Semi-leptonic $B_s$ decays

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(RBC-UKQCD collaborations)

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Jonathan M. Flynn, Ryan C. Hill, Andreas Jüttner, J. Tobias Tsang, Amarjit Soni
introduction
Motivation

- Determine CKM matrix elements, fundamental parameters of the Standard Model
- Predict processes to test Standard Model or discover new physics

[http://ckmfitter.in2p3.fr]
\[ |V_{ub}| \text{ from exclusive} \]

\[ B \rightarrow \pi \ell \nu \]

\[ B_s \rightarrow K \ell \nu \]

- \( B \rightarrow \pi \ell \nu \) and \( B \rightarrow D \ell \nu \) presented by Ryan C. Hill
- Only spectator quark differs
- Lattice QCD prefers \( s \) quark over \( u \) quark: statistically more precise, computationally cheaper
- \( B \) factories run at \( \Upsilon(4s) \) threshold \( \Rightarrow \) \( B \) mesons
- LHC collisions create many \( B \) and \( B_s \) mesons which decay \( \Rightarrow \) LHCb
  - LHCb prefers the ratio \( (B_s \rightarrow D_s \ell \nu)/(B_s \rightarrow K \ell \nu) \Rightarrow |V_{cb}/V_{ub}| \)
$|V_{ub}|$ from exclusive

$B \rightarrow \pi \ell \nu$

$B \rightarrow K \ell \nu$

$|V_{cb}|$ from exclusive

$B \rightarrow D \ell \nu$

$B_{s} \rightarrow D_{s} \ell \nu$
$V_{ub}$ from exclusive semi-leptonic $B_s \rightarrow K \ell \nu$ decay

\[
q^2 = M_{B_s}^2 + M_K^2 - 2M_{B_s}E_K
\]

- Conventionally parametrized by ($B_s$ meson at rest)

\[
d\Gamma(B_s \rightarrow K \ell \nu) \frac{dq^2}{dq^2} = \frac{G_F^2|V_{ub}|^2}{24\pi^3} \frac{(q^2 - m_\ell^2)^2 \sqrt{E_K^2 - M_K^2}}{q^4M_{B_s}^2} \left[ \left(1 + \frac{m_\ell^2}{2q^2}\right) M_{B_s}^2(E_K^2 - M_K^2)|f_+(q^2)|^2 + \frac{3m_\ell^2}{8q^2}(M_{B_s}^2 - M_K^2)^2|f_0(q^2)|^2 \right]
\]

nonperturbative input
Nonperturbative input

- Parametrizes interactions due to the (nonperturbative) strong force
- Use operator product expansion (OPE) to identify short distance contributions
- Calculate the flavor changing currents as point-like operators using lattice QCD
RBC-UKQCD’s set-up

- RBC-UKQCD’s 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - Three lattice spacings \( a \sim 0.11 \) fm, 0.08 fm, 0.07 fm; one ensemble with physical pions

RBC-UKQCD’s set-up

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* All mode averaging: 81 “sloppy” and 1 “exact” solve [Blum et al. PRD 88 (2012) 094503]

Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]

$\mathbf{a}: \sim 0.11 \text{ fm}, \sim 0.08 \text{ fm}, \sim 0.07 \text{ fm}$
RBC-UKQCD’s set-up

- RBC-UKQCD’s 2+1 flavor domain-wall fermion and Iwasaki gauge action ensembles
  - Three lattice spacings $a \sim 0.11$ fm, 0.08 fm, 0.07 fm; one ensemble with physical pions

- Unitary and partially quenched domain-wall up/down quarks

- Domain-wall strange quarks at/near the physical value

- Additional challenge $m_c = 1.28\text{GeV} \sim 270 \times m_d$
  $m_b = 4.18\text{GeV} \sim 1000 \times m_d$
RBC-UKQCD’s set-up

► Charm: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
→ Simulate 3 or 2 charm-like masses then extrapolate/interpolate

[Boyle et al. JHEP 1712 (2017) 008]
RBC-UKQCD’s set-up

- **Charm**: Möbius domain-wall fermions optimized for heavy quarks [Boyle et al. JHEP 1604 (2016) 037]
  → Simulate 3 or 2 charm-like masses then extrapolate/interpolate

