$K_L - K_S$ mass difference with Lattice QCD at physical masses

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$K^0 - \overline{K^0}$ Mixing and Δm_K

$K^0(S=-1)$ and $\overline{K}^0(S=+1)$ mix through second order weak interactions:

$$i\frac{d}{dt}\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix} = (M - \frac{i}{2}\Gamma)\begin{pmatrix} K^0(t) \\ \overline{K}^0(t) \end{pmatrix}, \qquad (1)$$

Long-lived (K_L) and short-lived (K_S) are the two eigenstates:

$$K_S \approx \frac{K^0 - \overline{K}^0}{\sqrt{2}}, \quad K_L \approx \frac{K^0 + \overline{K}^0}{\sqrt{2}}. \qquad (2)$$

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2ReM_{0\bar{0}}$$

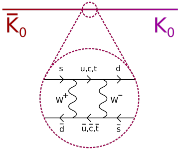


Figure: figure from wikipedia

Physics Motivation

$$\Delta m_K \equiv m_{K_L} - m_{K_S} = 2ReM_{0\bar{0}}$$

- This quantity is:
  1. Tiny, **sensitive to new physics**: FCNC via 2nd order weak interaction, precisely measured
  $$\Delta m_{K,exp} = 3.483(6) \times 10^{-12}\ \text{MeV}$$
  2. Significant contribution from scale of m_c(GIM mechanism)
  3. **Appears difficult to compute from QCD perturbation theory**: strong coupling at m_c scale; significant contributions from NNLO

  J. Brod and M. Gorbahn, Phys. Rev. Lett. 108, 121801 (2012)
- Lattice QCD:
  - from first principles
  - non-perturbative
  - systematic errors(FV, finite a, etc) could be controlled

From Correlators to Δm_K^{lat}

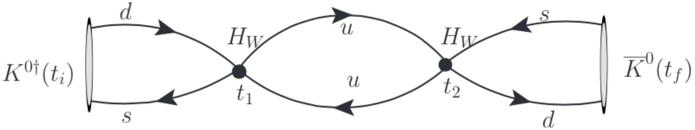
- Δm_K is given by:

$$\begin{aligned}\Delta m_K &\equiv m_{K_L} - m_{K_S} \\ &= 2\mathcal{P}\sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n}\end{aligned} \tag{3}$$

- What we measure on lattice are:

$$G(t_1, t_2, t_i, t_f) \equiv \langle 0|T\{\bar{K}^0(t_f)H_W(t_2)H_W(t_1)K^0(t_i)\}|0\rangle \tag{4}$$

$$\rightarrow G(\delta) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle e^{(m_K - E_n)\delta}$$



Extract Δm_K from Double-integrated Correlators

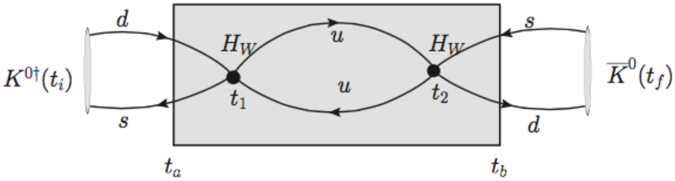
- The double-integrated correlator is defined as:

$$\mathcal{A} \equiv \frac{1}{2!} \sum_{t_2=t_a}^{t_b} \sum_{t_1=t_a}^{t_b} \langle 0 | T\{\bar{K}^0(t_f) H_W(t_2) H_W(t_1) K^0(t_i)\} | 0 \rangle \quad (5)$$

- If we insert a complete set of intermediate states, we find:

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{ -T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n} \} \quad (6)$$

with $T \equiv t_b - t_a + 1$.



