CPV and Mixing in the Neutral Kaon System

Past and Prospects

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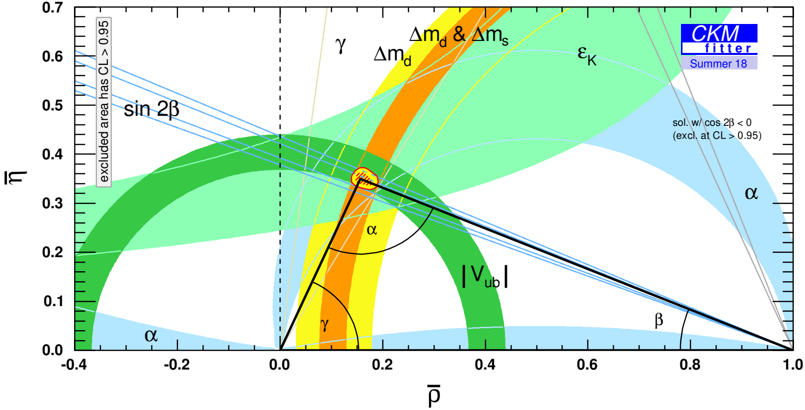
With Martin Gorbahn – Phys.Rev. D82 (2010) 094026, Phys.Rev.Lett. 108 (2012) 121801



0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
$\bar{\eta}$
$\bar{\rho}$
excluded area has CL > 0.95
sin 2β
γ
Δm_d
Δm_d & Δm_s
ε_K
CKM
fitter
Summer 18
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
α
$|V_{ub}|$
γ
β
α

Outline

- Kaon Mixing Overview
- $|\Delta S = 2|$ Weak Hamiltonian at NNLO
- Future Prospects

Kaon Mixing Overview

Neutral Kaon Mixing

- Neutral kaon mixing is described by

$$\boxed{i\frac{d}{dt}\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}=\left(M-i\frac{\Gamma}{2}\right)\begin{pmatrix}|K^0(t)\rangle\\|\bar{K}^0(t)\rangle\end{pmatrix}.}$$

- The Hamiltonian is diagonalized by

$$\boxed{\begin{aligned}|K_S\rangle &= p|K^0\rangle + q|\bar{K}^0(t)\rangle\,,\\ |K_L\rangle &= p|K^0\rangle - q|\bar{K}^0(t)\rangle\,.\end{aligned}}$$

- $|q| \neq |p| \quad \Rightarrow$ indirect CP violation

ϵ_K in the Standard Model

$$\langle K^0 | H^{|\Delta S|=2} | \bar{K}^0 \rangle$$

$$\boxed{\epsilon_K \equiv \frac{\langle (\pi\pi)_{I=0} | K_L \rangle}{\langle (\pi\pi)_{I=0} | K_S \rangle} = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta M_K} + \xi \right)}$$

$$\phi_\epsilon \equiv \arctan \frac{\Delta M_K}{\Delta\Gamma_K/2} \qquad \frac{\mathrm{Im}\langle (\pi\pi)_{I=0} | K^0 \rangle}{\mathrm{Re}\langle (\pi\pi)_{I=0} | K^0 \rangle}$$

- In "B-physics terminology" we have ($\lambda_0 = \frac{q}{p} \frac{\langle (\pi\pi)_{I=0} | \bar{K}^0 \rangle}{\langle (\pi\pi)_{I=0} | K^0 \rangle}$)

$$\boxed{\epsilon_K \approx \frac{1}{2} \left(1 - \left| \frac{q}{p} \right| - i\, \mathrm{Im}\lambda_0 \right)}$$

Long-distance Contributions – B_K

$$\langle H_{\text{eff}} \rangle = \underbrace{\langle Q^{|\Delta S=2|} \rangle (\mu_{\text{had}}) \quad U(\mu_{\text{had}}, \mu_c)} \quad \underbrace{U(\mu_c, \mu_W) \quad C(\mu_W)}$$

$$\boxed{\hat{B}_K = \frac{3}{2} b(\mu_{\text{had}}) \frac{\langle \bar{K}^0 | Q^{|\Delta S=2|} K^0 | \rangle}{f_K^2 M_K^2}}$$

$$\boxed{\eta_{ij} \, S(x_i, x_j)}$$

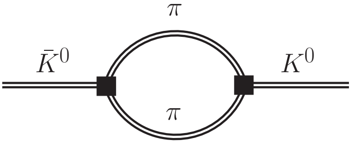
Bare lattice $\to$ RI-(S)MOM $\to$ $\overline{\text{MS}}$ [Aoki et al., 0712.1061]

