Long-distance contributions to $B \to K\, l^+ l^-$

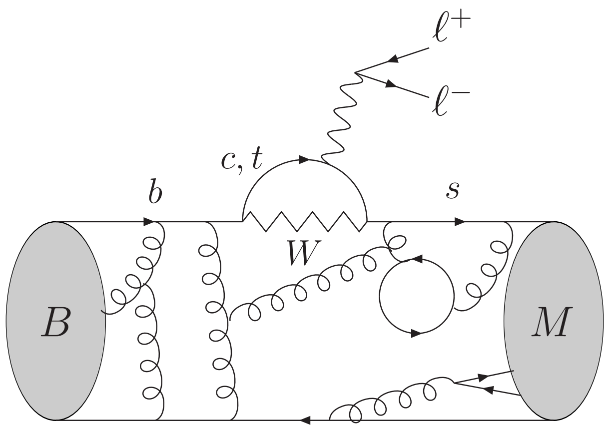
Shoji Hashimoto (KEK,SOKENDAI)

in collaboration with T. Ishikawa (SOKENDAI), T. Kaneko, (KEK,SOKENDAI), K. Nakayama (Osaka→DESY Zeuthen) and other members of JLQCD

Sep 24, 2019
@ BNL Lattice X Intensity Frontier Workshop

$B \to K\, l^+ l^-$

FCNC process, penguin induced:



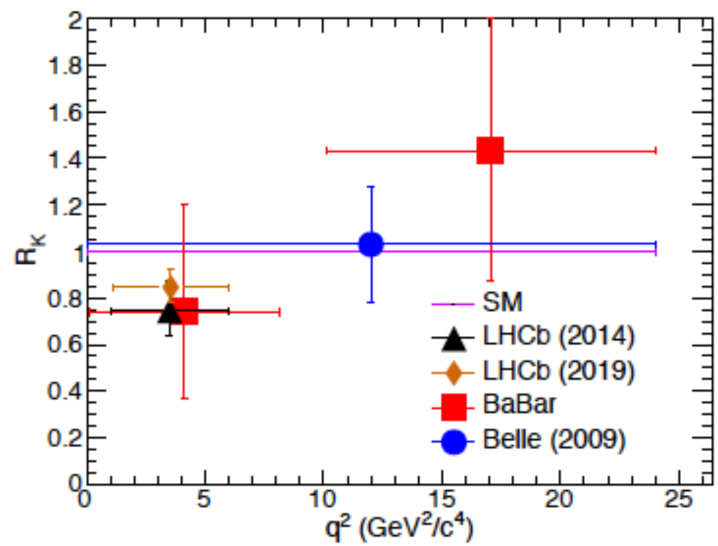
$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma_{\mu\nu} F^{\mu\nu} b_R,$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell),$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

Lepton Flavor Violation?

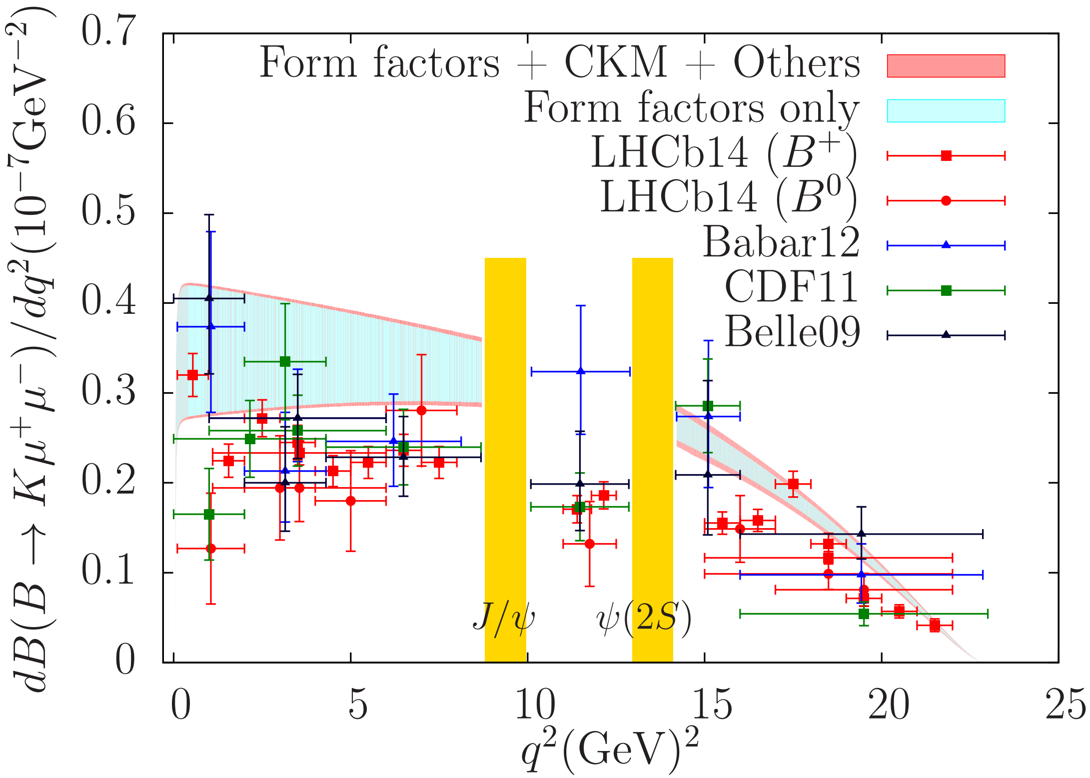
$$R_K = \frac{\Gamma(B \to K\mu^+\mu^-)}{\Gamma(B \to K e^+ e^-)}$$



$B \to K\, l^+ l^-$

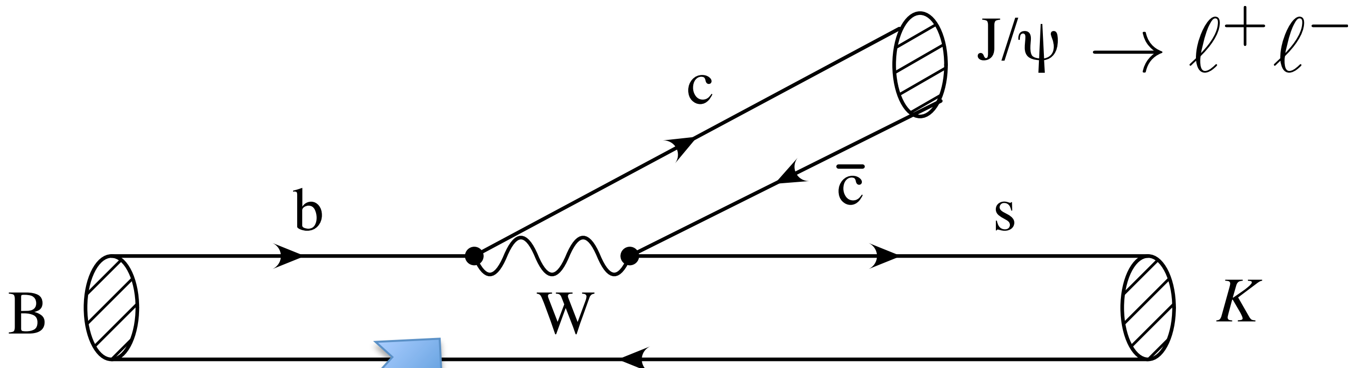
Already, at the level of BR…

Fermilab-MILC (Du et al.), PRD93, 034005 (2016).



