

β_{eff} benchmarks

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β_{eff} benchmark values

No.	Name	Reference	Experiment
1	SNEAK 9C1	Fischer (1977)	758 ± 24
2	FCA XIX-1	Okajima (2002)	742 ± 24
3	U9	JEFDOC-820	731 ± 15
4	Masurca R2	Okajima (2002)	721 ± 11
5	Big Ten	Kodeli (2017)	720 ± 7
6	U/Fe (ZPR9/34)	JEFDOC-820	671 ± 14
7	Topsy (25 Flattop)	Kodeli (2017)	665 ± 13
8	Godiva	Keepin (1965)	659 ± 28
9	SNEAK 7B	Ivanov (2009)	429 ± 22
10	SNEAK 9C2	Fischer (1977)	426 ± 19
11	SNEAK 7A	Ivanov (2009)	395 ± 22
12	C Ref (ZPPR21B)	JEFDOC-820	384 ± 8
13	FCA XIX-2	Okajima (2002)	364 ± 9
14	23 Flattop	Kodeli (2017)	360 ± 9
15	Masurca Zona 2	Okajima (2002)	349 ± 6
16	Skidoo (Jezebel 23)	Kodeli (2017)	290 ± 10
17	Popsy (49 Flattop)	Kodeli (2017)	276 ± 7
18	FCA XIX-3	Okajima (2002)	251 ± 4
19	P/C/SST (ZPR6-10)	JEFDOC-820	223 ± 5
20	Jezebel	Kodeli (2017)	195 ± 10
21	BFS-73-1	Manturov (2006)	735 ± 13
	TCA 1.83U	Meulekamp (2006)	761 ± 17

← 346 ± 11 per Kodeli (ref. needed)

← 194 ± 10 per Kodeli (ref. needed)

← No MC21 result for BFS-73-1

← No COG result for TCA 1.83U

References

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- E. Fort et al., "Recommended Values of the Delayed Neutron Yield for: U-235; U-238 and Pu 239," JEFDOC-820, 1999.
- Evgeny Ivanov, "SNEAK 7A and 7B Pu-Fueled Fast Critical Assemblies in the Karlsruhe Fast Critical Facility," SNEAK-LMFR-EXP-001, CRIT-BUCK-SPEC-COEF-KIN-RRATE-MISC (IRPhE), Rev. 0 (2009)
- Ivan Kodeli, "Combined Use of k-effective and beta-effective Measurements for Nuclear Data Validation and Improvement," EPJ Web of Conferences, **146**, 06018 (2017)
- G. R. Keepin, "Physics of Nuclear Kinetics," Addison-Wesley, Reading, Massachusetts (1965)
- Gennady Manturov et al., "BFS-73-1 Assembly: Experimental Model of Sodium-Cooled Fast Reactor with Core of Metal Uranium Fuel of 18.5% Enrichment and Depleted Uranium Dioxide Blanket," BFS1-LMFR-EXP-001, CRIT-SPEC-COEF-KIN-RRATE (IRPhE), Rev. 0 (2006)
- R.K. Meulekamp and S.C. van der Marck, "Calculating the Effective Delayed Neutron Fraction with Monte Carlo," *Nucl. Sci. Eng.*, **152**, pp.142-148 (2006)
- S. Okajima et al., "Summary on International Benchmark Experiments for Effective Delayed Neutron Fraction (β_{eff})," *Progress in Nuclear Energy*, Vol. 41, No. 1-4, pp. 285-301, 2002.

Computational methods

¹ **COG** beta-effective calculational method; reference:

LLNL-ABS-787340, “COG 11.3 New Features” – abstract submitted to SNA+MC 2020 (Japan)

² **MC21** next fission probability method; reference:

R.K. Meulekamp and S.C. van der Marck, “Calculating the Effective Delayed Neutron Fraction with Monte Carlo,” *Nucl. Sci. Eng.*, **152**, pp.142-148 (2006).

³ **MC21** correlated sampling method; reference:

D. P. Griesheimer and N.A. Gibson, “Simplified Method for Estimating the Effective Delayed Neutron Fraction with Monte Carlo Correlated Sampling,” *Proceedings of the International Conference on Mathematics and Computational Methods Applied to Nuclear Science and Engineering (M&C 2019)*, pp. 886-895 (2019).

