\( \Lambda_b \rightarrow \Lambda_c \ell \bar{\nu} \), Tests of HQS, and \( SU(3) \) breaking in \( B_s \rightarrow D_s^{*} \ell \bar{\nu} \)

Zoltan Ligeti

2019 Lattice X Intensity Frontier Workshop

BNL, Sep 23–25, 2019

See: Bernlochner, ZL, Robinson, Sutcliffe, arXiv:1808.09464 [PRL]; 1812.07593 [PRD]

Bernlochner and ZL, talk at LHCb analysis meeting, Sep 4, 2019 — LQCD connections
CKM fit: plenty of room for new physics

- SM dominates $CP$ viol. $\Rightarrow$ KM Nobel
- The implications of the consistency are often overstated
- Much larger allowed region if the SM is not assumed
- Tree-level (mainly $V_{ub} \& \gamma$) vs. loop
- $V_{ub} \& V_{cb}$: important SM measurements + essential for NP sensitivity
- In loop (FCNC) processes NP / SM $\sim 20\%$ still allowed (mixing, $B \rightarrow X\ell^+\ell^-$, $B \rightarrow X\gamma$, ...)

$ZL – p. 1$
Recent focus: $R(D)$ and $R(D^*)$

- BaBar, Belle, LHCb: enhanced $\tau$ rates, $R(D^{(*)}) = \frac{\Gamma(B \rightarrow D^{(*)}\tau\bar{\nu})}{\Gamma(B \rightarrow D^{(*)}l\bar{\nu})}$ ($l = e, \mu$)

Notation: $\ell = e, \mu, \tau$ and $l = e, \mu$

Future:

Belle II (50/ab, in SM): $\delta R(D^{(*)}) \sim 2(3)\%$

- Big improvements: even if central values change, plenty of room to establish NP
- Focus on the 3 modes that are expected to be most precise in the long term
Heavy quark symmetry 101

- Model independent from QCD, used both in some continuum & LQCD methods
- $Q\bar{Q}$: positronium-type bound state, perturbative in the $m_Q \gg \Lambda_{\text{QCD}}$ limit
- $Q\bar{q}$: wave function of the light degrees of freedom
  ("brown muck") insensitive to spin and flavor of $Q$
  
  (A $B$ meson is a lot more complicated than just a $b\bar{q}$ pair)

In the $m_Q \gg \Lambda_{\text{QCD}}$ limit, the heavy quark acts as a static color source with fixed four-velocity $v^\mu$ [Isgur & Wise]

$SU(2n)$ heavy quark spin-flavor symmetry at fixed $v^\mu$ [Georgi]

- Similar to atomic physics: $(m_e \ll m_N)$
  1. Flavor symmetry $\sim$ isotopes have similar chemistry [$\Psi_e$ independent of $m_N$]
  2. Spin symmetry $\sim$ hyperfine levels almost degenerate [$\vec{s}_e - \vec{s}_N$ interaction $\to 0$]

$ZL$ – p. 3
Spectroscopy of heavy-light mesons

- In $m_Q \gg \Lambda_{\text{QCD}}$ limit, spin of the heavy quark is a good quantum number, and so is the spin of the light d.o.f., since $\vec{J} = \vec{s}_Q + \vec{s}_l$ and
  
  angular momentum conservation: $[\vec{J}, \mathcal{H}] = 0$

  heavy quark symmetry: $[\vec{s}_Q, \mathcal{H}] = 0$ \implies\ [\vec{s}_l, \mathcal{H}] = 0

- For a given $s_l$, two degenerate states:
  
  $J_{\pm} = s_l \pm \frac{1}{2}$

  \implies\ $\Delta_i = \mathcal{O}(\Lambda_{\text{QCD}})$ — same in $B$ and $D$ sector

  Doublets are split by order $\frac{\Lambda_{\text{QCD}}^2}{m_Q}$, e.g.:

  $m_D^* - m_D \sim 140$ MeV

  $m_B^* - m_B \sim 45$ MeV

  ratio $\sim m_c/m_b$
Basics of $B \rightarrow D^{(*)}\ell\bar{\nu}$ or $\Lambda_b \rightarrow \Lambda_c\ell\bar{\nu}$