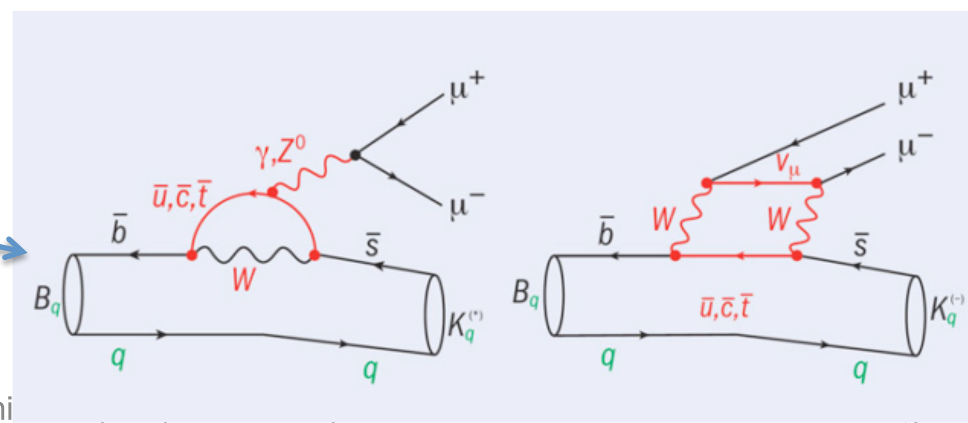
Complication due to…

enhancement due to resonances



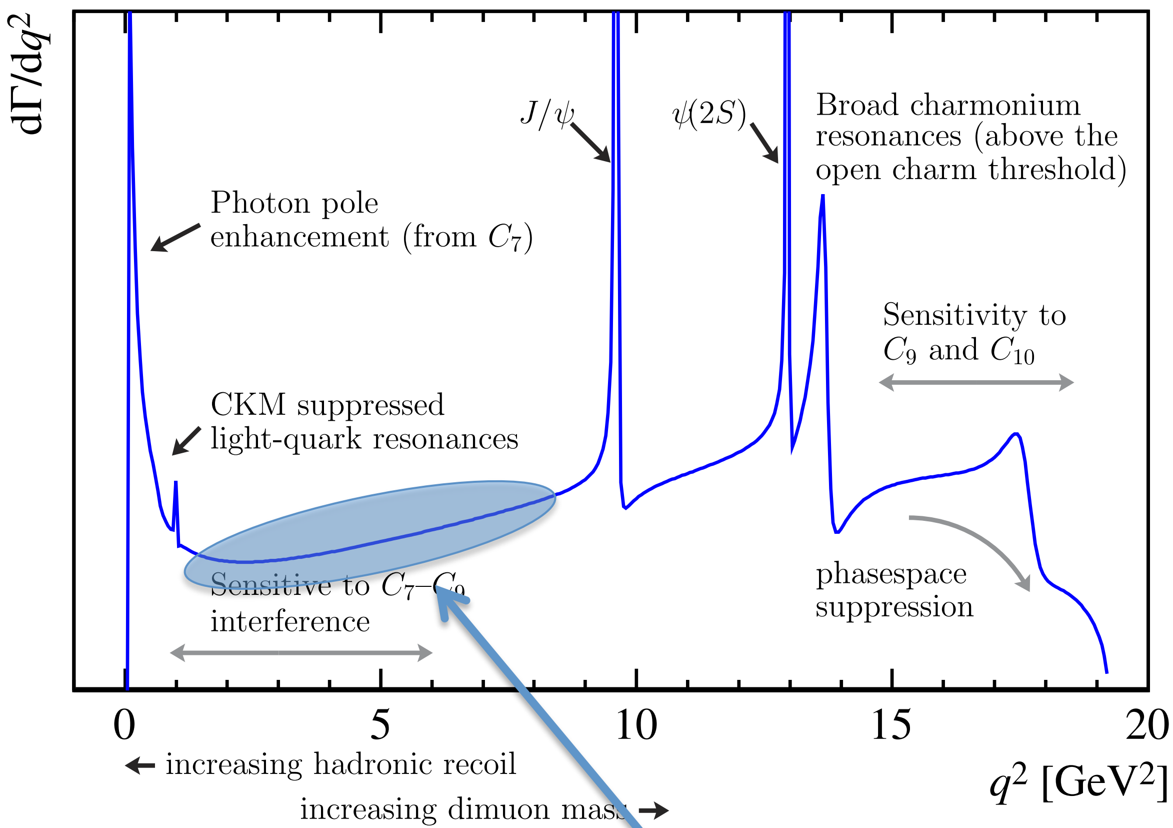
$$\mathcal{O}_2 = (\bar{s}_L\gamma_\mu c_L)(\bar{c}_L\gamma^\mu b_L)$$

tree vs loop



Long-distance effect?

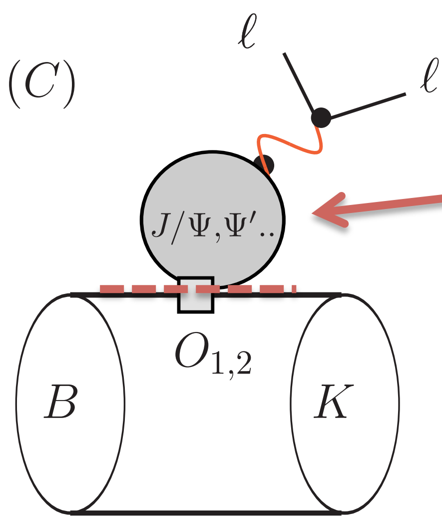
Lyon, Zwicky, arXiv:1406.0566



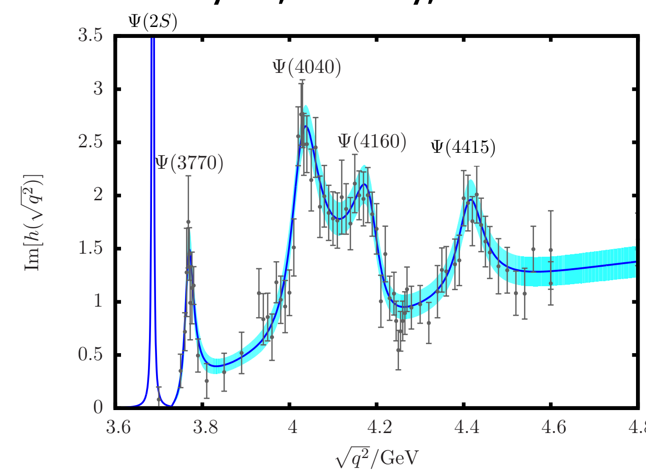
Insensitive to the resonance effects? Probably, but how much?