Computational models

No.	Name	COG Model	MC21 Model	Comment
1	SNEAK 9C1	Fischer (1977)	Fisher (1977)	
2	FCA XIX-1	Okajima (2002)	Okajima (2002)	
3	U9	ICSBEP	ICSBEP	
4	Masurca R2	Okajima (2002)	Okajima (2002)	
5	Big Ten	ICSBEP	Mosteller	
6	U/Fe (ZPR9/34)	ICSBEP	ICSBEP	
7	Topsy (25 Flattop)	ICSBEP	Mosteller	
8	Godiva	ICSBEP	Mosteller	
9	SNEAK 7B	IRPhE	Fisher (1977)	
10	SNEAK 9C2	Fischer (1977)	Fisher (1977)	
11	SNEAK 7A	IRPhE	Fisher (1977)	
12	C Ref (ZPPR21B)	ICSBEP	ICSBEP	
13	FCA XIX-2	Okajima (2002)	Okajima (2002)	
14	23 Flattop	ICSBEP	Mosteller	
15	Masurca Zona 2	Okajima (2002)	Okajima (2002)	
16	Skidoo (Jezebel 23)	ICSBEP	Mosteller	
17	Popsy (49 Flattop)	ICSBEP	Mosteller	
18	FCA XIX-3	Okajima (2002)	Okajima (2002)	
19	P/C/SST (ZPR6-10)	ICSBEP	ICSBEP	
20	Jezebel	ICSBEP	Mosteller	
21	BFS-73-1	IRPhE	–	
	TCA 1.83U	–	ICSBEP	With modifications

COG and MC21 ZPR/ZPPR models are the ICSBEP (homogenized) benchmark models.

ENDF/B-VII.1 β_{eff} results

No.	Model	Reference	LLNL (k_{eff})	LLNL ¹	MC21 (k_{eff})	MC21 (NFP) ²	MC21 (Corr.) ³
1	SNEAK 9C1	758(24)	1.0058(2)	721(2)	1.0049(2)	702	735(3)
2	FCA XIX-1	742(24)	1.0062(2)	736(2)	1.0025(2)	724	752(4)
3	ZPR U9	731(15)	0.9913(2)	736(2)	0.9963(1)	703	719(3)
4	Masurca R2	721(11)	0.9922(2)	737(2)	0.9918(2)	716	736(4)
5	Big Ten	720(7)	1.0002(2)	726(2)	0.9950(2)	705	726(4)
6	ZPR9/34 (U/Fe-Ref)	671(14)	1.0017(2)	692(2)	0.9992(2)	687	688(4)
7	Topsy (25 Flattop)	665(13)	1.0035(2)	657(2)	1.0029(2)	624	691(5)
8	Godiva	659(28)	1.0004(2)	678(2)	0.9999(2)	644	650(4)
9	SNEAK 7B	429(13)	1.0020(2)	411(1)	1.0015(2)	407	420(2)
10	SNEAK 9C2	426(19)	1.0012(2)	380(1)	1.0013(2)	370	383(3)
11	SNEAK 7A	395(12)	1.0070(2)	364(1)	1.0068(2)	360	376(3)
12	ZPR21B (C Ref)	384(8)	0.9921(2)	364(1)	0.9904(2)	353	353(3)
13	FCA XIX-2	364(9)	0.9876(2)	371(1)	0.9849(2)	360	369(2)
14	23 Flattop	360(9)	0.9993(2)	352(1)	0.9990(2)	333	374(4)
15	Masurca ZONA2	349(6)	0.9970(2)	342(1)	0.9966(2)	335	343(2)
16	Skidoo (Jezebel 23)	290(10)	1.0000(2)	303(1)	1.0000(2)	285	286(3)
17	Popsy (49 Flattop)	276(7)	1.0013(2)	268(1)	1.0005(2)	257	284(3)
18	FCA XIX-3	251(4)	0.9887(2)	258(1)	0.9859(2)	251	252(2)
19	ZPR6-10 (Pu/C/SST)	223(5)	1.0152(2)	231(1)	1.0134(4)	229	227(3)
20	Jezebel	195(10)	1.0002(2)	198(1)	1.0003(2)	183	180(2)
21	BFS-73-1	735(13)	0.9959(2)	729(2)	-	-	-
	TCA 1.83U	771(17)	-	-	1.0000(2)	764	762(6)

