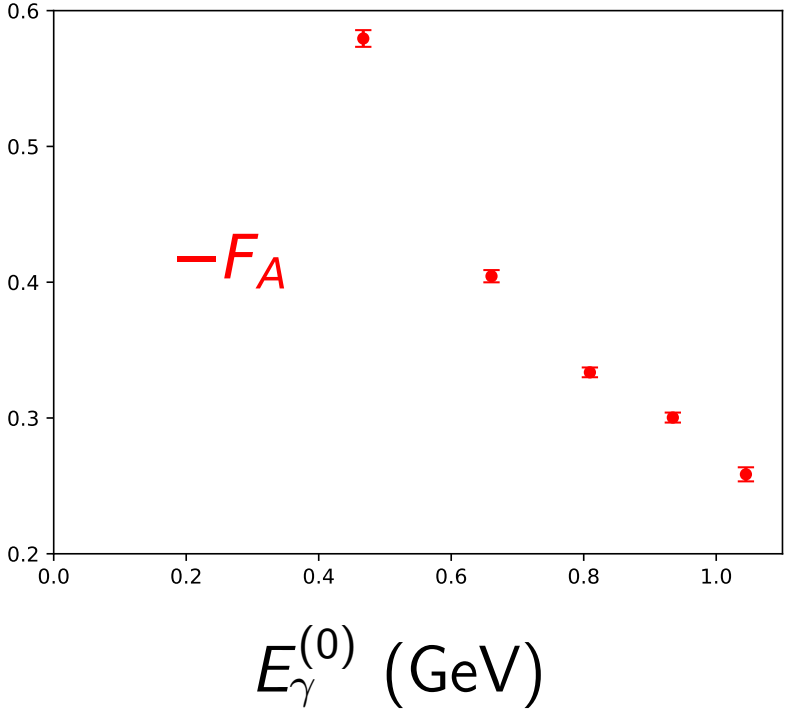
Extract $f_{D_{(s)}}$ as a cross-check

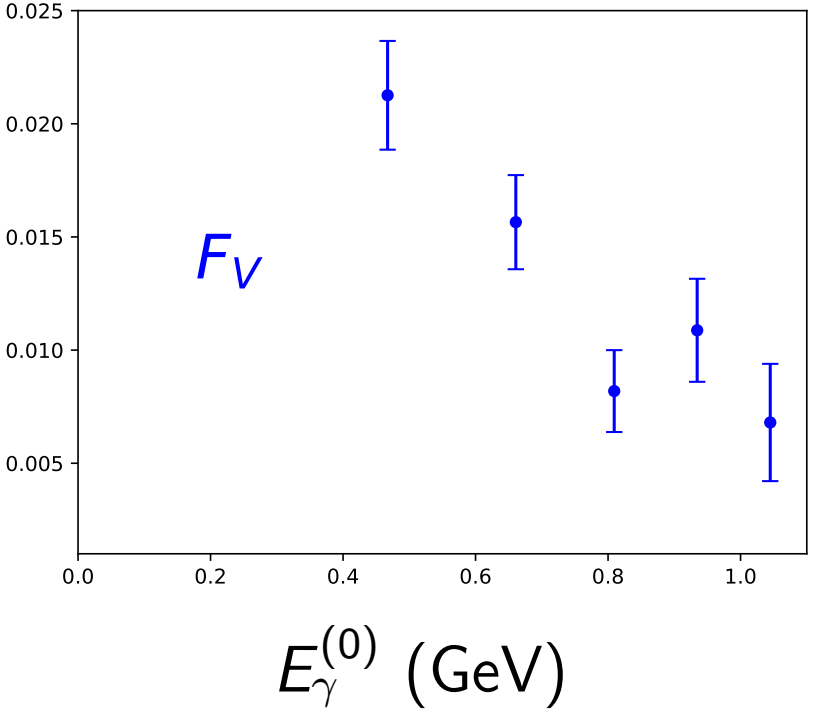
$D^+ \to \gamma \ell^+ \nu$: Decay constant vs T, $p_\gamma = (0,0,1)\frac{2\pi}{L}$