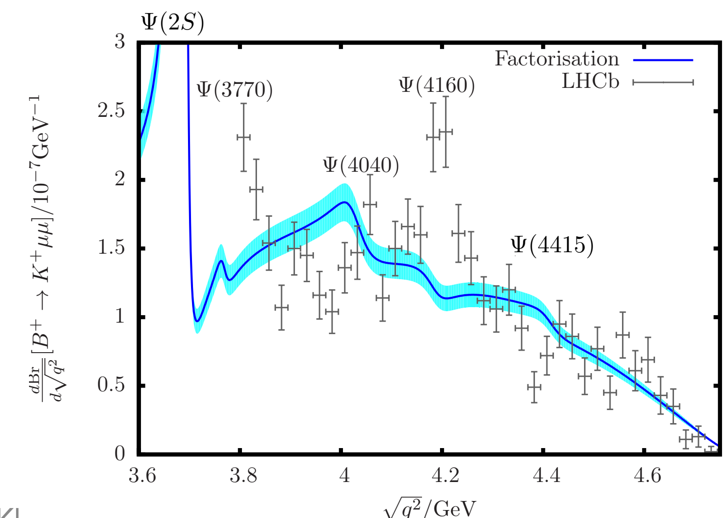
A way to estimate = factorization

vacuum polarization
from e⁺e⁻



Lyon, Zwicky, arXiv:1406.0566





It doesn’t look like a good approximation…

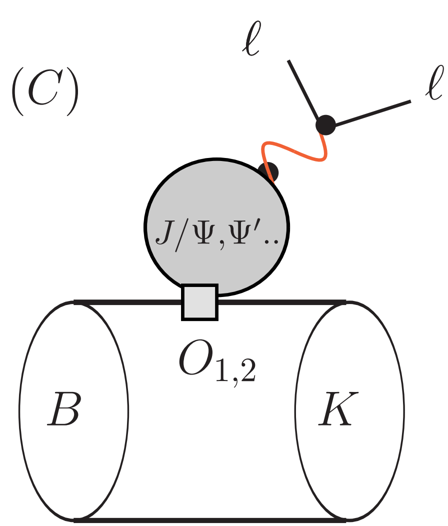
Analyticity:

C. Bobeth, M. Chrzaszcz, D. van Dyk, J. Virto, “Long-distance effects in $B \to K^* ll$ from analyticity,” Eur. Phys. J. C78, 451 (2018); arXiv:1707.07305.

Analyticity

$$\epsilon_\alpha^* \mathcal{H}^{\alpha\mu}(q,k) = i \int d^4x e^{iqx} \langle \bar{K}^*(k,\epsilon)|\mathcal{K}^\mu(x,0)|\bar{B}(p+k)\rangle,$$

$$\mathcal{K}^\mu(x,y) = T\{j^\mu_{\rm em}(x), C_1\mathcal{O}_1(y) + C_2\mathcal{O}_2(y)\}$$



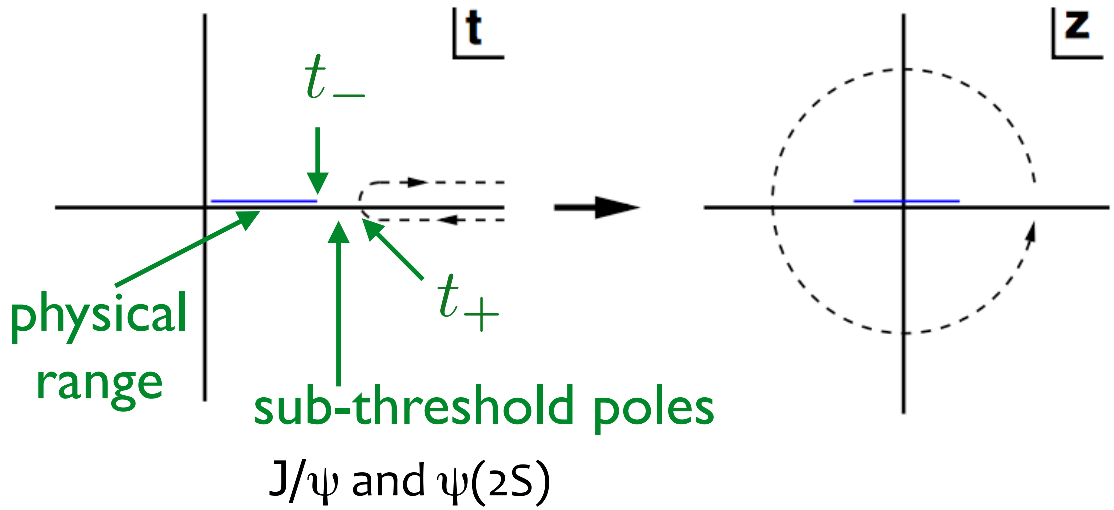
$$\mathcal{O}_1 = (\bar{s}_i c_j)_{V-A}(\bar{c}_j b_i)_{V-A}$$
$$\mathcal{O}_2 = (\bar{s}_i c_i)_{V-A}(\bar{c}_j b_j)_{V-A}$$

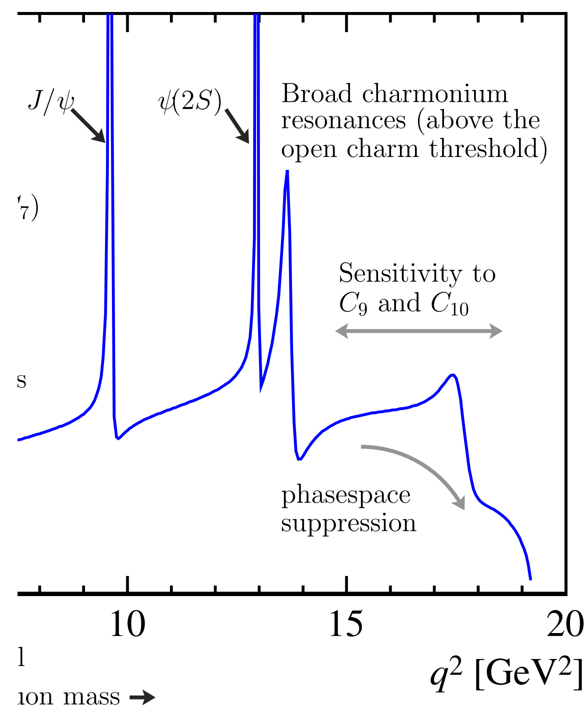
Try to parametrize the exp data in a systematically improvable form.

z-expansion

The amplitude is a smooth function of the “z” variable, once the singularities are taken out.

