$\Lambda_b \to \Lambda_c \ell\bar\nu$, Tests of HQS, and $SU(3)$ breaking in $B_{(s)} \to D^*_{(s)} \ell\bar\nu$

Zoltan Ligeti

2019 Lattice X Intensity Frontier Workshop

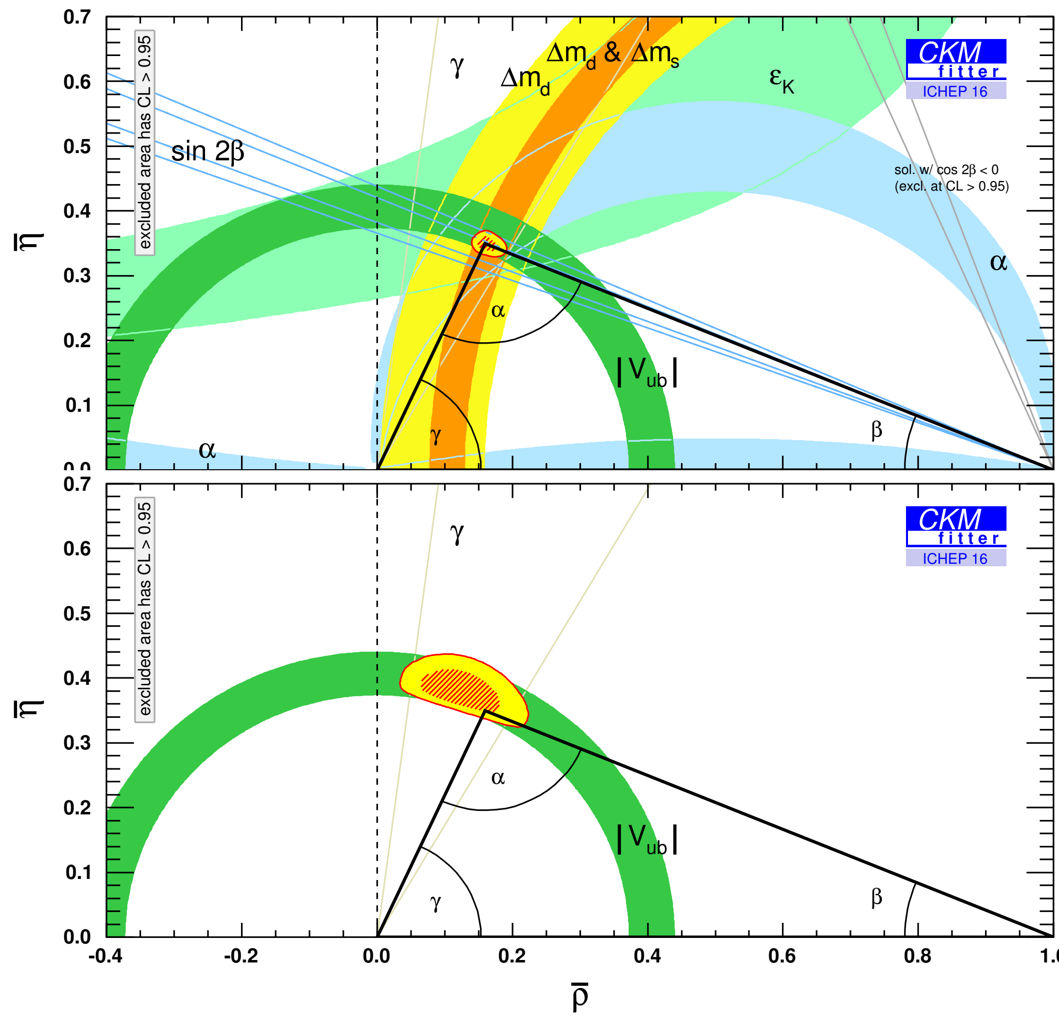
BNL, Sep 23–25, 2019

See: Bernlochner, ZL, Robinson, Sutcliffe, arXiv:1808.09464 [PRL]; 1812.07593 [PRD]

Bernlochner and ZL, talk at LHCb analysis meeting, Sep 4, 2019 — LQCD connections

CKM fit: plenty of room for new physics

- SM dominates CP viol. $\Rightarrow$ KM Nobel
- The implications of the consistency are often overstated
- Much larger allowed region if the SM is not assumed
- Tree-level (mainly V_{ub} & γ) vs. loop
- V_{ub} & V_{cb}: important SM measurements + essential for NP sensitivity



- In loop (FCNC) processes NP / SM $\sim 20\%$ still allowed (mixing, $B \to X\ell^+\ell^-$, $B \to X\gamma$, ...)

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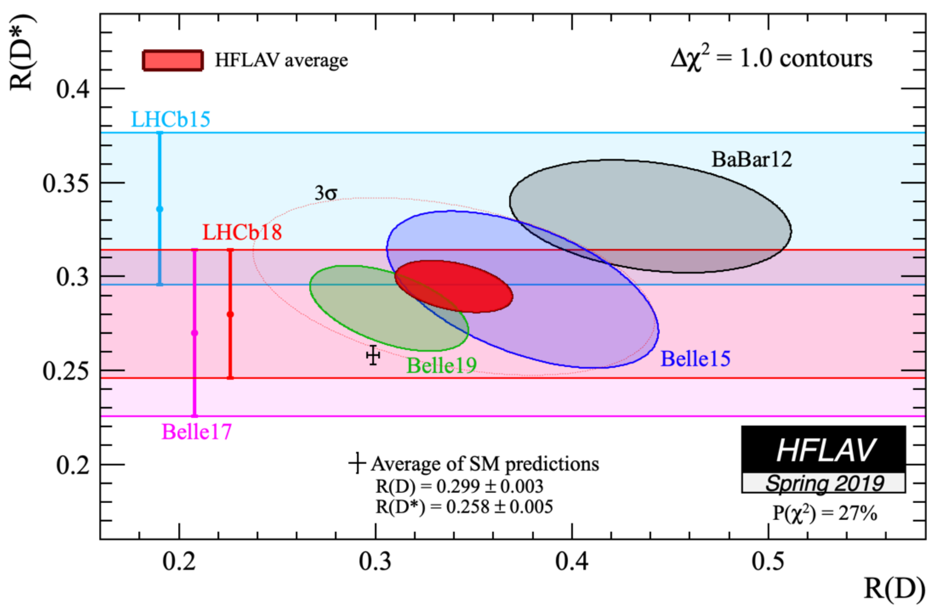
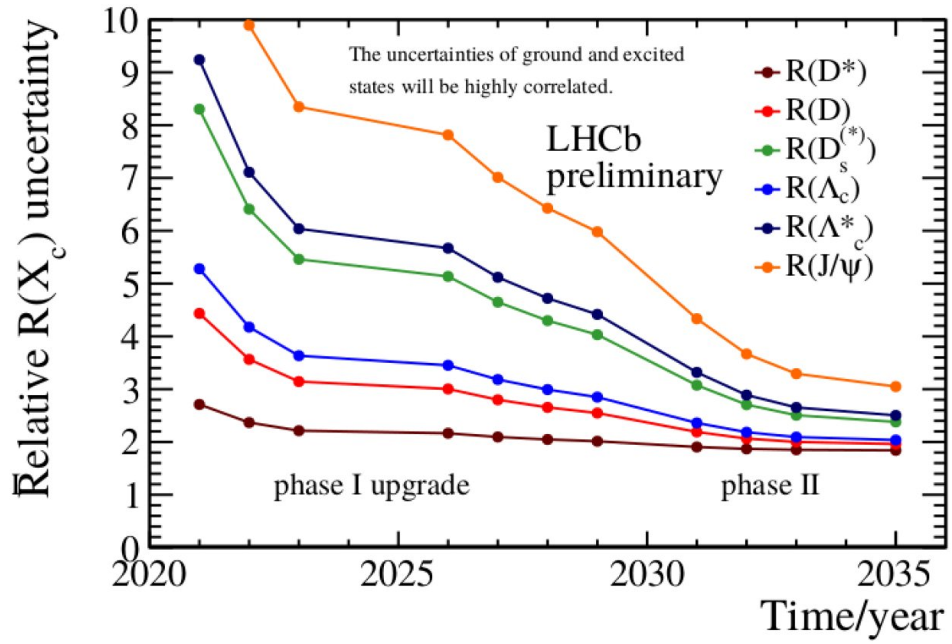
Recent focus: $R(D)$ and $R(D^*)$

