

EIC Kinematic Reconstruction Simulation Studies

Barak Schmookler

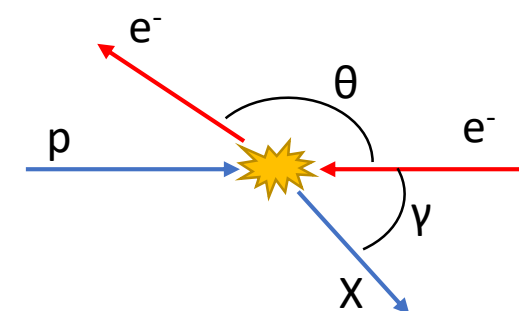
Kinematics for Electron-Proton Scattering

$$P_e = (E_e, 0, 0, -P_e)$$

$$P_p = (E_p, 0, 0, P_p)$$

$$P_{e'} = (E_{e'}, P_{e'} \sin \theta \cos \varphi, P_{e'} \sin \theta \sin \varphi, P_{e'} \cos \theta)$$

$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$



$$e + p \rightarrow e' + X$$

Kinematics for Electron-Proton Scattering

$$P_e = (E_e, 0, 0, -P_e)$$

$$P_p = (E_p, 0, 0, P_p)$$

$$P_{e'} = (E_{e'}, P_{e'} \sin \theta \cos \varphi, P_{e'} \sin \theta \sin \varphi, P_{e'} \cos \theta)$$

$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$

Kinematic Invariants:

$$s = (P_e + P_p)^2$$

$$Q^2 = -q^2 = -(P_e - P_{e'})^2$$

$$x = \frac{Q^2}{2P_p \cdot q}$$

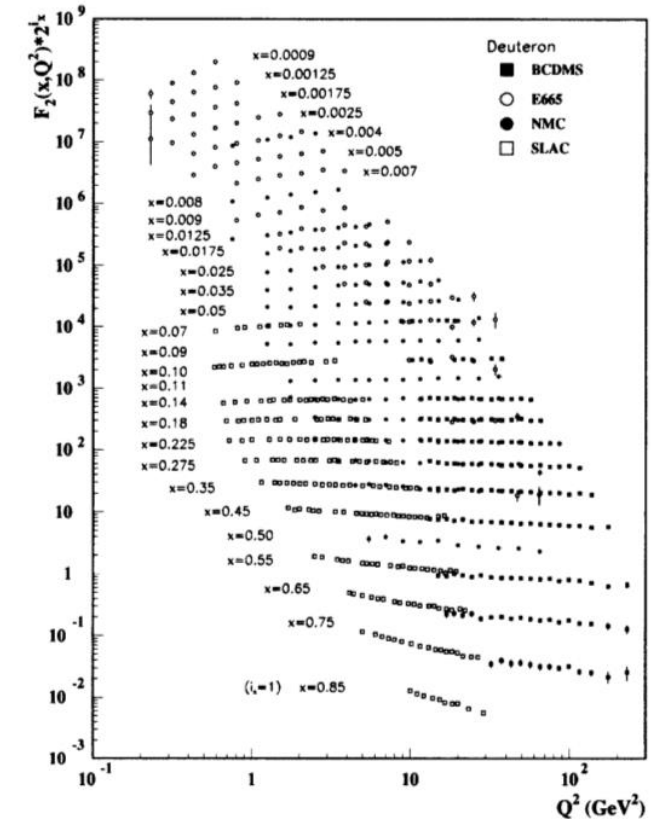
$$y = \frac{P_p \cdot q}{P_p \cdot P_e}$$

$$v = \frac{P_p \cdot q}{M_p}$$

$$W = M_X = \sqrt{(P_X)^2} = \sqrt{M_p^2 + 2M_p v - Q^2}$$

We Need to Accurately Reconstruct these Variables

$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[2 \frac{F_1(x_B, Q^2)}{M} \sin^2 \frac{\theta_e}{2} + \frac{F_2(x_B, Q^2)}{\nu} \cos^2 \frac{\theta_e}{2} \right]$$

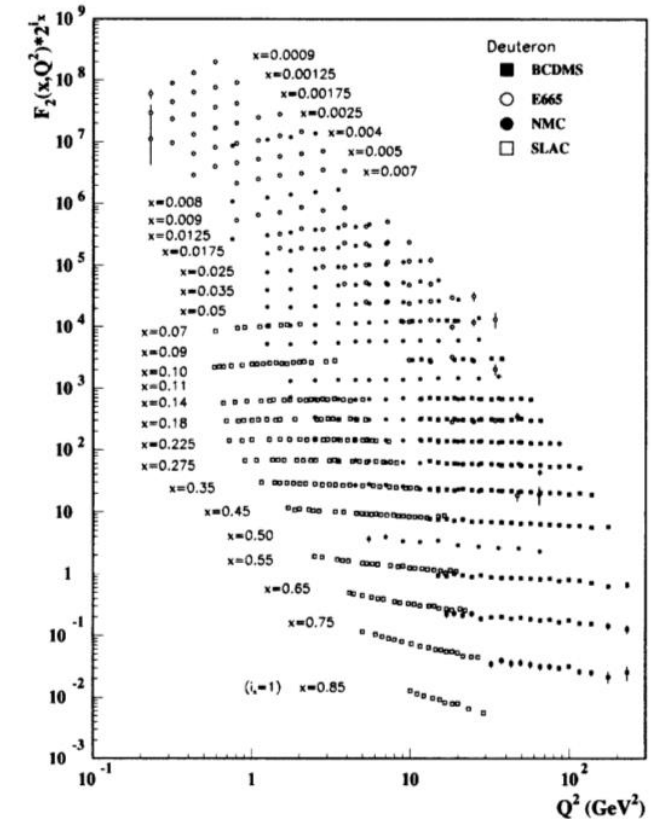


Particle Data Group collaboration, S. Eidelman, et al.
Physics Letters B, 592:1–1109, 2004.

We Need to Accurately Reconstruct these Variables

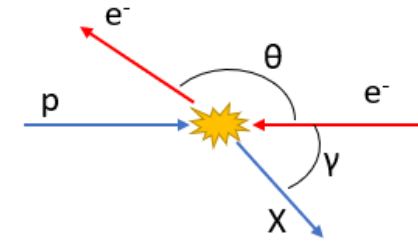
$$\frac{d^2\sigma}{d\Omega dE'} = \frac{4\alpha^2 E'^2}{Q^4} \left[2 \frac{F_1(x_B, Q^2)}{M} \sin^2 \frac{\theta_e}{2} + \frac{F_2(x_B, Q^2)}{\nu} \cos^2 \frac{\theta_e}{2} \right]$$

- We usually use the scattered electron to reconstruct these variables. But sometimes we cannot:
 - The scattered electron is not detected
 - There is no scattered electron (i.e. charged-current events)
 - The scattered electron is reconstructed poorly (consider y dependence, for example: $x = \frac{Q^2}{s} \times \frac{1}{y}$).
- In these cases, we must use the final hadronic state.
- This was done at *HERA* for electron-proton scattering, but hasn't been studied much for electron-nucleus scattering



Particle Data Group collaboration, S. Eidelman, et al.
Physics Letters B, 592:1–1109, 2004.

High-Energy Fixed-Target Scattering



$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (\cancel{E_p}, 0, 0, \cancel{E_p})$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin \theta \cos \varphi, \cancel{E_{e'}} \sin \theta \sin \varphi, \cancel{E_{e'}} \cos \theta)$$

$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$

Kinematic Invariants:

$$s = (P_e + P_p)^2$$

$$Q^2 = -q^2 = -(P_e - P_{e'})^2$$

$$x = \frac{Q^2}{2P_p \cdot q}$$

$$y = \frac{P_p \cdot q}{P_p \cdot P_e} = \frac{(E_e - E_{e'})}{E_e}$$

$$v = \frac{P_p \cdot q}{M_P} = (E_e - E_{e'})$$

$$W = M_X = \sqrt{(P_X)^2} = \sqrt{M_P^2 + 2M_P v - Q^2}$$

High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin \theta \cos \varphi, \cancel{E_{e'}} \sin \theta \sin \varphi, \cancel{E_{e'}} \cos \theta)$$

~~$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$~~

Scattered Electron Method

Kinematic Invariants:

$$s = (P_e + P_p)^2 = 4E_e E_p$$

High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin \theta \cos \varphi, \cancel{E_{e'}} \sin \theta \sin \varphi, \cancel{E_{e'}} \cos \theta)$$

~~$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$~~

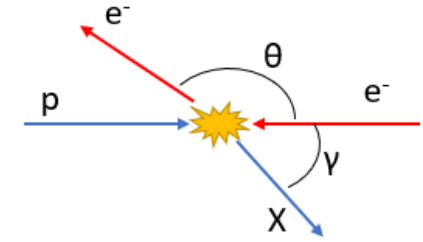
Scattered Electron Method

Kinematic Invariants:

$$y_e = \frac{P_p \cdot q}{P_p \cdot P_e} = 1 - \frac{\Sigma_e}{2E_e}$$

$$\Sigma_e = E_{e'} - P_{z,e'}$$

High-Energy Electron-Proton Collider



$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin \theta \cos \varphi, \cancel{E_{e'}} \sin \theta \sin \varphi, \cancel{E_{e'}} \cos \theta)$$

~~$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$~~

Scattered Electron Method

Kinematic Invariants:

$$Q_e^2 = -q^2 = -(P_e - P_{e'})^2$$

$$= 4E_e E_{e'} \cos^2 \frac{\theta}{2}$$

$$= \frac{p_{t,e}^2}{1 - y_e}$$

High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin \theta \cos \varphi, \cancel{E_{e'}} \sin \theta \sin \varphi, \cancel{E_{e'}} \cos \theta)$$

~~$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$~~

Scattered Electron Method

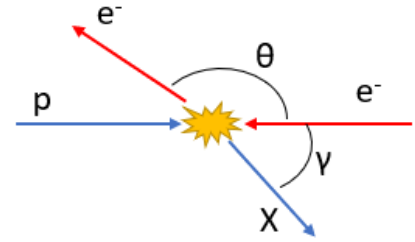
Kinematic Invariants:

$$x = \frac{Q^2}{(sy)}$$

$$v = \frac{Q^2}{2M_p x}$$

$$W = \sqrt{M_p^2 + 2M_p v - Q^2}$$

High-Energy Electron-Proton Collider



$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

Jacquet-Blondel Method

$$\cancel{P_{e'}} = (\cancel{E_{e'}}, \cancel{P_{e'} \sin \theta \cos \varphi}, \cancel{P_{e'} \sin \theta \sin \varphi}, \cancel{P_{e'} \cos \theta})$$

$$P_X = (\cancel{E_X}, \cancel{P_X \sin \gamma \cos(\varphi + \pi)}, \cancel{P_X \sin \gamma \sin(\varphi + \pi)}, \cancel{P_X \cos \gamma})$$

$$\sum_i E_i \quad \sum_i p_{x,i} \quad \sum_i p_{y,i} \quad \sum_i p_{z,i}$$

High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

Jacquet-Blondel Method

Kinematic Invariants:

$$y_h = \frac{P_p \cdot q}{P_p \cdot P_e} = \frac{\Sigma_h}{2E_e}$$

~~$$P_{e'} = (E_{e'}, P_{e'} \sin \theta \cos \varphi, P_{e'} \sin \theta \sin \varphi, P_{e'} \cos \theta)$$~~

~~$$P_X = (\cancel{E_X}, \cancel{P_X \sin \gamma \cos(\varphi + \pi)}, \cancel{P_X \sin \gamma \sin(\varphi + \pi)}, \cancel{P_X \cos \gamma})$$~~

$$\sum_i E_i \quad \sum_i p_{x,i} \quad \sum_i p_{y,i} \quad \sum_i p_{z,i}$$

$$\Sigma_h = \sum_i (E_i - p_{z,i}) = \sum_i E_i - \sum_i p_{z,i}$$

High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -E_e)$$

$$P_p = (E_p, 0, 0, E_p)$$

Jacquet-Blondel Method

Kinematic Invariants:

$$Q_h^2 = -q^2$$

$$= \frac{p_{t,h}^2}{1 - y_h}$$

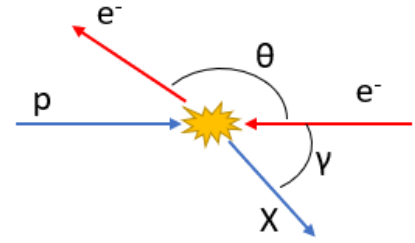
$$= \frac{(\sum_i p_{x,i})^2 + (\sum_i p_{y,i})^2}{1 - y_h}$$

~~$$P_{e'} = (E_{e'}, P_{e'} \sin \theta \cos \varphi, P_{e'} \sin \theta \sin \varphi, P_{e'} \cos \theta)$$~~

~~$$P_X = (E_X, P_X \sin \gamma \cos(\varphi + \pi), P_X \sin \gamma \sin(\varphi + \pi), P_X \cos \gamma)$$~~

$$\sum_i E_i \quad \sum_i p_{x,i} \quad \sum_i p_{y,i} \quad \sum_i p_{z,i}$$

High-Energy Electron-Proton Collider



Double-Angle Method

$$P_e = (E_e, 0, 0, -E_e)$$

$$P_p = (E_p, 0, 0, E_p)$$

$$P_{e'} = (E_{e'}, E_{e'} \sin \theta \cos \varphi, E_{e'} \sin \theta \sin \varphi, E_{e'} \cos \theta)$$

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High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

Double-Angle Method

$$\tan\left(\frac{\theta}{2}\right) = \frac{1 - \cos\theta}{\sin\theta} = \frac{1 - \frac{p_{z,e}}{P_{e'}}}{\frac{p_{t,e}}{P_{e'}}}$$

$$= \frac{P_{e'} - p_{z,e}}{p_{t,e}}$$

$$= \frac{\Sigma_e}{p_{t,e}}$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin\theta \cos\varphi, \cancel{E_{e'}} \sin\theta \sin\varphi, \cancel{E_{e'}} \cos\theta)$$

$$P_X = (\cancel{\Sigma_X}, \cancel{\Sigma_X \sin\gamma \cos(\varphi + \pi)}, \cancel{\Sigma_X \sin\gamma \sin(\varphi + \pi)}, \cancel{\Sigma_X \cos\gamma})$$

$$\sum_i E_i \quad \sum_i p_{x,i} \quad \sum_i p_{y,i} \quad \sum_i p_{z,i}$$

High-Energy Electron-Proton Collider

$$P_e = (E_e, 0, 0, -\cancel{E_e})$$

$$P_p = (E_p, 0, 0, \cancel{E_p})$$

Double-Angle Method

$$\tan\left(\frac{\gamma}{2}\right) = \frac{\Sigma_h}{p_{t,h}}$$

$$P_{e'} = (E_{e'}, \cancel{E_{e'}} \sin \theta \cos \varphi, \cancel{E_{e'}} \sin \theta \sin \varphi, \cancel{E_{e'}} \cos \theta)$$

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$$\sum_i E_i \quad \sum_i p_{x,i} \quad \sum_i p_{y,i} \quad \sum_i p_{z,i}$$

High-Energy Electron-Proton Collider

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$$\sum_i E_i \quad \sum_i p_{x,i} \quad \sum_i p_{y,i} \quad \sum_i p_{z,i}$$