$$z(q^2) \equiv \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$





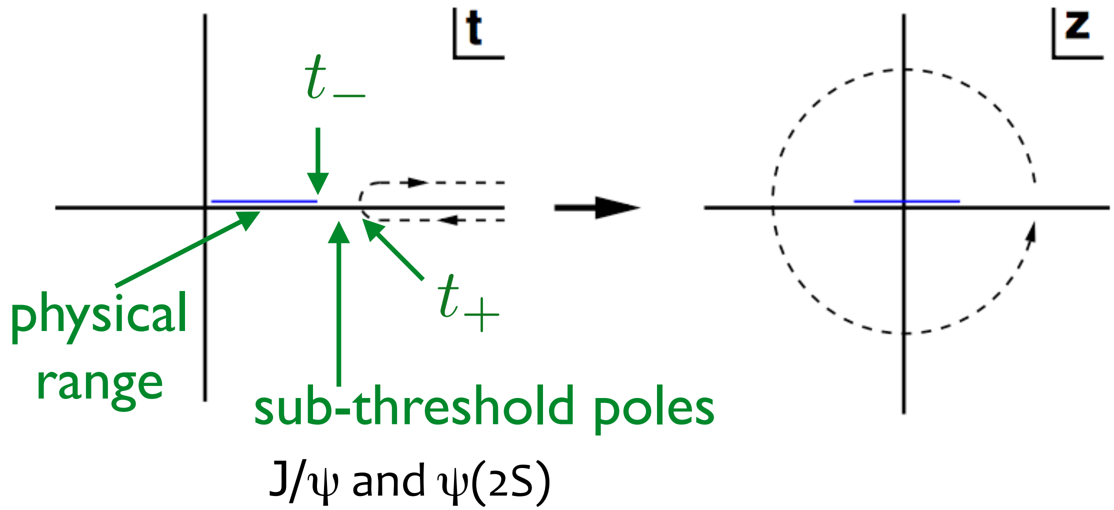
z-expansion

J/ψ and ψ(2S) pole singularities are taken out:

$$\mathcal{H}_\lambda(z) = \frac{1 - z z^*_{J/\psi}}{z - z_{J/\psi}} \frac{1 - z z^*_{\psi(2S)}}{z - z_{\psi(2S)}} \hat{\mathcal{H}}_\lambda(z),$$

$$\hat{\mathcal{H}}_\lambda(z) = \left[\sum_{k=0}^{K} \alpha_k^{(\lambda)} z^k \right] \mathcal{F}_\lambda(z)$$

Remaining function should be well-described by a polynomial of z.



Inputs

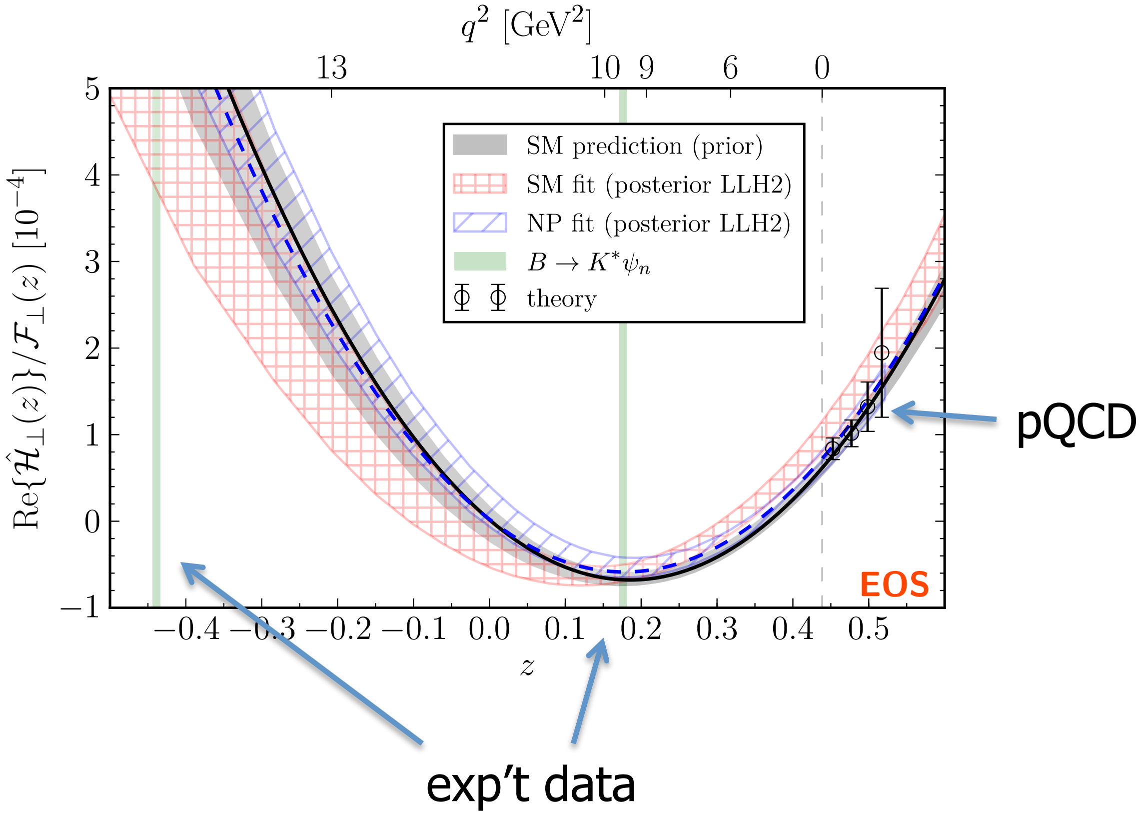
Need some inputs to determine the coefficients α_k

1. Known amplitudes of real decays $B \rightarrow K^* \psi_n$

$$\mathcal{H}_\lambda(q^2 \to M_{\psi_n}^2) \sim \frac{\langle 0 | j_{\mathrm{em}}^\mu | \psi_n(q,\varepsilon)\rangle \mathcal{A}^\mu_{\lambda,\psi_n}}{M_B^2(q^2 - M_{\psi_n}^2)} + \cdots$$

2. Perturbative calculation at $q^2<0$
   - QCD factorization at $O(\alpha_s)$
   - could be more than one points

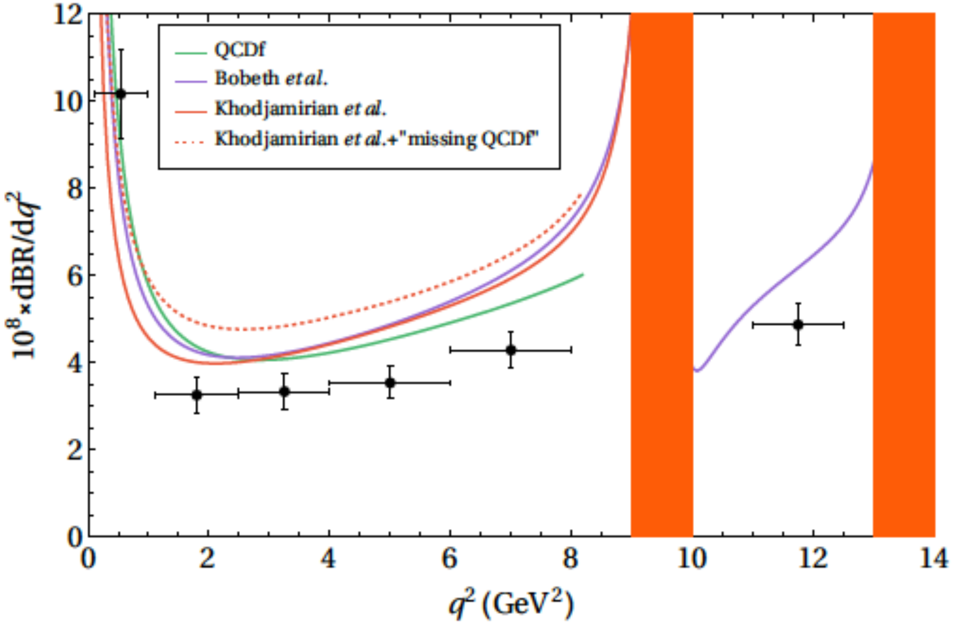
Polynomical $\sum_{k=0}^{2} \alpha_k^{(\lambda)} z^k$