- BaBar, Belle, LHCb: enhanced τ rates, $R(D^{(*)}) = \dfrac{\Gamma(B \to D^{(*)}\tau\bar\nu)}{\Gamma(B \to D^{(*)}l\bar\nu)}$ $\qquad (l = e, \mu)$

Notation: $\ell = e, \mu, \tau$ and $l = e, \mu$

Future:





Belle II (50/ab, in SM): $\delta R(D^{(*)}) \sim 2(3)\%$

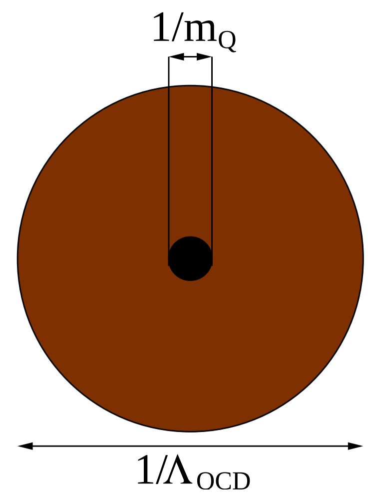
- Big improvements: even if central values change, plenty of room to establish NP
- Focus on the 3 modes that are expected to be most precise in the long trem

Heavy quark symmetry 101

- Model independent from QCD, used both in some continuum & LQCD methods
- $Q\,\overline{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{\rm QCD}$ limit
- $Q\,\bar{q}$: wave function of the light degrees of freedom

  ("brown muck") insensitive to spin and flavor of Q

  (A B meson is a lot more complicated than just a $b\bar{q}$ pair)



In the $m_Q \gg \Lambda_{\rm QCD}$ limit, the heavy quark acts as a static color source with fixed four-velocity v^μ [Isgur & Wise]

$SU(2n)$ heavy quark spin-flavor symmetry at fixed v^μ [Georgi]

- Similar to atomic physics: $(m_e \ll m_N)$

1. Flavor symmetry $\sim$ isotopes have similar chemistry [Ψ_e independent of m_N]
2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\to 0$]

Spectroscopy of heavy-light mesons

- In $m_Q \gg \Lambda_{\rm QCD}$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and

$$\left.\begin{array}{r} \text{angular momentum conservation: } [\vec{J}, \mathcal{H}] = 0 \\ \text{heavy quark symmetry: } [\vec{s}_Q, \mathcal{H}] = 0 \end{array}\right\} \Rightarrow [\vec{s}_l, \mathcal{H}] = 0$$

- For a given s_l, two degenerate states:

$$J_\pm = s_l \pm \tfrac{1}{2}$$

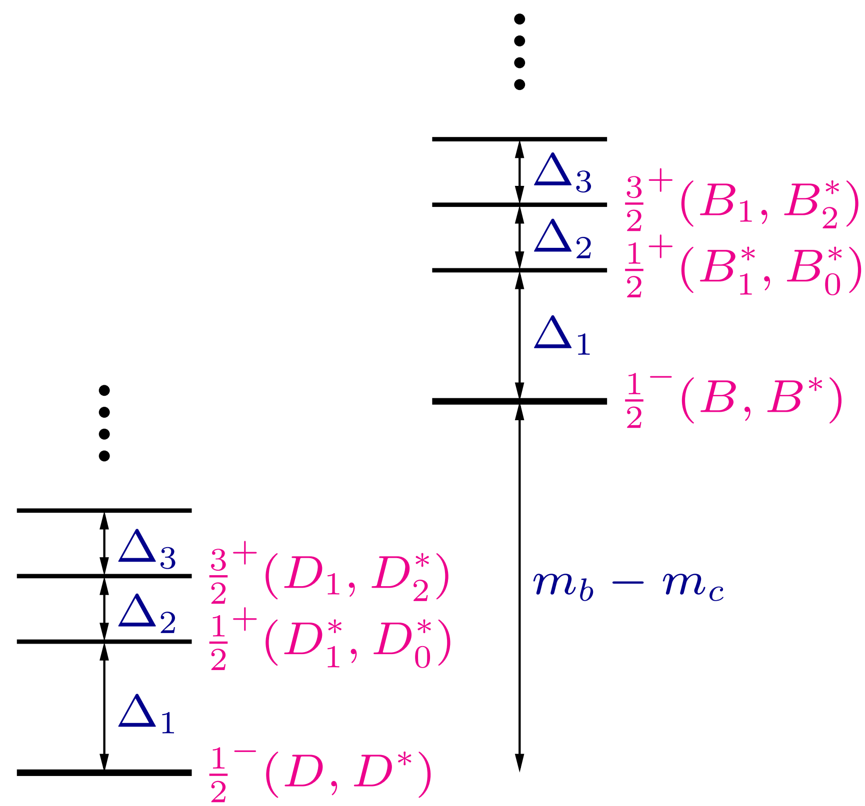
$\Rightarrow \Delta_i = \mathcal{O}(\Lambda_{\rm QCD})$ — same in B and D sector

Doublets are split by order $\Lambda^2_{\rm QCD}/m_Q$, e.g.:

$m_{D^*} - m_D \sim 140\,\mathrm{MeV}$

$m_{B^*} - m_B \sim 45\,\mathrm{MeV}$

ratio $\sim m_c/m_b$

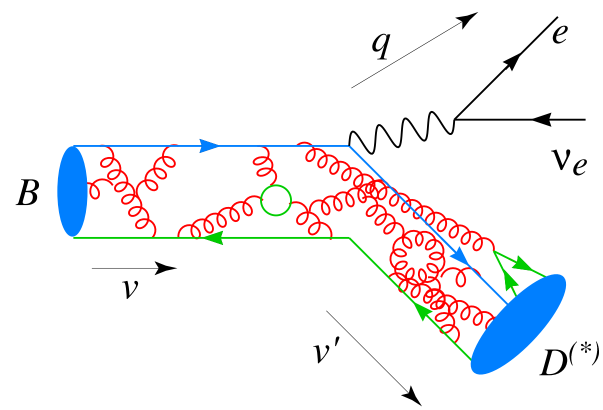


Basics of $B \to D^{(*)}\ell\bar\nu$ or $\Lambda_b \to \Lambda_c\ell\bar\nu$

- In the $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin

- On a time scale $\ll \Lambda_{\rm QCD}^{-1}$ weak current changes $b \to c$
  i.e.: $\vec p_b \to \vec p_c$ and possibly $\vec s_Q$ flips

  In $m_{b,c} \gg \Lambda_{\rm QCD}$ limit, brown muck only feels $v_b \to v_c$



- Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$

  $\Uparrow$

  Contains all nonperturbative low-energy hadronic physics

- $\xi(1) = 1$, because at "zero recoil" configuration of brown muck not changed at all

- Same holds for $\Lambda_b \to \Lambda_c\ell\bar\nu$, different Isgur-Wise fn, $\xi \to \zeta$ [also satisfies $\zeta(1) = 1$]

$$\boxed{\mathbf{\Lambda}_b \to \mathbf{\Lambda}_c \ell \bar{\nu}}$$

Ancient knowledge: baryons simpler than mesons

- Used to be well known — forgotten by experimentalists as well as theorists...