$|\beta_{\text{eff,c}} - \beta_{\text{eff,e}}| \leq 1\sigma$

 $1\sigma < |\beta_{\text{eff,c}} - \beta_{\text{eff,e}}| \leq 2\sigma$

 $2\sigma < |\beta_{\text{eff,c}} - \beta_{\text{eff,e}}| \leq 3\sigma$

 $3\sigma < |\beta_{\text{eff,c}} - \beta_{\text{eff,e}}|$

ENDF/B-VIII.0 β_{eff} results

No.	Model	Reference	LLNL (k_{eff})	LLNL	MC21 (k_{eff})	MC21 (NFP) ¹	MC21 (Corr.) ²
1	SNEAK 9C1	758(24)	1.0030(2)	726(2)	1.0044(2)	707	735(4)
2	FCA XIX-1	742(24)	1.0142(2)	735(2)	1.0113(2)	724	749(4)
3	ZPR U9	731(15)	0.9921(2)	741(2)	0.9963(2)	707	732(3)
4	Masurca R2	721(11)	0.9899(2)	737(2)	0.9922(2)	717	739(4)
5	Big Ten	720(7)	1.0002(2)	729(2)	0.9949(2)	708	732(4)
6	ZPR9/34 (U/Fe-Ref)	671(14)	0.9987(2)	668(2)	0.9960(4)	684	690(4)
7	Topsy (25 Flattop)	665(13)	1.0012(2)	660(2)	1.0008(2)	622	693(4)
8	Godiva	659(28)	1.0004(2)	676(2)	1.0001(2)	648	648(4)
9	SNEAK 7B	429(22)	1.0025(2)	410(1)	1.0017(1)	409	423(2)
10	SNEAK 9C2	426(19)	1.0008(2)	380(1)	1.0041(2)	376	393(3)
11	SNEAK 7A	395(22)	1.0063(2)	367(1)	1.0047(2)	361	379(3)
12	ZPR21B (C Ref)	384(8)	0.9906(2)	366(1)	0.9892(2)	353	352(3)
13	FCA XIX-2	364(9)	0.9864(2)	374(1)	0.9870(2)	363	373(3)
14	23 Flattop	360(9)	1.0003(2)	358(1)	1.0000(2)	340	386(4)
15	Masurca ZONA2	349(6)	0.9955(2)	343(1)	1.0018(2)	339	342(2)
16	Skidoo (Jezebel 23)	290(10)	1.0006(2)	312(1)	1.0005(2)	295	296(3)
17	Popsy (49 Flattop)	276(7)	0.9999(2)	271(1)	0.9986(2)	260	283(3)
18	FCA XIX-3	251(4)	0.9860(2)	257(1)	0.9827(3)	251	251(2)
19	ZPR6-10 (Pu/C/SST)	223(5)	1.0039(2)	205(1)	1.0020(6)	228	231(3)
20	Jezebel	195(10)	1.0000(2)	200(1)	0.9997(2)	184	187(2)
21	BFS-73-1	735(13)	0.9937(2)	730(2)	-	-	-
	TCA 1.83U	771(17)	-	-	0.9991(2)	769	758(6)