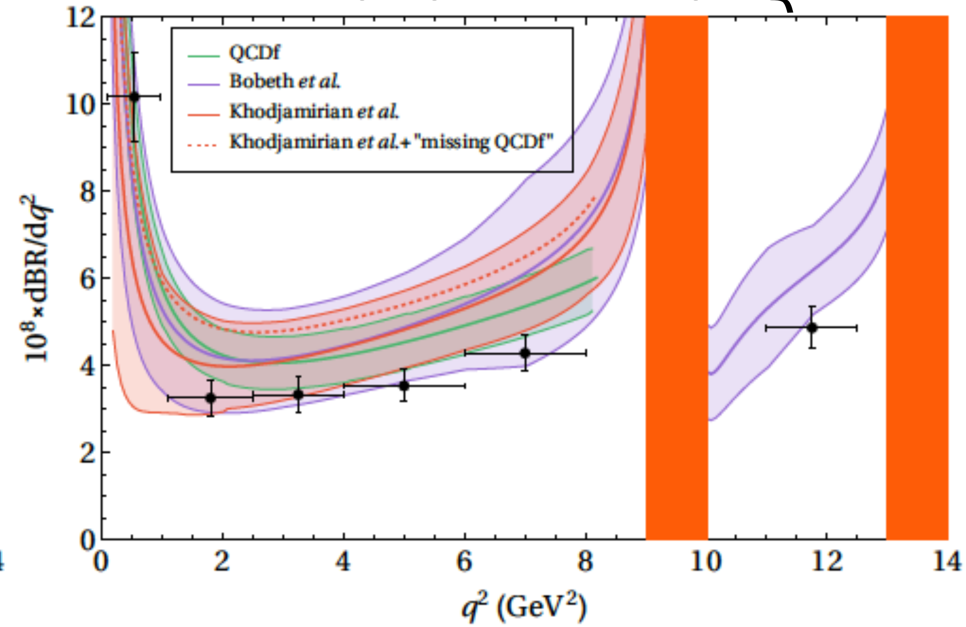
Comparison of various theoretical estimates:

Arbey, Hurth, Mahmoudi, Neshatpour, Phys. Rev. D98, 095027 (2018); arXiv:1805.06378

w/o error



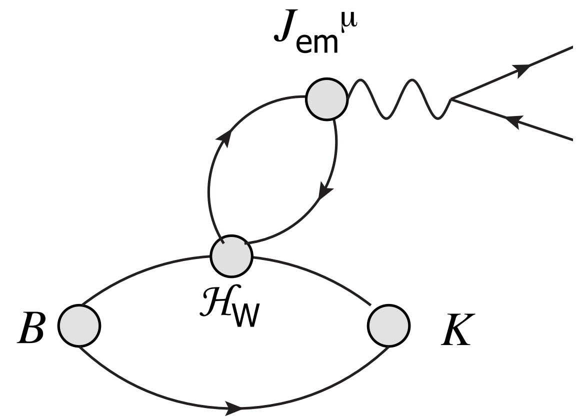
with error of “10%” for QCDF



Can lattice be of any help?

- Nakayama, Lattice 2018, 2019

Lattice calculation?



Nakayama @ Lattice 2018, 2019

Corresponding amplitude:

$$\mathcal{H}^\mu(p_B, p_K) = \int d^4x\, e^{iqx} \langle K(\mathbf{p}_K)|\mathrm{T}[J^\mu_{(\mathrm{em})}(x)\mathcal{H}_W(0)]|B(\mathbf{p}_B)\rangle$$

Euclidean

$$\underbrace{\int_0^\infty dt\, e^{\omega t}}_{\text{energy specified}} \int d^3\mathbf{x}\, \underline{e^{i\mathbf{q}\cdot\mathbf{x}}} \langle K(\mathbf{p}_K)|\underline{J^\mu_{(\mathrm{em})}(x)}\mathcal{H}_W(0)|B(\mathbf{p}_B)\rangle$$

energy specified

momentum inserted
for charmonium

$(\bar{c}\gamma_\mu P_L b)(\bar{s}\gamma_\mu P_L c)$

Limitation

- Internal charm quark loop has to be off-shell.

$$\int_0^\infty dt\, e^{\omega t} \int d^3\mathbf{x}\, e^{i\mathbf{q}\cdot\mathbf{x}} \langle K(\mathbf{p}_K)|J^\mu_{(\mathrm{em})}(x)\mathcal{H}_W(0)|B(\mathbf{p}_B)\rangle$$

= energy ω inserted to $J^\mu{}_{(\mathrm{em})}$ should be less than the corresponding ground state energy

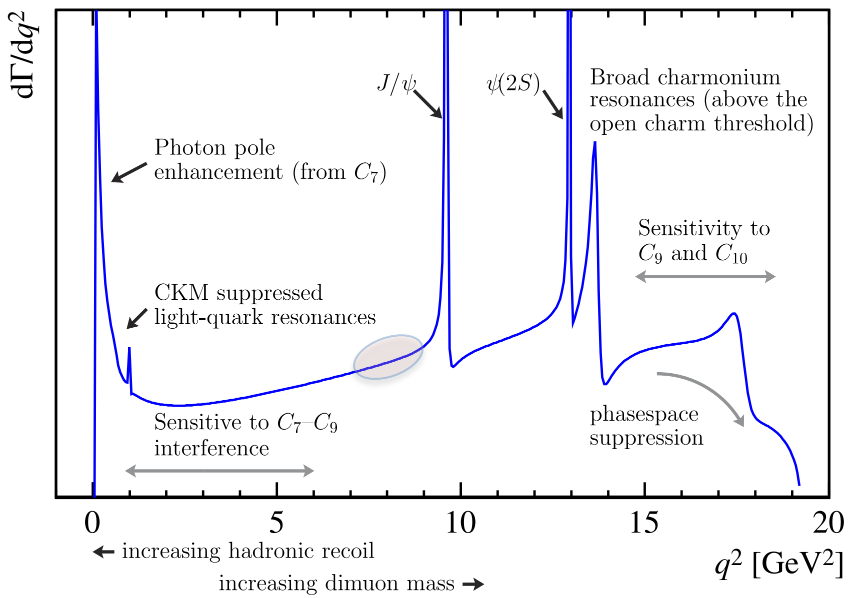
- Possible internal state must be heavier than the initial/final state. (Otherwise, the t integral diverges.)

$$\omega < m_{J/\psi} - (E_K - m_K)$$

  - Treating the physical kinematics is very challenging because of the large recoil momentum ~ 1.7 GeV/c.
  - Maybe the method is more realistic for D→πϕ→πll ?