VOLUME 75, NUMBER 4 PHYSICAL REVIEW LETTERS 24 JULY 1995

Form Factor Ratio Measurement in $\Lambda_c^+ \to \Lambda e^+ \nu_e$

G. Crawford,[1] C. M. Daubenmier,[1] R. Fulton,[1] D. Fujino,[1] K. K. Gan,[1] K. Honscheid,[1] H. Kagan,[1] R. Kass,[1] J. Lee,[1]

[CLEO]

element $|V_{cs}|$ is known from unitarity [1]. Within heavy quark effective theory (HQET) [2], Λ-type baryons are more straightforward to treat than mesons as they consist of a heavy quark and a spin and isospin zero light diquark.

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Combine LHCb measurement of $\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu})/\mathrm{d}q^2$ shape [1709.01920] with LQCD results for (axial-)vector form factors [1503.01421]

[Bernlochner, ZL, Robinson, Sutcliffe, 1808.09464; 1812.07593]

Intro to $\Lambda_b \to \Lambda_c \ell \bar\nu$

- Ground state baryons are simpler than mesons: brown muck in (iso)spin-0 state
- SM: 6 form factors, functions of $w = v \cdot v' = (m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2)/(2m_{\Lambda_b} m_{\Lambda_c})$

$$\langle \Lambda_c(p', s') | \bar c \gamma_\nu b | \Lambda_b(p, s)\rangle \;=\; \bar u_c(v', s') \Big[f_1 \gamma_\mu + f_2 v_\mu + f_3 v'_\mu \Big] u_b(v, s)$$

$$\langle \Lambda_c(p', s') | \bar c \gamma_\nu \gamma_5 b | \Lambda_b(p, s)\rangle \;=\; \bar u_c(v', s') \Big[g_1 \gamma_\mu + g_2 v_\mu + g_3 v'_\mu \Big] \gamma_5\, u_b(v, s)$$

Heavy quark limit: $f_1 = g_1 = \zeta(w)$ Isgur-Wise fn, and $f_{2,3} = g_{2,3} = 0$ $\quad$ $[\zeta(1) = 1]$

- Include α_s , $\varepsilon_{b,c}$, $\alpha_s \varepsilon_{b,c}$, ε_c^2 : $\qquad m_{\Lambda_{b,c}} = m_{b,c} + \bar\Lambda_\Lambda + \ldots, \qquad \varepsilon_{b,c} = \bar\Lambda_\Lambda/(2m_{b,c})$

($\bar\Lambda_\Lambda \sim 0.8\,\mathrm{GeV}$ larger than $\bar\Lambda$ for mesons, enters via eq. of motion $\Rightarrow$ expect worse expansion?)

$$f_1 = \zeta(w) \bigg\{ 1 + \frac{\alpha_s}{\pi} C_{V_1} + \varepsilon_c + \varepsilon_b + \frac{\alpha_s}{\pi} \Big[C_{V_1} + 2(w-1) C'_{V_1} \Big] (\varepsilon_c + \varepsilon_b) + \frac{\hat b_1 - \hat b_2}{4m_c^2} + \ldots \bigg\}$$

- No $\mathcal{O}(\Lambda_{\rm QCD}/m_{b,c})$ subleading Isgur-Wise function, only 2 at $\mathcal{O}(\Lambda_{\rm QCD}^2/m_c^2)$

[Falk & Neubert, hep-ph/9209269]

- HQET is more constraining than in meson decays!

$B \to D^{(*)} \ell \bar\nu$: 6 sub-subleading Isgur-Wise functions at $\mathcal{O}(\Lambda_{\rm QCD}^2/m_c^2)$ [w/ LCSR, 1908.09398]

Fits and form factor definitions

- Standard HQET form factor definitions: $\{f_1,\, g_1\} = \zeta(w)\, \big[1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\big]$
  $\{f_{2,3},\, g_{2,3}\} = \zeta(w)\, \big[0 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\big]$

  Form factor basis in LQCD calculation: $\{f_{0,+,\perp},\, g_{0,+,\perp}\} = \zeta(w)\, \big[1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b})\big]$

  LQCD results published as fits to 11 or 17 BCL parameters, including correlations

  All 6 form factors computed in LQCD $\sim$ Isgur-Wise fn $\Rightarrow$ despite good precision, limited constraints on subleading terms and their w dependence [Detmold, Lehner, Meinel, 1503.01421]

- Only 4 parameters (and m_b^{1S}): $\{\zeta',\, \zeta'',\, \hat{b}_1,\, \hat{b}_2\}$

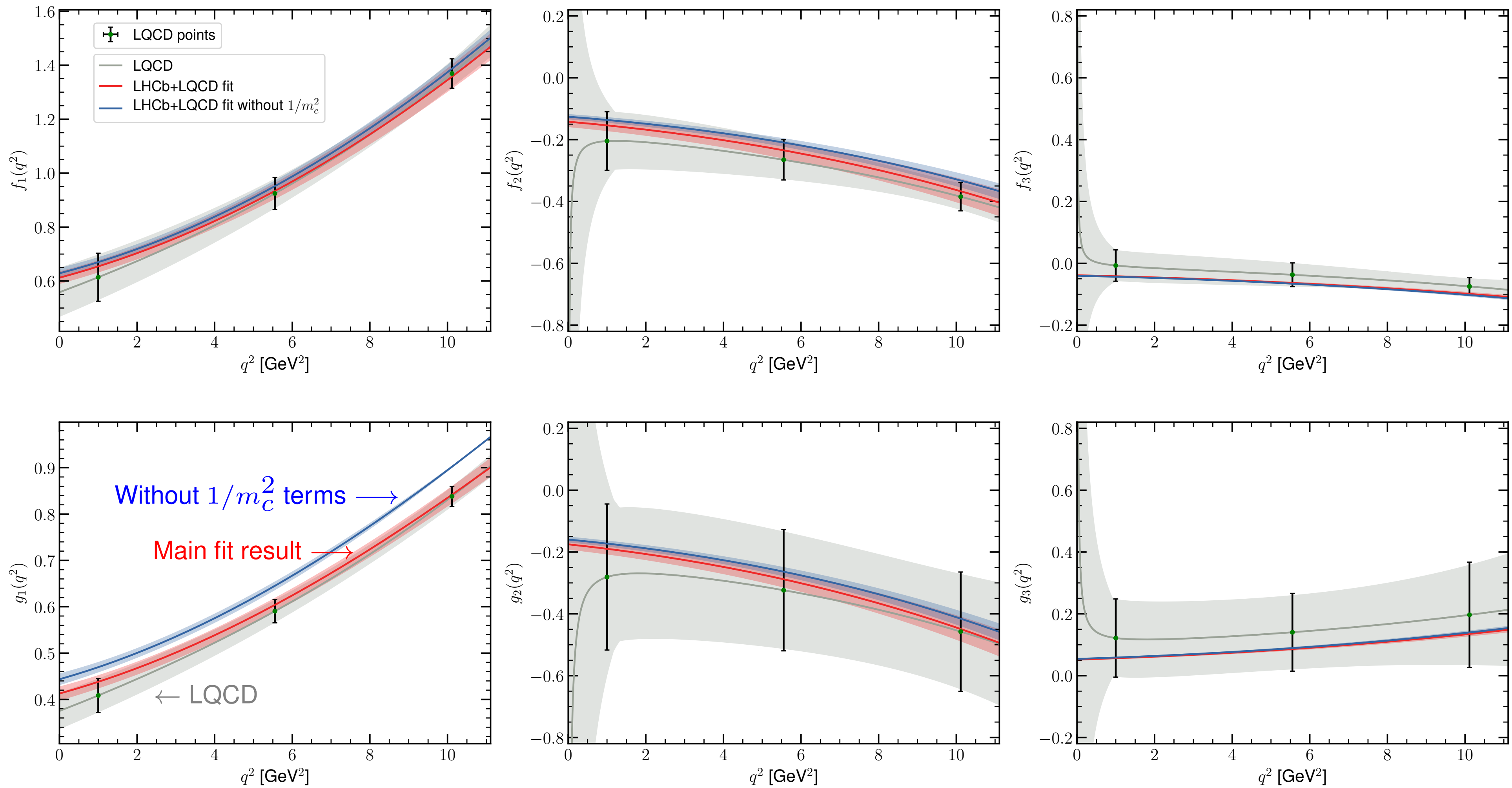
  $\zeta(w) = 1 + (w-1)\,\zeta' + \frac{1}{2}(w-1)^2\,\zeta'' + \dots \qquad b_{1,2}(w) = \zeta(w)\,\big(\hat{b}_{1,2} + \dots\big)$