LD Contributions from $|\Delta S = 1|$ Hamiltonian

$$\epsilon_K = e^{i\phi_\epsilon} \sin\phi_\epsilon \left(\frac{\mathrm{Im}(M_{12})}{\Delta M_K} + \xi \right)$$

Dispersive (real) and

absorptive (imaginary) part of



$$\int d^4x \langle \bar{K}^0 | H^{|\Delta S=1|}(x) H^{|\Delta S=1|}(0) | K^0 \rangle$$

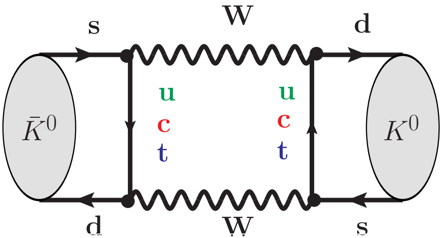
- Estimate ξ from ϵ'/ϵ: -6% [Nierste, 0201071; Buras et al., 0805.3887]
- Estimate absorptive part in ChPT: $+$ 2.4% [Buras et al., 1002.3612]

$$\boxed{\text{Combine with prefactor to } \kappa_\epsilon = \mathbf{0.94(2)}}$$

- See also recent lattice results [Blum et al., 1502.00263; Bai et al. 1505.07863]

Weak Hamiltonian at NNLO

CKM structure of weak Hamiltonian



	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_c \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$

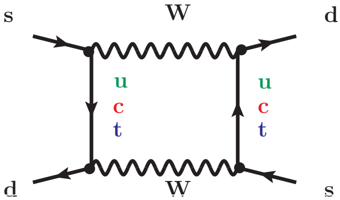
- $\lambda_i \equiv V_{is}^* V_{id}$
- $\lambda_u = -\lambda_c - \lambda_t$
- $\lambda \equiv |V_{us}| \approx 0.2$

$$\mathrm{Im}(M_{12}) \to \epsilon_K \qquad \mathrm{Re}(M_{12}) \to \Delta M_K$$

$|\Delta S| = 2$ Hamiltonian

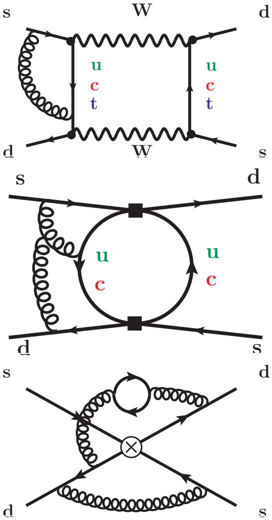
$$H^{|\Delta S|=2} \propto \left[\lambda_t^2 \eta_{tt} S\left(\frac{m_t^2}{M_W^2}\right) + \lambda_c^2 \eta_{cc} S\left(\frac{m_c^2}{M_W^2}\right) + 2\lambda_c \lambda_t \eta_{ct} S\left(\frac{m_c^2}{M_W^2}, \frac{m_t^2}{M_W^2}\right)\right] Q^{|\Delta S|=2}$$

ϵ_K : 77% $\quad - 17\% \quad + 40\%$



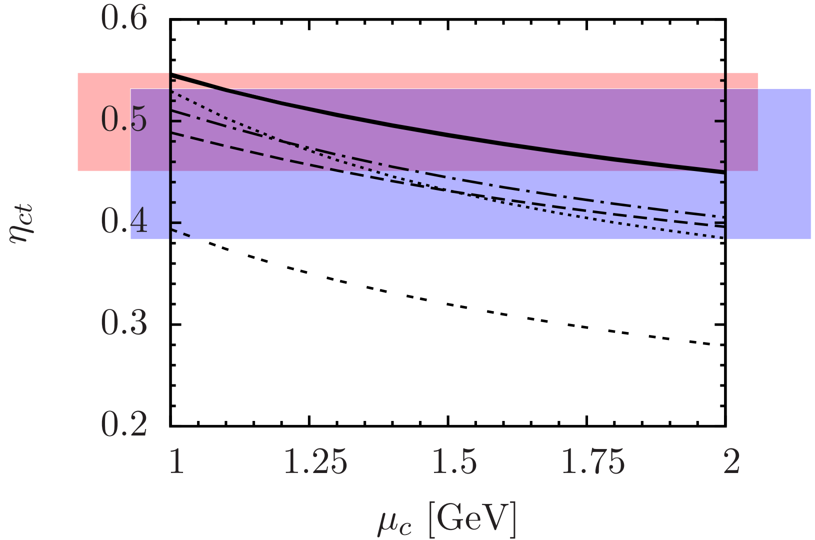
- $Q^{|\Delta S|=2} = (\bar{s}_L \gamma_\mu d_L)(\bar{s}_L \gamma^\mu d_L)$