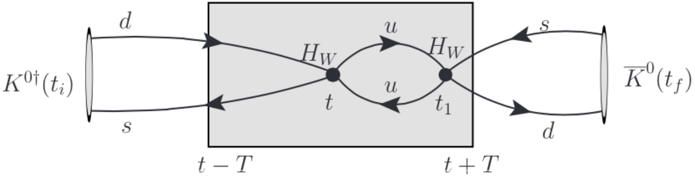
Extract Δm_K from Single-integrated Correlators

- The single-integrated correlator is defined as:

$$\mathcal{A}^s(t,T) \equiv \frac{1}{2!}\sum_{t_1=t-T}^{t+T} \langle 0| T\{\bar{K}^0(t_f) H_W(t_1) H_W(t) K^0(t_i)\} |0\rangle \quad (7)$$

- If we insert a complete set of intermediate states, we find:

$$\mathcal{A}^s = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0|H_W|n\rangle\langle n|H_W|K^0\rangle}{m_K - E_n}\left(-1+e^{(m_K-E_n)(T+1)}\right) \quad (8)$$



Subtraction of the light states

- Either Double- or Single-integrated Method requires subtraction of the terms from light states:

$$\mathcal{A} = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K-E_n)T} - 1}{m_K - E_n}\} \tag{9}$$

$$\mathcal{A}^s = N_K^2 e^{-m_K(t_f-t_i)} \sum_n \frac{\langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle}{m_K - E_n} \{-1 + e^{(m_K-E_n)(T+1)}\} \tag{10}$$

- For $|n\rangle$(in our case $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$, $|\pi\rangle$) with $E_n < m_K$ or $E_n \sim m_K$: the exponential terms will be significant. We can:
  - freedom of adding $c_s \bar{s} d$, $c_p \bar{s} \gamma^5 d$ operators to the weak Hamiltonian Here we choose:

$$\langle 0 | H_W - c_p \bar{s} \gamma_5 d | K^0 \rangle = 0, \langle \eta | H_W - c_s \bar{s} d | \bar{K}^0 \rangle = 0$$

  - subtract contributions from other states($|\pi\rangle$, $|\pi\pi\rangle$) explicitly

Operators of Δm_K^{lat} calculation

- The $\Delta S = 1$ effective Weak Hamiltonian:

$$H_W = \frac{G_F}{\sqrt{2}} \sum_{q,q'=u,c} V_{qd} V^*_{q's} (C_1 Q_1^{qq'} + C_2 Q_2^{qq'}) \qquad (11)$$

where the $Q_i^{qq'}{}_{i=1,2}$ are current-current opeartors, defined as:

$$Q_1^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma^5) d_i)(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_j)$$

$$Q_2^{qq'} = (\bar{s}_i \gamma^\mu (1-\gamma^5) d_j)(\bar{q}_j \gamma^\mu (1-\gamma^5) q'_i)$$

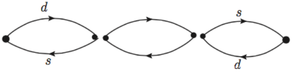
- There are four states need to subtracted: $|0\rangle$, $|\pi\pi\rangle$, $|\eta\rangle$, $|\pi\rangle$. We add $c_s \bar{s} d$, $c_p \bar{s}\gamma^5 d$ operators to weak operators to make:

$$\langle 0|Q_i - c_{pi}\bar{s}\gamma_5 d|K^0\rangle = 0, \langle \eta|Q_i - c_{si}\bar{s} d|K^0\rangle = 0 \qquad (12)$$

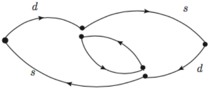
$$Q'_i = Q_i - c_{pi}\bar{s}\gamma_5 d - c_{si}\bar{s} d \qquad (13)$$

Diagrams in the Calculation of Δm_K^{lat}

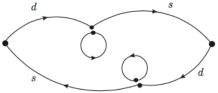
- For contractions among Q_i, there are four types of diagrams to be evaluated.



type 1



type 2



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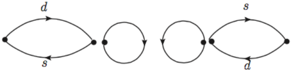
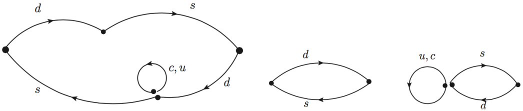


type 4

- In addition, there are "mixed" diagrams from the contractions between the $c_s \bar{s} d$ $c_p \bar{s}\gamma^5 d$ operators and Q_i operators.