  (Expanding in $w-1$ or in conformal parameter, z, makes negligible difference)

- Current LHCb and LQCD data do not yet allow constraining ζ''' and/or $\hat{b}'_{1,2}$

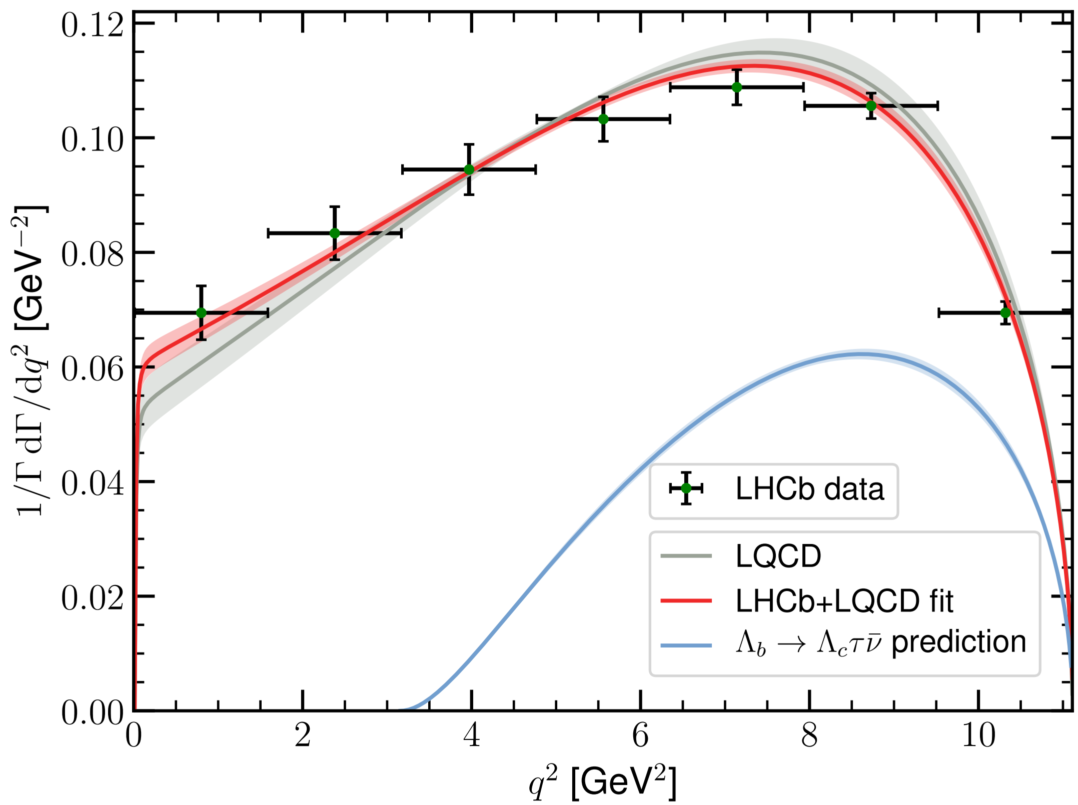
Fit to lattice QCD form factors and LHCb (1)

- Fit 6 form factors w/ 4 parameters: $\zeta'(1)$, $\zeta''(1)$, $\hat{b}_1$, $\hat{b}_2$ [LQCD: Detmold, Lehner, Meinel, 1503.01421]



Fit to lattice QCD form factors and LHCb (2)

- Our fit, compared to the LQCD fit to LHCb:



- Obtain: $R(\Lambda_c) = 0.324 \pm 0.004$

A factor of ~ 3 more precise than LQCD prediction — data constrains combinations of form factors relevant for predicting $R(\Lambda_c)$

We do not follow: "In order to determine the shape of the Isgur-Wise function $\xi_B(w)$, we use the square root of dN_{corr}/dw ... evaluated at the midpoint in the seven unfolded w bins." [LHCb, 1709.01920]

The fit requires the $1/m_c^2$ terms

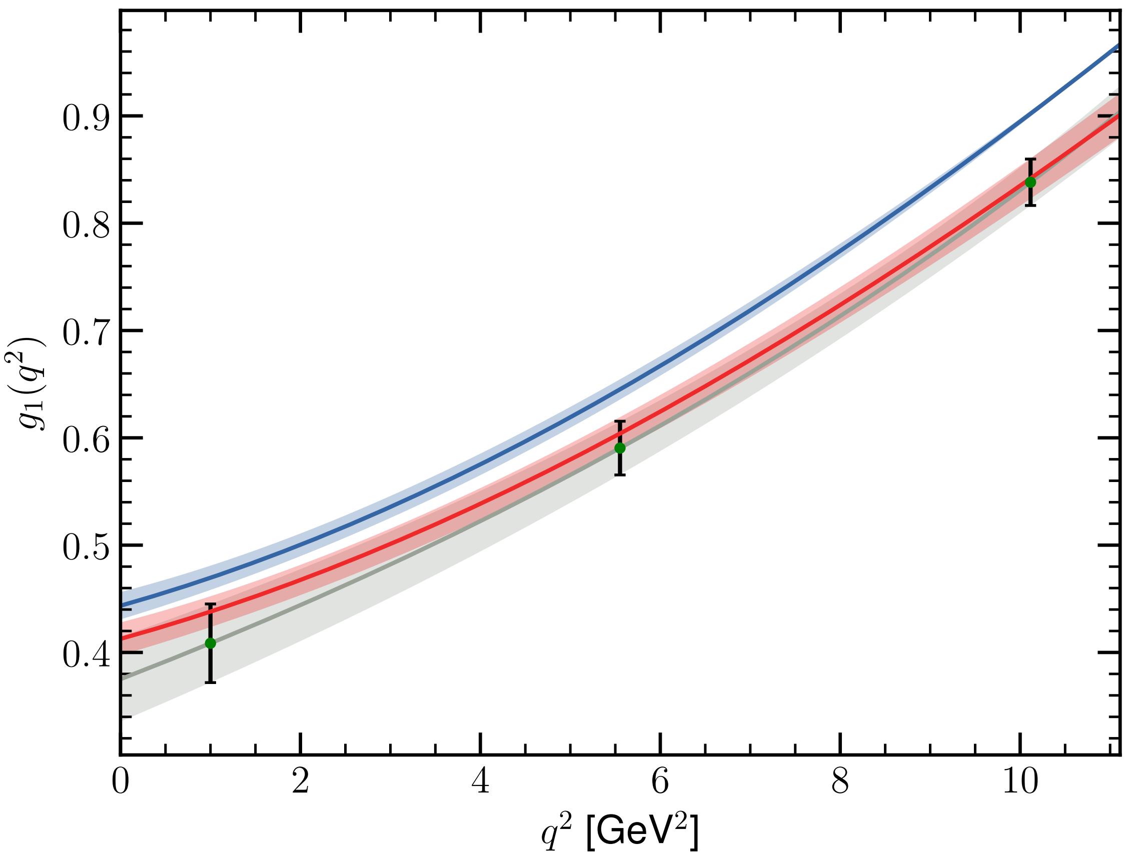
- E.g., fit results for g_1
  blue band shows fit with $\hat{b}_{1,2} = 0$