Limitation

- Instead, we consider the case of artificially small B meson mass (then, small recoil momentum, say 0.5 GeV/c). Maximum possible q^2 is 1.5 GeV^2 below $m_{J/\psi}^2$.



Can we learn something??

- Test of the factorization approximation

Similar problem

Formulation borrowed from "long-distance effects to $K \rightarrow \pi ll$"

Christ et al (RBC/UKQCD), PRD92, 094512 (2015); PRD94, 114516 (2016).

Our case is simpler, because

- Interested in only one diagram (charm-loop), compared to many possible diagrams to $K \rightarrow \pi ll$.
- We don't have to subtract unphysical contribution due to the states of lower energy. (We avoid by limiting the kinematics.)
  - $K \rightarrow \pi^* \rightarrow \pi ll$, $K \rightarrow (\pi\pi)^* \rightarrow \pi ll$

Our case is harder, on the other hand, due to the kinematics.

A pilot lattice study:

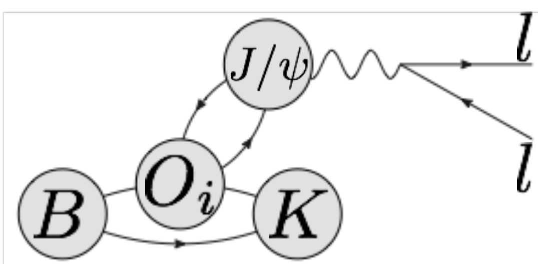
- on a 2+1 flavor domain-wall ensemble
- valence domain-wall, tuned charm, too light bottom

β	a^{-1} [GeV]	$L^3 \times T(\times L_s)$	am_{val}	am_c	am_b
4.35	3.610(9)	$48^3 \times 96(\times 8)$	0.025	0.27287	0.66619

ap	# Conf.	m_π[MeV]	E_K [MeV]	$E_{J/\psi}$ [GeV]	m_B [GeV]
$-\frac{2\pi}{L}$ (1,0,0)	390	714(1)	854(3)	3.128(1)	3.44(1)
$-\frac{2\pi}{L}$ (1,1,0)	400	714(1)	969(9)	3.158(1)	3.44(1)

Energies don't match.
Assuming HQET, may adjust m_B.