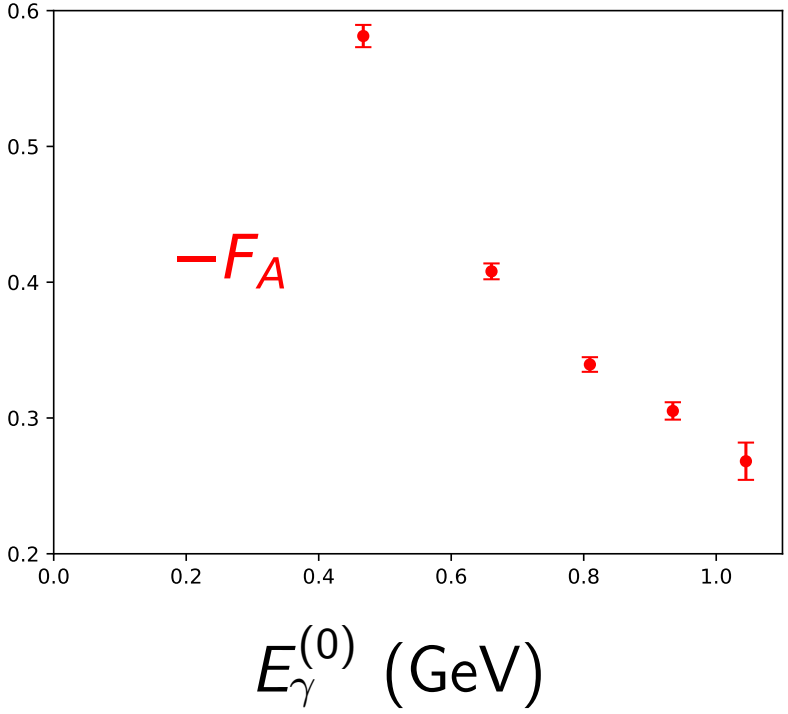
Green line shows the Flag average for af_{D_s}
[arXiv:1902.08191]

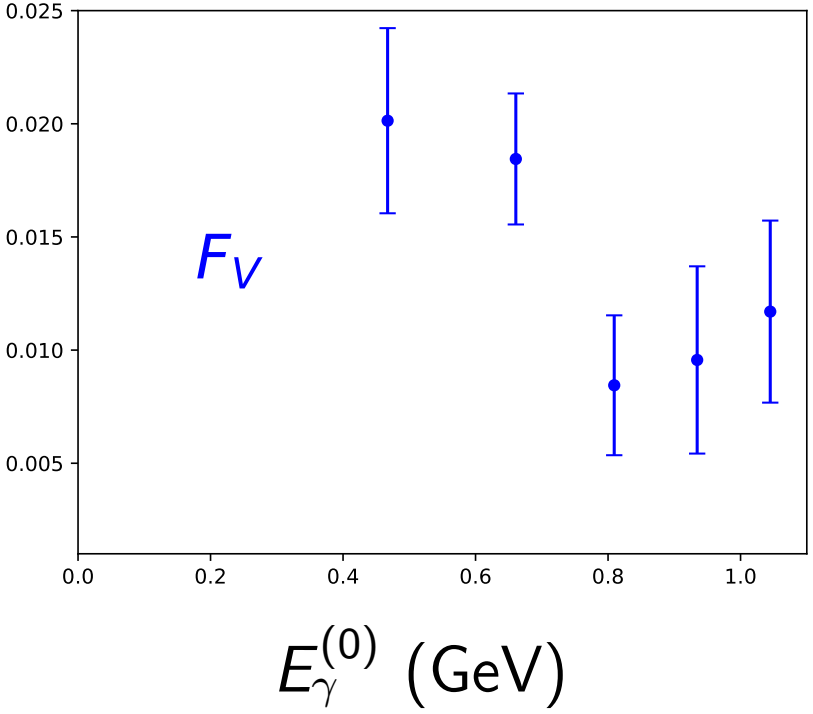
$D_s^+ \to \gamma \ell^+ \nu$: Form factors vs $E^{(0)}$, $t_{D_{(s)}}/a = -12$, $T/a = 10$





$D_s^+ \to \gamma \ell^+ \nu$: Form factors vs $E^{(0)}$, $t_{D_{(s)}}/a = -15$, $T/a = 12$





Differential Decay Rate:

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}E_\gamma^{(0)}} = \frac{\alpha_{em} G_F^2 |V_{ub}|^2}{6\pi^2} m_B (E_\gamma^{(0)})^3 (1 - \frac{2E_\gamma^{(0)}}{m_B}) \left(\left|F_V\right|^2 + \left| F_A - \left(\underbrace{-\frac{e_\ell f_B}{E_\gamma^{(0)}}}_{point-like} \right) \right|^2 \right)$$

[See e.g., Beneke, Braun, Ji, Wei, arXiv:1804.04962/JHEP 2018]

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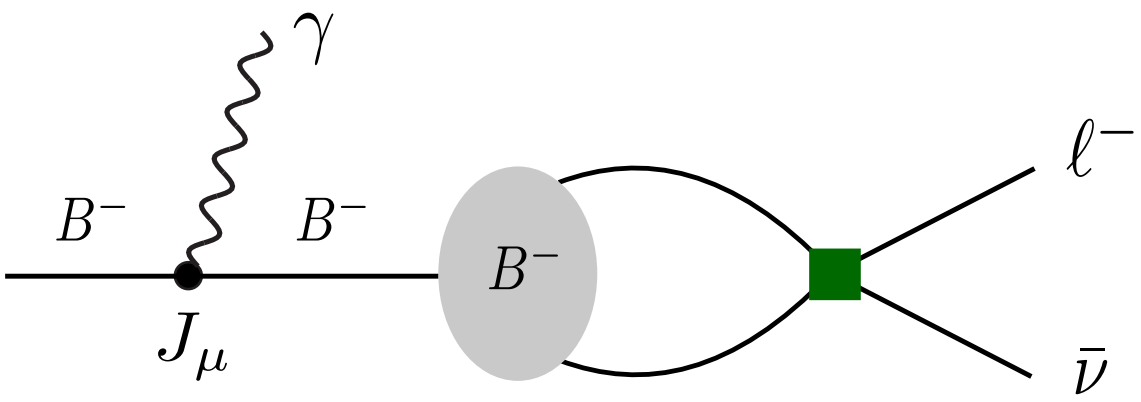
[See e.g., Beneke, Braun, Ji, Wei, arXiv:1804.04962/JHEP 2018]