- Find: $\hat{b}_1 = -(0.46 \pm 0.15)\,\mathrm{GeV}^2$
  ... of the expected magnitude

  Well below the model-dependent estimate: $\hat{b}_1 = -3\bar{\Lambda}_\Lambda^2 \simeq -2\,\mathrm{GeV}^2$

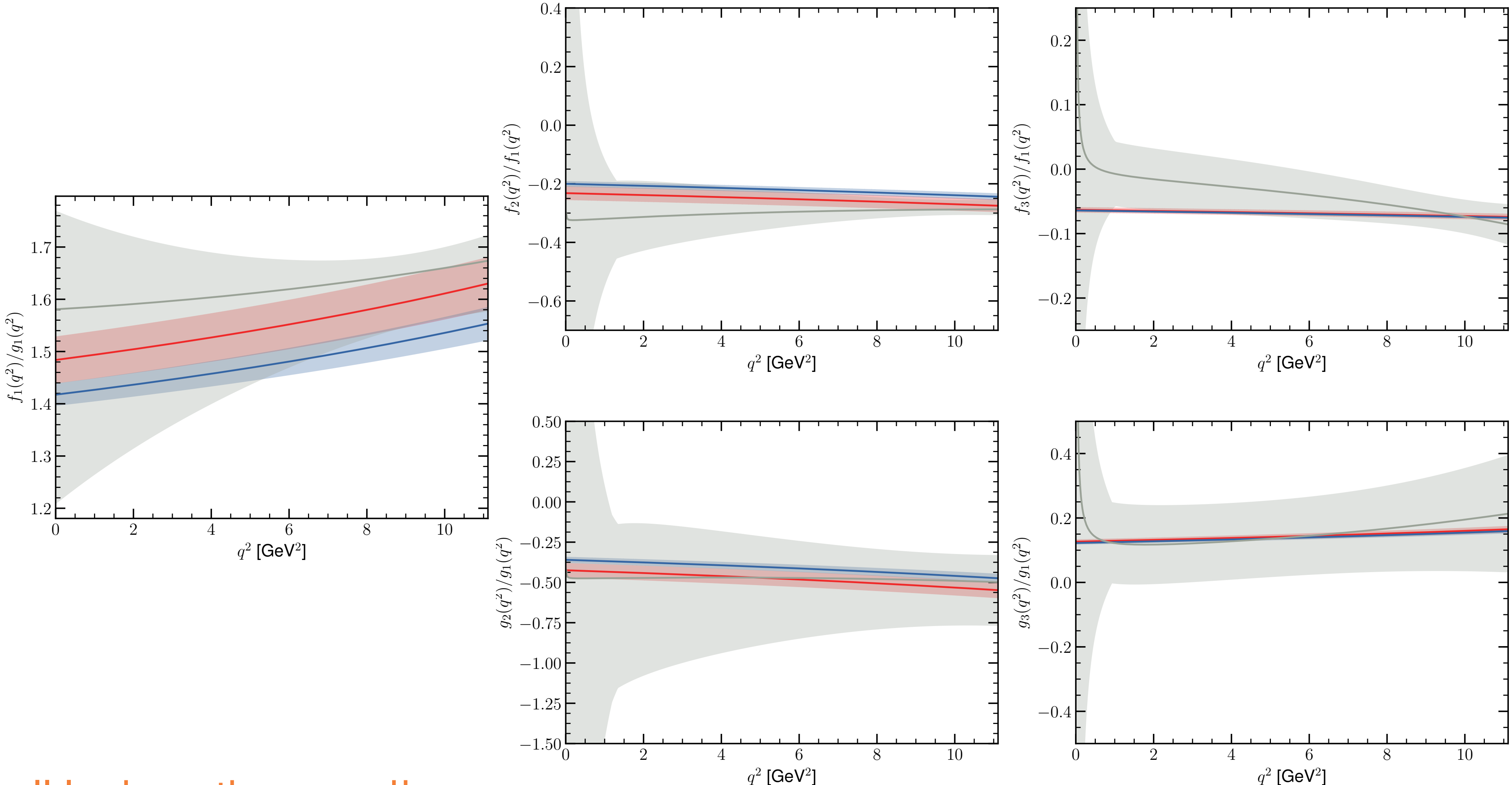
  [Falk & Neubert, hep-ph/9209269]

- Expansion in $\Lambda_{\mathrm{QCD}}/m_c$
  appears well behaved
  (contrary to some claims in literature)



Ratios of form factors

- $f_1(q^2)/g_1(q^2) = \mathcal{O}(1)$, whereas $\{f_{2,3}(q^2)/f_1(q^2),\ g_{2,3}(q^2)/g_1(q^2)\} = \mathcal{O}(\alpha_s, \varepsilon_{c,b})$



- It all looks rather good!

BSM: tensor form factors — issues?

- There are 4 form factors

  We get parameter free predictions!