η_{ct} @ NNLO – Calculation

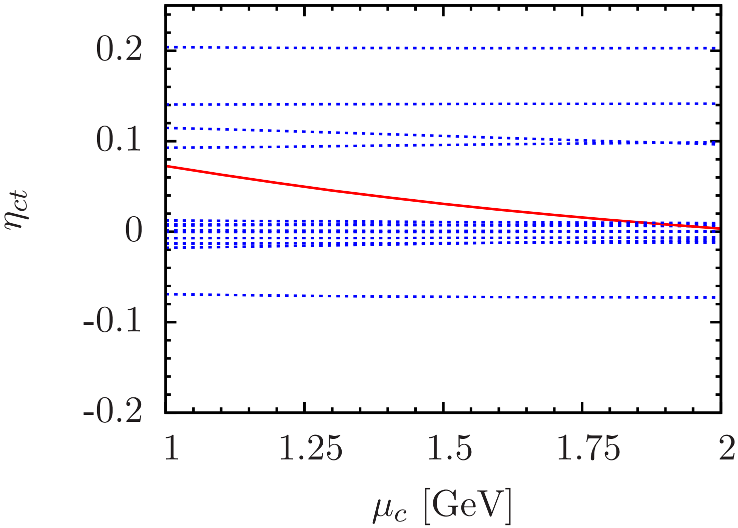


- Initial conditions: Matching at M_W
- Running to m_c
  - $\mathcal{O}(100\,000)$ Feynman diagrams
  - RGE for double insertion
  - Include threshold corrections at m_b
- Matching at m_c
- RGE in three-flavor EFT

η_{ct} @ NNLO – Scale Dependence



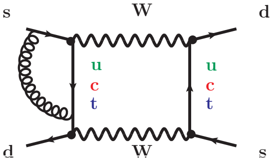
η_{ct} @ NNLO – Scale Dependence

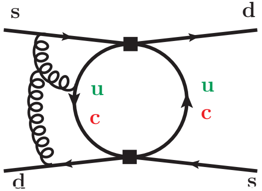


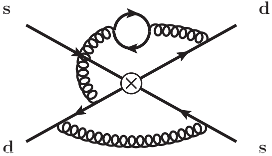
η_{ct} @ NNLO – Result

$$\eta_{ct} = \mathbf{0.496} \pm \mathbf{0.047}$$

η_{cc} @ NNLO – Calculation







- Initial conditions at M_W vanish by GIM
  [E. Witten, Nucl.Phys. B122 (1977) 109-143]
- Running to m_c
  - Only $|\Delta S| = 1$ operators contribute
  - Double insertions are finite (GIM)
- Matching at m_c
- $\mathcal{O}(100\,000)$ Feynman diagrams
- Including finite pieces

η_{cc} @ NNLO – Result

$$
\begin{aligned}
\langle Q_1 Q_1 \rangle &= \ldots \\
\langle Q_2 Q_2 \rangle &= \frac{69738523}{113400} + \frac{47407}{8505}\pi^2 - \frac{1733}{810}\pi^4 + \frac{1872}{35}\sqrt{3}\,\mathrm{Im}\,\mathrm{Li}_2((-1)^{1/3}) \\
&\quad + 24(\mathrm{Im}\,\mathrm{Li}_2((-1)^{1/3}))^2 - \frac{32}{27}\log(2)^2 + \frac{32}{27}\log(2)^4 \\
&\quad + \frac{563}{18}\log\frac{\mu_c^2}{m_c^2} + \frac{32}{3}\pi^2\log\frac{\mu_c^2}{m_c^2} + \frac{193}{3}\log^2\frac{\mu_c^2}{m_c^2} \\
&\quad + \frac{256}{9}\mathrm{Li}_4(1/2) - \frac{15145}{54}\zeta_3 \\
\langle Q_1 Q_2 \rangle &= \ldots
\end{aligned}
$$