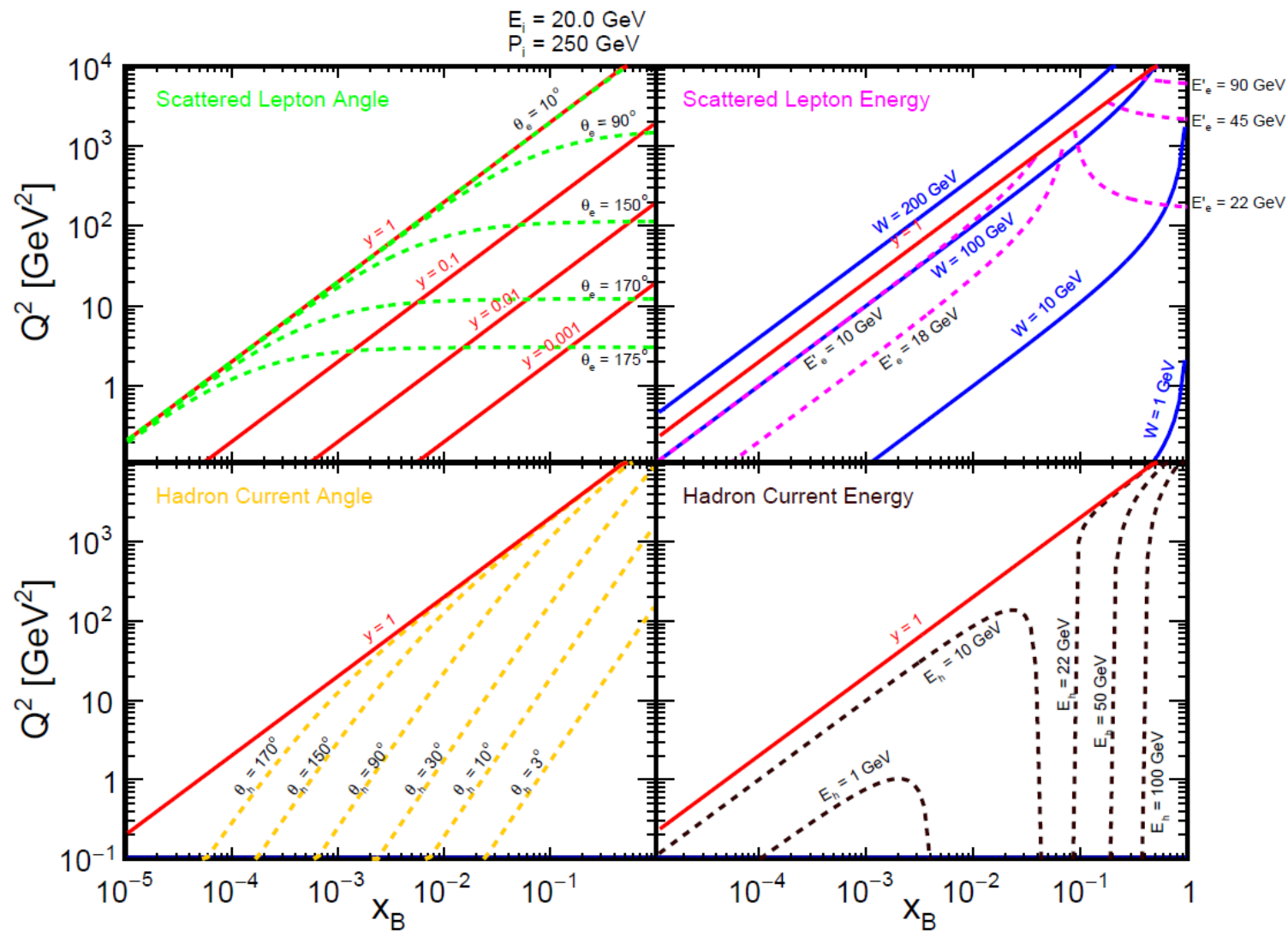
Double-Angle Method

Kinematic Invariants:

$$y_{DA} = \frac{\tan \gamma/2}{\tan \theta/2 + \tan \gamma/2}$$

$$Q_{DA}^2 = 4E_e^2 \frac{\cot \theta/2}{\tan \theta/2 + \tan \gamma/2}$$

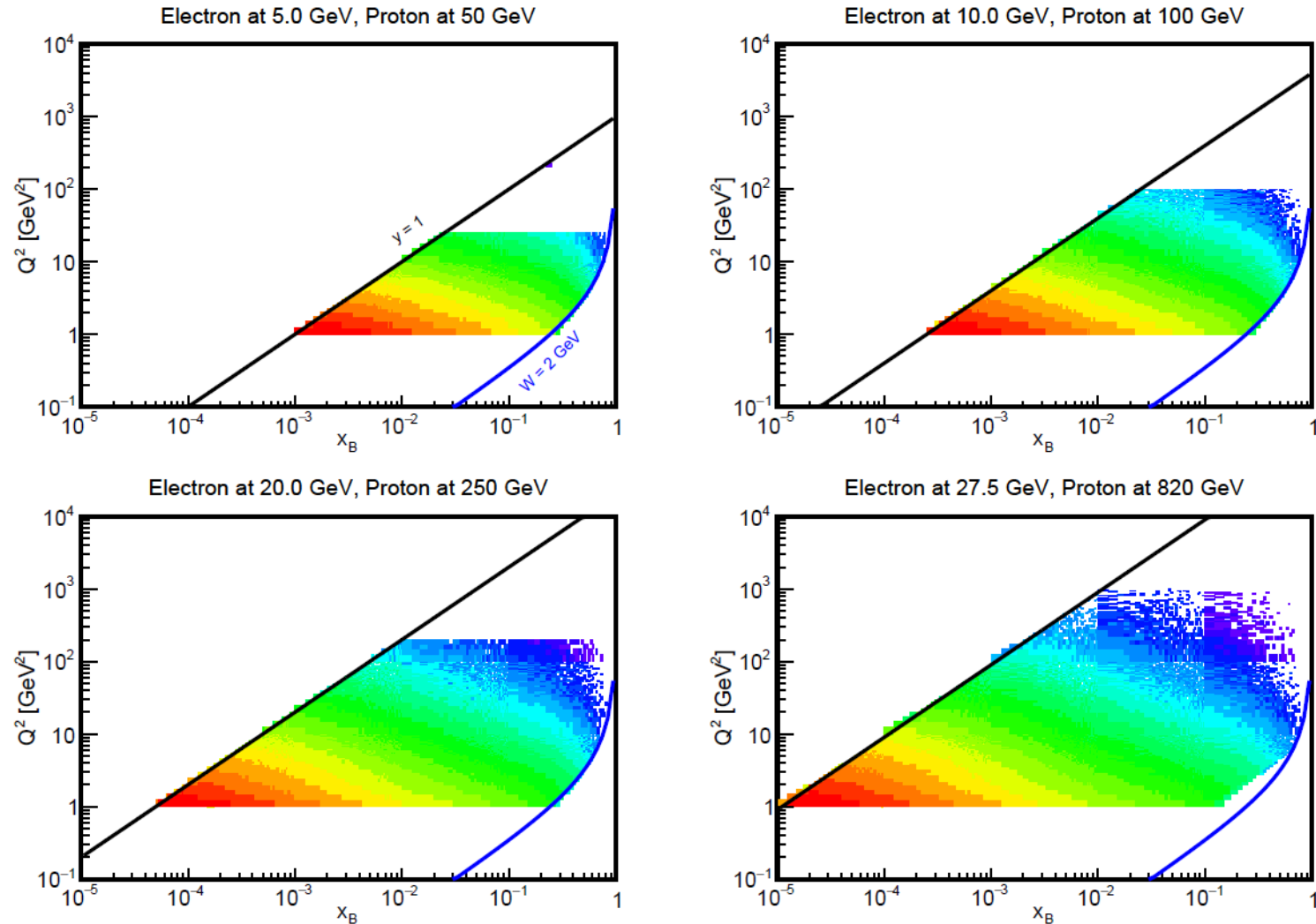
Kinematic Phase Space: 20 GeV, 250 GeV

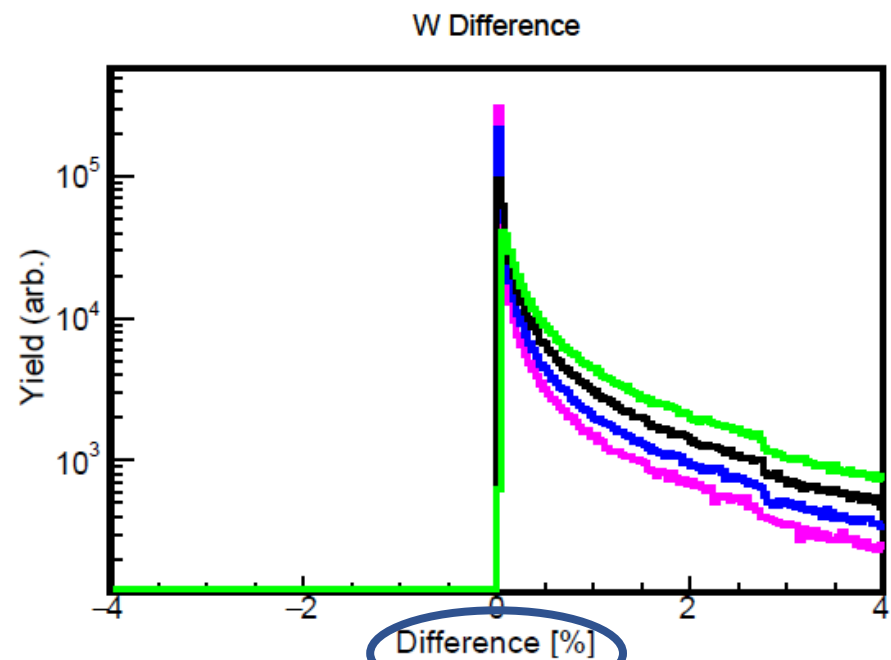
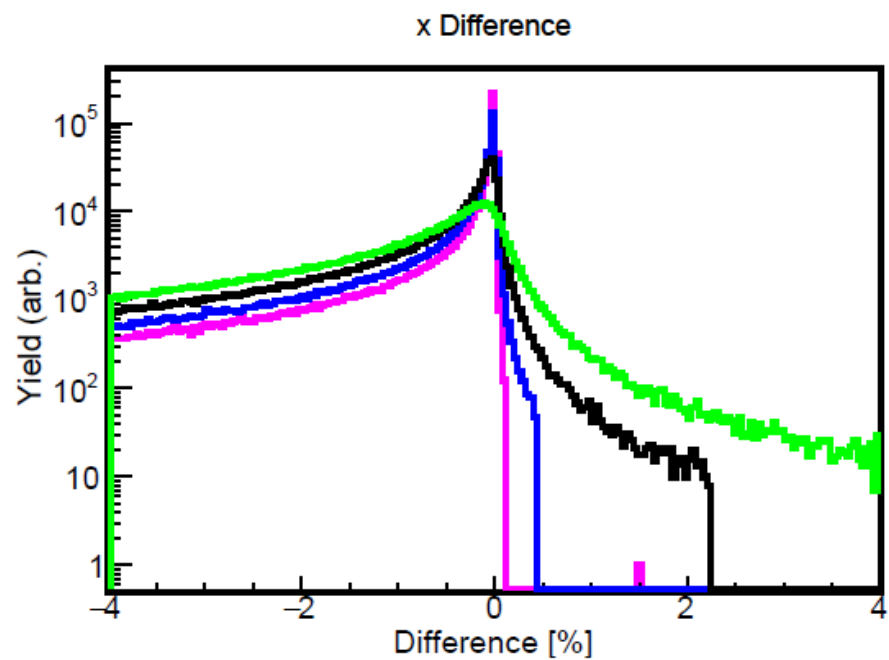
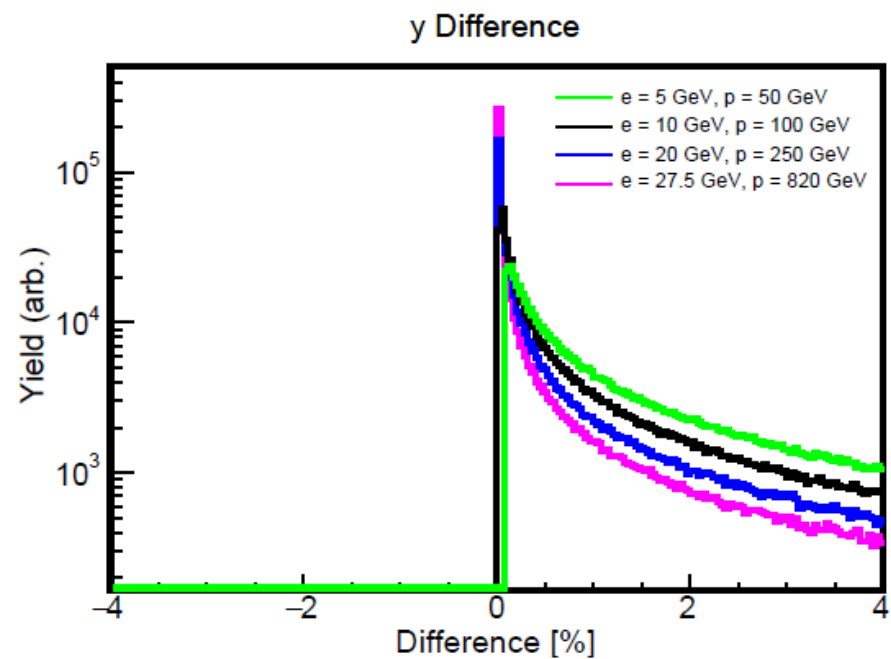
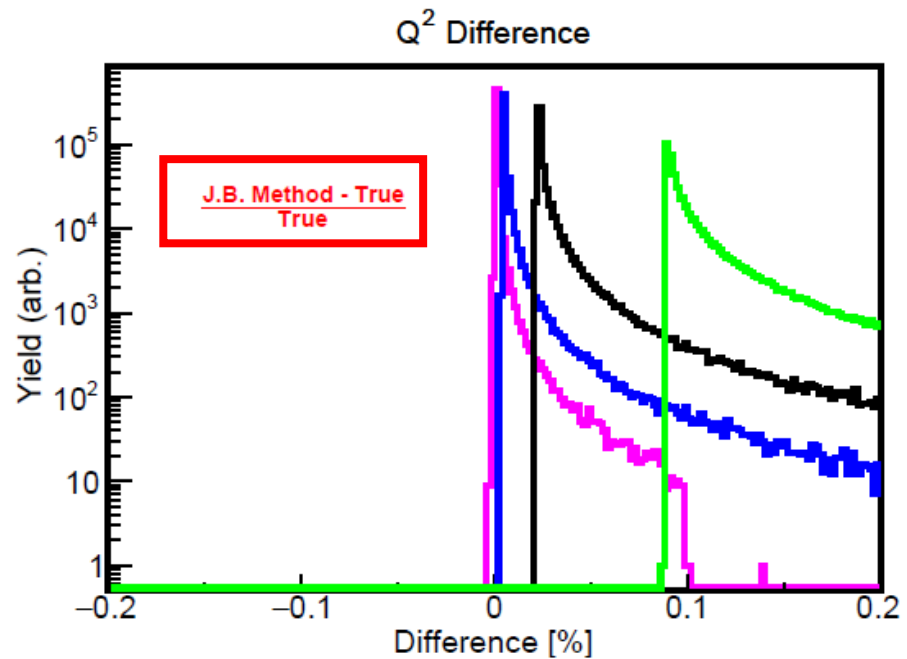