Finite lattice spacing effects

Short distance correction?

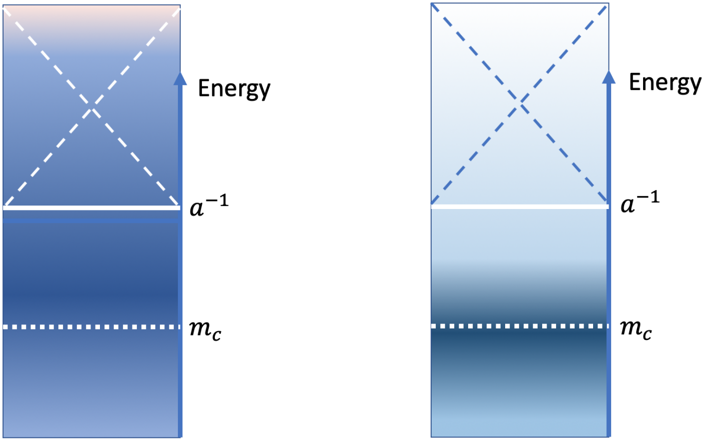
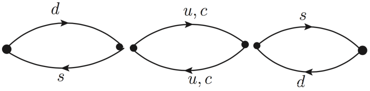


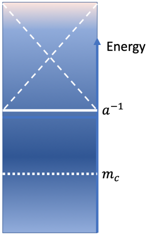
Figure: Different cases about physics on lattice with respect to energy scales. The shaded area represents where the contributions are important.

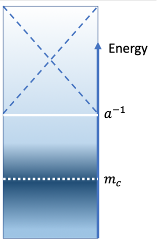
Finite lattice spacing effects



Quadratic divergences as the two H_W approach each other:
cutoff effect $\propto (1/a)^2$
needs short-distance correction.

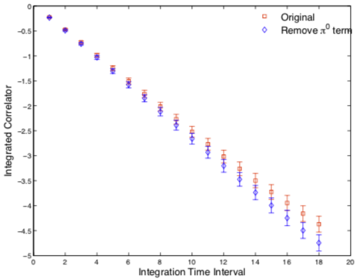
GIM mechanism + LL structure removes both quadratic and logarithmic divergences:
$\sim (m_c a)^2$



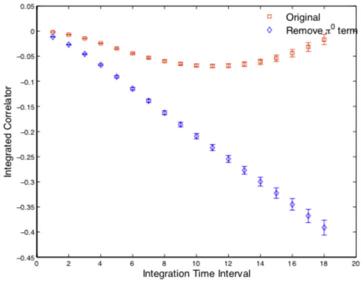


Finite lattice spacing effects

- Ultraviolet divergences as the two H_W approach each other:$\sim (1/a)^2$
- GIM mechanism $\rightarrow$ up minus charm quark propagators(for valence charm we used $am_c \simeq 0.31$)
  $16^3 \times 32$ lattice: $Q_1 Q_1$ correlator amplitude reduction by a factor of 10 after introducing valence charm with mass 863 MeV (Jianglei Yu's PhD thesis, 2014).



