Proposal to CSEWG
Clarification of R-Matrix Limited format
Relativistic flag `KRL` for `LRF=7` formatting*

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I. SUMMARY & PURPOSE

The purpose of this proposal is to specify a clarification or extension of the resonance parameter file `MF=2, MT=151` for the R-Matrix Limited (RML) format, specified in Sec. (2.2.1.6) of the `ENDF-6` manual.

In order to allow the direct distribution of R-Matrix parameters that have been generated by evaluators at Los Alamos National Laboratory (using the evaluation code EDA), we must specify a previously unused (but defined in the ENDF-6 manual) parameter, `KRL` in the RML format. These RML evaluations employ the following `ENDF-6` specification: `MF=2, MT=151, LRU=1, LRF=7, KRM=4`.[1]

Our understanding is that no previous evaluations, in either `ENDF/B-VIII.0` or earlier `ENDF/B` releases, employs `KRL=1`; all previous evaluations use `KRL=0`.

The following sections give the details of the specification.

We use natural units ($\hbar = 1 = c$) throughout.

II. `KRL=1` RML R-MATRIX PARAMETRIZATION

This section specifies the parametrization of the R-Matrix; the next section will address kinematic variables.

The expression for the R-Matrix for a given spin group (J, π) is the usual one:

$$R_{c'c} = \sum_{\lambda=1}^{N_\lambda} \frac{\gamma_{\lambda,c'}\gamma_{\lambda,c}}{E_\lambda - E(s)}, \tag{2.1}$$

where c denotes the channel (α, ℓ, s, J; α is the particle-pair or partition). Here, $E(s)$ is related to the relativistic invariant Mandelstam variable s, described below.

The parameters E_λ (the level energies in eV units) and $\gamma_{\lambda,c}$ (the reduced widths in $\mathrm{eV}^{1/2}$ units) are the resonance energies `ER` and reduced widths `GAM` taken directly from the `MF=2, MT=151, LRU=1, LRF=7, KRM=4, KRL=1` RML section

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[1] Incidentally, we know of no evaluation that uses `KRM=4`. Even if this is not the case, the present proposal is unchanged.

```
[MAT,2,151/ 0.0,      0.0,       0,       NRS,     6*NX,       NX/
             ER_1,    GAM_{1,1},   GAM_{2,1},    GAM_{3,1},   GAM_{4,1},   GAM_{5,1},
           GAM_{6,1}, ------------------------- GAM_{NCH,1},
             ER_2,    GAM_{1,2},   GAM_{2,2},    GAM_{3,2},   GAM_{4,2},   GAM_{5,2},
           GAM_{6,2}, -------------------------- GAM_{NCH,2},
             ---------------------------------------------
          ER_{NRS}, GAM_{1,NRS}, GAM_{2,NRS}, GAM_{3,NRS}, GAM_{4,NRS}, GAM_{5,NRS},
       GAM_{6,NRS}, ------------------------ GAM_{NCH,NRS}           ]LIST
```

as described in the ENDF-6 mantual RML Sec. 2.2.1.6. The parameter NRS $= N_\lambda$ in Eq.(2.1).

The E_λ are defined with respect to the first partition, which is taken as the reference partition mentioned in this LRF=7 section, and depend on the form chosen for $E(s)$ (see below). (This removes the need to explicitly define the reference section; further, by keeping KRL$\neq 0, 1$ free, future clarifications – like different forms for $E(s)$ – are possible.)

Currently, only a single form for the relativistic energy parameter $E(s)$ has been employed in existing relativistic parametrizations:

$$E(s) = \frac{s - m_{c_0}^2}{2m_{c_0}}, \qquad\qquad \text{KRL} = 1, \tag{2.2}$$

where m_{c_0} is the channel mass for c_0 the first, reference partition:

$$m_{c_0} = m_{c_0,1} + m_{c_0,2}. \tag{2.3}$$

Future clarifications to LRF=7 might use different values of KRL > 1.

III. KRL=1 KINEMATIC VARIABLES

Since the Mandelstam variable s is both relativistically invariant and partition-pair invariant, we may readily connect the kinematic variables in different channels through s. We have

$$s = (p_{c,1} + p_{c,2})^2 = m_c^2 + 2m_{c,2}E_c, \tag{3.1}$$

where $p_{c,1}$ ($p_{c,2}$) is the four-momentum of the projectile (target) in the partition c, and E_c is the kinetic energy of the projectile in the lab from for partition c:

$$E_c = \left(m_{c,1}^2 + |\mathbf{p}_{c,1}|^2\right)^{1/2} - m_{c,1}. \tag{3.2}$$

The relationship between the incident lab energies of the projectiles in channels c and c' are:

$$E_{c'} = \frac{1}{m_{c',2}}\Big(m_{c,2}E_c + \overline{m}_{c,c'}Q_{c,c'}\Big), \tag{3.3}$$

where, for channel masses m_c and $m_{c'}$, we have:

$$\overline{m}_{c,c'} = \frac{1}{2}(m_c + m_{c'}), \tag{3.4}$$

$$Q_{c,c'} = m_c - m_{c'}. \tag{3.5}$$

The wave number vector $\mathbf{k}_c$ in the center of mass frame, the magnitude of which (k_c) appears in the argument for the shift $S_\ell(k_c a_c)$ and penetrability $P_\ell(k_c a_c)$ factors, can be expressed as

$$k_c^2 = \frac{1}{4s}(s - m_c^2)(s - \Delta_c^2), \tag{3.6}$$

where

$$\Delta_c = m_{c,1} - m_{c,2}. \tag{3.7}$$

The relative velocity, which appears in the Schrödinger equation for (pointlike-particles) Coulomb scattering is

$$\beta_c = \frac{|\mathbf{p}_{c,1}|}{p^0_{c,1}}, \tag{3.8}$$

$$= \frac{[(E_c + 2m_{c,1})E_c]^{1/2}}{m_{c,1} + E_c}, \tag{3.9}$$

where $p^0_{c,1}$ is the time component of the projectile four-momentum.

For completeness, we mention the relativistic Sommerfeld parameter, in which β_c appears, that features in Coulomb scattering wave function:

$$\eta_c = Z_{c,1} Z_{c,2} \frac{\alpha_{\rm em}}{\beta_c}, \tag{3.10}$$

$$= Z_{c,1} Z_{c,2} \alpha_{\rm em} \frac{s - m^2_{c,1} - m^2_{c,2}}{2k_c\sqrt{s}}. \tag{3.11}$$

Here, $\alpha^{-1}_{\rm em} \approx 137$ and $Z_{c,i}$ is the charge of nuclide i and we have used the expressions

$$|\mathbf{p}_{c,1}| = \frac{1}{2m_{c,2}}[(s - m^2_c)(s - \Delta^2_c)]^{1/2} = \frac{\sqrt{s}}{m_{c,2}} k_c, \tag{3.12}$$

$$p^0_{c,1} = \frac{1}{2m_{c,2}}(s - m^2_{c,1} - m^2_{c,2}). \tag{3.13}$$