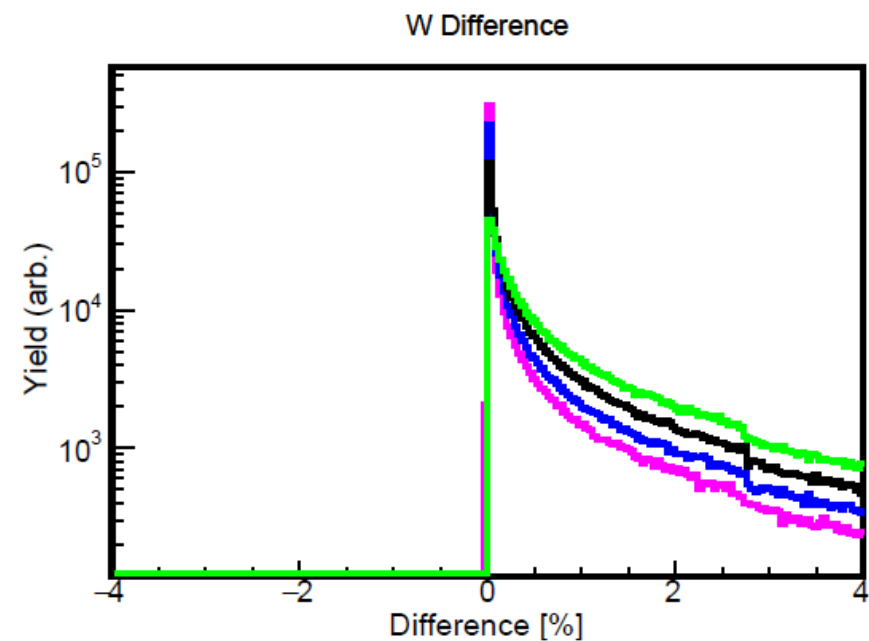
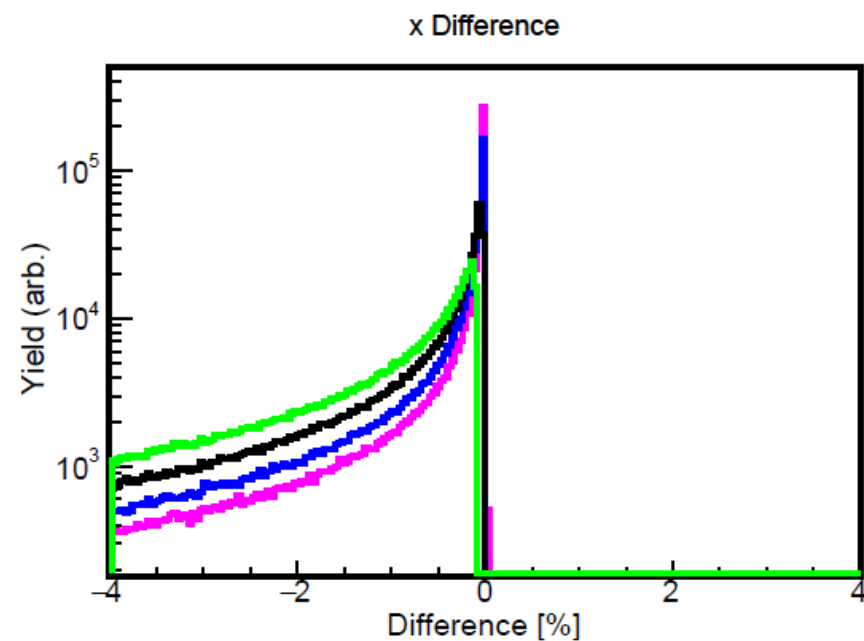
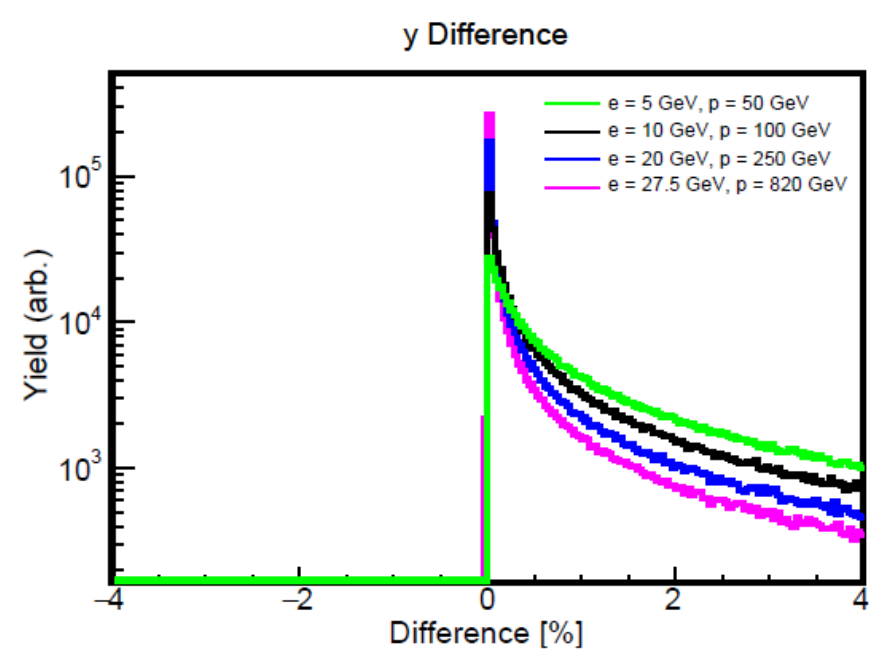
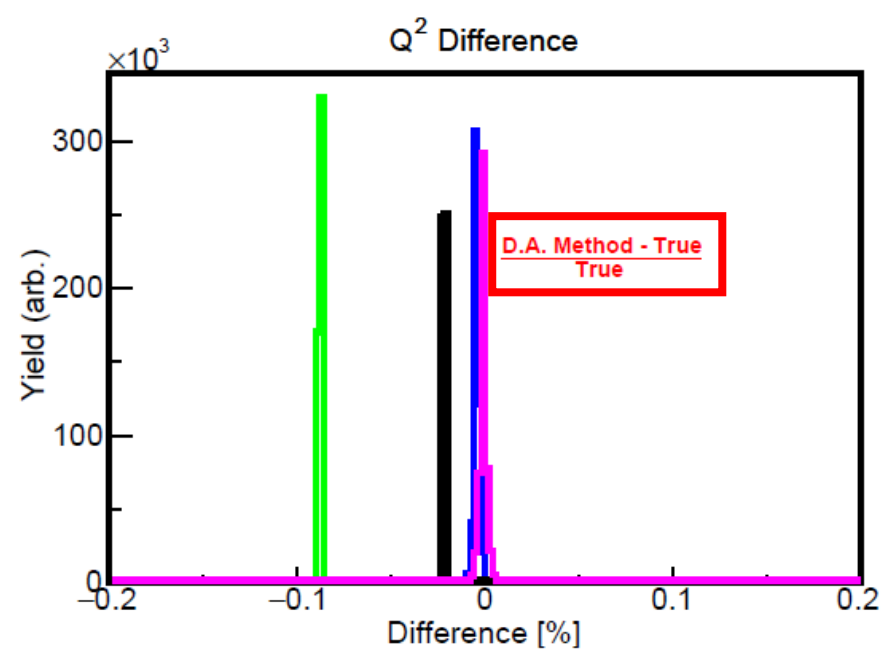
Initial Simulation Studies

- Used *BeAGLE* to study electron-proton scattering with the following energies:
 1. e at 5 GeV; p at 50 GeV
 2. e at 10 GeV; p at 100 GeV
 3. e at 20 GeV; p at 250 GeV
 4. e at 27.5 GeV; p at 820 GeV
- Assume perfect detector acceptance, efficiency and resolution. Radiation is turned OFF.
- Final Hadronic State $\equiv$ Sum over all final state particles other than the Scattered Electron
- Aside: I compared one of the simulation runs to a *PYTHIA 6 ep* simulation at the same settings. The results were quite similar...

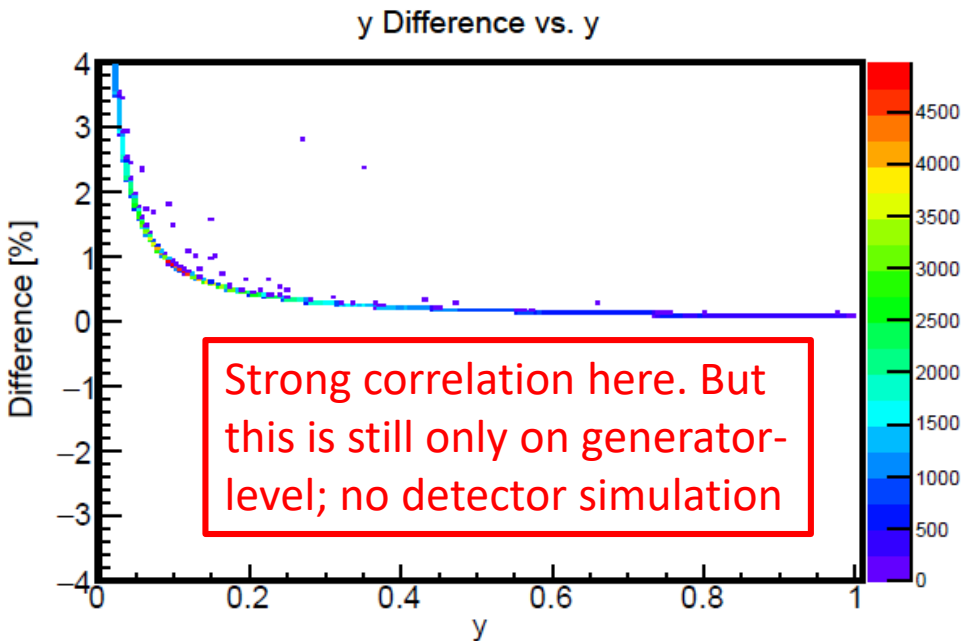
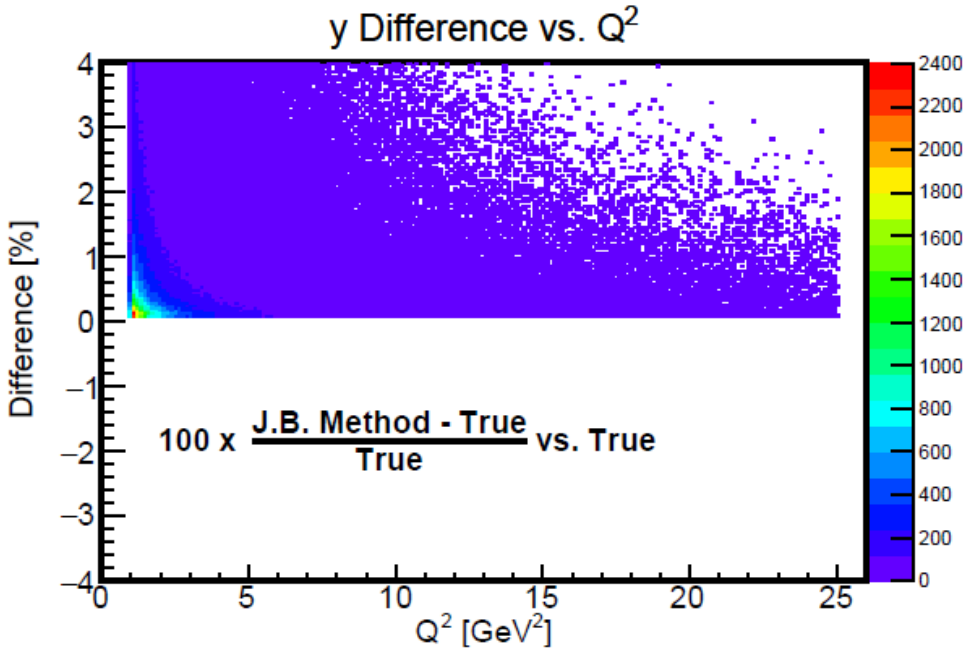
Electron-Proton Kinematic Phase Space



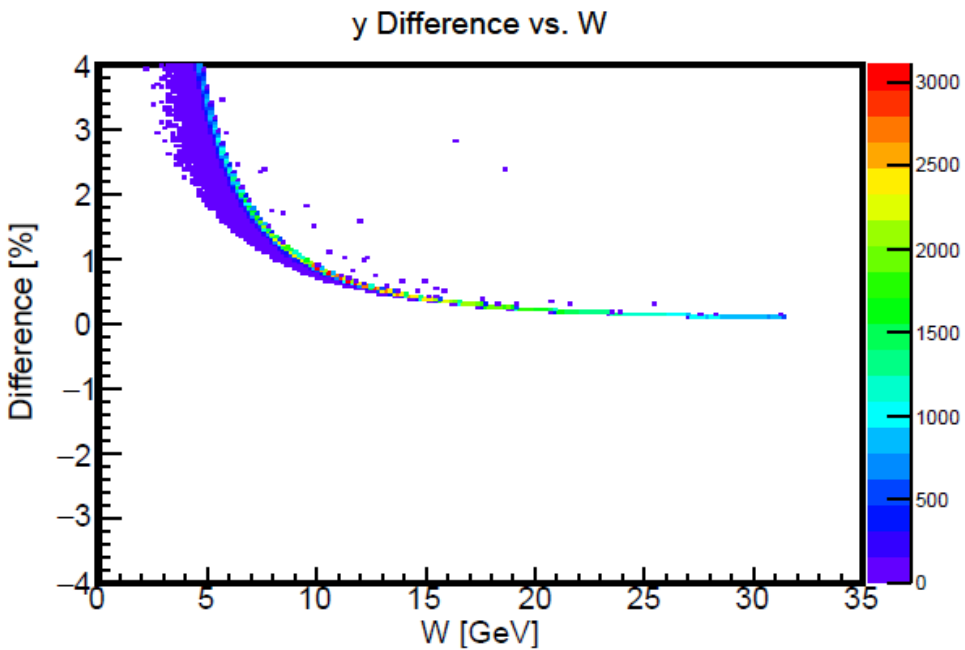
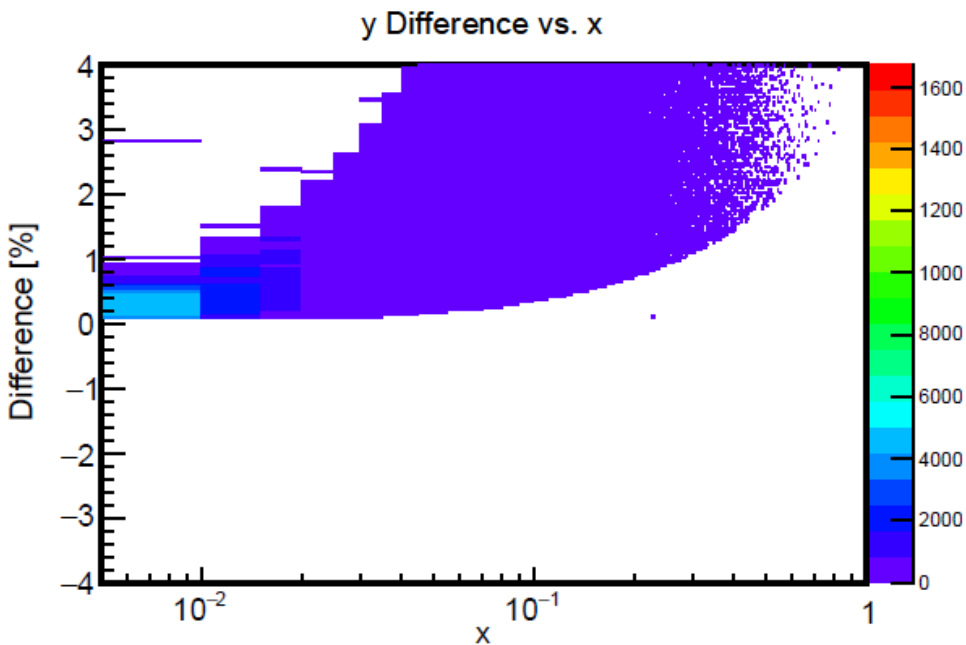




Use all particles
to characterize
hadronic final-
state



e at 5 GeV;
p at 50 GeV

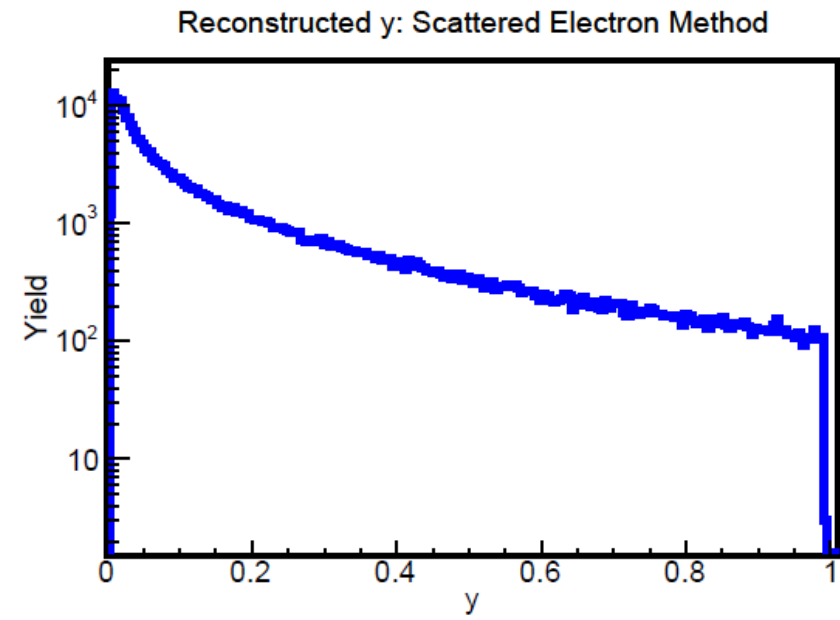
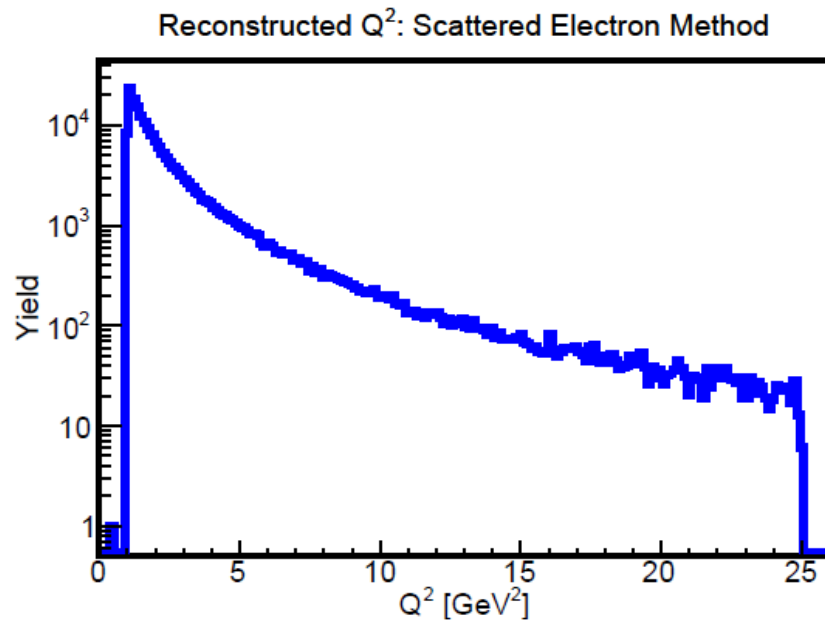


What about Electron-Nucleus Scattering?