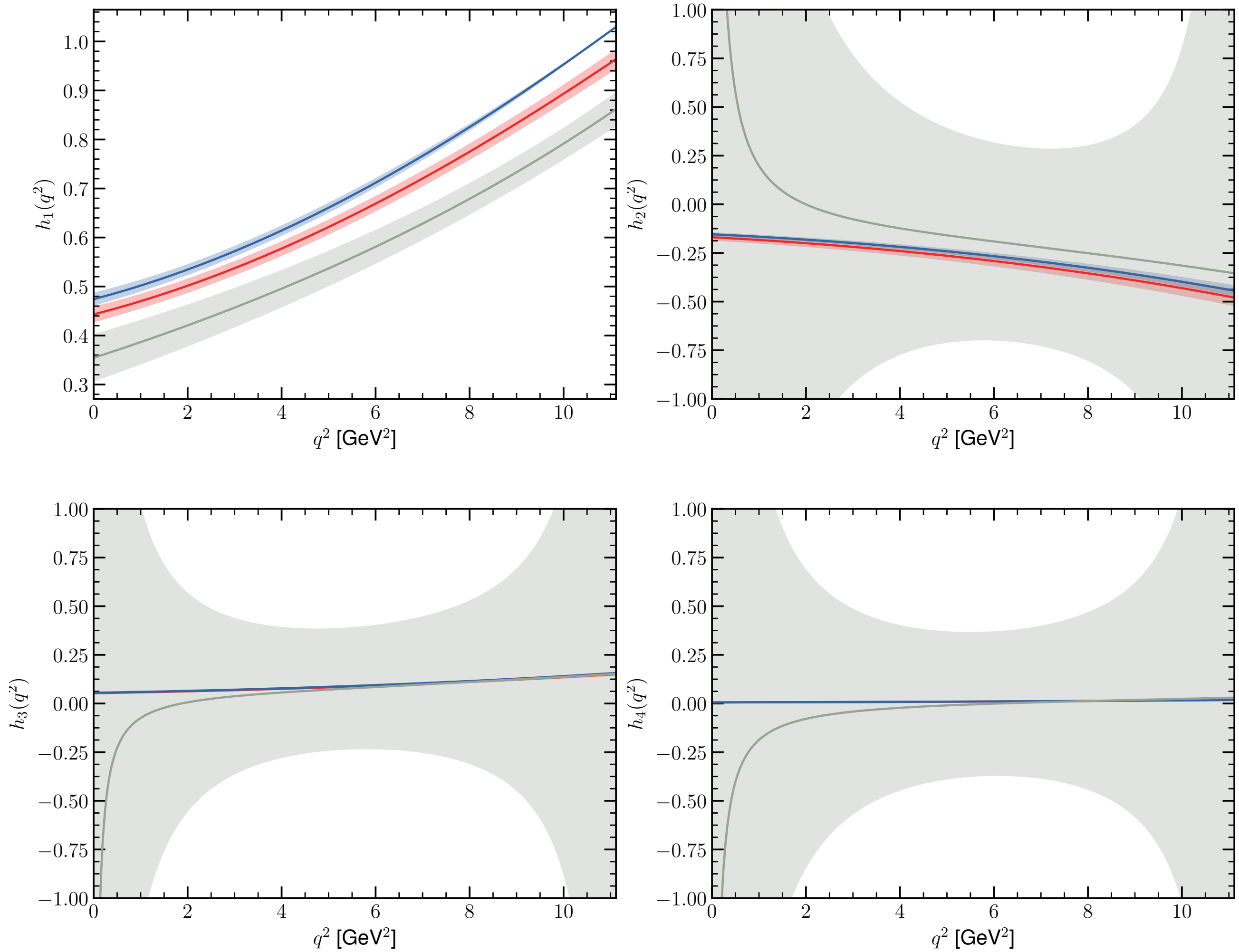
  HQET: $h_1\,(=\widetilde{h}_+) = \mathcal{O}(1)$

  $$h_{2,3,4} = \mathcal{O}(\alpha_s, \varepsilon_{c,b})$$

  LQCD basis: all 4 form factors calculated are $\mathcal{O}(1)$

  [Datta, Kamali, Meinel, Rashed, 1702.02243]

  Compare at $\mu = \sqrt{m_b m_c}$



- Heavy quark symmetry breaking terms consistent (weakly constrained by LQCD)

- If tensions between data and SM remain, we'll have to sort out this difference

More to measure...

- What is the maximal information that the $\Lambda_b \to \Lambda_c \mu \bar{\nu}$ decay can give us?

  $\Lambda_c \to pK\pi$ complicated, $\Lambda_c \to \Lambda\pi\,(\to p\pi\pi)$ looses lots of statistics

- If Λ_c decay distributions are integrated over, but θ is measured (angle between the $\vec{p}_\mu$ and $\vec{p}_{\Lambda_c}$ in $\mu\bar{\nu}$ rest frame), then maximal info one can get:

$$\frac{\mathrm{d}^2\Gamma(\Lambda_b \to \Lambda_c \mu\bar{\nu})}{\mathrm{d}w\,\mathrm{d}\cos\theta} = \frac{3}{8}\Big[(1+\cos^2\theta)\,H_T(w) + 2\cos\theta\,H_A(w) + 2(1-\cos^2\theta)\,H_L(w)\Big]$$

(forward-backward asym.)

Measuring the 3 terms would give more information than just $\mathrm{d}\Gamma(\Lambda_b \to \Lambda_c \mu\bar{\nu})/\mathrm{d}q^2$

- These results will be included in `Hammer` [Bernlochner, Duell, ZL, Papucci, Robinson, soon]

$SU(3)$ breaking in $B_{(s)} \to D^*_{(s)} \ell \bar{\nu}$

$SU(3)$ breaking in $B_{(s)} \to D_{(s)}\ell\bar{\nu}$

- We know little directly from the data about $SU(3)$ breaking in semileptonic decays
- Isgur-Wise fn: "The correction is velocity dependent, but vanishes at zero recoil as required by heavy quark symmetry", about 5% at $w_{\rm max}$ [Jenkins, PLB 281 (1992) 331]

  Calculations showing that $\mathcal{O}(20\%)$ corrections to $SU(3)$ symmetry are possible

  [e.g: Boyd & Grinstein, hep-ph/9502311; Eeg, Fajfer, Kamenik, arXiv:0807.0202]
- LQCD mostly at $w = 1$ so far; FLAG review, Sec.8.4, results for both: [1902.08191]

$$\mathcal{G}_{B\to D}(1) = 1.035 \pm 0.040 \qquad \mathcal{G}_{B_s\to D_s}(1) = 1.068 \pm 0.040$$

$$R(D) = 0.300 \pm 0.008 \qquad R(D_s) = 0.301 \pm 0.006$$

[1703.09728 $\leftrightarrow$ FLAG]

$$\mathcal{F}_{B\to D^*}(1) = 0.895 \pm 0.026 \qquad \mathcal{F}_{B_s\to D_s^*}(1) = 0.883 \pm 0.030$$

  For decay constants, $SU(3)$ breaking is substantial: $f_{B_s}/f_B \approx 1.21 \pm 0.01$

$SU(3)$ breaking in $B_{(s)} \to D_{(s)}\ell\bar{\nu}$ (cont.)

- Some new/old considerations suggesting possibly sizable effects:

Bjorken and Voloshin sum rules relate the behavior of $B_{(s)} \to D^{(*)}_{(s)}$ ground state transition to decays to excited states; e.g., Voloshin sum rule [PRD 46 (1992) 3062]

$$\rho^2 = -\frac{\mathrm{d}}{\mathrm{d}w}\,\frac{\mathrm{d}\Gamma}{\mathrm{d}w}\bigg|_{w=1} < \frac{1}{4} + \frac{m_M - m_Q}{2(m_{M_1} - m_M)} + \dots$$

where $m_{M_1} - m_M$ is the gap to the first excited meson state above $D^{(*)}_{(s)}$

- Expect: slope parameter, ρ^2, increases, if $B_{(s)} \to D^{**}_{(s)}$ rates increase
  if $m_{M_1} - m_M$ decreases

Discovered in 2003: $m_{D^{*\pm}_{s0}} - m_{D^\pm_s} \approx 206\,\mathrm{MeV}$, but $m_{D^{*\pm}_0} - m_{D^\pm} \approx 484\,\mathrm{MeV}$

- It will be interesting to see if these arguments for a steeper fall-off play out, or are compensated by some other effects — will (eventually) measure $SU(3)$ breaking

Some probes of $SU(3)$ breaking

- Compare shapes of $\mathrm{d}\Gamma/\mathrm{d}w$

- Factorization may work better in $B_s \to D_s^{(*)}\pi$ than $B \to D^{(*)}\pi$, tells us $\mathrm{d}\Gamma/\mathrm{d}w\big|_{w_{\max}}$