- **Effective relativistic heavy quark (RHQ) action for bottom quarks**
  → Builds upon Fermilab approach [El-Khadra et al. PRD 55 (1997) 3933]
  → Allows to tune the three parameters \((m_0 a, c_P, \zeta)\) nonperturbatively [PRD 86 (2012) 116003]
  → Smooth continuum limit; heavy quark treated to all orders in \((m_b a)^n\)
  → Mostly nonperturbative renormalization [Hashimoto et al. PRD61 (1999) 014502]
  [El-Khadra et al. PRD64 (2001) 014502]

\[
Z_{V}^{bl} = \rho \sqrt{Z_{V}^{ll} Z_{V}^{bb}}
\]
\[ B_s \rightarrow K \ell \nu \]
\( B_s \rightarrow K \ell \nu \) form factors

- Parametrize the hadronic matrix element for the flavor changing vector current \( V^\mu \) in terms of the form factors \( f_+(q^2) \) and \( f_0(q^2) \)

\[
\langle K | V^\mu | B_s \rangle = f_+(q^2) \left( p_{B_s}^\mu + p_K^\mu - \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu \right) + f_0(q^2) \frac{M_{B_s}^2 - M_K^2}{q^2} q^\mu
\]

- Calculate 3-point function by
  
  - Inserting a quark source for a strange quark propagator at \( t_0 \)
  - Allow it to propagate to \( t_{\text{sink}} \), turn it into a sequential source for a \( b \) quark
  - Use a light quark propagating from \( t_0 \) and contract both at \( t \) with \( t_0 \leq t \leq t_{\text{sink}} \)
$B_s \rightarrow K\ell\nu$ form factors: F1 ensemble

Comparison of fit to the ground state only with fit including one excited state term for $K$ and $B_s$
Chiral-continuum extrapolation using SU(2) hard-kaon $\chi$PT

- Updating calculation [PRD 91 (2015) 074510] with improved values for $a^{-1}$ and RHQ parameters
- $f_{pole}(M_K, E_K, a^2) = \frac{1}{E_K+\Delta} c^{(1)} \times \left[ 1 + \frac{\delta f}{(4\pi f)^2} + c^{(2)} \frac{M_K^2}{\Lambda^2} + c^{(3)} \frac{E_K}{\Lambda} + c^{(4)} \frac{E_K^2}{\Lambda^2} + c^{(5)} \frac{a^2}{\Lambda^2 a_{g_2}^4} \right]
- $\delta f$ non-analytic logs of the kaon mass and hard-kaon limit is taken by $M_K/E_K \to 0$
Estimate systematic errors due to

- Chiral-continuum extrapolation
  - Use alternative fit functions, vary pole mass, etc.
  - Impose different cuts on the data

- Discretization errors of light and heavy quarks
  - Estimate via power-counting

- Uncertainty of the renormalization factors
  - Estimate effect of higher loop corrections

- Finite volume, iso-spin breaking, . . .

- Uncertainty due to RHQ parameters and lattice spacing ($a^{-1}$)
  - Carry out additional simulations to test effects on form factors

- Uncertainty of strange quark mass
  - Repeat simulation with different valence quark mass

⇒ full error budget
PRELIMINARY error budget $B_s \to K \ell \nu$

$\delta f = \frac{|f_{\text{variation}} - f_{\text{central}}|}{f_{\text{central}}}$
PRELIMINARY error budget $B_s \rightarrow K \ell \nu$

“Other”: 3% placeholder to cover higher order corrections, lattice spacing, finite volume, ...
Kinematical extrapolation (z-expansion)

- Map $q^2$ to $z$ with minimized magnitude in the semi-leptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+} - \sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+} + \sqrt{1-t_0/t_+}}$$

with

$$t_\pm = (M_B \pm M_\pi)^2$$

$$t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Express $f_+$ as convergent power series