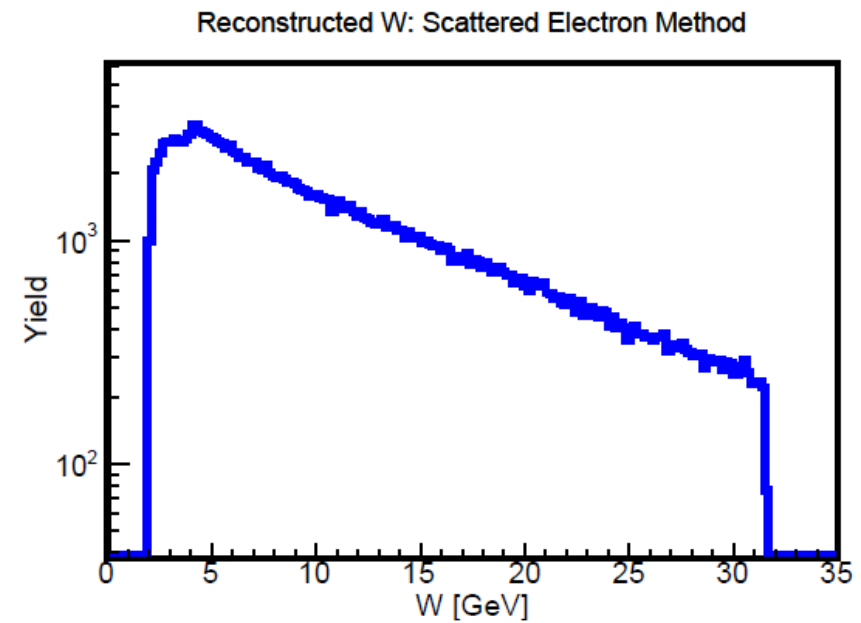
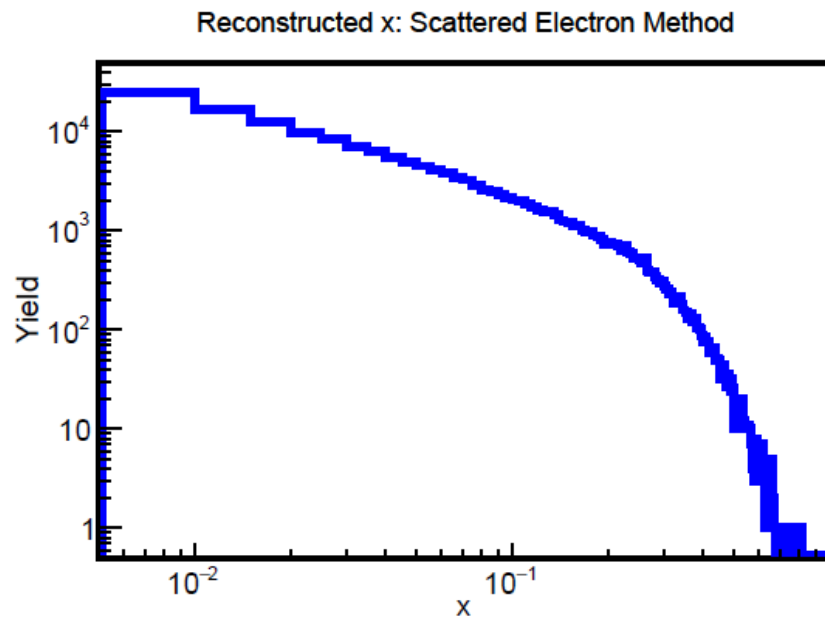
- For an electron-nucleus collider, we would define then define the initial proton's momentum to give the same result as the rest frame.
- For example, if we have a ^{197}Au beam with momentum 100 GeV/u, we assume (when calculating the kinematic variables) that we scatter off a proton with energy

$$E_p = 100 \times \frac{1.0073}{0.99983} = 100.75 \text{ GeV}$$

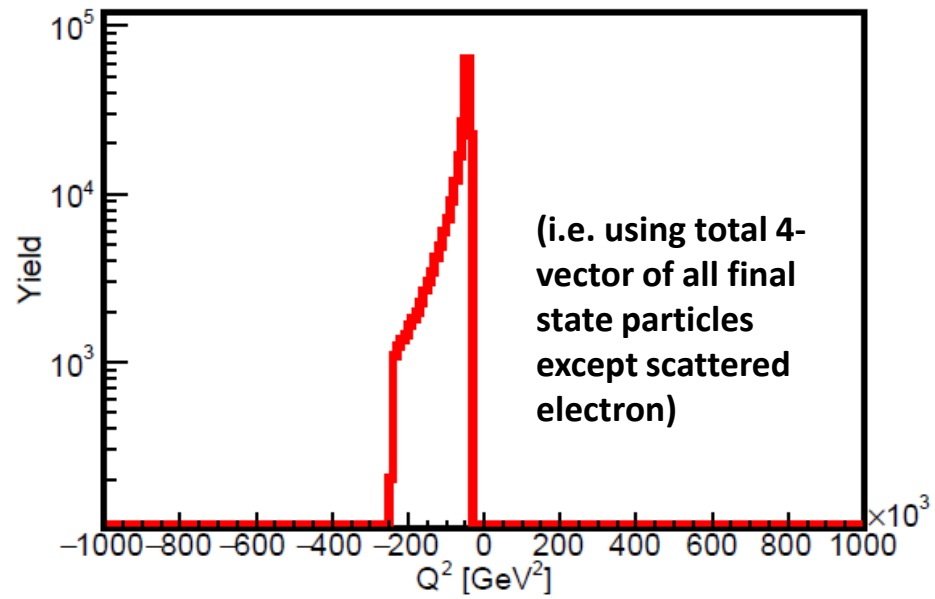
- The scattered electron method still serves as a baseline to test the hadronic reconstruction methods.
- Of course, for the hadronic methods, some of the detected particles may originate from the remnant nucleus. However, if their momenta are primarily along the beam line, they will have a minimal effect on the kinematic variables (which depend on Σ_h).



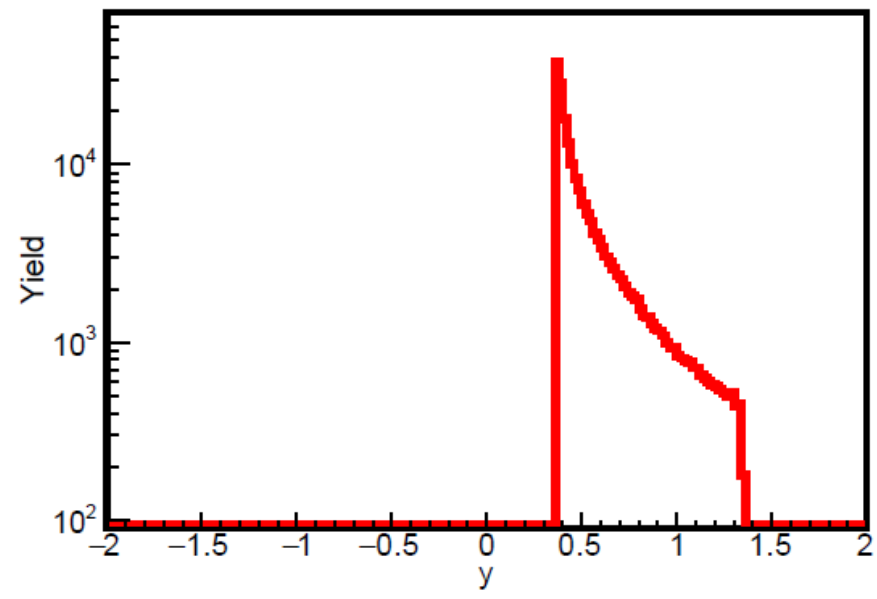
**e at 5GeV;
Pb at 50 GeV/u**



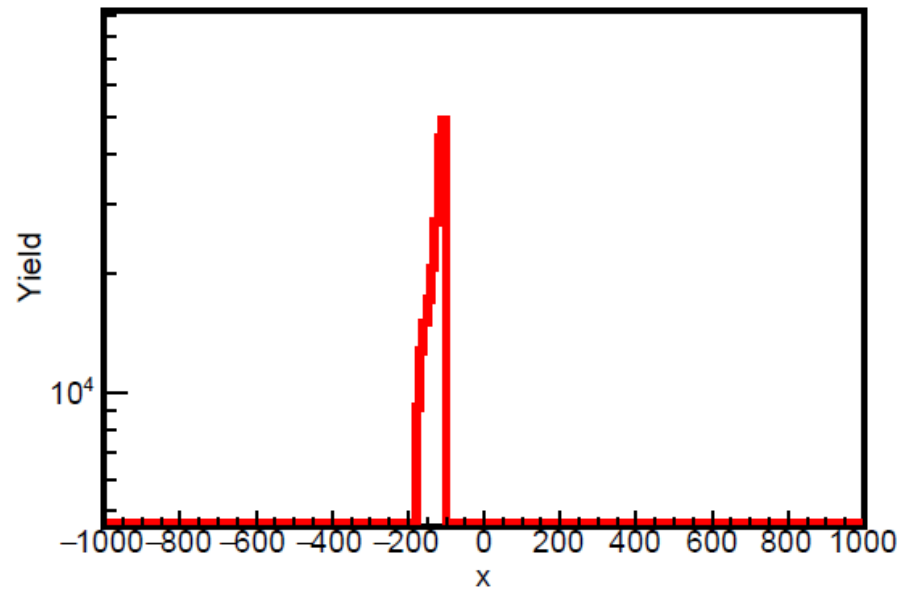
Reconstructed Q^2 : Final State Hadron Method



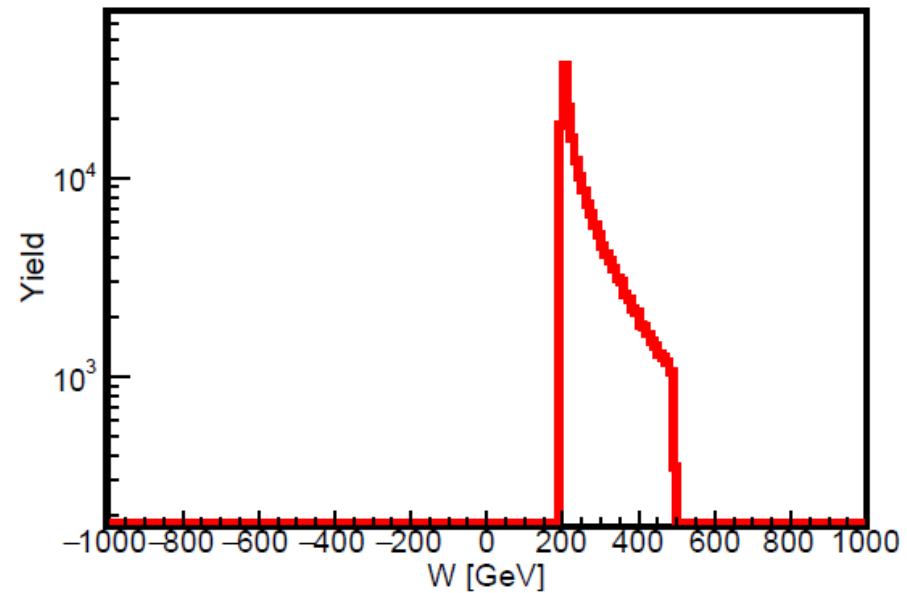
Reconstructed y : Final State Hadron Method



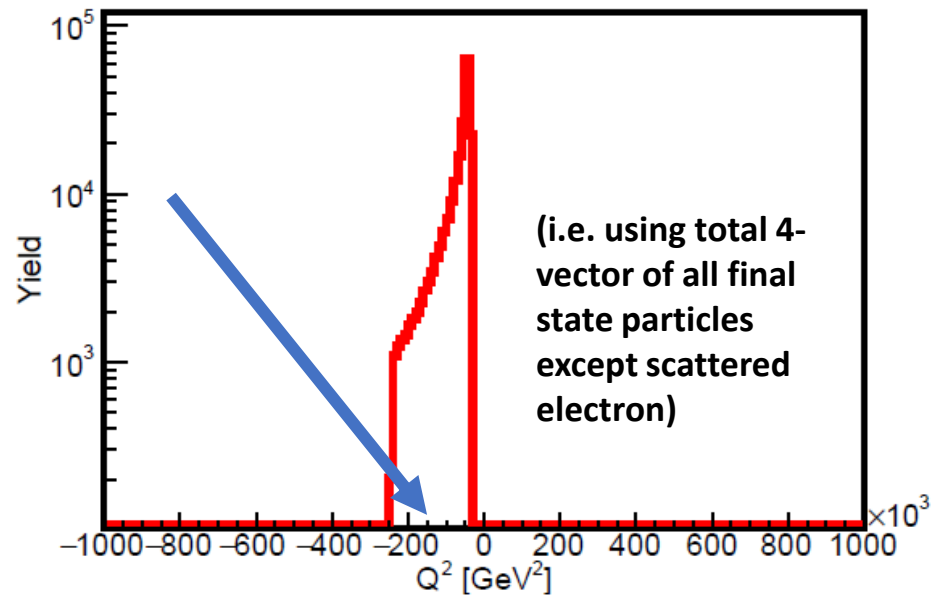
Reconstructed x : Final State Hadron Method



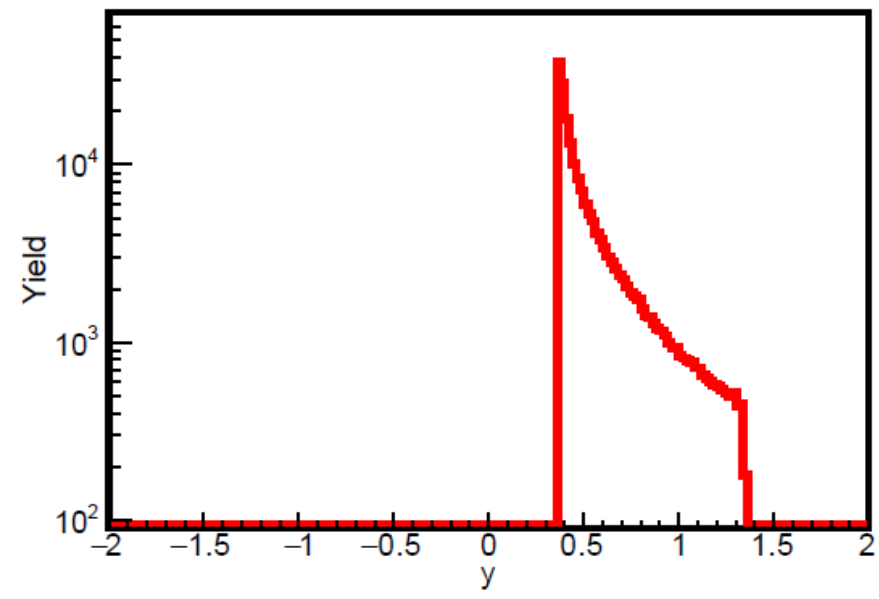
Reconstructed W : Final State Hadron Method



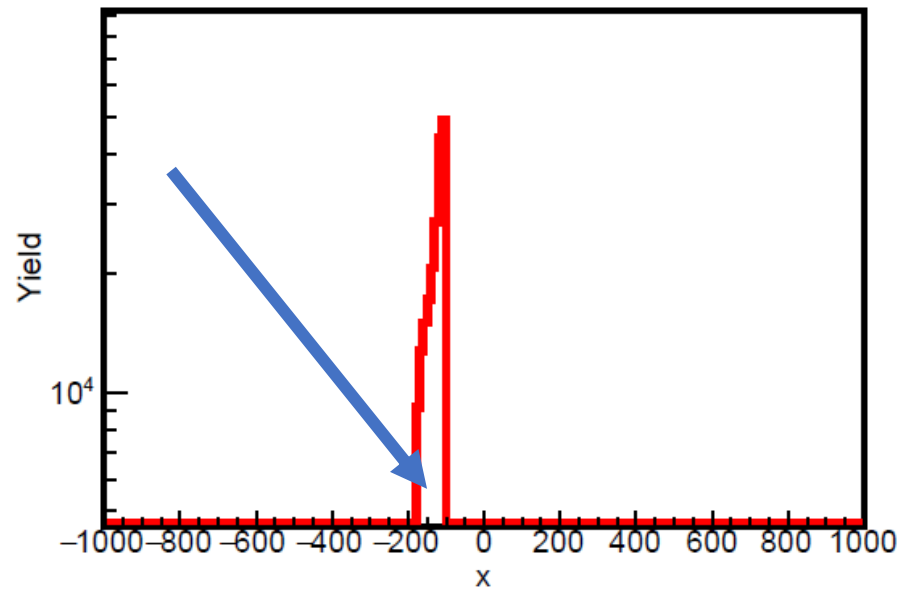
Reconstructed Q^2 : Final State Hadron Method