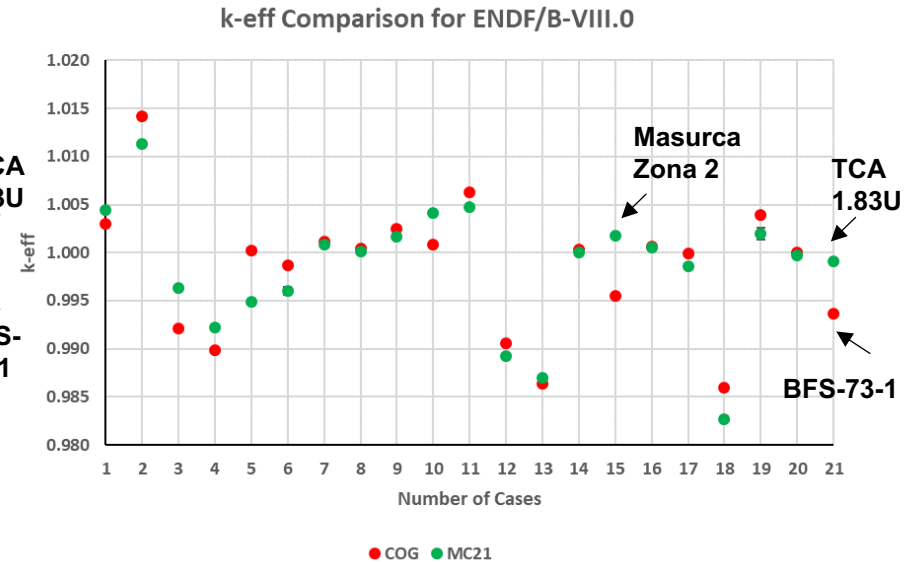
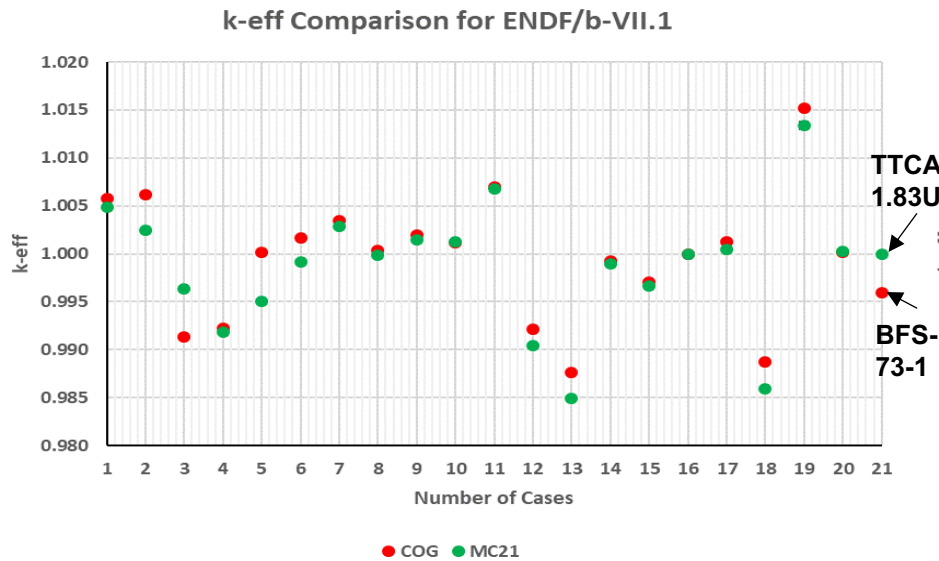
$|\beta_{\text{eff,c}} - \beta_{\text{eff,e}}| \leq 1\sigma$

 $1\sigma < |\beta_{\text{eff,c}} - \beta_{\text{eff,e}}| \leq 2\sigma$

 $2\sigma < |\beta_{\text{eff,c}} - \beta_{\text{eff,e}}| \leq 3\sigma$

 $3\sigma < |\beta_{\text{eff,c}} - \beta_{\text{eff,e}}|$

Observations



Statistically significant differences in COG and MC21 k-eff results are observed.