  Interesting for hadronic dynamics as well, to better understand: [hep-ph/0312319]

  $$|A(\bar{B}^0 \to D^+\pi^-)| = |T+E|, \quad |A(B^- \to D^0\pi^-)| = |T+C|, \quad |A(B_s \to D_s^-\pi^+)| = |T|$$

  Since $\tau_{B^0} \approx \tau_{B_s}$, we can compare directly the branching ratios:

  $[1]\ \ \mathcal{B}(B^0 \to D\pi) = (2.52 \pm 0.13) \times 10^{-3}$

  $[2]\ \ \mathcal{B}(B^0 \to D^*\pi) = (2.74 \pm 0.13) \times 10^{-3}$

  $[3]\ \ \mathcal{B}(B_s \to D_s\pi) = (3.00 \pm 0.23) \times 10^{-3}$ [LHCb, only $0.37/\mathrm{fb}$]

  $[4]\ \ \mathcal{B}(B_s \to D_s^*\pi) = (2.0 \pm 0.5) \times 10^{-3}$

  Central values: $[1] < [3]$ and $[2] > [4]$ seem puzzling, warrants more precise measurements

- Improvements in $B_{(s)} \to D_{(s)}^{**}\pi$ and $B_{(s)} \to D_{(s)}^{**}\ell\bar{\nu}$ rate measurements

$D^{**}_{(s)}$ states: surprises in 1606.09300 (for me?)

- Mass splitting: $m_{D_1^*} - m_{D_0^*} \sim m_{D^*} - m_D$?

  Poor consistency of $m_{D_0^*}$ measurements

Parameter	$\bar\Lambda$	$\bar\Lambda'$	$\bar\Lambda^*$
Value [GeV]	0.40	0.80	0.76

Particle	$s_l^{\pi l}$	J^P	m (MeV)	Γ (MeV)
D_0^*	$\frac{1}{2}^+$	0^+	2349	236
D_1^*	$\frac{1}{2}^+$	1^+	2427	384
D_1	$\frac{3}{2}^+$	1^+	2421	31
D_2^*	$\frac{3}{2}^+$	2^+	2461	47

- $\mathcal{B}(B \to D_0^*\pi)$ puzzling: $\ll D_1\pi$ and $D_2^*\pi$

  breakdown of factorization?

  Small fraction of BaBar & Belle data $+$ LHCb

Decay mode	Branching fraction
$B^0 \to D_2^{*-}\pi^+$	$(0.59 \pm 0.13) \times 10^{-3}$
$B^0 \to D_1^-\pi^+$	$(0.75 \pm 0.16) \times 10^{-3}$
$B^0 \to D_0^{*-}\pi^+$	$(0.12 \pm 0.02) \times 10^{-3}$

- $D_{s0}^*(2317)$: orbitally excited state or "molecule"? Nice for LHCb, $\Gamma_{D_{s0}^*} < 4\,$MeV

  If D_{s0}^* is excited $c\bar s$ state, predict $\mathcal{B}(D_{s0}^* \to D_s^*\gamma)/\mathcal{B}(D_{s0}^* \to D_s\pi)$ above CLEO bound, < 0.059 [Mehen & Springer, hep-ph/0407181; Colangelo & De Fazio, hep-ph/0305140; Godfrey, hep-ph/0305122]

  CLEO used 13.5/fb, the Belle bound < 0.18 used $87/$fb, the BaBar bound < 0.16 used $232/$fb

Final comments

Conclusions

- Measurable NP contribution to $b \to c\ell\bar\nu$ would imply NP at a fairly low scale
- $\Lambda_b \to \Lambda_c \ell\bar\nu$ will provide important cross checks, ultimate uncertainty near $R(D^{(*)})$
- HQET: model independent, more predictive in $\Lambda_b \to \Lambda_c \ell\bar\nu$ than in $B \to D^{(*)}\ell\bar\nu$
- Clear evidence for $\Lambda_{\rm QCD}/m_c^2$ term in an exclusive decay (independent of $|V_{cb}|$)
- The expansion in $\Lambda_{\rm QCD}/m_c^2$ appears well behaved
- LQCD important: all form factors in full phase space, $SU(3)$ breaking (LHCb)
- $B \to D^*\ell\bar\nu$ and $|V_{cb}|$: Lots of progress, many open issues, feel free to ask...
- Belle II and LHCb data $+$ theory progress
  $\Rightarrow$ great improvements in SM measurements and in sensitivity to new physics

Extra slides

$|V_{cb}|$ from $B \to D^* \ell \bar{\nu}$

Making the most of heavy quark symmetry

- "Idea": fit 4 functions (1 leading-order $+$ 3 subleading Isgur-Wise functions) from
  $B \to D^{(*)}\, l\bar\nu \;\Rightarrow\; \mathcal{O}(\Lambda^2_{\rm QCD}/m^2_{c,b}\,,\, \alpha_s^2)$ uncertainties

  [Bernlochner, ZL, Papucci, Robinson, 1703.05330]

- Observables: in $B \to Dl\bar\nu$: $\mathrm{d}\Gamma/\mathrm{d}w$ (Only Belle published fully corrected distributions)

  in $B \to D^*l\bar\nu$: $\mathrm{d}\Gamma/\mathrm{d}w$

  $R_{1,2}(w)$ form factor ratios

  – Systematically improvable with more data

  – $\mathcal{O}(\Lambda^2_{\rm QCD}/m^2_{c,b})$ uncertainties can be constrained comparing w/ lattice form fact.

- Considered many fit scenarios, with/without LQCD and/or QCD sum rule inputs

With all LQCD and no QCDSR input:

Fitting only unfolded Belle data $\quad |V_{cb}|_{\rm BLPR} = (39.1 \pm 1.1) \times 10^{-3}$

SM predictions for $R(D)$ and $R(D^*)$

- Small variations: heavy quark symmetry & phase space leave little wiggle room

Scenario	$R(D)$	$R(D^*)$	Correlation
$\mathrm{L}_{w=1}$	0.292 ± 0.005	0.255 ± 0.005	41%
$\mathrm{L}_{w=1}$+SR	0.291 ± 0.005	0.255 ± 0.003	57%
NoL	0.273 ± 0.016	0.250 ± 0.006	49%
NoL+SR	0.295 ± 0.007	0.255 ± 0.004	43%
$\mathbf{L}_{w\geq1}$	$\mathbf{0.298 \pm 0.003}$	$\mathbf{0.261 \pm 0.004}$	19%
$\mathbf{L}_{w\geq1}$+SR	$\mathbf{0.299 \pm 0.003}$	$\mathbf{0.257 \pm 0.003}$	44%
th:$\mathrm{L}_{w\geq1}$+SR	0.306 ± 0.005	0.256 ± 0.004	33%
Data [HFLAV]	0.340 ± 0.030	0.295 ± 0.014	-38%
Fajfer et al. '12	—	0.252 ± 0.003	—
Lattice [FLAG]	0.300 ± 0.008	—	—
Bigi, Gambino '16	0.299 ± 0.003	—	—
Bigi, Gambino, Schacht '17	—	0.260 ± 0.008	—
Jaiswal, Nandi, Patra '17	0.302 ± 0.003	0.257 ± 0.005	13%
SM [HFLAV]	0.299 ± 0.003	0.258 ± 0.005	—