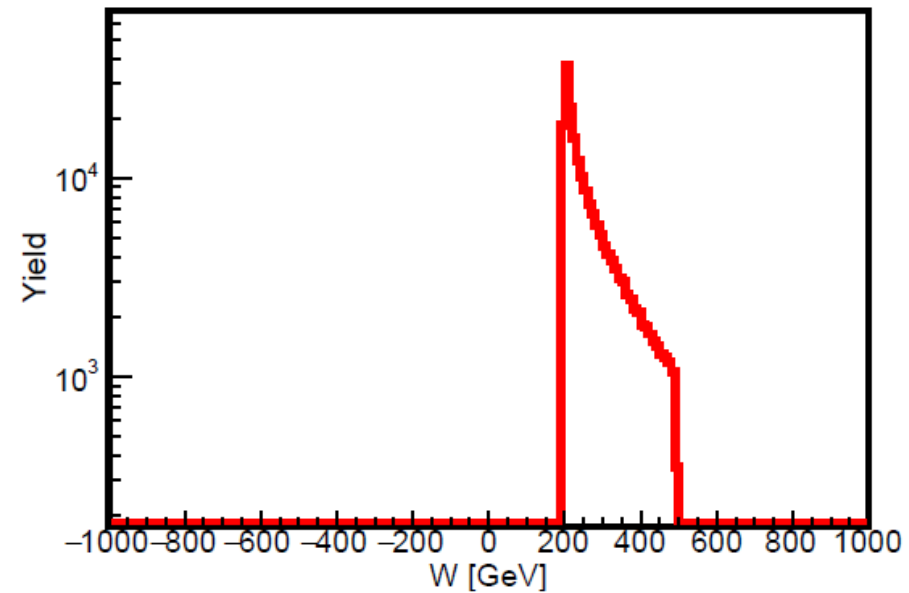
Reconstructed y : Final State Hadron Method

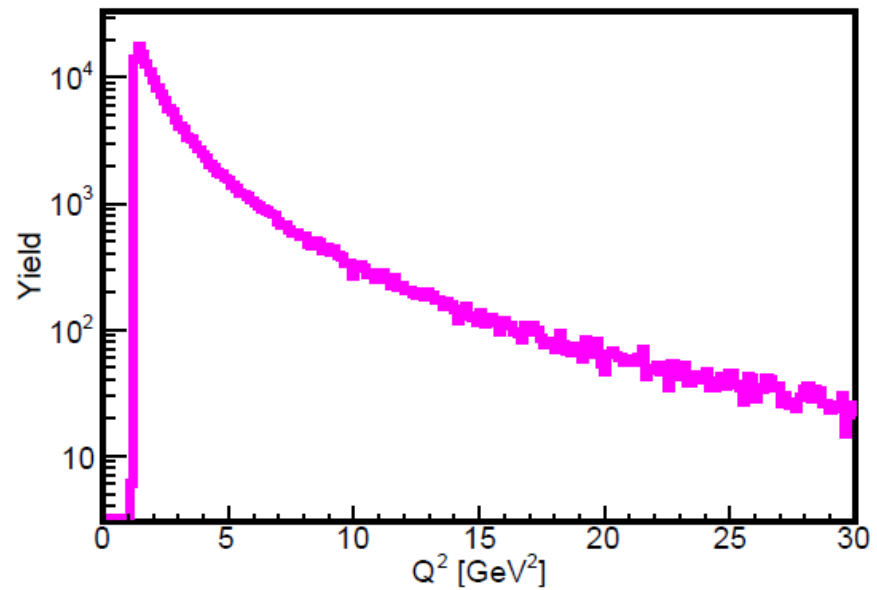
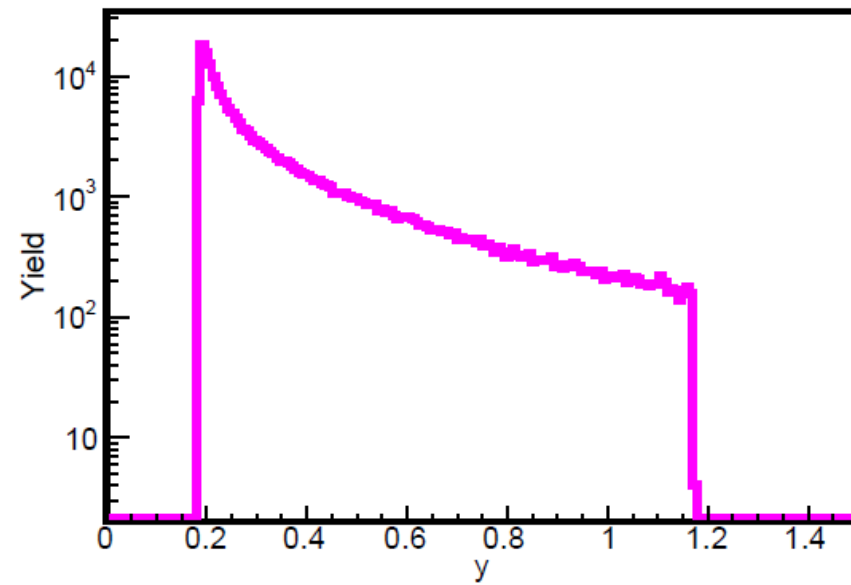
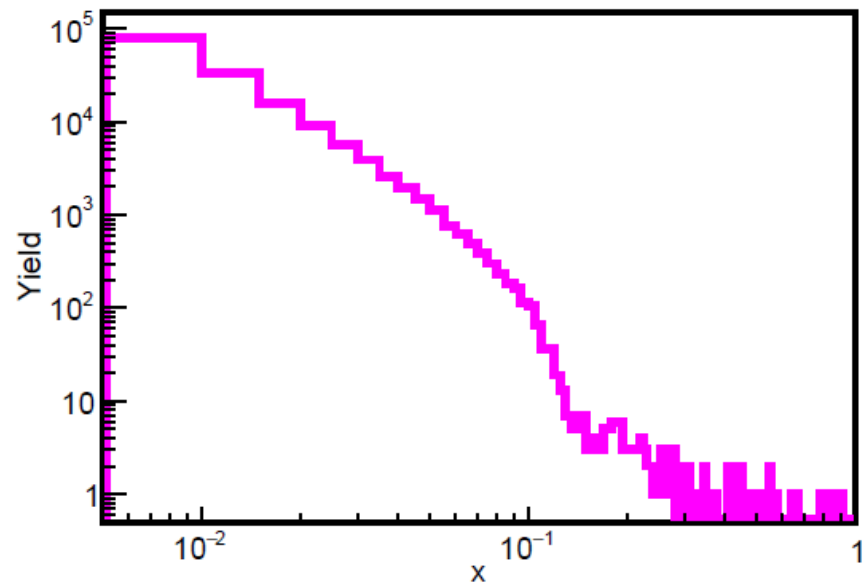
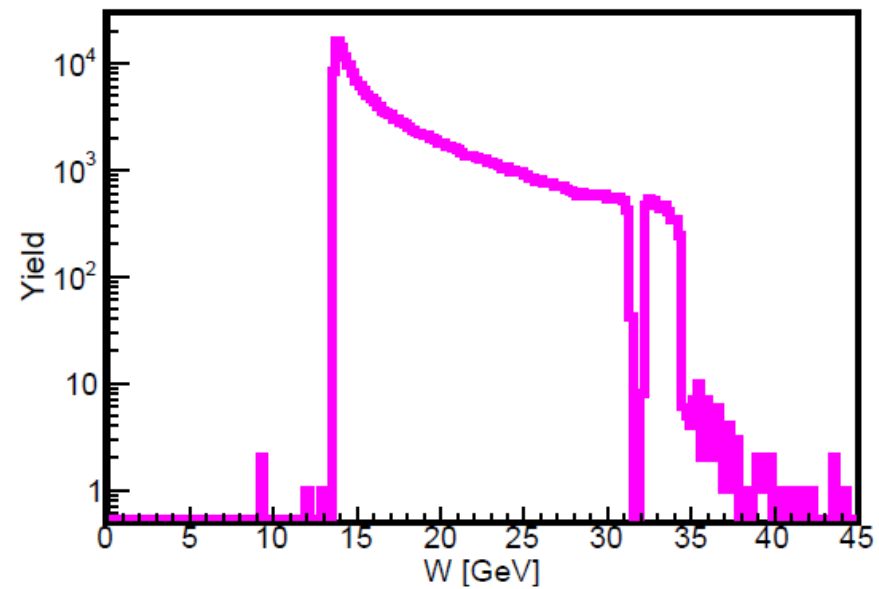


Reconstructed x : Final State Hadron Method



Reconstructed W : Final State Hadron Method

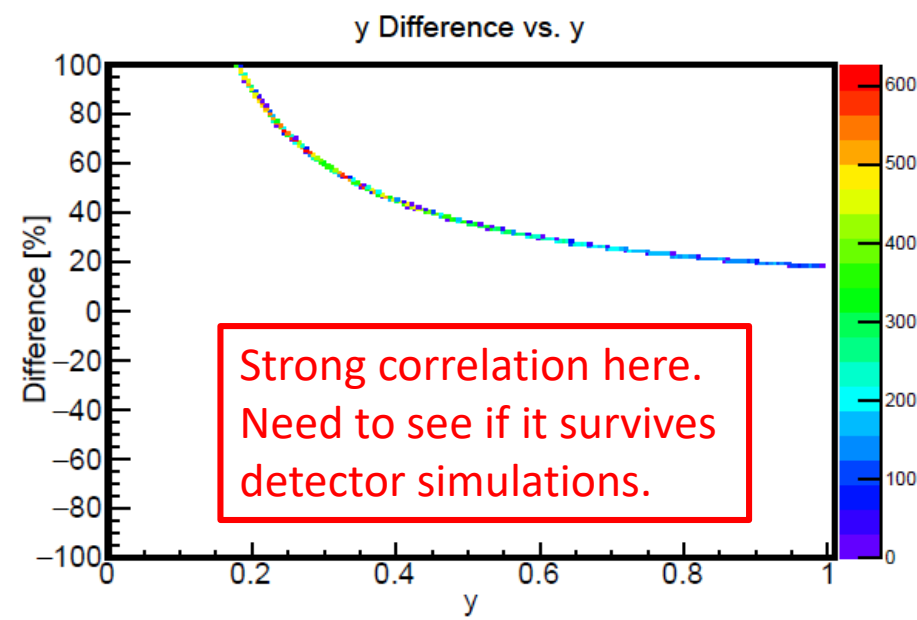
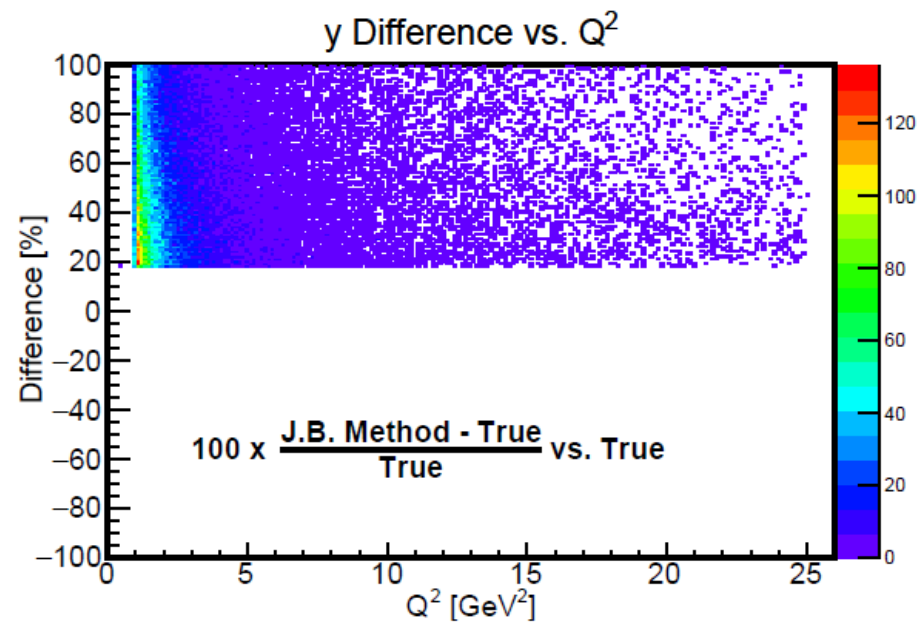


Reconstructed Q^2 : J.B. MethodReconstructed y : J.B. MethodReconstructed x : J.B. MethodReconstructed W : J.B. Method

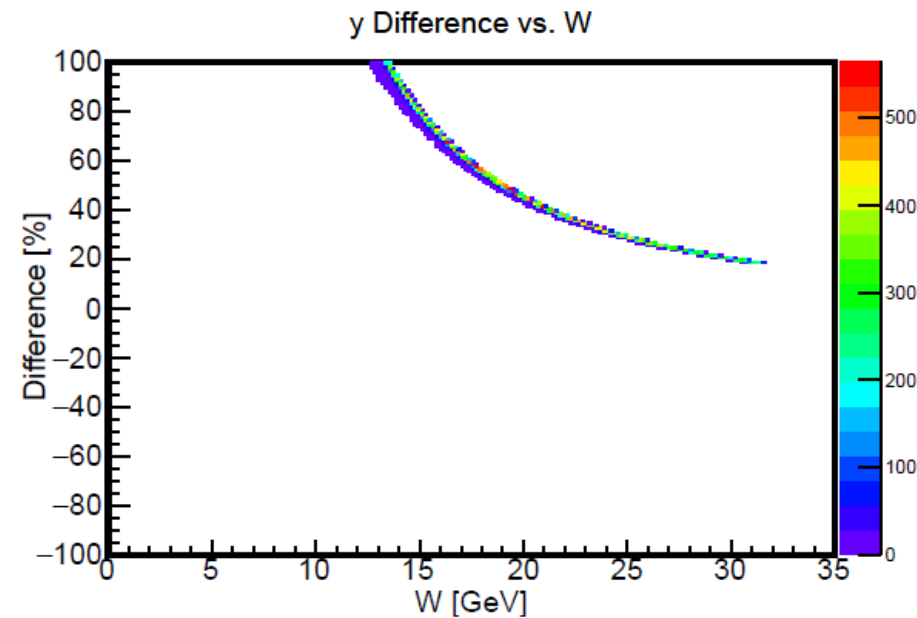
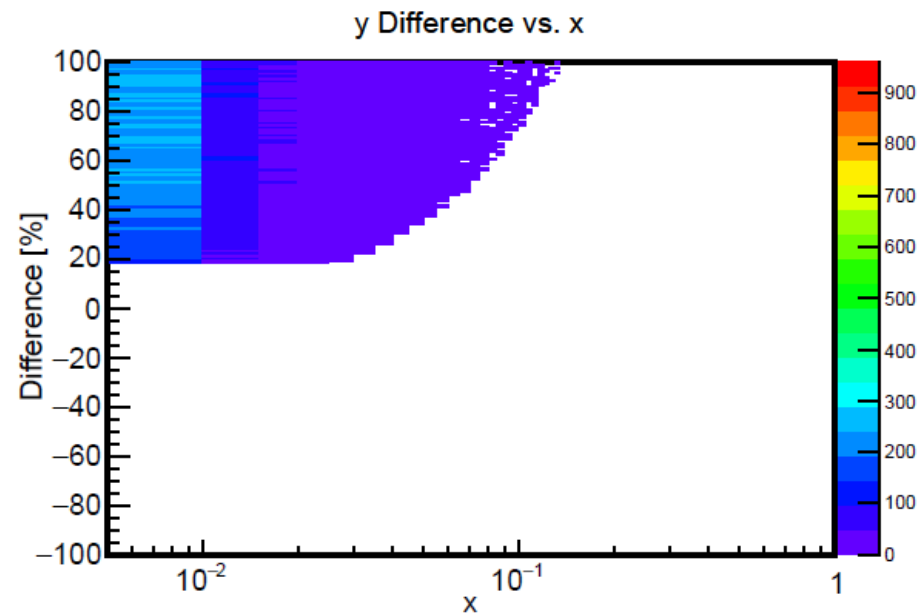
Why does the J.B. Method work Better than using the 4-vector Directly?