- In the $m_{b,c} \gg \Lambda_{QCD}$ limit, configuration of brown muck only depends on the four-velocity of the heavy quark, but not on its mass and spin.
- On a time scale $\ll \Lambda_{QCD}^{-1}$ weak current changes $b \rightarrow c$ i.e.: $\vec{p}_b \rightarrow \vec{p}_c$ and possibly $\bar{s}_Q$ flips.
- In $m_{b,c} \gg \Lambda_{QCD}$ limit, brown muck only feels $v_b \rightarrow v_c$
- Form factors independent of Dirac structure of weak current $\Rightarrow$ all form factors related to a single function of $w = v \cdot v'$, the Isgur-Wise function, $\xi(w)$
  \begin{align*}
  \uparrow \\
  \text{Contains all nonperturbative low-energy hadronic physics}
  \end{align*}
- $\xi(1) = 1$, because at “zero recoil” configuration of brown muck not changed at all.
- Same holds for $\Lambda_b \rightarrow \Lambda_c\ell\bar{\nu}$, different Isgur-Wise fn, $\xi \rightarrow \zeta$ [also satisfies $\zeta(1) = 1$]
\[ \Lambda_b \rightarrow \Lambda_c \ell \bar{\nu} \]
Ancient knowledge: baryons simpler than mesons

- Used to be well known — forgotten by experimentalists as well as theorists...

Form Factor Ratio Measurement in $\Lambda_c^+ \to \Lambda e^+ \nu_e$

G. Crawford,¹ C. M. Daubenmier,¹ R. Fulton,¹ D. Fujino,¹ K. K. Gan,¹ K. Honscheid,¹ H. Kagan,¹ R. Kass,¹ J. Lee,¹

[CLEO]

Element $|V_{cs}|$ is known from unitarity [1]. Within heavy quark effective theory (HQET) [2], $\Lambda$-type baryons are more straightforward to treat than mesons as they consist of a heavy quark and a spin and isospin zero light diquark.
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Combine LHCb measurement of $d\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu})/dq^2$ shape \[1709.01920\] with LQCD results for (axial-)vector form factors \[1503.01421\]

[Bernlochner, ZL, Robinson, Sutcliffe, 1808.09464; 1812.07593]
Intro to $\Lambda_b \to \Lambda_c \ell \bar{\nu}$

- Ground state baryons are simpler than mesons: brown muck in (iso)spin-0 state

- SM: 6 form factors, functions of $w = v \cdot v' = (m_{\Lambda_b}^2 + m_{\Lambda_c}^2 - q^2)/(2m_{\Lambda_b}m_{\Lambda_c})$

$$
\langle \Lambda_c(p', s')|\bar{c}\gamma_\nu b|\Lambda_b(p, s)\rangle = \bar{u}_c(v', s')\left[f_1\gamma_\mu + f_2v_\mu + f_3v'_\mu\right]u_b(v, s)
$$

$$
\langle \Lambda_c(p', s')|\bar{c}\gamma_\nu \gamma_5 b|\Lambda_b(p, s)\rangle = \bar{u}_c(v', s')\left[g_1\gamma_\mu + g_2v_\mu + g_3v'_\mu\right]\gamma_5 u_b(v, s)
$$

Heavy quark limit: $f_1 = g_1 = \zeta(w)$ Isgur-Wise fn, and $f_{2,3} = g_{2,3} = 0$ [\(\zeta(1) = 1\)]

- Include $\alpha_s, \varepsilon_{b,c}, \alpha_s\varepsilon_{b,c}, \varepsilon_c^2$:

$$
m_{\Lambda_{b,c}} = m_{b,c} + \bar{\Lambda}_\Lambda + \ldots, \quad \varepsilon_{b,c} = \bar{\Lambda}_\Lambda/(2m_{b,c})
$$

($\bar{\Lambda}_\Lambda \sim 0.8$ GeV larger than $\bar{\Lambda}$ for mesons, enters via eq. of motion $\Rightarrow$ expect worse expansion?)