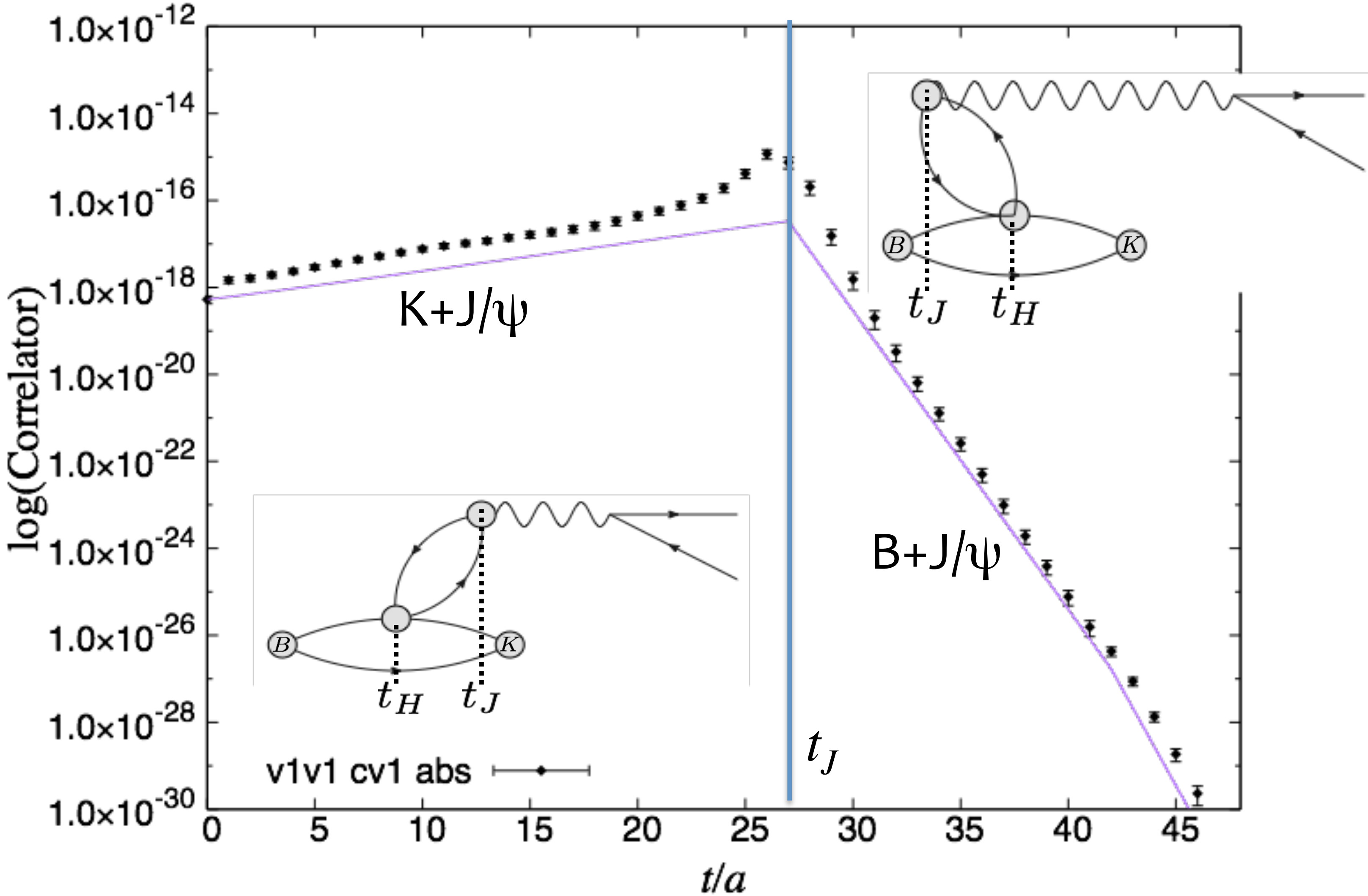
- four-point function calculated for



with

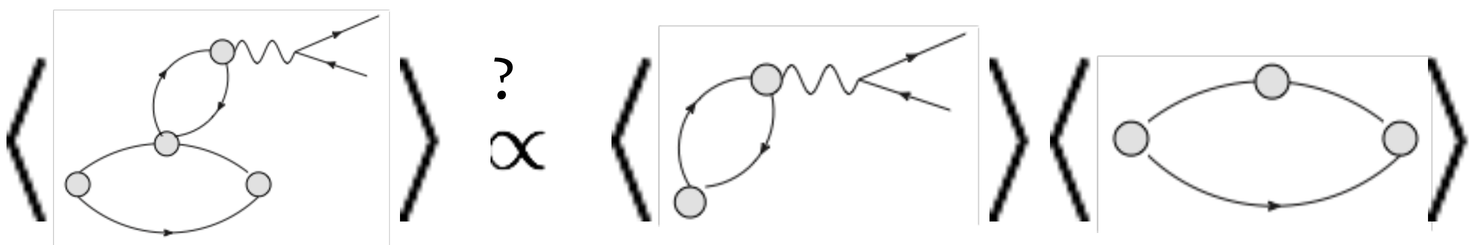
$$O_1^c = (\bar{s}_i \gamma_\mu P_- c_j)(\bar{c}_j \gamma_\mu P_- b_i)$$
$$O_2^c = (\bar{s}_i \gamma_\mu P_- c_i)(\bar{c}_j \gamma_\mu P_- b_j)$$

$$\Gamma^{(4)}_{\mu}\left(t_H, t_J, \mathbf{p}, \mathbf{k}\right) = \int \mathrm{d}^3\mathbf{x}\mathrm{d}^3\mathbf{y} e^{-i\mathbf{q}\cdot\mathbf{y}} \left\langle \phi_K\left(t_K, \mathbf{k}\right) \mathrm{T}\left[J_\mu\left(t_J, \mathbf{y}\right) H_{\text{eff}}\left(t_H, \mathbf{x}\right)\right] \phi^\dagger_B(0, \mathbf{p}) \right\rangle$$



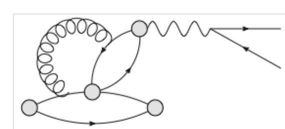
Factorization?

Is this a good approximation?



$$\langle P_K | J_\nu^{\bar{c}c} (\bar{c}_i \gamma_\mu P_- c_i)(\bar{s}_j \gamma_\mu P_- b_j) | P_B \rangle = \frac{1}{(\mathrm{Vol.})} \langle 0 | J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} | 0 \rangle \langle P_K | V_\mu | P_B \rangle$$

- gluon exchange is missing:
- any rescattering is missing.



➔ Test with the lattice calculation.

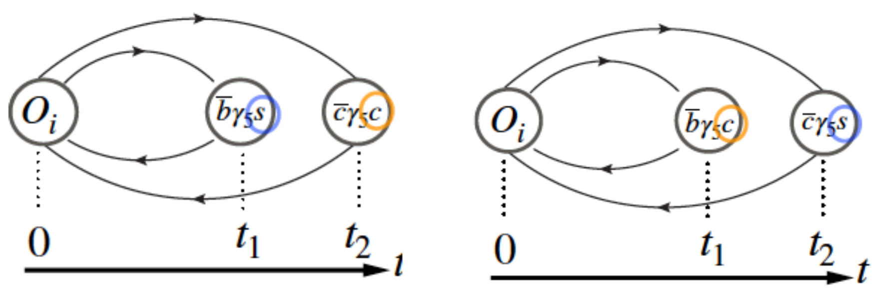
Renormalization constants:

Ishikawa @ Lattice 2019

- determined in a scheme to match charmonium time moments

$$\langle O_1\rangle_R = Z_{11}\langle O_1\rangle + Z_{12}\langle O_2\rangle \qquad Z_{11} = Z_{22} = 0.669(11)$$

$$\langle O_2\rangle_R = Z_{21}\langle O_1\rangle + Z_{22}\langle O_2\rangle \qquad Z_{12} = Z_{21} = 0.093(4)$$



Renormalization condition:

- These amplitudes become equal to their tree value at a certain distance.

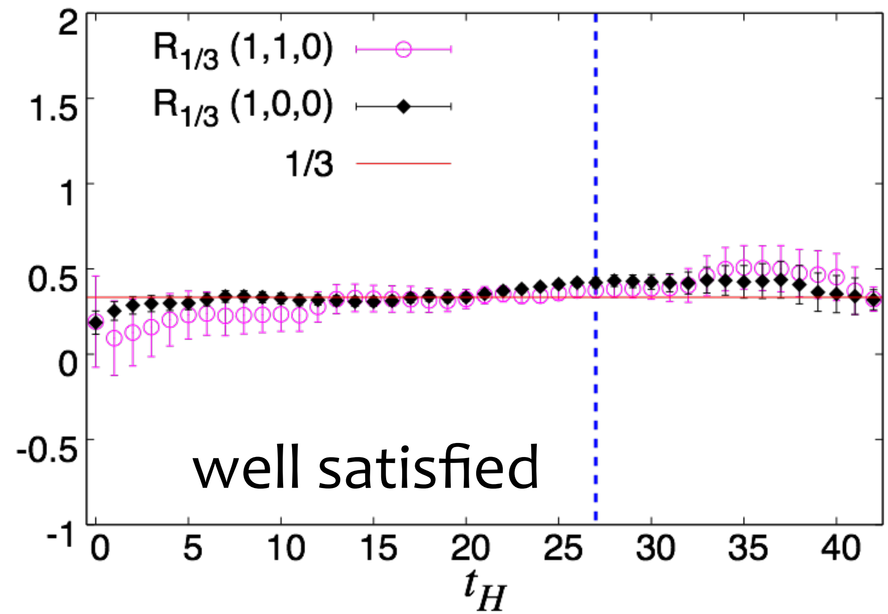
Under the factorization approximation

$$R_{1/3} \equiv \frac{\langle O_2\rangle_R}{\langle O_1\rangle_R} \quad \rightarrow 1/3$$

$$R_1 \equiv \frac{\langle O_1\rangle_R}{\langle J_\nu^{\bar{c}c} J_\mu^{\bar{c}c}\rangle_R \langle P_K|V_\mu|P_B\rangle_R} \quad \rightarrow 1$$