Need to look at the axial form factor with the point-like contribution subtracted

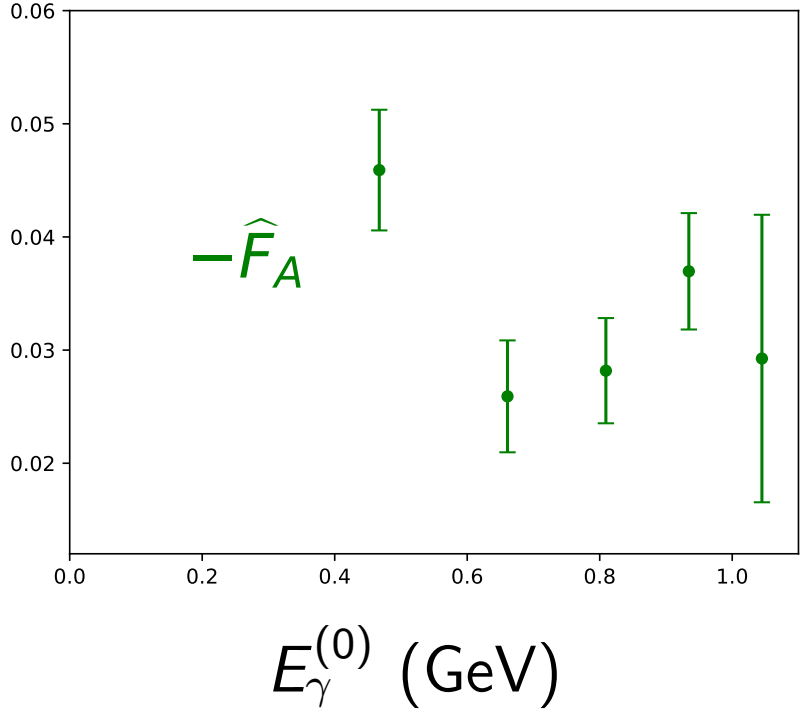
Point-like contribution to F_A

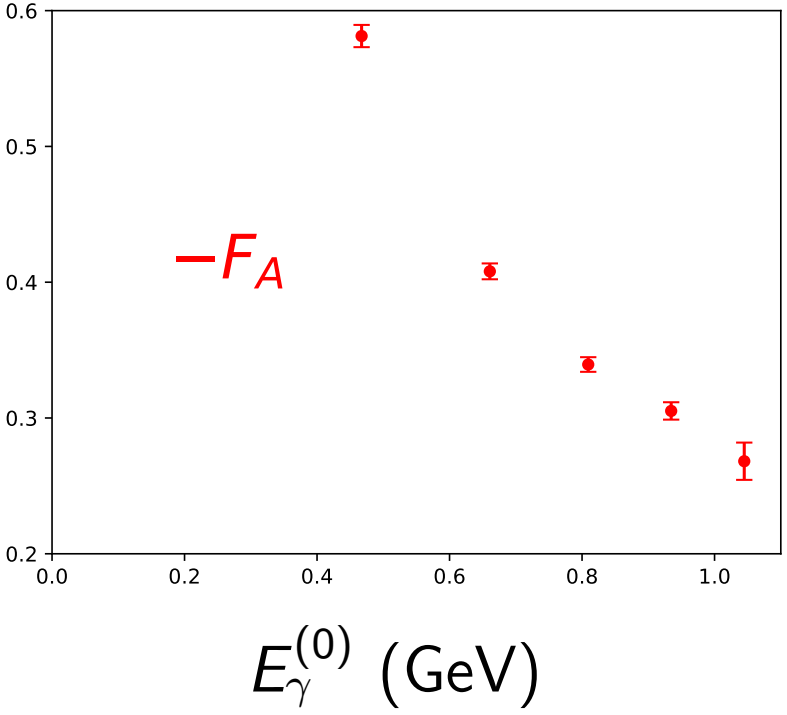


- Point-like contribution: $-\frac{e_\ell f_B}{E_\gamma^{(0)}}$
- Photon does not probe structure of B meson
- Investigate F_A after subtracting point-like contribution:

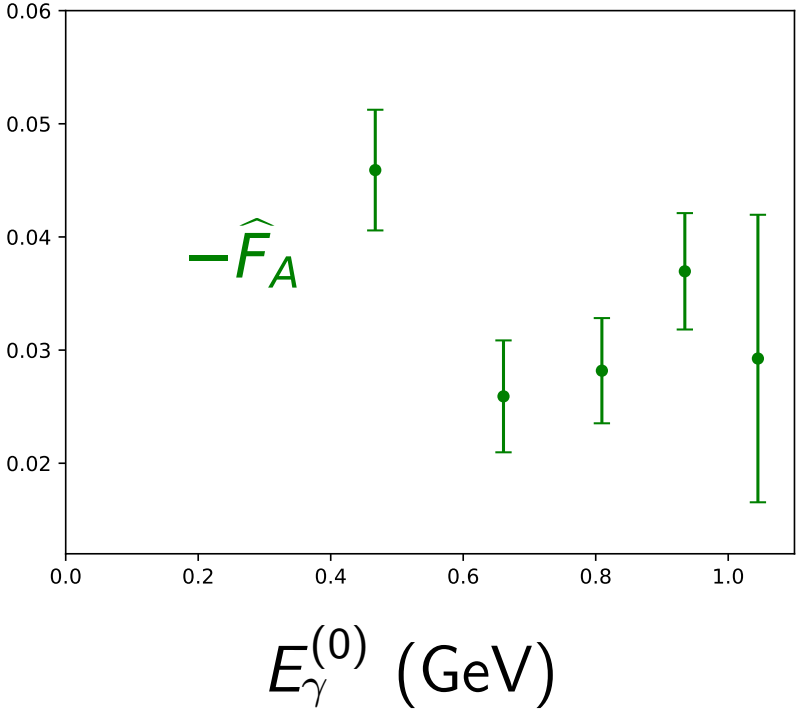
$$\widehat{F}_A \equiv F_A - \left(\frac{-e_\ell f_B}{E_\gamma^{(0)}}\right)$$

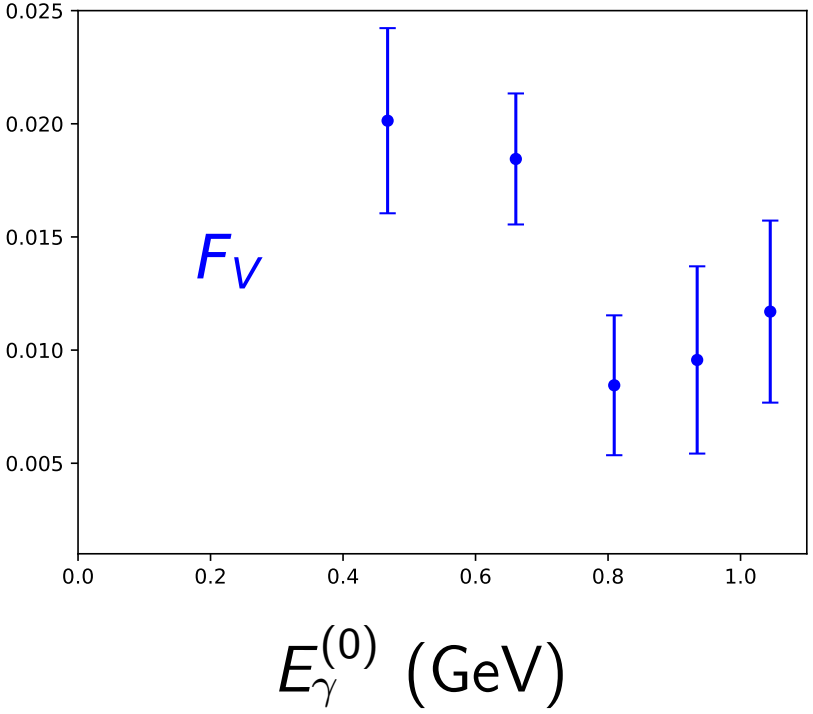
$D_s^+ \to \gamma \ell^+ \nu$: Form factors vs $E_\gamma^{(0)}$, $t_{D_{(s)}}/a = -15$, $T/a = 12$





$D_s^+ \to \gamma \ell^+ \nu$: Form factors vs $E_\gamma^{(0)}$, $t_{D_{(s)}}/a = -15$, $T/a = 12$





With $\mathbf{p}_{D_{(s)}} = \mathbf{0}$ on $24^3 \times 64$, lowest photon energy was $E_\gamma^{(0)} \approx 450$ MeV

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$\to$ use larger spacial lattice volumes

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Need $E_\gamma^{(0)} \sim 200$ MeV to get to physical values for $K^- \to \gamma \ell^- \bar{\nu}$

$\to$ use larger spacial lattice volumes
$\to$ use moving frames, i.e. $\mathbf{p}_K \neq 0$

$K^- \to \gamma \ell^- \bar{\nu}$ runs

Show preliminary results from two RBC/UKQCD ensembles:

- Dimensions: $24^3 \times 64$, $m_\pi \approx 340$ (MeV), $a \approx 0.11$(fm)
- Dimensions: $32^3 \times 64$, $m_\pi \approx 340$ (MeV), $a \approx 0.11$(fm)