$$
f_1 = \zeta(w)\left\{1 + \frac{\alpha_s}{\pi}C_V + \varepsilon_c + \varepsilon_b + \frac{\alpha_s}{\pi}\left[C_V + 2(w - 1)C'_V\right](\varepsilon_c + \varepsilon_b) + \frac{\hat{b}_1 - \hat{b}_2}{4m_c^2} + \ldots\right\}
$$

- No $\mathcal{O}(\Lambda_{QCD}/m_{b,c})$ subleading Isgur-Wise function, only 2 at $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ [Falk & Neubert, hep-ph/9209269]

- HQET is more constraining than in meson decays!

$B \to D^{(*)}\ell\bar{\nu}$: 6 sub-subleading Isgur-Wise functions at $\mathcal{O}(\Lambda^2_{QCD}/m_c^2)$ [w/ LCSR, 1908.09398]

ZL – p. 7
Fits and form factor definitions

- Standard HQET form factor definitions: 
  \[ \{ f_1, g_1 \} = \zeta(w) \left[ 1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b}) \right] \]
  \[ \{ f_{2,3}, g_{2,3} \} = \zeta(w) \left[ 0 + \mathcal{O}(\alpha_s, \varepsilon_{c,b}) \right] \]

- Form factor basis in LQCD calculation: 
  \[ \{ f_{0,+}, g_{0,+} \} = \zeta(w) \left[ 1 + \mathcal{O}(\alpha_s, \varepsilon_{c,b}) \right] \]

LQCD results published as fits to 11 or 17 BCL parameters, including correlations

All 6 form factors computed in LQCD \sim \text{Isgur-Wise fn} \Rightarrow \text{despite good precision, limited constraints on subleading terms and their } w \text{ dependence} \quad \text{[Detmold, Lehner, Meinel, 1503.01421]}

- Only 4 parameters (and } m_b^{1S} \text{): } \{ \zeta', \zeta'', \hat{b}_1, \hat{b}_2 \}

  \[ \zeta(w) = 1 + (w - 1) \zeta' + \frac{1}{2}(w - 1)^2 \zeta'' + \ldots \]  
  \[ b_{1,2}(w) = \zeta(w) \left( \hat{b}_{1,2} + \ldots \right) \]

  (Expanding in } w - 1 \text{ or in conformal parameter, } z, \text{ makes negligible difference)

- Current LHCb and LQCD data do not yet allow constraining } \zeta''' \text{ and/or } \hat{b}'_{1,2} \text{)
Fit to lattice QCD form factors and LHCb (1)

- Fit 6 form factors w/ 4 parameters: $\zeta'(1), \zeta''(1), \hat{b}_1, \hat{b}_2$ [LQCD: Detmold, Lehner, Meinel, 1503.01421]
Our fit, compared to the LQCD fit to LHCb:

Obtain: $R(\Lambda_c) = 0.324 \pm 0.004$

A factor of $\sim 3$ more precise than LQCD prediction — data constrains combinations of form factors relevant for predicting $R(\Lambda_c)$

We do not follow: “In order to determine the shape of the Isgur-Wise function $\xi_B(w)$, we use the square root of $dN_{\text{corr}}/dw$ ... evaluated at the midpoint in the seven unfolded $w$ bins.” [LHCb, 1709.01920]
The fit requires the $1/m_c^2$ terms

- E.g., fit results for $g_1$
  blue band shows fit with $\hat{b}_{1,2} = 0$

- Find: $\hat{b}_1 = -(0.46 \pm 0.15)$ GeV$^2$
  ... of the expected magnitude

Well below the model-dependent estimate:
$$\hat{b}_1 = -3\bar{\Lambda}_\Lambda^2 \simeq -2 \text{ GeV}^2$$

- Expansion in $\Lambda_{QCD}/m_c$
  appears well behaved
  (contrary to some claims in literature)
Ratios of form factors

- $f_1(q^2)/g_1(q^2) = \mathcal{O}(1)$, whereas $\{f_{2,3}(q^2)/f_1(q^2), g_{2,3}(q^2)/g_1(q^2)\} = \mathcal{O}(\alpha_s, \varepsilon_{c,b})$

- It all looks rather good!
There are 4 form factors
We get parameter free predictions!