(a) Without charm: ~ 1



(b) With charm: ~ 0.1

Finite lattice spacing effects

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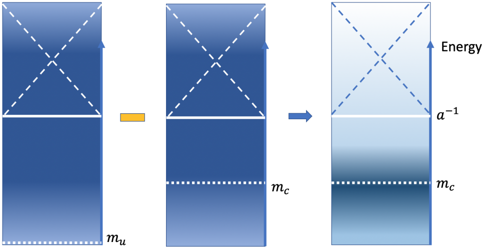

Finite lattice spacing effects

- Ultraviolet divergences as the two H_W approach each other:$\sim (1/a)^2$
- GIM mechanism $\rightarrow$ charm minus up quark propagators(for valence charm we used $am_c \simeq 0.31$)
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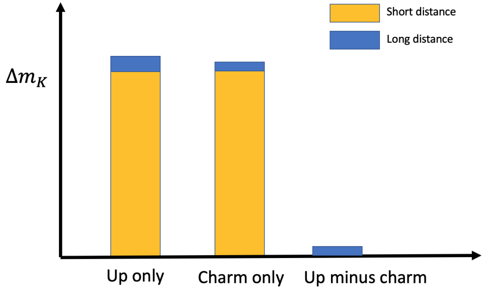

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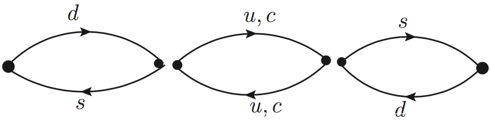
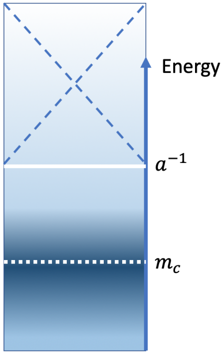


Thus in our calculation of Δm_K, GIM mechanism + LL structure removes both quadratic and logarithmic divergences:

- short distance contribution greatly suppressed.
- Major contribution to Δm_K from scale $\sim m_c$



Operator Renormalizations

- Renormalization of Lattice operator $Q_{1,2}$ in 3 steps:

$$C_i^{lat} = C_a^{\overline{MS}}(1 + \Delta r)_{ab}^{RI \to \overline{MS}} Z_{bi}^{lat \to RI}$$

  - Non-perturbative Renormalization: from lattice to RI-SMOM

$$Z^{lat \to RI} = \begin{bmatrix} 0.6266 & -0.0437 \\ -0.0437 & 0.6266 \end{bmatrix} \tag{14}$$

  - Perturbation theory: from RI-SMOM to $\overline{MS}$

C. Lehner, C. Sturm, Phys. Rev. D 84(2011), 014001

$$\Delta r^{RI \to \overline{MS}} = 10^{-3} \times \begin{bmatrix} -2.28 & 6.85 \\ 6.85 & -2.28 \end{bmatrix} \tag{15}$$

  - Use Wilson coefficients in the $\overline{MS}$ scheme

G. Buchalla, A.J. Buras and M.E. Lautenbacher, arXiv:hep-ph/9512380

$$C^{\overline{MS}} = 10^{-3} \times \begin{bmatrix} -0.260 & 1.118 \end{bmatrix} \tag{16}$$

Status of RBC-UKQCD Calculations of Δm_k

- **"Long-distance contribution of the $K_L - K_S$ mass difference"**, **N. H. Christ, T. Izubuchi, C. T. Sachrajda, A. Soni and J. Yu**

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  All diagrams included on a $64^3 \times 128$ lattice with **physical mass** on 59 configurations: $\Delta m_k = (5.5 \pm 1.7_{stat}) \times 10^{-12} MeV$

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- Here I present an update of the analysis **methods** used and **results** having smaller statistical errors with **152** configurations.

Details of the Calculation

- $64^3 \times 128 \times 12$ lattice with Möbius DWF and the Iwasaki gauge action with physical pion mass (136 MeV) and $a^{-1} = 2.36$GeV

N_f	β	am_l	am_h	$\alpha = b + c$	L_s
2+1	2.25	0.0006203	0.02539	2.0	12

- Data:
  - Sample AMA Correction and Super-jackknife Method

data type	CG stop residual
sloppy	$1e-4$
exact	$1e-8$

Data Set	# of Sloppy	# of Correction	# of Type 1&2
Total	116	**36**	**36**

  - Disconnected Type4 diagrams: save left- and right-pieces separately and use multiple source-sink separation for fitting.