$$\begin{aligned}
 y &= \frac{P_p \cdot q}{P_p \cdot P_e} \approx \frac{P_p \cdot (P_X - P_p)}{2E_p E_e} \approx \frac{E_p E_X - |P|_p P_{z,X}}{2E_p E_e} \approx \frac{E_p (E_X - P_{z,X})}{2E_p E_e} \\
 &= \frac{\sum_i E_i - \sum_i p_{z,i}}{2E_e} \\
 &= \frac{\sum_i (E_i - p_{z,i})}{2E_e} = \frac{\Sigma_h}{2E_e}
 \end{aligned}$$

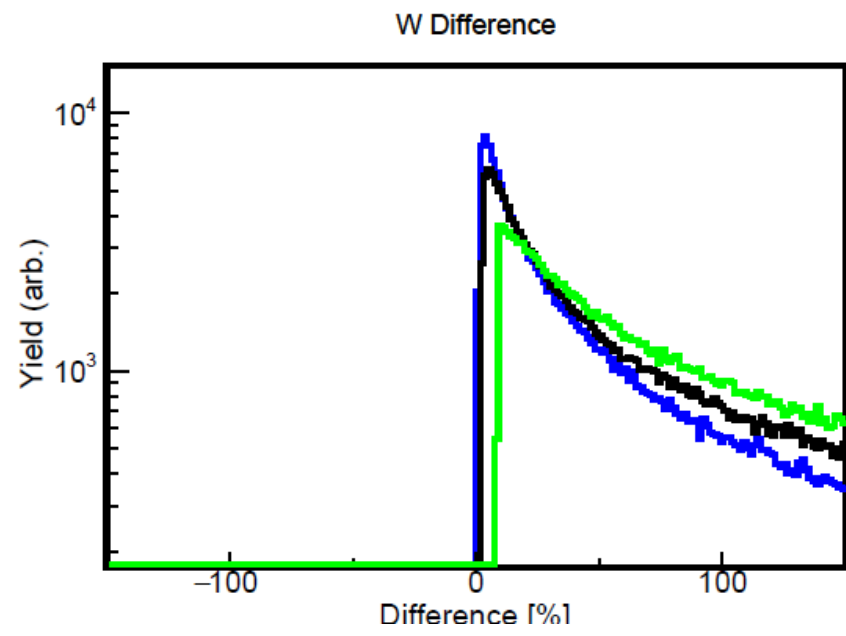
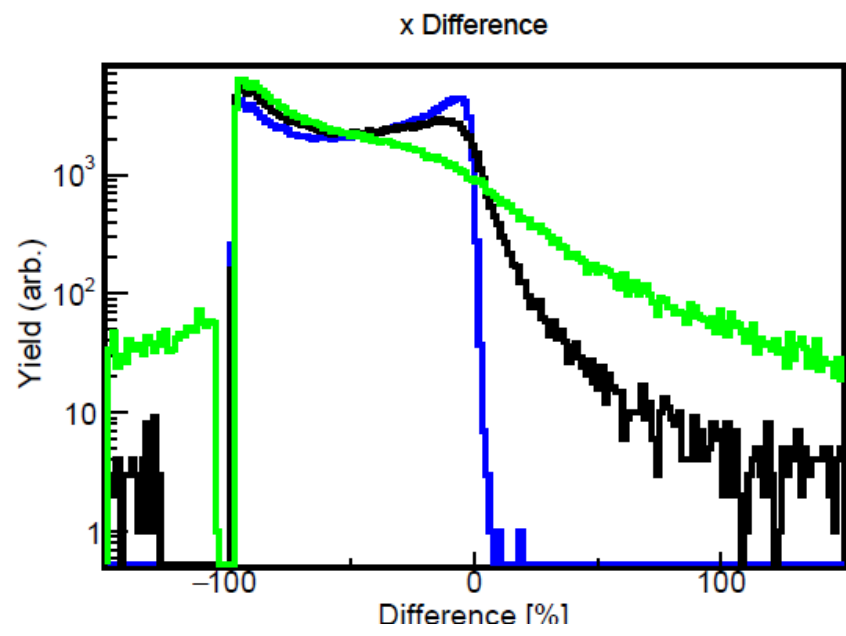
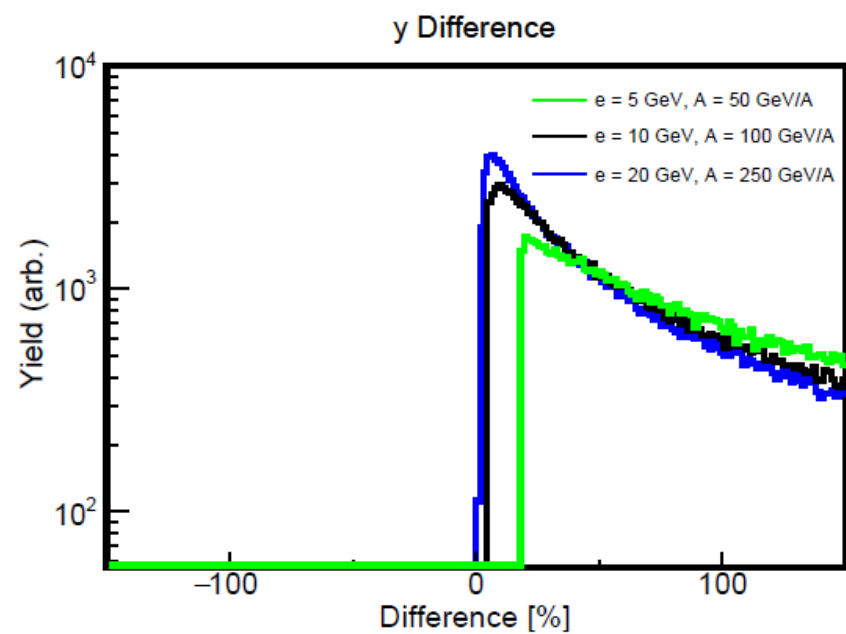
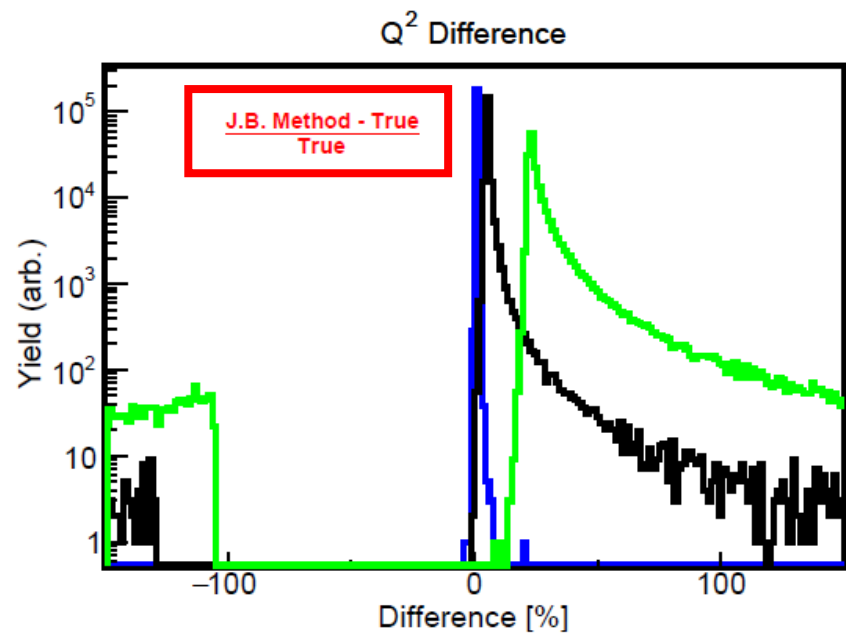
Using all particles – including spectators - to characterize hadronic final-state



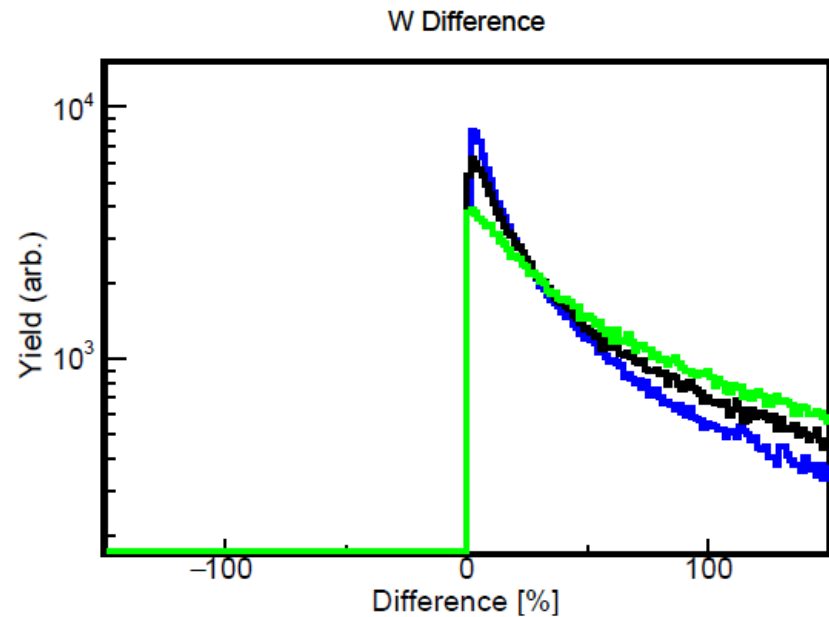
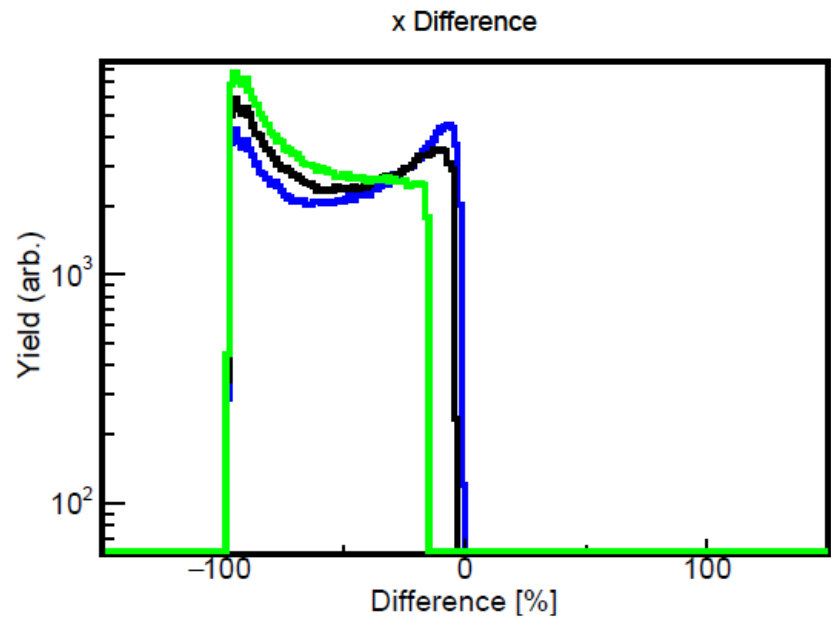
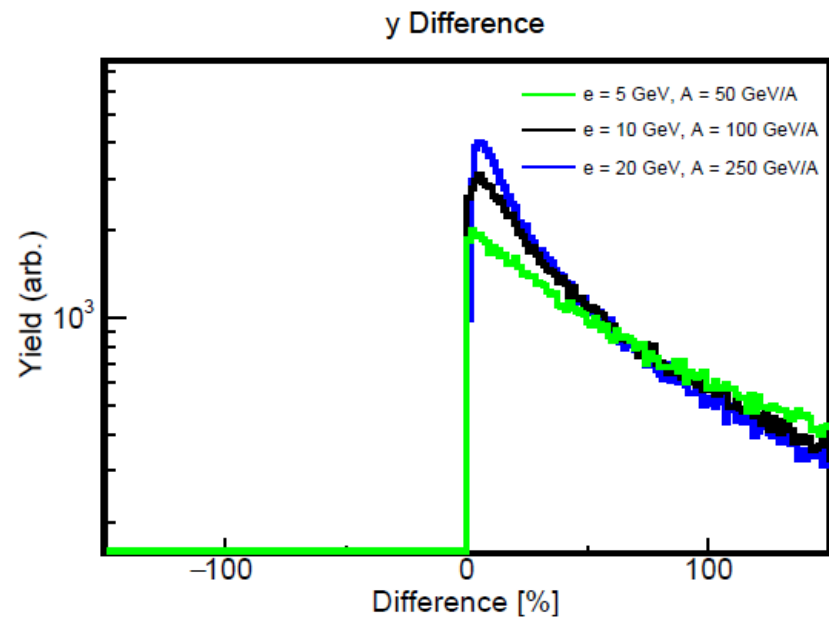
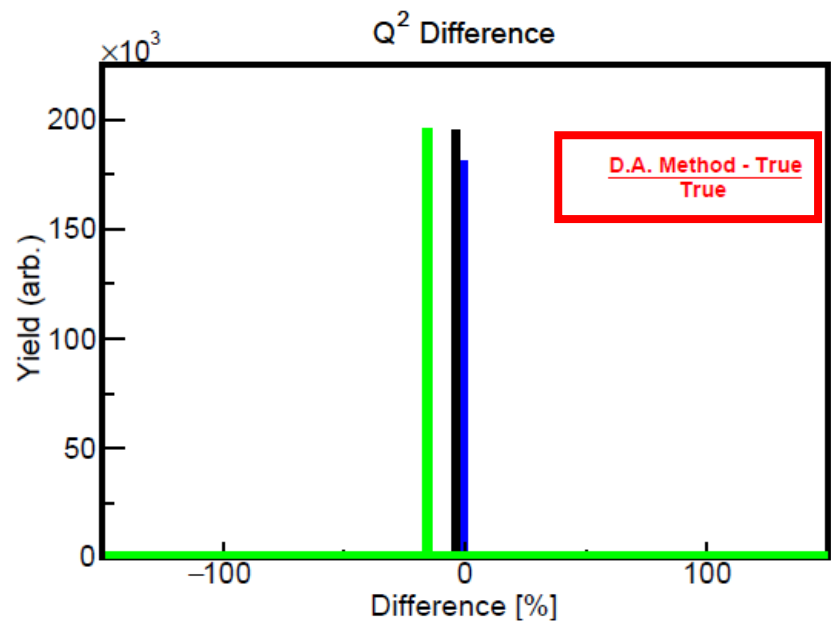
e at 5GeV;
Pb at 50 GeV/A



Using all particles –
including
spectators - to
characterize
hadronic final-
state



Using all particles –
including
spectators - to
characterize
hadronic final-
state