HQET: $h_1 (\equiv \tilde{h}_+ ) = \mathcal{O}(1)$
$h_{2,3,4} = \mathcal{O}(\alpha_s, \varepsilon_{c,b})$

LQCD basis: all 4 form factors calculated are $\mathcal{O}(1)$
[Datta, Kamali, Meinel, Rashed, 1702.02243]

Compare at $\mu = \sqrt{m_b m_c}$

Heavy quark symmetry breaking terms consistent (weakly constrained by LQCD)

If tensions between data and SM remain, we’ll have to sort out this difference
What is the maximal information that the $\Lambda_b \to \Lambda_c \mu \bar{\nu}$ decay can give us?

$\Lambda_c \to pK\pi$ complicated, $\Lambda_c \to \Lambda\pi \to p\pi\pi$ looses lots of statistics

If $\Lambda_c$ decay distributions are integrated over, but $\theta$ is measured (angle between the $\vec{p}_\mu$ and $\vec{p}_{\Lambda_c}$ in $\mu\bar{\nu}$ rest frame), then maximal info one can get:

$$\frac{d^2\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu})}{dw \, d\cos \theta} = \frac{3}{8} \left[ (1 + \cos^2 \theta) \, H_T(w) + 2 \cos \theta \, H_A(w) + 2(1 - \cos^2 \theta) \, H_L(w) \right]$$

(forward-backward asym.)

Measuring the 3 terms would give more information than just $d\Gamma(\Lambda_b \to \Lambda_c \mu \bar{\nu})/dq^2$

These results will be included in Hammer

[Bernlochner, Duell, ZL, Papucci, Robinson, soon]
$SU(3)$ breaking in $B_{(s)} \rightarrow D^{*}_{(s)} \ell \bar{\nu}$
SU(3) breaking in \( B_s \to D_s \ell \bar{\nu} \)

- We know little directly from the data about \( SU(3) \) breaking in semileptonic decays

- Isgur-Wise fn: “The correction is velocity dependent, but vanishes at zero recoil as required by heavy quark symmetry”, about 5% at \( w_{\text{max}} \) [Jenkins, PLB 281 (1992) 331]

Calculations showing that \( \mathcal{O}(20\%) \) corrections to \( SU(3) \) symmetry are possible
  
  [e.g: Boyd & Grinstein, hep-ph/9502311; Eeg, Fajfer, Kamenik, arXiv:0807.0202]

- LQCD mostly at \( w = 1 \) so far; FLAG review, Sec.8.4, results for both: [1902.08191]

\[
\begin{align*}
  G_{B \to D}(1) &= 1.035 \pm 0.040 \\
  R(D) &= 0.300 \pm 0.008 \\
  F_{B \to D^*}(1) &= 0.895 \pm 0.026 \\
  G_{B_s \to D_s}(1) &= 1.068 \pm 0.040 \\
  R(D_s) &= 0.301 \pm 0.006 \\
  F_{B_s \to D_s^*}(1) &= 0.883 \pm 0.030
\end{align*}
\]

For decay constants, \( SU(3) \) breaking is substantial: \( f_{B_s}/f_B \approx 1.21 \pm 0.01 \)
Some new/old considerations suggesting possibly sizable effects:

Bjorken and Voloshin sum rules relate the behavior of $B(s) \rightarrow D(s)^{*} \ell \bar{\nu}$ ground state transition to decays to excited states; e.g., Voloshin sum rule \cite{PRD 46 (1992) 3062}

$$\rho^2 = -\frac{d}{dw} \frac{d\Gamma}{dw} \bigg|_{w=1} < \frac{1}{4} + \frac{m_M - m_Q}{2(m_{M_1} - m_M)} + \ldots$$

where $m_{M_1} - m_M$ is the gap to the first excited meson state above $D(s)^{*}$