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Rest frame:

Lattice	t_K/a	$\mathbf{p}_K(\frac{2\pi}{L})$	$\mathbf{p}_\gamma^2(\frac{2\pi}{L})^2$	# configs
$24^3 \times 64$	(-12, -15)	(0,0,0)	$\{1,2,3,4,5\}$	25
$32^3 \times 64$	(-12, -15)	(0,0,0)	$\{1,2\}$	31

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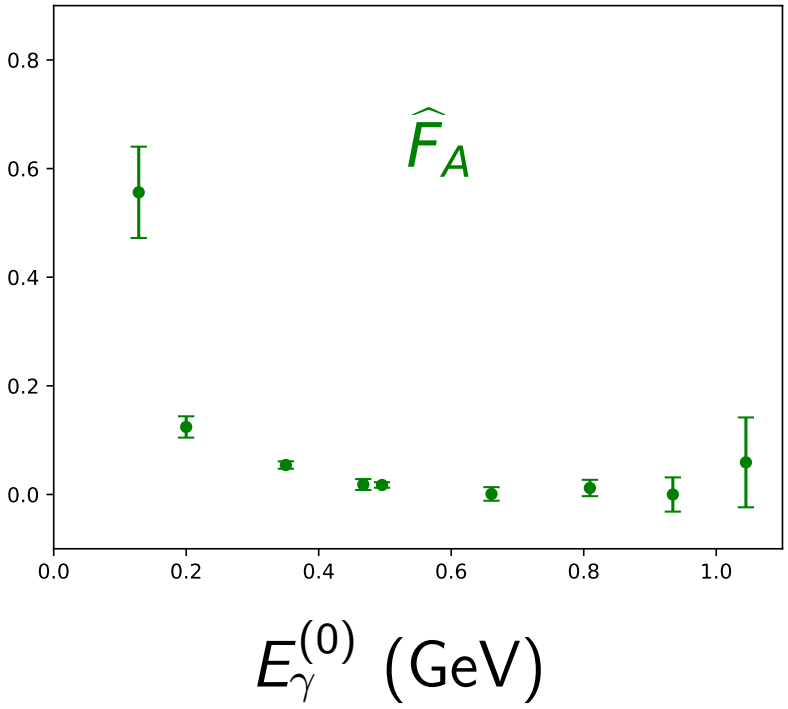
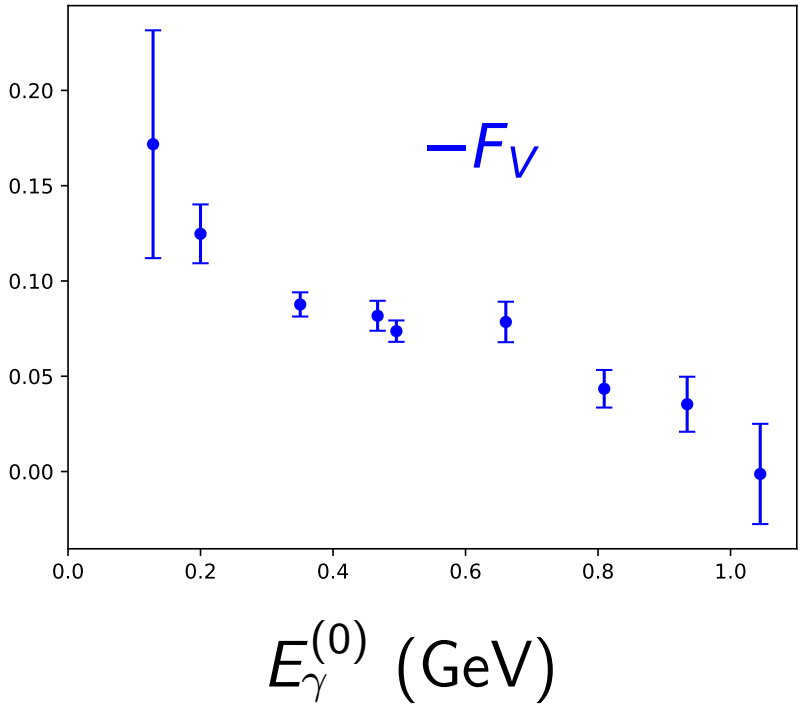
Moving frame:

Lattice	t_K/a	$\mathbf{p}_K(\frac{2\pi}{L})$	$\mathbf{p}_\gamma(\frac{2\pi}{L})$	# configs
$32^3 \times 64$	(-12, -15)	(0,0,1), (0,0,2)	(0,0,1)	31

$K^- \to \gamma \ell^- \bar{\nu}$: Form factors vs $E_\gamma^{(0)}$, $t_{D_{(s)}}/a = -12$, $T/a = 10$