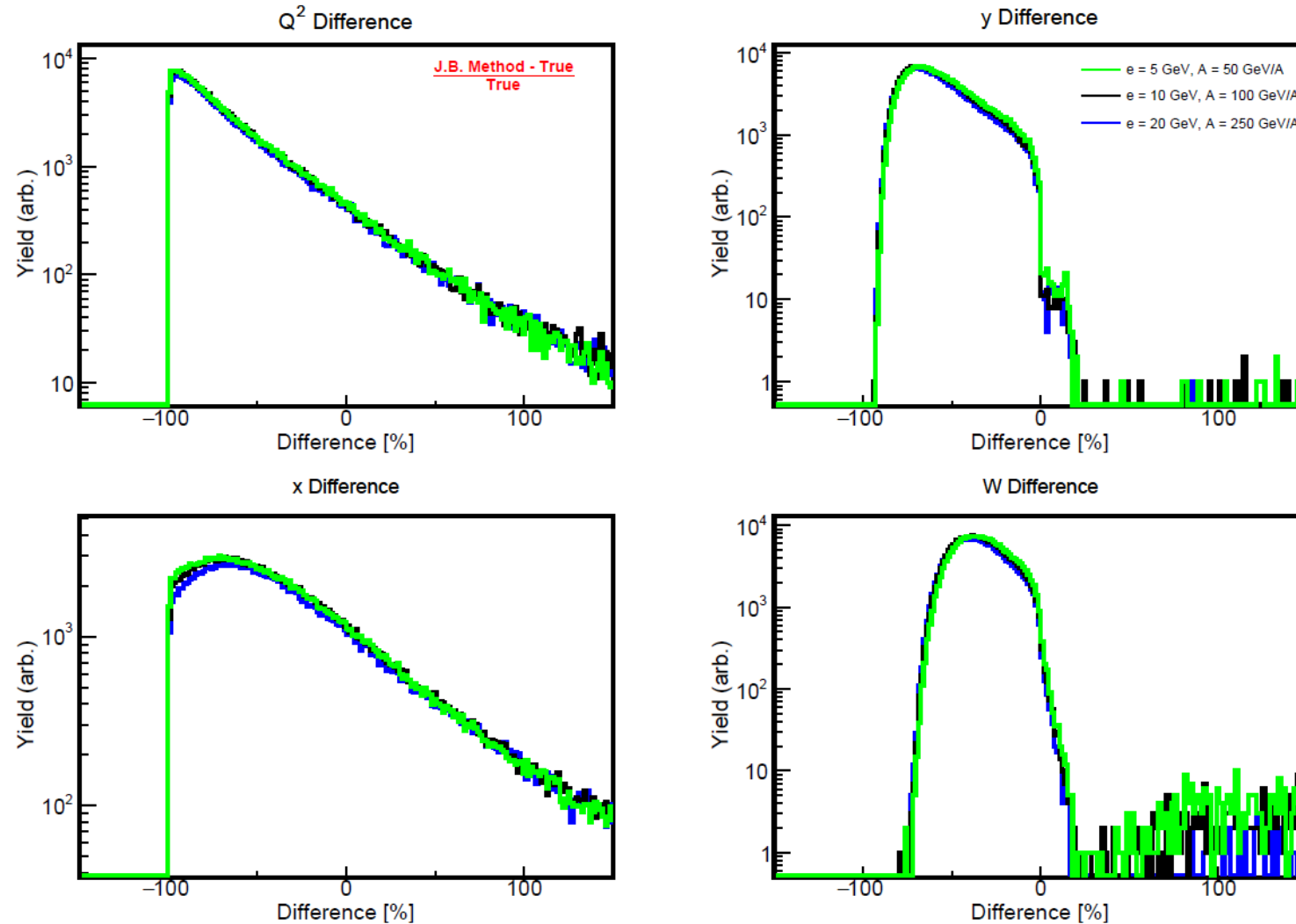


Next Method: Use only Part of the Hadronic Final State

1. Use only the particle with the highest transverse momentum when characterizing the hadronic final state ✓
2. Include particles which are close by (in η vs. ϕ space) to the highest transverse momentum particle ✓

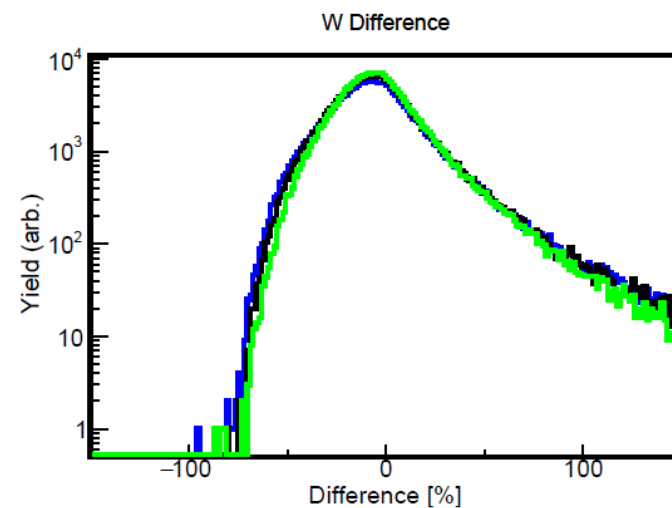
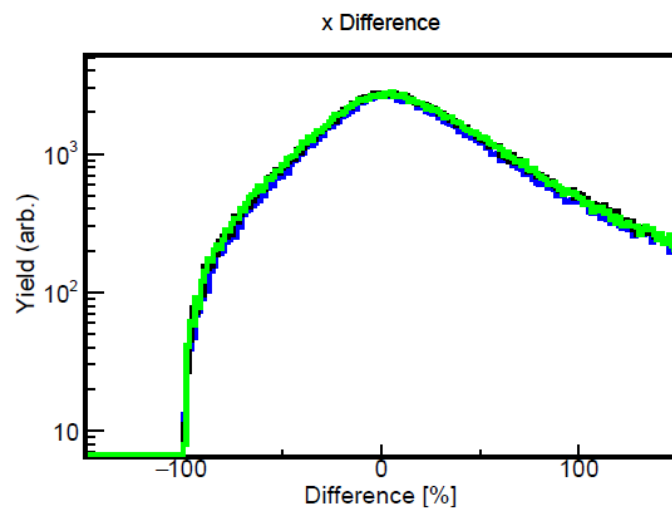
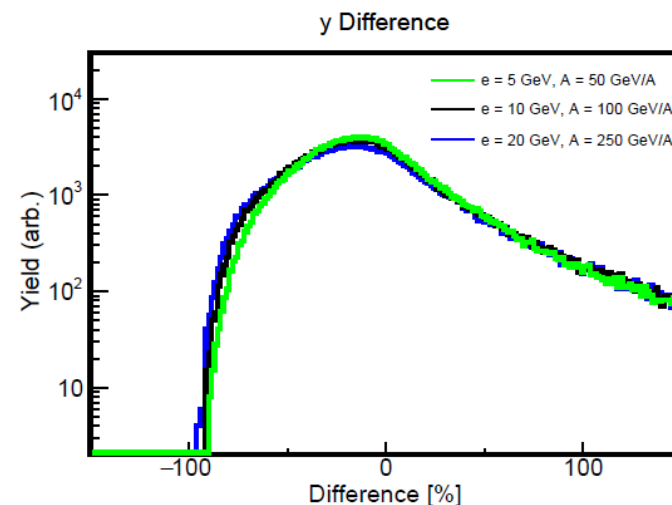
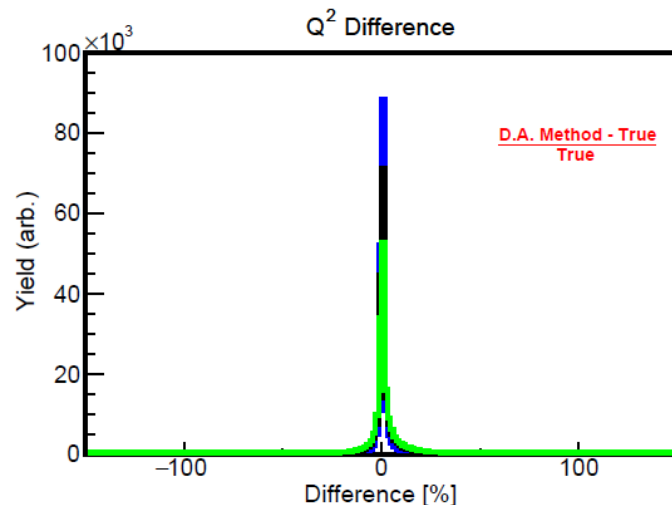
Use only Highest-Momentum particle in IRF

Note how results
'flip sides' from "all
particles" case.



Use only Highest-Momentum particle in IRF

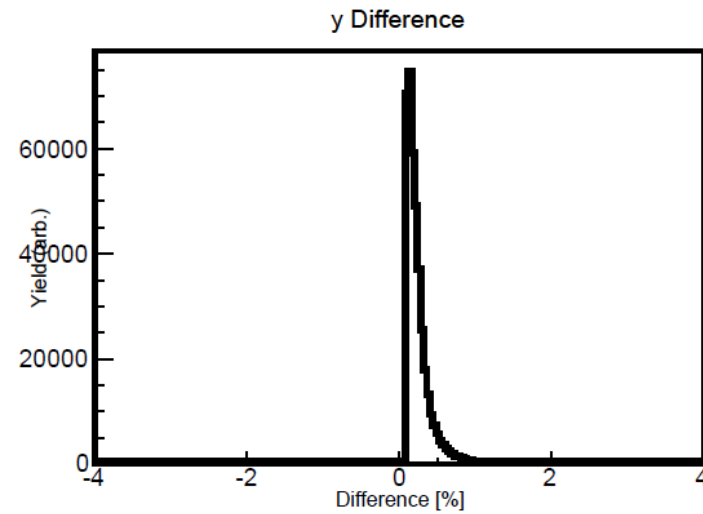
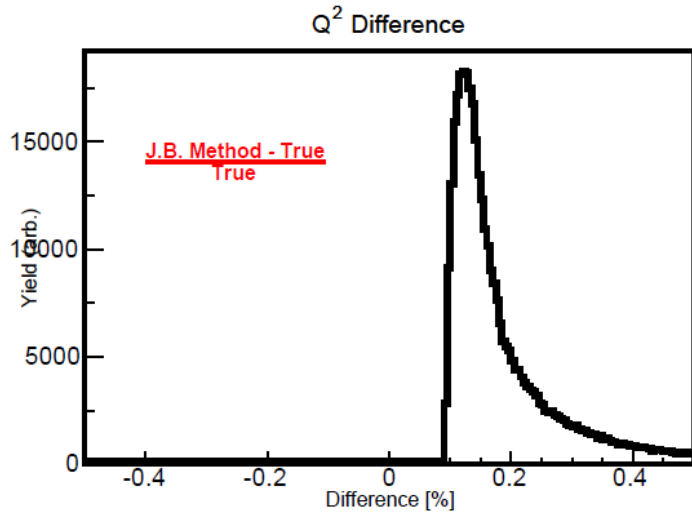
Note how results
'flip sides' from "all
particles" case.



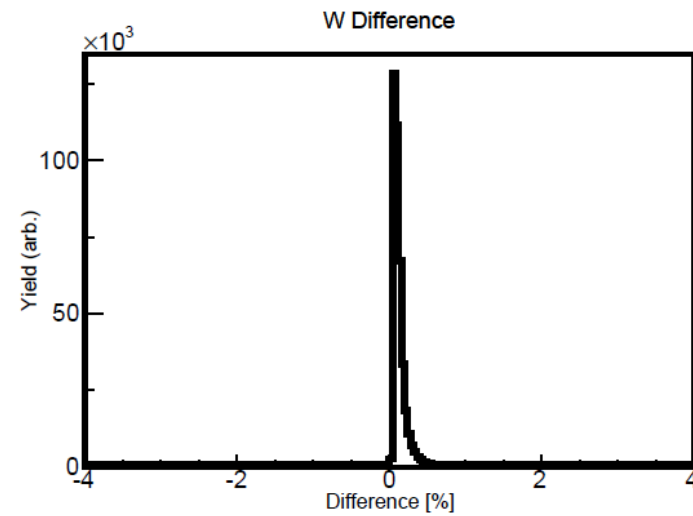
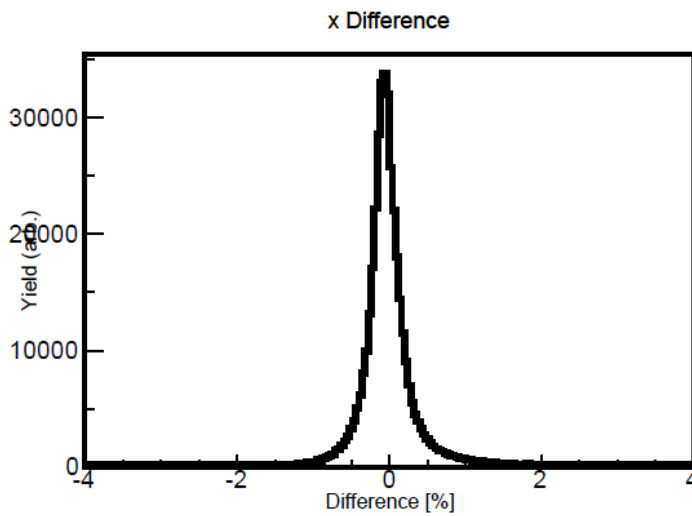
Study Simpler Final State: Elastic Vector Meson Production

$$e + p \rightarrow e' + p' + J/\psi$$
$$J/\psi \rightarrow e^+ e^-$$

Elastic Vector Meson Production

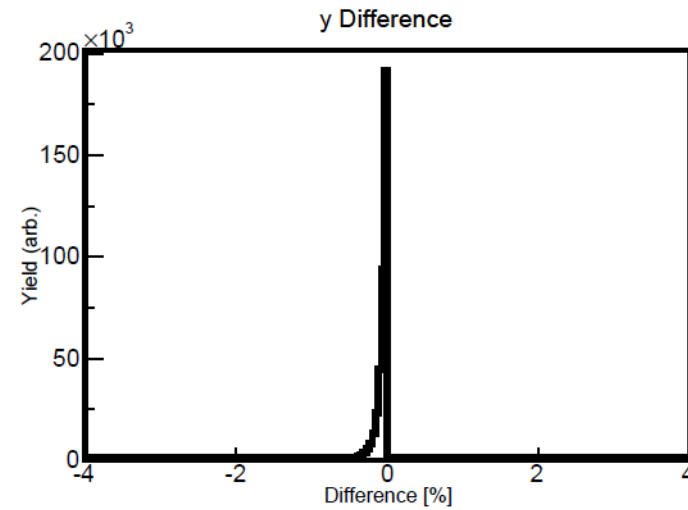
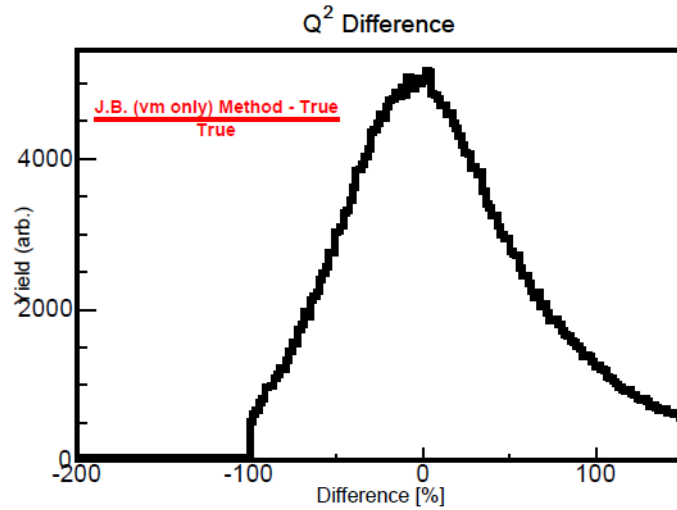


$$e + p \rightarrow e' + p' + J/\psi$$
$$J/\psi \rightarrow e^+ e^-$$



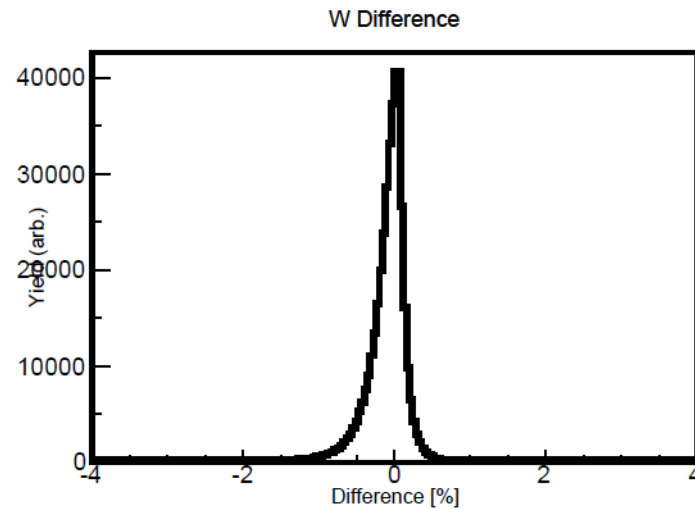
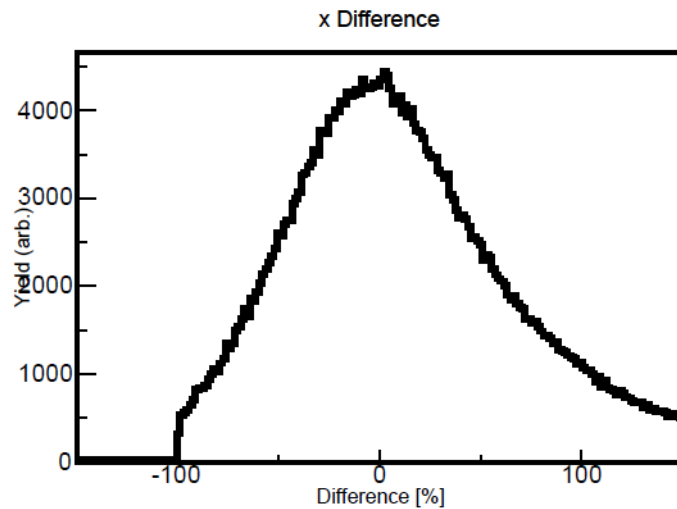
Using *Sartre* Generator