Expect: slope parameter, $\rho^2$, increases, if $B(s) \rightarrow D(s)^{*}$ rates increase

if $m_{M_1} - m_M$ decreases

Discovered in 2003: $m_{D(s_0^*)^\pm} - m_{D(s)^\pm} \approx 206$ MeV, but $m_{D_0^* \pm} - m_{D^\pm} \approx 484$ MeV

It will be interesting to see if these arguments for a steeper fall-off play out, or are compensated by some other effects — will (eventually) measure $SU(3)$ breaking
Some probes of $SU(3)$ breaking

- Compare shapes of $d\Gamma/dw$
- Factorization may work better in $B_s \to D_s^{(*)}\pi$ than $B \to D^{(*)}\pi$, tells us $d\Gamma/dw|_{w_{\text{max}}}$

Interesting for hadronic dynamics as well, to better understand:

$$|A(\bar{B}^0 \to D^+\pi^-)| = |T + E|, \quad |A(B^- \to D^0\pi^-)| = |T + C|, \quad |A(B_s \to D_s^-\pi^+)| = |T|$$

Since $\tau_{B^0} \approx \tau_{B_s}$, we can compare directly the branching ratios:

[1] $\mathcal{B}(B^0 \to D\pi) = (2.52 \pm 0.13) \times 10^{-3}$
[2] $\mathcal{B}(B^0 \to D^{*}\pi) = (2.74 \pm 0.13) \times 10^{-3}$
[3] $\mathcal{B}(B_s \to D_s\pi) = (3.00 \pm 0.23) \times 10^{-3}$ \quad [LHCb, only 0.37/fb]
[4] $\mathcal{B}(B_s \to D^{*}_s\pi) = (2.0 \pm 0.5) \times 10^{-3}$

Central values: [1] < [3] and [2] > [4] seem puzzling, warrants more precise measurements

- Improvements in $B_{(s)} \to D^{**}_{(s)}\pi$ and $B_{(s)} \to D^{**}_{(s)}\ell\bar{\nu}$ rate measurements
$D^{**}(s)$ states: surprises in 1606.09300 (for me?)

- Mass splitting: $m_{D^*_1} - m_{D^*_0} \sim m_{D^*} - m_D$?
  Poor consistency of $m_{D^*_0}$ measurements

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\bar{\Lambda}$</th>
<th>$\bar{\Lambda}'$</th>
<th>$\bar{\Lambda}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value [GeV]</td>
<td>0.40</td>
<td>0.80</td>
<td>0.76</td>
</tr>
</tbody>
</table>

- $\mathcal{B}(B \to D_{0}^*\pi)$ puzzling: $\ll D_1\pi$ and $D_2^*\pi$
  breakdown of factorization?

Small fraction of BaBar & Belle data + LHCb

- $D_s^*(2317)$: orbitally excited state or “molecule”?
  Nice for LHCb, $\Gamma_{D_{s0}^*} < 4$ MeV

If $D_{s0}^*$ is excited $c\bar{s}$ state, predict $\mathcal{B}(D_{s0}^* \to D_{s0}^*\gamma)/\mathcal{B}(D_{s0}^* \to D_s\pi)$ above CLEO bound, $< 0.059$


CLEO used 13.5/fb, the Belle bound $< 0.18$ used 87/fb, the BaBar bound $< 0.16$ used 232/fb
Final comments
Conclusions

- Measurable NP contribution to $b \to c\ell\bar{\nu}$ would imply NP at a fairly low scale
- $\Lambda_b \to \Lambda_c\ell\bar{\nu}$ will provide important cross checks, ultimate uncertainty near $R(D(\ast))$
- HQET: model independent, more predictive in $\Lambda_b \to \Lambda_c\ell\bar{\nu}$ than in $B \to D(\ast)\ell\bar{\nu}$
- Clear evidence for $\Lambda_{QCD}/m_c^2$ term in an exclusive decay (independent of $|V_{cb}|$)
- The expansion in $\Lambda_{QCD}/m_c^2$ appears well behaved
- LQCD important: all form factors in full phase space, $SU(3)$ breaking (LHCb)
- $B \to D^*\ell\bar{\nu}$ and $|V_{cb}|$: Lots of progress, many open issues, feel free to ask...
- Belle II and LHCb data + theory progress
  $\Rightarrow$ great improvements in SM measurements and in sensitivity to new physics
$|V_{cb}|$ from $B \to D^* \ell \bar{\nu}$
Making the most of heavy quark symmetry