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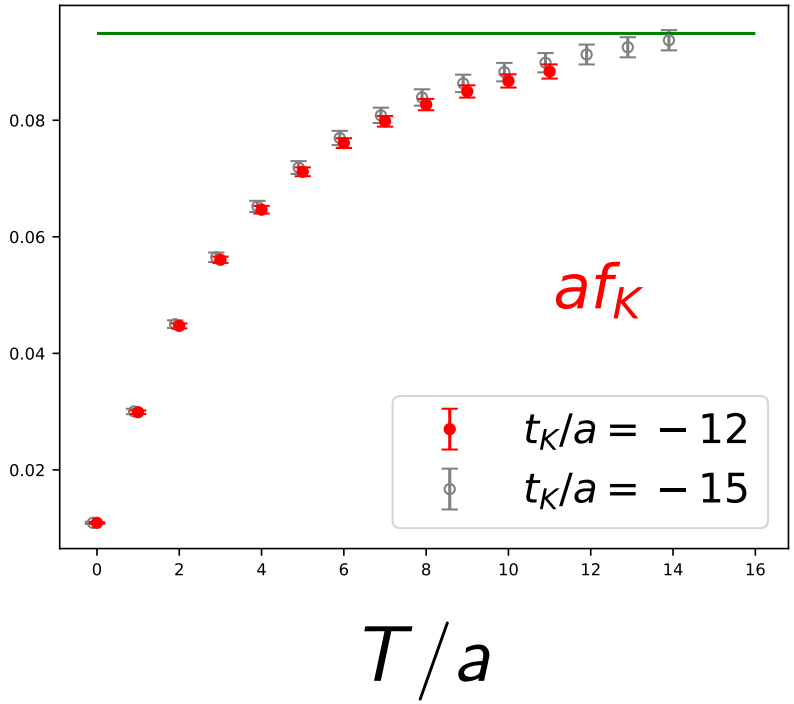
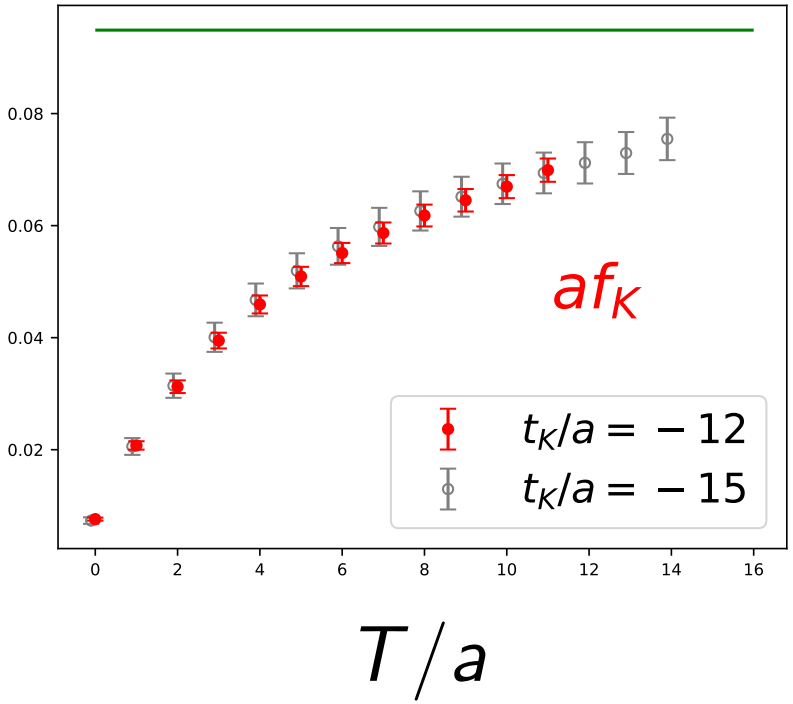
$f_K \to$ bad plateaus ✗ $\to$ bad plateaus in $\widehat{F}_A = F_A - \left(- \frac{e_\ell f_K}{E_\gamma^{(0)}} \right)$

$K^- \to \gamma \ell^- \bar{\nu}$: Decay constant vs T, $p_\gamma = (0,0,1)\frac{2\pi}{L}$