Update of the results

2-point and 3-point results **preliminary**

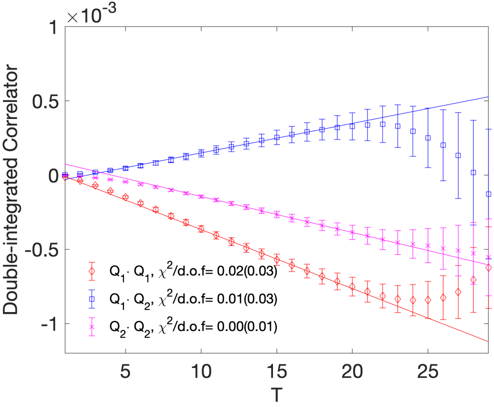
- Meson masses are consistent with physical values

m_π	m_K	m_η	$m_{\pi\pi,I=0}$
0.0574(1)	0.2104(1)	0.258(16)	0.1138(5)
135.5(2)	496.5(2)	609.9(37.8)	268.5(1.3)

- c_s's and c_p's will be multilpied by the "mixing" diagrams and the errors from c_s's and c_p's will be carried all along.

$c_{s1,\eta}$	$c_{s2,\eta}$	$c_{p1,vac}$	$c_{p2,vac}$
2.13(33) $\times 10^{-4}$	-3.16(25)$\times 10^{-4}$	1.472(2) $\times 10^{-4}$	2.807(2) $\times 10^{-4}$
$\langle \pi\pi_{I=0} \vert Q_1' \vert K^0 \rangle$	$\langle \pi\pi_{I=0} \vert Q_2' \vert K^0 \rangle$	$\langle \pi \vert Q_1' \vert K^0 \rangle$	$\langle \pi \vert Q_2' \vert K^0 \rangle$
-8.7(1.5)$\times 10^{-5}$	9.5(1.5) $\times 10^{-5}$	7.7(2.5)$\times 10^{-4}$	-4.1(1.6)$\times 10^{-4}$

Double-integrated correlators **preliminary**



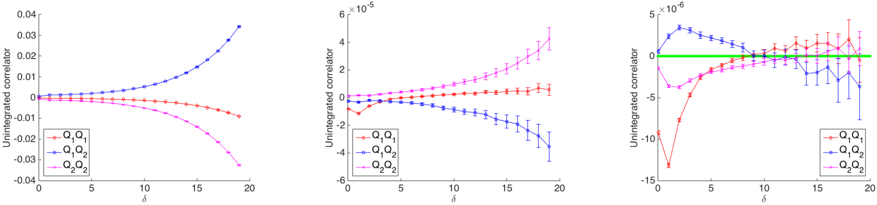
- Fitting range: 10:20
- All diagrams, uncorrelated fit
- $\Delta m_K =$ $8.1(1.2) \times 10^{-12}$MeV

$$\mathcal{A} = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \frac{\langle K^0 | H_W | n \rangle \langle n | H_W | \bar{K}^0 \rangle}{m_K - E_n} \{-T + \frac{e^{(m_K - E_n)T} - 1}{m_K - E_n}\} \tag{17}$$

Single-integrated correlators preliminary

$$G(\delta) = N_K^2 e^{-m_K(t_f - t_i)} \sum_n \langle \bar{K}^0 | H_W | n \rangle \langle n | H_W | K^0 \rangle e^{(m_K - E_n)\delta} \qquad (18)$$

Check: Unintegrated $\rightarrow \langle 0 | Q_i' | K^0 \rangle = 0$, $\langle \eta | Q_i' | K^0 \rangle = 0 \rightarrow$ Subtract $\langle \pi | Q_i' | K^0 \rangle$