- $f_0$ is analytic, except for $B^*$ pole

- BCL with poles $M_+ = B^* = 5.33$ GeV and $M_0 = 5.63$ GeV

- Exploit kinematic constraint $f_+ = f_0\big|_{q^2=0}$

- Include HQ power counting to constrain size of $f_+$ coefficients

- Systematic errors subject to changes!
Kinematical extrapolation (z-expansion)

- Map $q^2$ to $z$ with minimized magnitude in the semi-leptonic region: $|z| \leq 0.146$

$$z(q^2, t_0) = \frac{\sqrt{1-q^2/t_+}-\sqrt{1-t_0/t_+}}{\sqrt{1-q^2/t_+}+\sqrt{1-t_0/t_+}}$$

with

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$$t_0 \equiv t_{\text{opt}} = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

[Bourrely, Caprini, Lellouch, PRD 79 (2009) 013008]

- Allows to compare shape of form factors
  - Obtained by other lattice calculations
    [Bouchard et al. PRD 90 (2014) 054506]
  - Predicted by QCD sum rules and alike

- Combination with experiment leads to the overall normalization: $|V_{ub}|$

- Systematic errors subject to changes!

- Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = $\mu$ or $\tau$

- Predict SM differential branching fractions using \(|V_{ub}|\) as input for lepton = \(\mu\) or \(\tau\)

- Predict ratio of branching fractions \(\sim\) LFUV

\[
\begin{align*}
R_{\pi}^{\tau}/\mu &= 0.69(19) \\
R_{K}^{\tau}/\mu &= 0.77(12)
\end{align*}
\]

- Predict SM differential branching fractions using $|V_{ub}|$ as input for lepton = $\mu$ or $\tau$

- Predict ratio of branching fractions $\sim$ LFUV

- Predict forward-backward asymmetries using $|V_{ub}|$ as input for lepton = $\mu$ or $\tau$
$B_s \rightarrow D_s \ell \nu$
$|V_{cb}|$ from exclusive semi-leptonic $B_s \rightarrow D_s \ell \nu$ decay

\[ q^2 = M_{B_s}^2 + M_{D_s}^2 - 2M_{B_s}E_{D_s} \]

- Conventionally parametrized by ($B_s$ meson at rest)

\[
\frac{d\Gamma(B_s \rightarrow D_s \ell \nu)}{dq^2} = \frac{G_F^2 |V_{cb}|^2 (q^2 - m_\ell^2)^2 \sqrt{E_{D_s}^2 - M_{D_s}^2}}{24\pi^3 \ q^4 M_{B_s}^2} \times \left[ \left( 1 + \frac{m_\ell^2}{2q^2} \right) M_{B_s}^2 (E_{D_s}^2 - M_{D_s}^2) f_+(q^2)^2 + \frac{3m_\ell^2}{8q^2} (M_{B_s}^2 - M_{D_s}^2)^2 |f_0(q^2)|^2 \right]
\]

Experiment known CKM

Nonperturbative input
Global fit $B_s \to D_s \ell \nu$

$$f(q^2, a, M_\pi, M_{D_s}) = \left[ \alpha_1 + \alpha_2 M_\pi^2 + \sum_{j=1}^{n_{D_s}} \alpha_{3,j} \left[ \Delta M_{D_s}^{-1} \right]^j + \alpha_4 a^2 \right] P_{a,b} \left( \frac{q^2}{M_{B_s}^2} \right)$$

with $\Delta M_{D_s}^{-1} \equiv \left( \frac{1}{M_{D_s}} - \frac{1}{M_{D_s}^{\text{phys}}} \right)$, $P_{a,b}(x) = \frac{1 + \sum_{i=1}^{N_a} a_i x^i}{1 + \sum_{i=1}^{N_b} b_i x^i}$.
Global fit $B_s \rightarrow D_s \ell \nu$

$\begin{align*}
\text{Global fit } B_s \rightarrow D_s \ell \nu \\
\begin{array}{c}
\text{Graph 1} \\
\text{Graph 2}
\end{array}
\end{align*}$

- $f(q^2, a, M_\pi, M_{D_s}) = \left[ \alpha_1 + \alpha_2 M_\pi^2 + \sum_{j=1}^{n_{D_s}} \alpha_{3,j} \left[ \Delta M_{D_s}^{-1} \right]^j + \alpha_4 a^2 \right] P_{a,b} \left( \frac{q^2}{M_{B_s}^2} \right)$