The CLN fits used 1997–2017

- Role of QCD SR in CLN: $R_{1,2}(w) = \underbrace{R_{1,2}(1)}_{\text{fit}} + \underbrace{R'_{1,2}(1)}_{\text{fixed}}\,(w-1) + \underbrace{R''_{1,2}(1)}_{\text{fixed}}\,(w-1)^2/2$

In HQET: $R_{1,2}(1) = 1 + \mathcal{O}(\Lambda_{\rm QCD}/m_{c,b}\,,\alpha_s)$ $\qquad R^{(n)}_{1,2}(1) = 0 + \mathcal{O}(\Lambda_{\rm QCD}/m_{c,b}\,,\alpha_s)$

The $\mathcal{O}(\Lambda_{\rm QCD}/m_{c,b})$ terms are determined by 3 subleading Isgur-Wise functions

- Inconsistent fits: same param's determine $R_{1,2}(1) - 1$ (fit) and $R^{(1,2)}_{1,2}(1)$ (QCDSR)

Sometimes calculations using QCD sum rules are called the HQET predictions

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- Devised fits to "interpolate" between BGL and CLN [Bernlochner, ZL, Robinson, Papucci, 1708.07134]

form factors	BGL	CLN	CLNnoR	noHQS
axial $\propto \epsilon^*_\mu$	$b_0,\ b_1$	$h_{A_1}(1),\ \rho^2_{D^*}$	$h_{A_1}(1),\ \rho^2_{D^*}$	$h_{A_1}(1),\ \rho^2_{D^*},\ c_{D^*}$
vector	$a_0,\ a_1$	$R_1(1)$	$R_1(1),\ R'_1(1)$	$R_1(1),\ R'_1(1)$
axial $(\mathcal{F}_1)$	$c_1,\ c_2$	$R_2(1)$	$R_2(1),\ R'_2(1)$	$R_2(1),\ R'_2(1)$

Relaxing constraints on $R'_{1,2}(1)$, fit results similar to BGL

Nested hypothesis tests

- Optimal BGL fit parameter choice, given available data? (upper: χ^2, lower: $|V_{cb}| \times 10^3$)

$n_c \backslash n_a$	1	2	3	1	2	3	1	2	3
1	33.2 38.6 ± 1.0	31.6 38.6 ± 1.0	31.2 38.6 ± 1.0	33.0 39.0 ± 1.5	29.1 40.7 ± 1.6	28.9 40.7 ± 1.6	30.4 40.7 ± 1.7	29.1 40.6 ± 1.8	28.9 40.6 ± 1.8
2	32.9 38.8 ± 1.1	31.3 38.7 ± 1.1	31.1 38.8 ± 1.0	32.7 39.5 ± 1.7	**27.7** $\mathbf{41.7 \pm 1.8}$	27.7 41.6 ± 1.8	29.2 41.8 ± 2.0	27.7 41.8 ± 2.0	27.7 41.7 ± 2.0
3	31.7 39.0 ± 1.1	31.3 38.6 ± 1.2	31.0 38.6 ± 1.1	29.1 41.9 ± 2.0	27.7 41.8 ± 2.0	27.6 41.7 ± 2.0	29.2 41.8 ± 2.0	27.6 41.7 ± 1.9	23.2 41.4 ± 2.0
	$n_b = 1$			$n_b = 2$			$n_b = 3$		

- Fit w/ 1 param added / removed: $\mathrm{BGL}_{(n_a\pm1)n_bn_c}$, $\mathrm{BGL}_{n_a(n_b\pm1)n_c}$, $\mathrm{BGL}_{n_an_b(n_c\pm1)}$
- Accept descendant (parent) if $\Delta\chi^2$ is above (below) a boundary, say, $\Delta\chi^2 = 1$
- Repeat until "stationary" fit is found, preferred over its parents and descendants
- If multiple stationary fits, choose smallest N, then smallest χ^2 (333 is an overfit!)

Start from small N, to avoid overfitting e.g.: $\begin{cases} 111 \to 211 \to 221 \to 222 \\ 121 \to 131 \to 231 \to 232 \to 222 \end{cases}$

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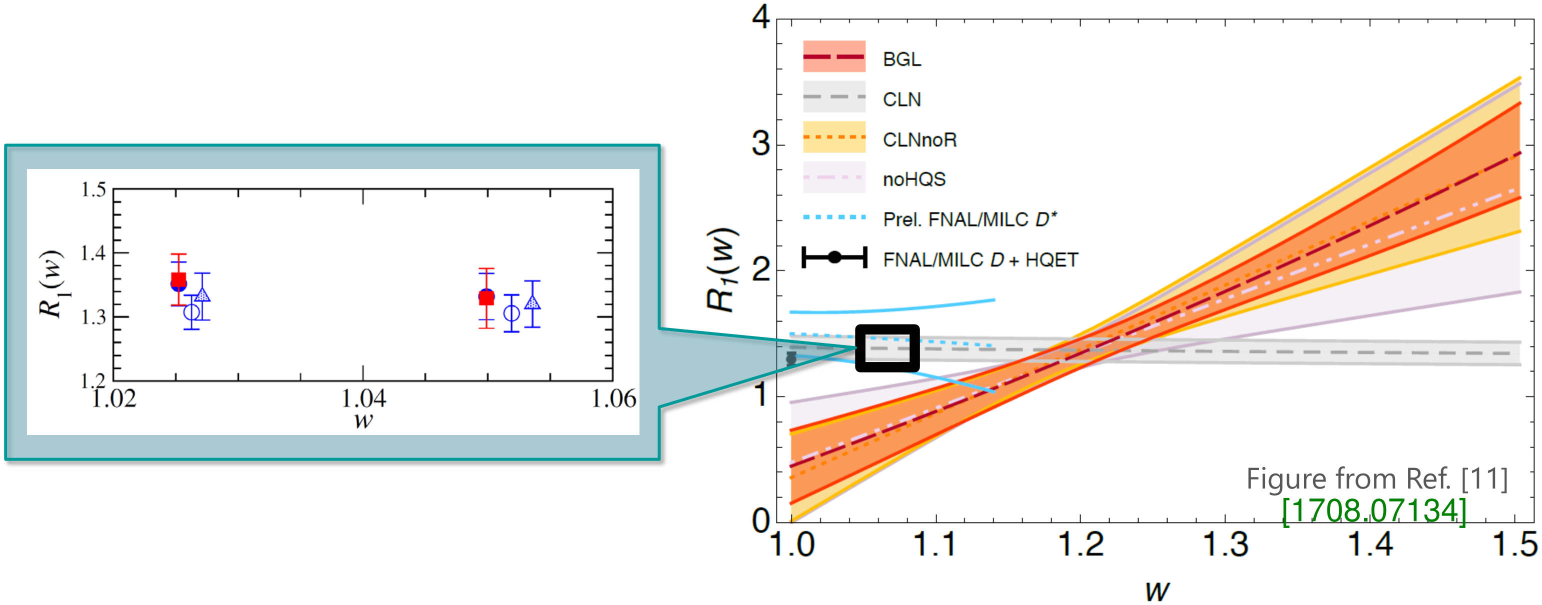
Lattice QCD, preliminary results

- FNAL/MILC and JLQCD are both working on the $B \to D^* \ell \bar{\nu}$ form factors
  Independent formulations: staggered vs. Mobius domain-wall actions



Therefore, this issue is still open. These parametrizations should be eventually replaced by a lattice-based parametrization.

[T. Kaneko, JLQCD poster at Lattice 2018, 1811.00794; also Fermilab/MILC, 1710.09817l]

- No qualitative difference between LQCD calculation at $w = 1$, or slightly above