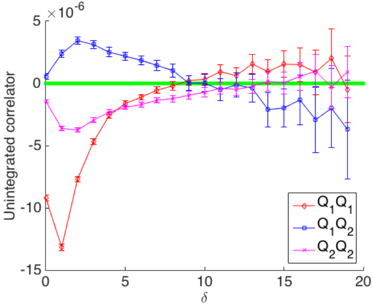


$$Q_i' = Q_i - c_{pi} \bar{s} \gamma_5 d - c_{si} \bar{s} d$$

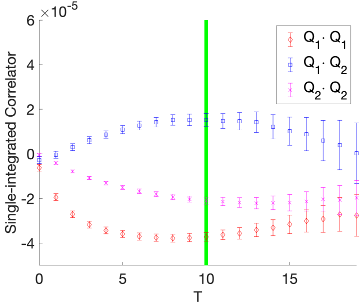
Next step: integrate and obtain Δm_K

Note: Need to add back contributions to Δm_K from subtracted states.

Single-integrated correlators: All diagrams, uncorrelated, **preliminary**



(a) unintegrated results with π subtraction



(b) After integrating to large T, converged

Choosing T=10, as the integration upper limit:

$$\Delta m_K = 6.9(0.6) \times 10^{-12}\text{MeV}$$

Sources of Error

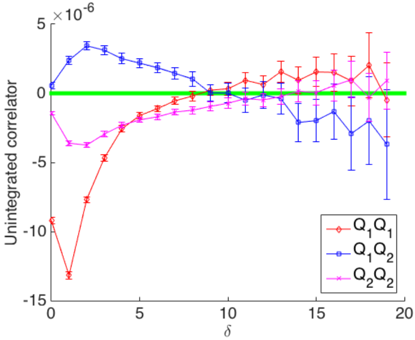
- Statistical Error
  - Less statistics for large operator separation
- Systematic errors:
  - **Finite-volume corrections**: small compared to statistical errors
    "Effects of finite volume on the $K_L - K_S$ mass difference"
    N.H. Christ, X. Feng, G. Martinelli and C.T. Sachrajda, arXiv:1504.01170

$$\Delta m_K(FV) = -0.22(7) \times 10^{-12} MeV \tag{19}$$

  - **Discretization effects** are the **largest** sources of systematic error
    - $\mathcal{O}(a)$: No contributions from DWF;
      Insure that no O(a) error is introduced by lattice summation.(please see next slide)
    - $\mathcal{O}(a^2)$: No short distance correction needed due to GIM cancellation
      Instead, $\sim (m_c a)^2$

Systematic errors

- **Discretization effects** are the **largest** source of systematic error:
  - $\mathcal{O}(a)$: No corrections needed: integrand's boundary values goes to zero



  - $\mathcal{O}(a^2)$:
    - Heavy charm quark, $\sim (m_c a)^2$ gives 25% Extrapolation needed.
    - Another estimate based on HVP calculation is $\sim$ 15%

Results **preliminary**

- Using single-integration method, we could:
  1. Manually **avoid including noise around zero** for large enough operator separations
  2. Smaller error in subtraction:
     $e^{-(E_n - m_K)t}$ rather than $\frac{1}{E_n - m_K} e^{-(E_n - m_K)t}$
- Δm_K values obtained from 2 analysis methods

Method	Double-int	Single-int
$\Delta m_K / 10^{-12}$ MeV	$8.1(1.2)_{stat}$	$6.9(0.6)_{stat}$

consistent within uncertainties

Conclusion and Outlook

- Our **preliminary** result based on 152 configurations is

$$\Delta m_K = 6.7(0.6)_{stat}(1.7)_{sys} \times 10^{-12} MeV$$

  to be compared to the experimental value

$$(\Delta m_K)^{exp} = 3.483(6) \times 10^{-12} MeV$$

- Outlook
  - Better estimate of the discretization error:
    Continue the calculation of Δm_K on Summit:
    - On finer lattice($96^3 \times 192$, $a^{-1} = 2.8$ GeV) $\rightarrow$ smaller $m_c a$.
  - Continue the check of the measurement on lattice and data analysis(coefficients and renormalization factors), though the code was checked by Jianglei, Ziyuan and myself before.