• “Idea”: fit 4 functions (1 leading-order + 3 subleading Isgur-Wise functions) from 

\[ B \rightarrow D^{(*)} l\bar{\nu} \Rightarrow \mathcal{O}(\Lambda_{QCD}^2/m_{c,b}^2, \alpha_s^2) \] 

uncertainties

[Bernlochner, ZL, Papucci, Robinson, 1703.05330]

• Observables: in \( B \rightarrow Dl\bar{\nu} \): \( \frac{d\Gamma}{dw} \)

(Only Belle published fully corrected distributions)

in \( B \rightarrow D^*l\bar{\nu} \): \( \frac{d\Gamma}{dw} \)

\( R_{1,2}(w) \) form factor ratios

– Systematically improvable with more data

– \( \mathcal{O}(\Lambda_{QCD}^2/m_{c,b}^2) \) uncertainties can be constrained comparing w/ lattice form fact.

• Considered many fit scenarios, with/without LQCD and/or QCD sum rule inputs

With all LQCD and no QCDSR input:

Fitting only unfolded Belle data

\[ |V_{cb}|_{BLPR} = (39.1 \pm 1.1) \times 10^{-3} \]

ZL – p. i
SM predictions for $R(D)$ and $R(D^*)$

- Small variations: heavy quark symmetry & phase space leave little wiggle room

<table>
<thead>
<tr>
<th>Scenario</th>
<th>$R(D)$</th>
<th>$R(D^*)$</th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{w=1}$</td>
<td>0.292 ± 0.005</td>
<td>0.255 ± 0.005</td>
<td>41%</td>
</tr>
<tr>
<td>$L_{w=1} + SR$</td>
<td>0.291 ± 0.005</td>
<td>0.255 ± 0.003</td>
<td>57%</td>
</tr>
<tr>
<td>NoL</td>
<td>0.273 ± 0.016</td>
<td>0.250 ± 0.006</td>
<td>49%</td>
</tr>
<tr>
<td>NoL + SR</td>
<td>0.295 ± 0.007</td>
<td>0.255 ± 0.004</td>
<td>43%</td>
</tr>
<tr>
<td>$L_{w \geq 1}$</td>
<td>0.298 ± 0.003</td>
<td>0.261 ± 0.004</td>
<td>19%</td>
</tr>
<tr>
<td>$L_{w \geq 1} + SR$</td>
<td>0.299 ± 0.003</td>
<td>0.257 ± 0.003</td>
<td>44%</td>
</tr>
<tr>
<td>th:$L_{w \geq 1} + SR$</td>
<td>0.306 ± 0.005</td>
<td>0.256 ± 0.004</td>
<td>33%</td>
</tr>
<tr>
<td>Data [HFLAV]</td>
<td>0.340 ± 0.030</td>
<td>0.295 ± 0.014</td>
<td>−38%</td>
</tr>
<tr>
<td>Fajfer et al. ’12</td>
<td>—</td>
<td>0.252 ± 0.003</td>
<td>—</td>
</tr>
<tr>
<td>Lattice [FLAG]</td>
<td>0.300 ± 0.008</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Bigi, Gambino ’16</td>
<td>0.299 ± 0.003</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Bigi, Gambino, Schacht ’17</td>
<td>—</td>
<td>0.260 ± 0.008</td>
<td>—</td>
</tr>
<tr>
<td>Jaiswal, Nandi, Patra ’17</td>
<td>0.302 ± 0.003</td>
<td>0.257 ± 0.005</td>
<td>13%</td>
</tr>
<tr>
<td>SM [HFLAV]</td>
<td>0.299 ± 0.003</td>
<td>0.258 ± 0.005</td>
<td>—</td>
</tr>
</tbody>
</table>