Elastic Vector Meson Production



$$e + p \rightarrow e' + p' + J/\psi$$

$$J/\psi \rightarrow e^+ e^-$$



Using *Sartre* Generator

Fermi Motion of Struck Nucleon in LO DIS

Using *BeAGLE* with Fermi Motion OFF:

$$p_e + p_N + p_{A-1} = p_{e'} + p_{\text{current-jet}} + p_{N'} + p_{A-1'}$$

Fermi Motion of Struck Nucleon in LO DIS

Using *BeAGLE* with Fermi Motion OFF:

$$p_e + p_N + \cancel{p_{A-1}} = p_{e'} + p_{\text{current-jet}} + p_{N'} + \cancel{p_{A-1}'}$$
$$p_N = p_{\text{current-jet}} + p_{N'} - q_e$$

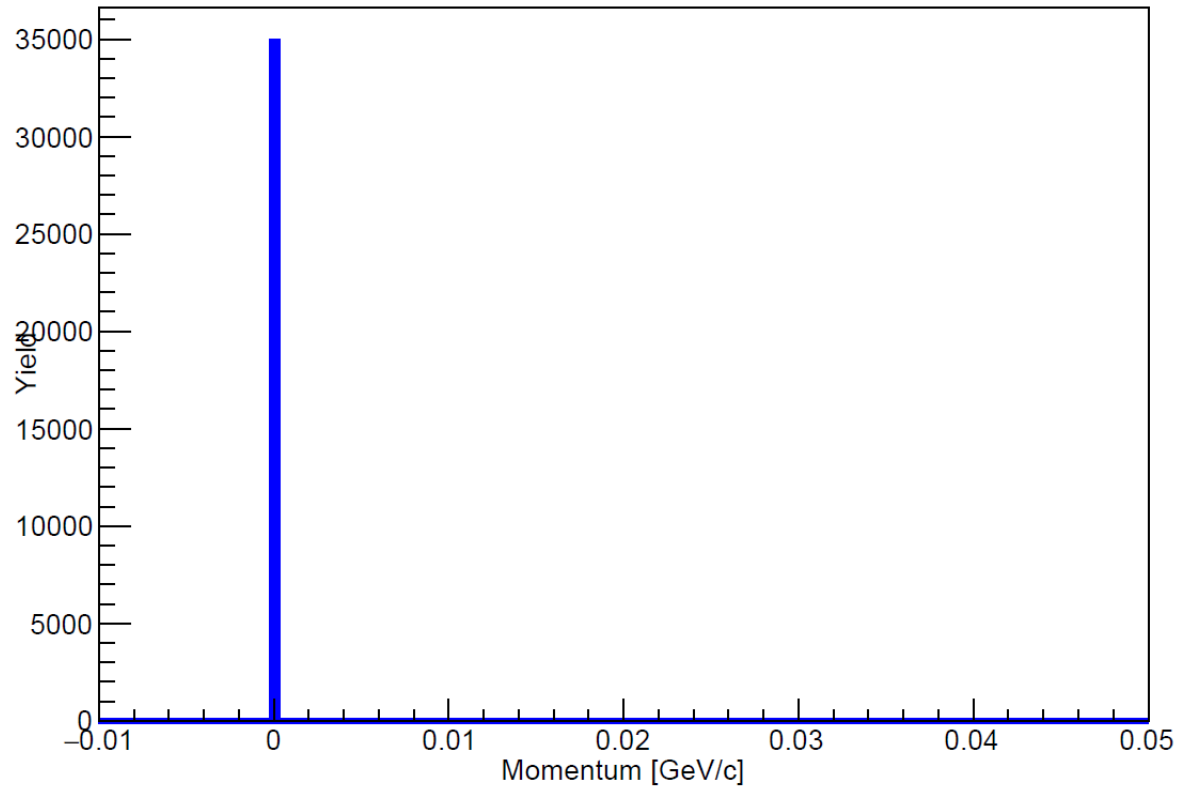
Fermi Motion of Struck Nucleon in LO DIS

Using *BeAGLE* with Fermi Motion OFF:

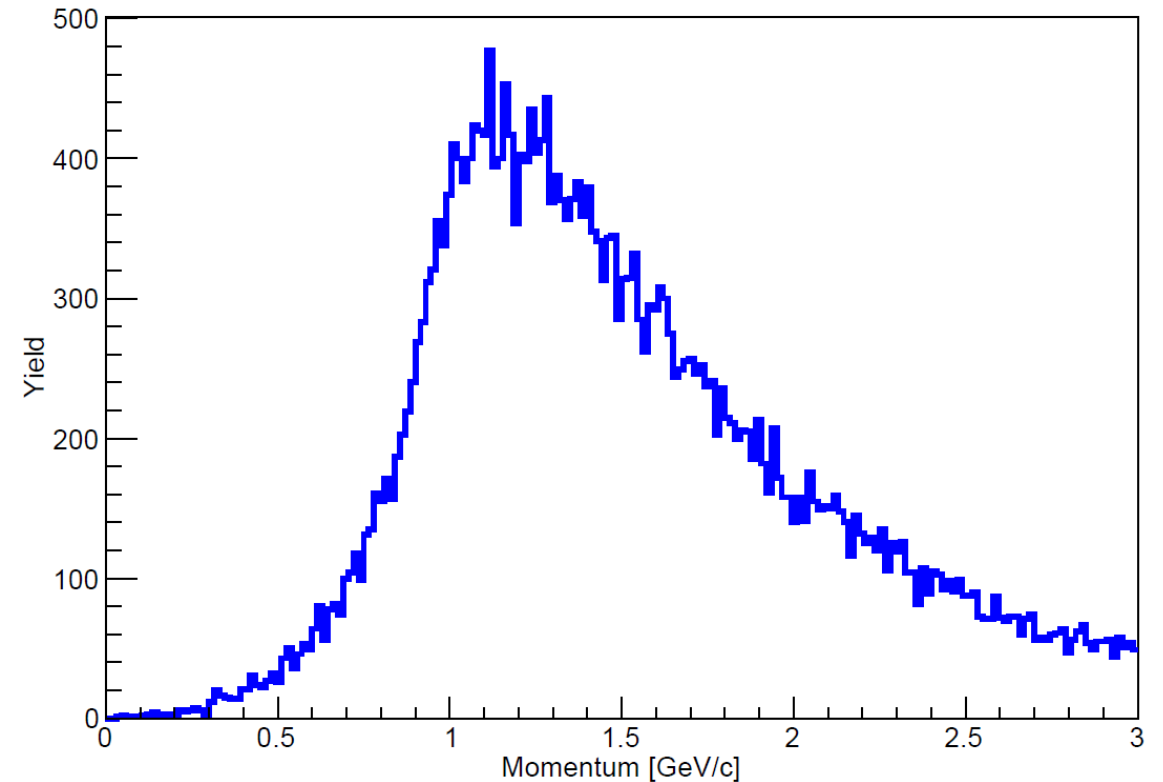
$$p_e + p_N + \cancel{p_{A-1}} = p_{e'} + p_{\text{current-jet}} + p_{N'} + \cancel{p_{A-1}'}$$
$$p_N = p_{\text{current-jet}} + p_{N'} - q_e = 0?$$

Fermi Motion of Struck Nucleon in LO DIS

Momentum of Struck Nucleon

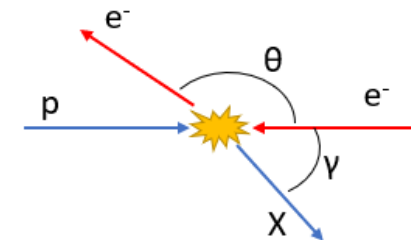


Reconstructed Momentum of Struck Nucleon



Backup Slides

Interpretation of Hadron Side Variables



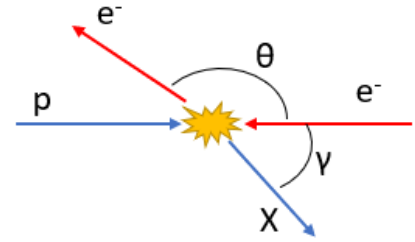
On the electron side, we have:

$$y = 1 - \left(\frac{E_{e'}}{2E_e} \right) (1 - \cos \theta)$$

$$Q^2 = 4E_e E_{e'} \cos^2 \frac{\theta}{2}$$

$$x = E_{e'}(1 + \cos \theta)/(2yE_p)$$

Interpretation of Hadron Side Variables



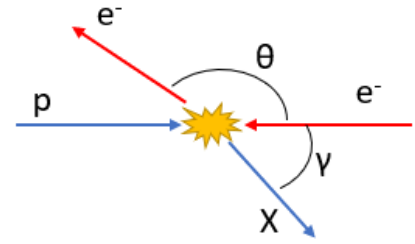
On the hadron side, for massless X , we have:

$$y = \frac{E_X}{2E_e} (1 - \cos \gamma)$$

$$Q^2 = \frac{E_X^2 \sin^2 \gamma}{1 - y}$$

$$x = \frac{E_X(1 + \cos \gamma)}{(1 - y)(2E_e)}$$

Interpretation of Hadron Side Variables

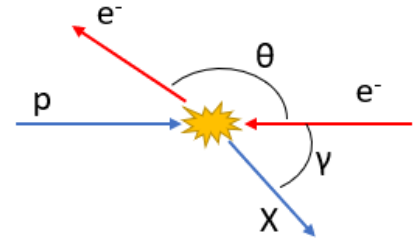


Inverting the above, we have:

$$E_X = yE_e + (1 - y)xE_p$$

$$\cos \gamma = \frac{-yE_e + (1 - y)xE_p}{yE_e + (1 - y)xE_p}$$

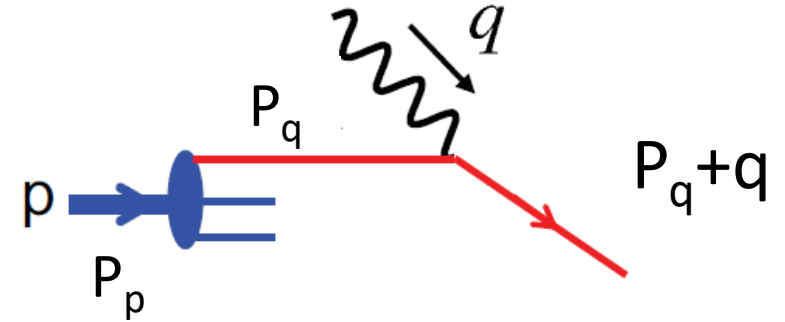
Interpretation of Hadron Side Variables



Inverting the above, we have:

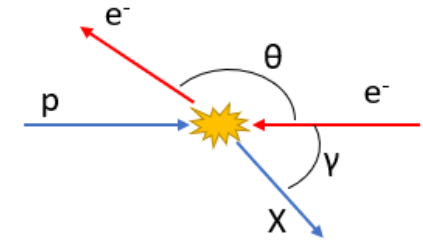
$$E_X = yE_e + (1 - y)xE_p$$

$$\cos \gamma = \frac{-yE_e + (1 - y)xE_p}{yE_e + (1 - y)xE_p}$$



$$\begin{aligned} P_q &= (xE_p, x\mathbf{P}_p) \\ x_q &= 1 \\ y_q &= y \end{aligned}$$

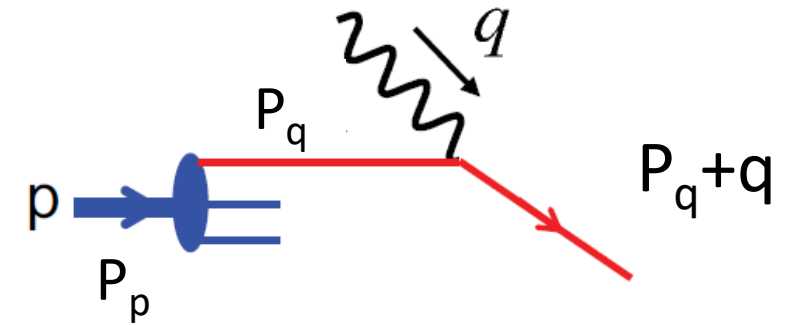
Interpretation of Hadron Side Variables



Inverting the above, we have:

$$E_X = y_q E_e + (1 - y_q) E_q$$

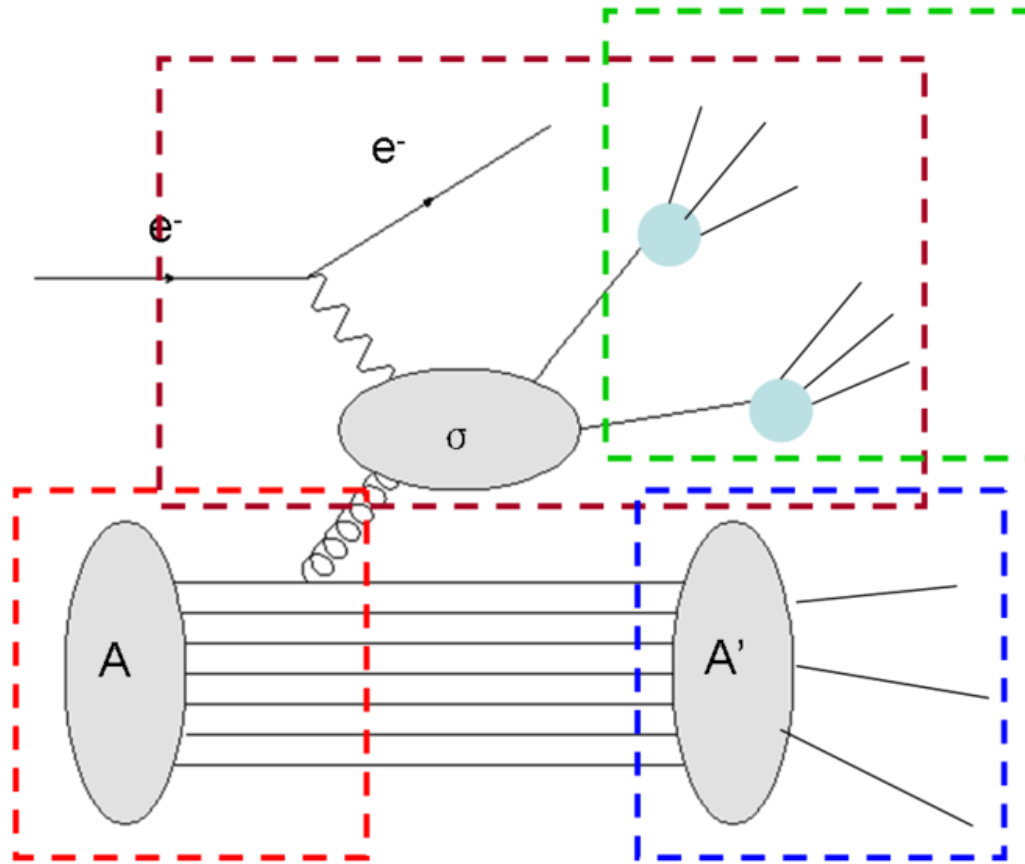
$$\cos \gamma = \frac{-y_q E_e + (1 - y_q) E_q}{y_q E_e + (1 - y_q) E_q}$$



$$\begin{aligned} P_q &= (xE_p, x\mathbf{P}_p) \\ x_q &= 1 \\ y_q &= y \end{aligned}$$

- Energy and angle of struck quark in naïve quark-parton model

BeAGLE Generator



A hybrid model consisting of DPMJet and PYTHIA with nPDF EPS09.

Nuclear geometry by DPMJet and nPDF provided by EPS09.

Parton level interaction and jet fragmentation completed in PYTHIA.

Nuclear evaporation (gamma dexcitation/nuclear fission/fermi break up) treated by DPMJet

Energy loss effect from routine by Salgado&Wiedemann to simulate the nuclear fragmentation effect in cold nuclear matter

What about Electron-Nucleus Scattering?

$$e + A \rightarrow e' + X$$

For fixed-target scattering, we have:

$$P_A = (M_A, 0, 0, 0) \quad ,$$

and we calculate our kinematic variables assuming scattering off a stationary proton:

$$P_p = (M_p, 0, 0, 0)$$

What about Electron-Nucleus Scattering?

For example, we would still write the invariant-mass squared in terms of the electron angle and energy as:

$$\begin{aligned} W^2 &= M_P^2 + 2M_P(E_e - E_{e'}) - 4E_e E_{e'} \cos^2 \frac{\theta}{2} \\ &= M_P^2 - Q^2(1 - 1/x) \end{aligned}$$