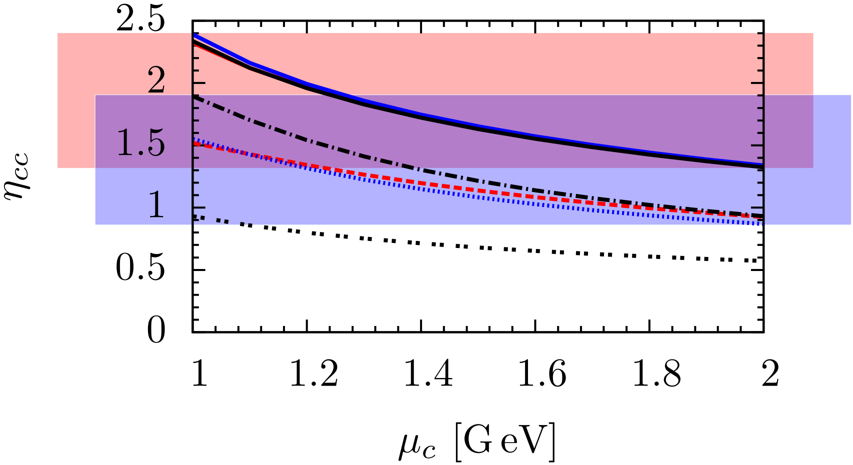
η_{cc} @ NNLO – Result

$$\eta_{cc} = \mathbf{1.87}$$

η_{cc} @ NNLO – Result

$$\eta_{cc} = \mathbf{1.87} \pm \mathbf{???}$$

η_{cc} @ NNLO – Scale Dependence



η_{cc} @ NNLO – Convergence

$$\frac{\eta_{cc}}{\alpha_s^{2/9}} = 1 + \alpha_s\,(0.25 + 0.32 L_c) + \alpha_s^2\,(1.20 + 0.22 L_c + 0.27 L_c^2)$$

- $L_c = \log(m_c^2/M_W^2) = -8.18$
- $\alpha_s = \alpha_s(m_c) = 0.35$

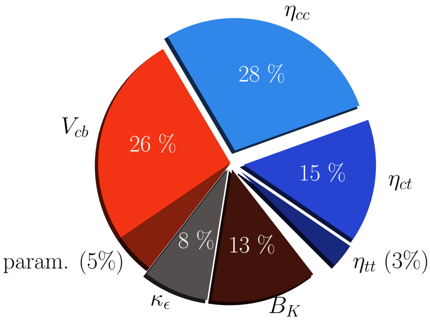
η_{cc} @ NNLO – Uncertainty and Progress

- Bad convergence of perturbation series
- $\Rightarrow$ Make use of RI-SMOM schemes at NNLO?
  [Gorbahn et al., work in progress]
- $\Rightarrow$ Dynamical charm on the lattice
  - Match at higher scales
- Our *preliminary prescription*:
  - Add NNLO shift and scale uncertainty in quadrature

$$\eta_{cc} = \mathbf{1.87} \pm \mathbf{0.76}$$

$|\epsilon_K|$ – Result and Error Budget

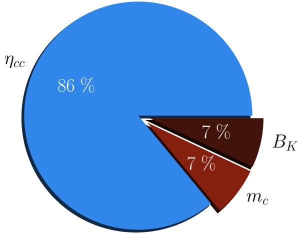
$$|\epsilon_K| \propto \kappa_\epsilon B_K |V_{cb}|^2 \sin\beta \Big(|V_{cb}|^2 \cos\beta\, \eta_{tt} S(x_t) + \eta_{ct} S(x_c, x_t) - \eta_{cc} S(x_c) \Big)$$



- $\hat{B}_K = 0.717(18)(16)$
  [FLAG 2019, 1902.08191
  (ETM 1505.06639)]
- $|\epsilon_K| = 2.06(23) \times 10^{-3}$
- $|\epsilon_K^{\mathrm{exp}}| = 2.228(11) \times 10^{-3}$

ΔM_K^{SD} – Result and Error Budget

$$\Delta M_K^{\mathrm{SD}} \propto B_K\Big((\mathrm{Re}\lambda_c)^2\eta_{cc}S(x_c) + \big(\mathrm{Re}\lambda_t^2 - \mathrm{Im}\lambda_t^2\big)\eta_{ct}S(x_c, x_t) + 2\mathrm{Re}\lambda_c\mathrm{Re}\lambda_t\eta_{tt}S(x_t)\Big)$$