Observations

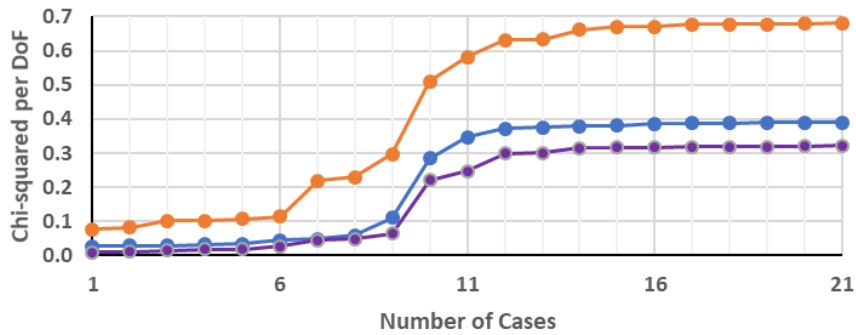
Cases	ENDF/B-VII.1			ENDF/B-VIII.0		
	COG	MC21 (Corr.)	MC21 (NFP)	COG	MC21 (Corr.)	MC21 (NFP)
21	Orange	Purple	Purple	Purple	Purple	Purple
20	Orange	Yellow	Purple	Orange	Purple	Purple
19	Orange	Yellow	Orange	Orange	Orange	Orange
18	Yellow	Yellow	Orange	Orange	Yellow	Orange
17	Yellow	Yellow	Orange	Yellow	Yellow	Orange
16	Yellow	Yellow	Orange	Yellow	Yellow	Orange
15	Yellow	Yellow	Orange	Yellow	Yellow	Yellow
14	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow
13	Yellow	Yellow	Yellow	Yellow	Yellow	Yellow
12	Yellow	Green	Yellow	Green	Yellow	Yellow
11	Green	Green	Yellow	Green	Green	Yellow
10	Green	Green	Yellow	Green	Green	Green
9	Green	Green	Yellow	Green	Green	Green
8	Green	Green	Yellow	Green	Green	Green
7	Green	Green	Green	Green	Green	Green
6	Green	Green	Green	Green	Green	Green
5	Green	Green	Green	Green	Green	Green
4	Green	Green	Green	Green	Green	Green
3	Green	Green	Green	Green	Green	Green
2	Green	Green	Green	Green	Green	Green
1	Green	Green	Green	Green	Green	Green

ENDF/B-VII.1 and VIII.0 produce similar results. COG and MC21 (Corr.) are in good agreement with MC21 (NFP) a bit worse.

Observations

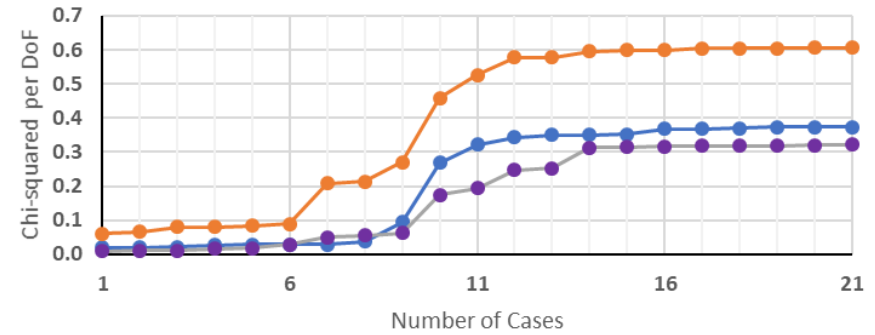
$$\chi^2 = \frac{1}{\nu} \sum_{i=1}^N \frac{(\beta_{c,i} - \beta_{b,i})^2}{\sigma_i^2}$$

Cumulative χ^2
for ENDF/B-VII.1



COG MC21 (NFP) MC21 (Corr.)

Cumulative χ^2
for ENDF/B-VIII.0



COG MC21 (NFP) MC21 (Corr.)

Note σ_i is a little larger for MC21 (Corr.)

Conclusions

- COG and MC21 calculated β_{eff} results are consistent with experimental (benchmark) values
- COG and MC21 (Corr.) results are in good agreement
- MC21 (NFP) results are a bit discrepant

Future work

- Compare details of the COG and MC21 models
 - Resolve k-eff discrepancies
 - Impact of the large “transformation biases” between some ZPR/ZPPR benchmark models and plate-by-plate models
- Expand benchmark suite to include additional benchmarks

MISTRAL-1	790(12)	
TCA 1.83U	771(17)	~LEU-COMP-THERM-006
Georgia Tech. Research Reactor	769	713 (neutron) 56 (photon)
BFS-73-1	735(13)	
SHE-8	696(32)	
ZPR U/Fe-Leak	676(14)	
MISTRAL-2	373(6)	
BFS-61	371(60)	
ZPR RSR	337(7)	

- ‘Alpha’ benchmarks

- Including: $-\alpha_{DC} = \beta_{\text{eff}} / \ell$

Acknowledgement

The contribution of Dr. Ivo Kodeli, Institut “Jožef Stefan”, Ljubljana, Slovenia, in identifying β_{eff} benchmarks including best values and (revised) uncertainties is gratefully acknowledged.



Questions?