$\mathbf{p}_K = (0,0,1)\frac{2\pi}{L}$



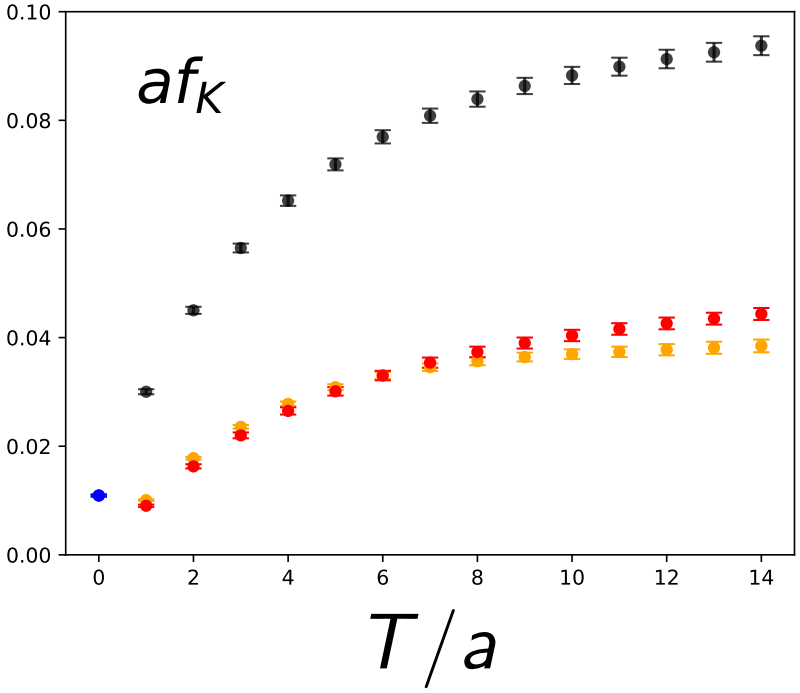
$\mathbf{p}_K = (0,0,2)\frac{2\pi}{L}$

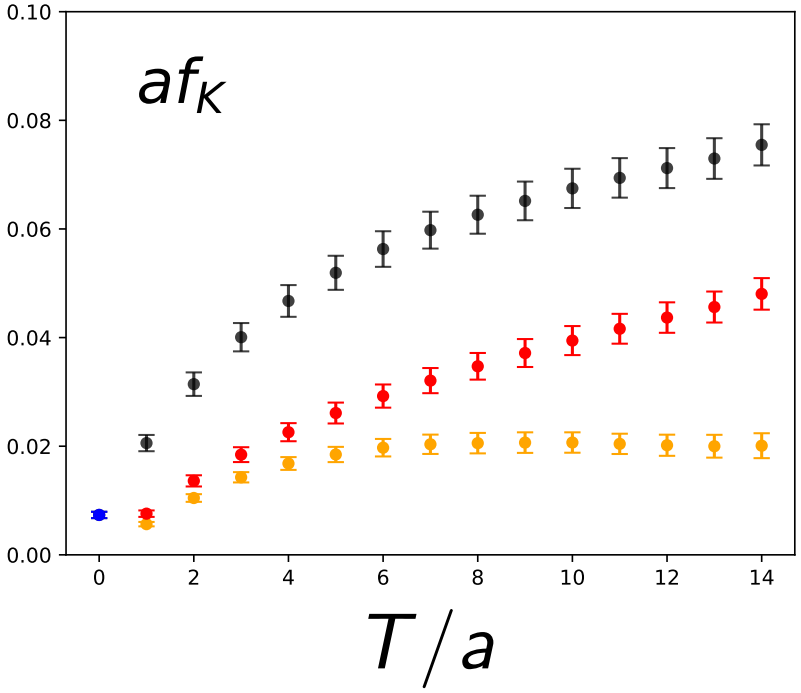


(Green line is known average for af_K on the RBC/UKQCD $24^3 \times 64$ lattice)

[arXiv:1908.10160]

$K^- \to \gamma \ell^- \bar{\nu}$: Separate t-order contributions to f_K





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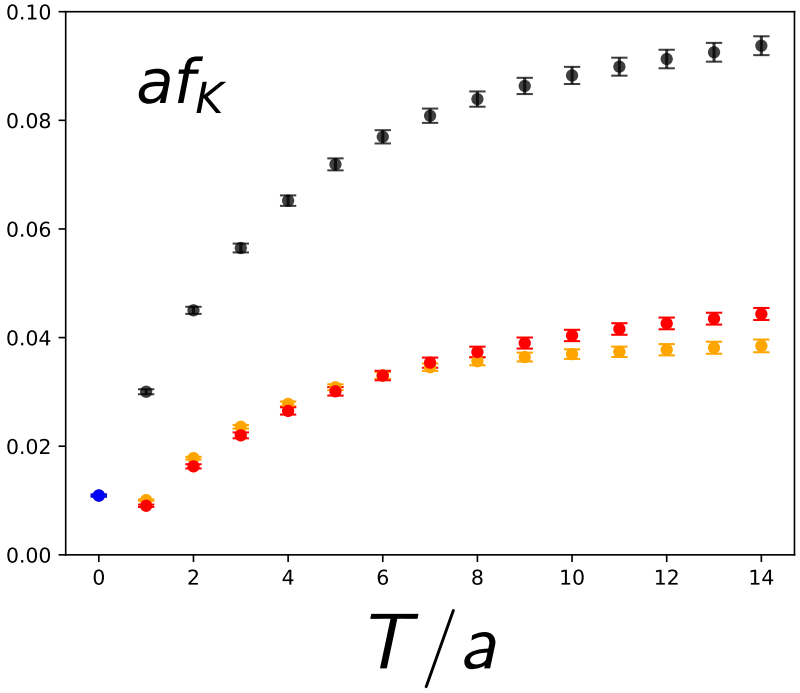