- Extrapolation to the continuum limit with physical quark masses

\[\text{PRELIMINARY}\]
PRELIMINARY error budget $B_s \rightarrow D_s \ell \nu$

\[ \delta f = \frac{|f_{\text{variation}} - f_{\text{central}}|}{f_{\text{central}}} \]
PRELIMINARY error budget $B_s \rightarrow D_s \ell \nu$

“Other”: 3% placeholder to cover higher order corrections, lattice spacing, finite volume, …
**z-expansion**

- **BCL with poles** $M_+ = B_c^* = 6.33$ GeV and $M_0 = 6.42$ GeV
  
  kinematical constraint $f_0^{B_s \to D_s}(0) = f_+^{B_s \to D_s}(0)$
Status $B_s \rightarrow K \ell \nu$ and $B_s \rightarrow D_s \ell \nu$

- $B_s \rightarrow K \ell \nu$ chiral-continuum extrapolation
- $B_s \rightarrow D_s \ell \nu$ global fit ($M_\pi$, $M_{D_s}$, $a^2$, $q^2$)
- Extract synthetic data points
- Full systematic error budget
  - RHQ parameter tuning
  - Continuum extrapolation:
    - cut to data set, different fit functions, . . .
  - Charm extrapolation
  - FV, higher order disc. effects, isospin, $s$-quark mass tuning, . . .

- $z$-expansion over full $q^2$ range
  - BGL vs. BCL
  - Test CLN for $B_s \rightarrow D_s \ell \nu$
  - Number of synthetic data points
  - Different truncations
  - Incl. vs. excluding $f_+ = f_0 \bigg|_{q^2=0}$

- Phenomenology: $R(K)$, $R(D_s)$, . . .
Flavor Lattice Averaging Group

[FLAG 2019]
**$B_s \rightarrow K\ell\nu$**

\[
\frac{B(q^2)}{B(0^+)} f_{B_s\rightarrow K\ell\nu}(q^2) = \frac{f_0}{f_0^0} f_{B_s\rightarrow K\ell\nu}(q^2)
\]

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- **New FNAL/MILC** [arXiv:1901.02561]
- **Please do cite calculations feeding into FLAG averages**

[FLAG 2019] 23 / 26
B → D_{(s)}^{(*)}\ell\nu

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New HPQCD $B_s \rightarrow D_s \ell\nu$ [arXiv:1906.00701]
New HPQCD $B_s \rightarrow D_s^{*}\ell\nu$ [arXiv:1904.02046]

Please do cite calculations feeding into FLAG averages
outlook
Outlook

▶ Second (third) entirely independent analysis completed

▶ In the final stages to complete $B_s \rightarrow K \ell \nu$ and $B_s \rightarrow D_s \ell \nu$ form factor calculation
  → As usual, carefully estimating all systematic uncertainties is tedious

▶ Our lattice calculation also includes
  → $B \rightarrow \pi \ell \nu$, $B \rightarrow \pi^+ \ell^-$
  → $B \rightarrow K^* \ell^+ \ell^-$
  → $B \rightarrow D(\ast) \ell \nu$
  → $B_s \rightarrow K^* \ell^+ \ell^-$
  → $B_s \rightarrow D_s^* \ell \nu$
  → $B_s \rightarrow \phi \ell^+ \ell^-$
  → ...

▶ Current status $B_s \rightarrow K \ell \nu$ and $B_s \rightarrow D_s \ell \nu$:
  [arXiv:1903.02100]

▶ Future
  → Add $48^3 \times 96$ ensemble with physical pions

▶ Parallel efforts: SU(3) breaking ratios
  [arXiv:1812.08791]
  → Talk by J. Tobias Tsang
Resources for RBC-UKQCD’s calculation

**USQCD:** Ds, Bc, and pi0 cluster (Fermilab), qcd12s cluster (Jlab), skylake cluster (BNL)

**RBC qcdcl** (RIKEN) and cuth (Columbia U)

**UK:** ARCHER, Cirrus (EPCC) and DiRAC (UKQCD)