ZL – p. ii
The CLN fits used 1997–2017

- **Role of QCD SR in CLN**: \( R_{1,2}(w) = \underbrace{R_{1,2}(1)}_{\text{fit}} + \underbrace{R'_{1,2}(1)(w - 1)}_{\text{fixed}} + \underbrace{R''_{1,2}(1)(w - 1)^2/2}_{\text{fixed}} \)

In HQET:
\[
R_{1,2}(1) = 1 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s) \quad R_{1,2}^{(n)}(1) = 0 + \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}, \alpha_s)
\]
The \( \mathcal{O}(\Lambda_{\text{QCD}}/m_{c,b}) \) terms are determined by 3 subleading Isgur-Wise functions

- **Inconsistent fits**: same param’s determine \( R_{1,2}(1) - 1 \) (fit) and \( R_{1,2}^{(1,2)}(1) \) (QCDSR)

Sometimes calculations using QCD sum rules are called the HQET predictions

- **Devised fits to “interpolate” between BGL and CLN** [Bernlochner, ZL, Robinson, Papucci, 1708.07134]

<table>
<thead>
<tr>
<th>form factors</th>
<th>BGL</th>
<th>CLN</th>
<th>CLNnoR</th>
<th>noHQS</th>
</tr>
</thead>
<tbody>
<tr>
<td>axial ( \propto \epsilon^*_\mu )</td>
<td>( b_0, b_1 )</td>
<td>( h_{A_1}(1), \rho^2_D )</td>
<td>( h_{A_1}(1), \rho^2_D )</td>
<td>( h_{A_1}(1), \rho^2_D^<em>, c_D^</em> )</td>
</tr>
<tr>
<td>vector</td>
<td>( a_0, a_1 )</td>
<td>( R_1(1) )</td>
<td>( R_1(1), R'_1(1) )</td>
<td>( R_1(1), R'_1(1) )</td>
</tr>
<tr>
<td>axial ( (F_1) )</td>
<td>( c_1, c_2 )</td>
<td>( R_2(1) )</td>
<td>( R_2(1), R'_2(1) )</td>
<td>( R_2(1), R'_2(1) )</td>
</tr>
</tbody>
</table>

Relaxing constraints on \( R'_{1,2}(1) \), fit results similar to BGL

ZL – p. iii
Nested hypothesis tests

- Optimal BGL fit parameter choice, given available data?  
  (upper: $\chi^2$, lower: $|V_{cb}| \times 10^3$)

<table>
<thead>
<tr>
<th>$n_a$</th>
<th>$n_c$</th>
<th>1</th>
<th>2</th>
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<td>38.6 ± 1.2</td>
<td>41.9 ± 2.0</td>
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<td>27.6</td>
<td>29.2</td>
<td>27.6</td>
<td>23.2</td>
</tr>
</tbody>
</table>

- Fit w/ 1 param added / removed: $\text{BGL}_{(n_a\pm 1)n_b n_c}$, $\text{BGL}_{n_a(n_b\pm 1)n_c}$, $\text{BGL}_{n_a n_b (n_c\pm 1)}$

- Accept descendant (parent) if $\Delta \chi^2$ is above (below) a boundary, say, $\Delta \chi^2 = 1$

- Repeat until “stationary” fit is found, preferred over its parents and descendants

- If multiple stationary fits, choose smallest $N$, then smallest $\chi^2$  
  (333 is an overfit!)

Start from small $N$, to avoid overfitting  
  e.g.: $\{111 \rightarrow 211 \rightarrow 221 \rightarrow 222, 121 \rightarrow 131 \rightarrow 231 \rightarrow 232 \rightarrow 222\}$

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Lattice QCD, preliminary results

- FNAL/MILC and JLQCD are both working on the $B \rightarrow D^* \ell \bar{\nu}$ form factors
  Independent formulations: staggered vs. Mobius domain-wall actions

Therefore, this issue is still open. These parametrizations should be eventually replaced by a lattice-based parametrization.

[T. Kaneko, JLQCD poster at Lattice 2018, 1811.00794; also Fermilab/MILC, 1710.09817]

- No qualitative difference between LQCD calculation at $w = 1$, or slightly above