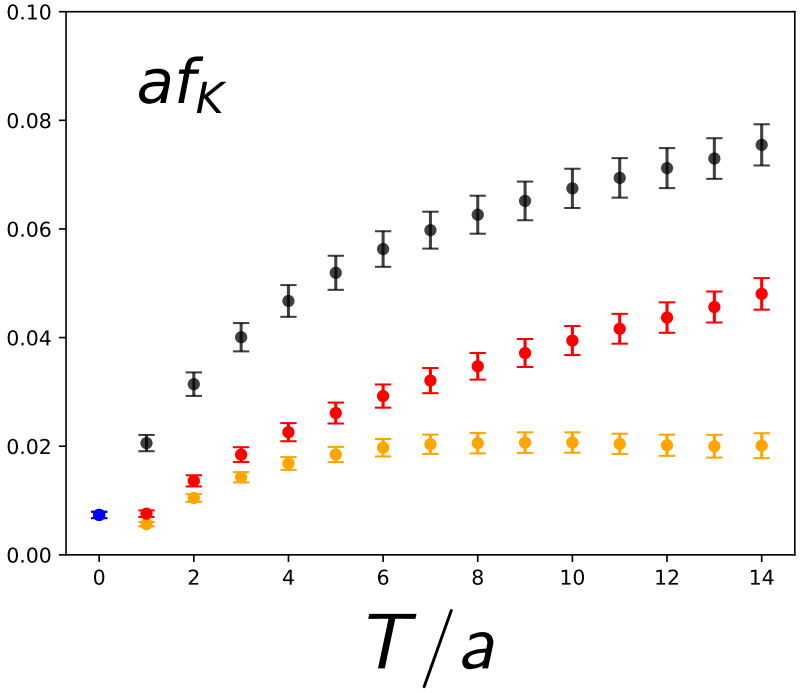

$$\int_{-T}^{T} = \int_{t=0} + \int_{t<0} + \int_{t>0}$$

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Time ordering $t < 0$:

$$
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I^{<}_{\mu\nu}(t_B, T) &= \int_{-T}^{0} \mathrm{d}t \; e^{E_\gamma t} C_{\mu\nu}(t, t_B) \\
&= \langle B(\vec{\mathbf{p}}_B) | \, \phi_B^\dagger(0) \, | 0 \rangle \, \frac{1}{2E_{B,\vec{\mathbf{p}}_B}} e^{-E_B t_B} \\
&\times \sum_n \frac{1}{2E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma}} \frac{\langle 0 | \, J_\nu^{weak}(0) \, | n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma) \rangle \, \langle n(\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma) | \, J_\mu^{em}(0) \, | B(\vec{\mathbf{p}}_B) \rangle}{E_\gamma + E_{n,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{B,\vec{\mathbf{p}}_B}} \\
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States $\left| n(\mathbf{p}_B - \mathbf{p}_\gamma) \right\rangle$ has same flavor quantum numbers as B meson

$\rightarrow$ Lowest energy gap is $\left| n(\mathbf{p}_B - \mathbf{p}_\gamma) \right\rangle = \left| K(\mathbf{p}_K - \mathbf{p}_\gamma) \right\rangle$

For $|n(\vec{\mathbf{p}}_K - \vec{\mathbf{p}}_\gamma)\rangle = |K(\vec{\mathbf{p}}_K - \vec{\mathbf{p}}_\gamma)\rangle$, the unwanted exponential is

$$\exp\left\{ -\left(E_\gamma + E_{K,\vec{\mathbf{p}}_K - \vec{\mathbf{p}}_\gamma} - E_{K,\vec{\mathbf{p}}_K} \right) T \right\}$$

Moving frame parameters:

- $\mathbf{p}_\gamma = (0,0,1)\frac{2\pi}{L}$
- $am_K \approx 0.33$
- For $T/a = 15$ (largest possible separation)
  - $\mathbf{p}_K = (0,0,1)\frac{2\pi}{L}$: $e^{-(E_\gamma + E_{K,\vec{\mathbf{p}}_B - \vec{\mathbf{p}}_\gamma} - E_{K,\vec{\mathbf{p}}_K})T} \approx 0.12$
  - $\mathbf{p}_K = (0,0,2)\frac{2\pi}{L}$: $e^{-(E_\gamma + E_{n,\vec{\mathbf{p}}_K - \vec{\mathbf{p}}_\gamma} - E_{K,\vec{\mathbf{p}}_K})T} \approx 0.36$

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Large systematic errors present in moving frames needed to get to physical photon energy for Kaon.

Summary and Outlook

- Possible to study radiative decays on the lattice
  [see also G. Martinelli arXiv:1908.10160]
- Lower photon energy needed for $K^- \to \gamma \ell^- \bar{\nu}$ have large systematic errors
  $\to$ fit the exponential of the lowest lying energy gap
- To use DWF action to study $B_{(s)}$ meson radiative leptonic decays, need to extrapolate the mass
- Considering also using the "RHQ" action to do calculations directly at $B_{(s)}$ mass, but action is only on-shell improved