Thanks for your attention!

Finite lattice spacing error effects

- Ultraviolet divergences as the two H_W approach each other:

$$\int_{m_u}^{a^{-1}} d^4p\gamma^\mu(1-\gamma_5)\frac{p-m_u}{p^2+m_u^2}\gamma^\nu(1-\gamma_5)\frac{p-m_u}{p^2+m_u^2} \propto (1/a)^2 \tag{20}$$

- GIM mechanism removes both quadratic and logarithmic divergences $\rightarrow$ charm quark propagators(for valence charm we used $am_c \simeq 0.31$)

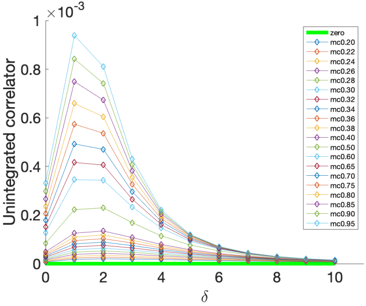
$$\int d^4p\gamma^\mu(1-\gamma_5)(\frac{p-m_u}{p^2+m_u^2}-\frac{p-m_c}{p^2+m_c^2})\gamma^\nu(1-\gamma_5)(...-...) \tag{21}$$

$$\int d^4p\gamma^\mu(1-\gamma_5)(\frac{p(m_c^2-m_u^2)}{(p^2+m_u^2)(p^2+m_c^2)})\gamma^\nu(1-\gamma_5)(...-...) \tag{22}$$

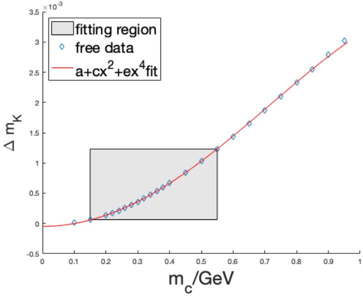
And "short distance" now comming from $\sim 1/m_c$, with $\sim (m_c a)^2$ finite lattice spacing error relevant, rather than $\sim (a^{-1})^2$ divergence.

Finite lattice spacing effects

- Ultraviolet divergences as the two H_W approach each other:$\sim (1/a)^2$
- GIM mechanism $\rightarrow$ charm minus up quark propagators(for valence charm we used $am_c \simeq 0.31$)
  removes both quadratic and logarithmic divergences:$\sim m_c^2$



(a) Unintegrated correlators

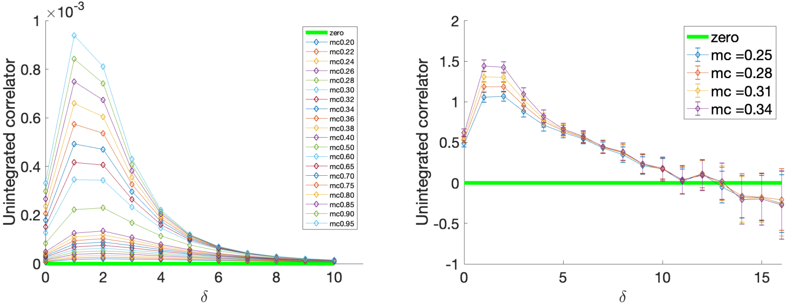


(b) m_c dependence

Figure: GIM effect in the QCD-free case on lattice quadratic m_c dependence.

Finite lattice spacing effects

- GIM mechanism $\rightarrow$ 64I lattice charm quark propagators(for valence charm we used $am_c \simeq 0.31$)
  Similar behavior



(a) Without QCD (b) With Iwasaki gauge action

Figure: GIM effect on $64^3 \times 128$ lattice.