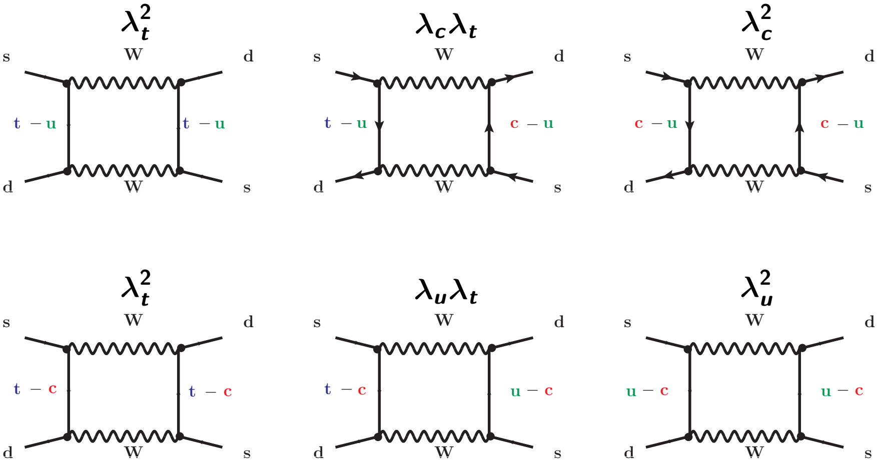


- $\Delta M_K^{\mathrm{SD}} = 2.9(1.2) \times 10^{-15}$ GeV
- $\Delta M_K^{\mathrm{exp}} = 3.482(6) \times 10^{-15}$ GeV
- $(83 \pm 34)\%$ short distance

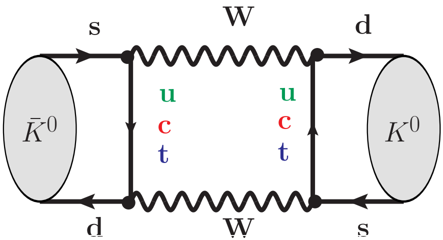
Future Prospects

"CISSY" Form of GIM Mechanism

[Christ Izubuchi Sachrajda Soni Yu, 1212.5931]



CKM structure in the "CISSY" Scheme



Conventional

	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_c \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_c^2	$\sim \lambda^6$	$\sim \lambda^2$

"CISSY"

	Im	Re
λ_t^2	$\sim \lambda^{10}$	$\sim \lambda^{10}$
$\lambda_u \lambda_t$	$\sim \lambda^6$	$\sim \lambda^6$
λ_u^2	$\mathbf{0}$	$\sim \lambda^2$

$\mathrm{Im}(M_{12}) \to \epsilon_K \qquad \mathrm{Re}(M_{12}) \to \Delta M_K$

Inami-Lim Boxes – Traditional Form

$$\lambda_c^2 : S(x_c) = x_c \,,$$

$$\lambda_t^2 : S(x_t) = \frac{4x_t - 11{x_t}^2 + {x_t}^3}{4(1-x_t)^2} - \frac{3{x_t}^3 \log x_t}{2(1-x_t)^3} \,,$$

$$\lambda_c \lambda_t : S(x_c, x_t) = x_c \left(\frac{{x_t}^2 - 8x_t + 4}{4(1-x_t)^2} \log x_t - \frac{3x_t}{4(1-x_t)} \right) - x_c \log x_c \,.$$

Inami-Lim Boxes – "CISSY" Form

$$\lambda_u^2 : S(x_c) = x_c \,,$$

$$\lambda_t^2 : S(x_t) = \frac{4x_t - 11{x_t}^2 + {x_t}^3}{4(1-x_t)^2} - \frac{3{x_t}^3 \log x_t}{2(1-x_t)^3} \qquad -0.2\% \,,$$

$$\lambda_u \lambda_t : S(x_c, x_t) = x_c \log x_c + x_c \left(\frac{4 - x_t}{4(1-x_t)} - \frac{{x_t}^2 - 8x_t + 4}{4(1-x_t)^2} \log x_t \right) .$$

Prospects for ϵ_K

- Calculate "CISSY" coefficients at NNLO ($N^3LO\ldots?$)
  [Brod, Stamou, work in progress]
  - λ_u^2 real $\Rightarrow$ does not contribute to ϵ_K
  - λ_t^2 is the same as in usual convention
  - $\lambda_u \lambda_t$ very similar to η_{ct} calculation;
    $|S^{\text{CISSY}}(x_c, x_t)/S^{\text{trad}}(x_c, x_t)| = 88\%$
  - Technology for four-loop calculation exists

Prospects for ΔM_K

- Mainly a project for the lattice
  - Necessary for $\boldsymbol{\Delta M_K}$ in any case
  - Scheme conversion for lattice renormalization
  - Analoguous to three-flavor case [Lehner, Sturm 1104.4948; Christ et al. 1212.5931]
  - Two-loop scheme conversion?

Summary

- ϵ_K is a powerful constraint on BSM physics
- NNLO calculation yields $+7\%$ shift of charm-top-quark contribution to ϵ_K, leading to $\boldsymbol{\eta_{ct} = 0.496(47)}$
- NNLO calculation yields $+36\%$ shift of charm-quark contribution to ϵ_K, leading to $\boldsymbol{\eta_{cc} = 1.87(76)}$
- Small parametric uncertainties – worth putting some effort into ϵ_K!
- I expect the perturbative error to go down significantly