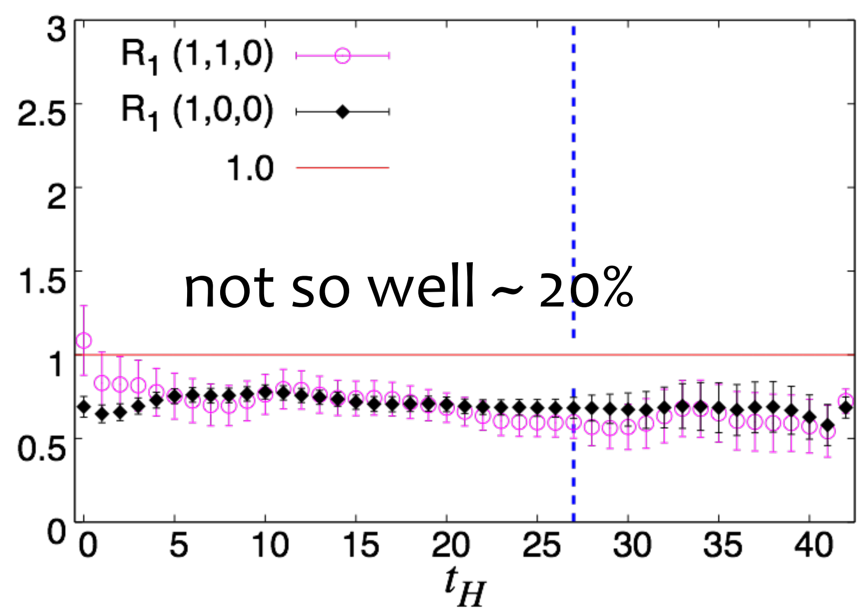
$$R_{1/3} \equiv \frac{\langle O_2 \rangle_R}{\langle O_1 \rangle_R}$$

$$O_2^c = \frac{1}{3} O_F^{(1)} + 2 O_{NF}^{(8)}$$



$$R_1 \equiv \frac{\langle O_1 \rangle_R}{\langle J_\nu^{\bar{c}c} J_\mu^{\bar{c}c} \rangle_R \langle P_K | V_\mu | P_B \rangle_R}$$

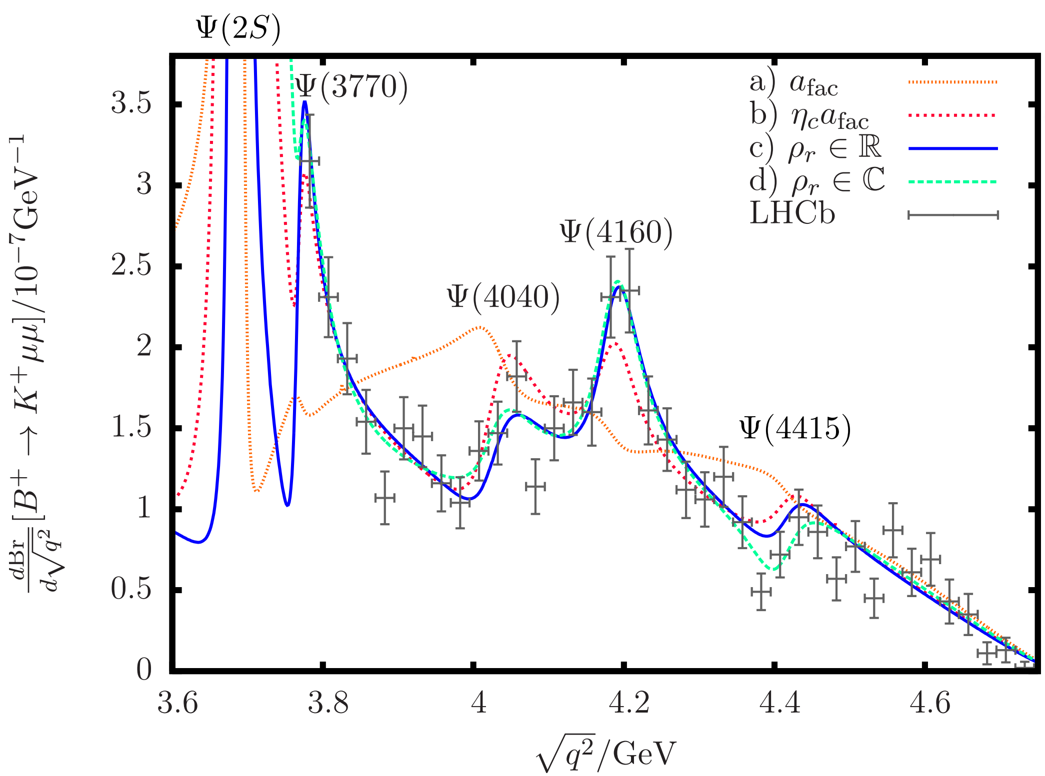
$$O_1^c = O_F^{(1)}$$



Factorization?

Not always satisfied well.

- But the deviation seems to be a constant.
- Pheno suggests that the deviation is to be x(−2.4) or energy (or time) dependent.

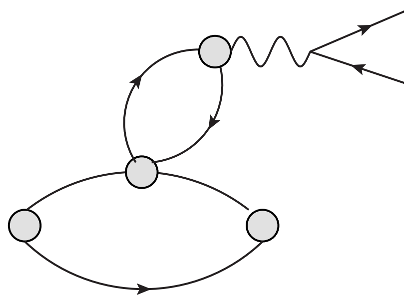


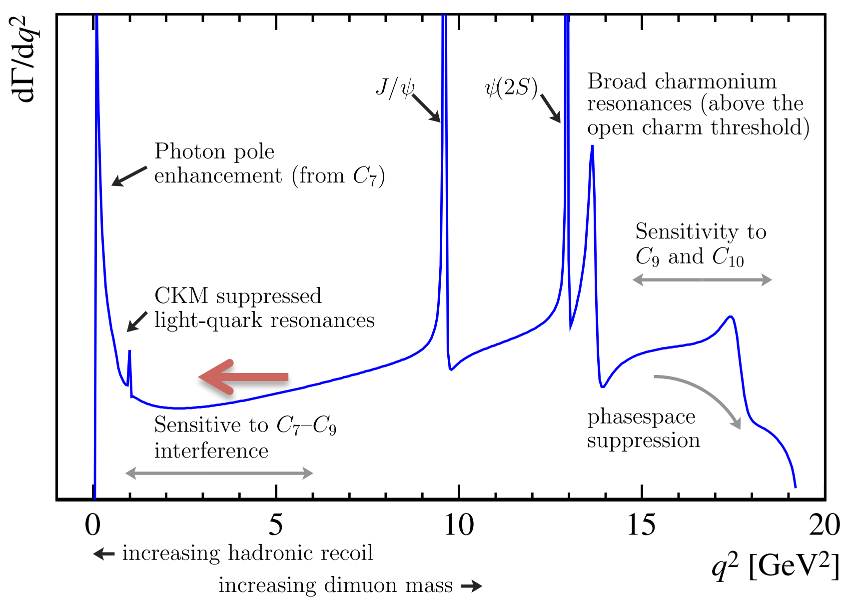
x(−2.4)

different factors for each resonance

Lyon, Zwicky, arXiv:1406.0566

Lattice is not a solution, alone





- Only the region far below J/ψ is accessible.
  - Otherwise one need to subtract the lower energy states (as in K→πll)
- Recoil momentum too low ~ 0.5 GeV compared to > 2.5 GeV (physical)
  - How does the amplitude depend on the momentum?
  - Real calculation is still too hard.

Problems…

- Many issues…
  - What happens for larger recoil momenta
  - Energy (or q^2) range is (too) far from the region of interest.
  - Spectator is strange: η_s rather than K.

- Possible extensions?
  - Can the region between 1S and 2S be analyzed in a similar manner?
  - Maybe even the higher excited states, where recoil momentum is small. Sort of "inclusive measurements" possible?