

# *Verification of R-matrix calculations for charged-particle reactions*

CSEWG, Nuclear Data Week,  
November 2019

Ian Thompson

with R.J. deBoer, P. Dimitriou, S. Kunieda,  
M.T. Pigni, G. Arbanas, H. Leeb, Th. Srdinko,  
G. Hale, P. Tamagno, and P. Archier

 Lawrence Livermore  
National Laboratory

LLNL-PRES-796424

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC



Review

# Verification of R-matrix calculations for charged-particle reactions in the resolved resonance region for the $^7\text{Be}$ system

Ian J. Thompson<sup>1</sup>, R.J. deBoer<sup>2,3</sup>, P. Dimitriou<sup>4,a</sup>, S. Kunieda<sup>5</sup>, M.T. Pigni<sup>6</sup>, G. Arbanas<sup>6</sup>, H. Leeb<sup>7</sup>, Th. Srdinko<sup>7</sup>, G. Hale<sup>8</sup>, P. Tamagno<sup>9</sup>, and P. Archier<sup>9</sup>

<sup>1</sup> Lawrence Livermore National Laboratory, L-414, Livermore, CA 94551, USA

<sup>2</sup> The Joint Institution of Nuclear Astrophysics, University Notre Dame, Notre Dame, IN 46556, USA

<sup>3</sup> Department of Physics, University Notre Dame, Notre Dame, IN 46556, USA

<sup>4</sup> Division of Physical and Chemical Sciences, International Atomic Energy Agency, Wagramerstrasse 5, Vienna A-1400, Austria

<sup>5</sup> Japan Atomic Energy Agency, Tokai 319-1195, Japan

<sup>6</sup> Oak Ridge National Laboratory, PO Box 2008, Oak Ridge, TN 37831, USA

<sup>7</sup> Atominstitut, Technische Universität Wien, Wiedner Hauptstrasse 8-10, A-1040 Vienna, Austria

<sup>8</sup> Los Alamos National Laboratory, Theoretical Division, Los Alamos, NM 87545, USA

<sup>9</sup> CEA-DEN Cadarache, DER/SPRC/LEPh, F-13108 Saint-Paul-lez-Durance, France

Received: 15 February 2019 / Revised: 4 April 2019

Published online: 17 June 2019

# Need to validate R-matrix codes

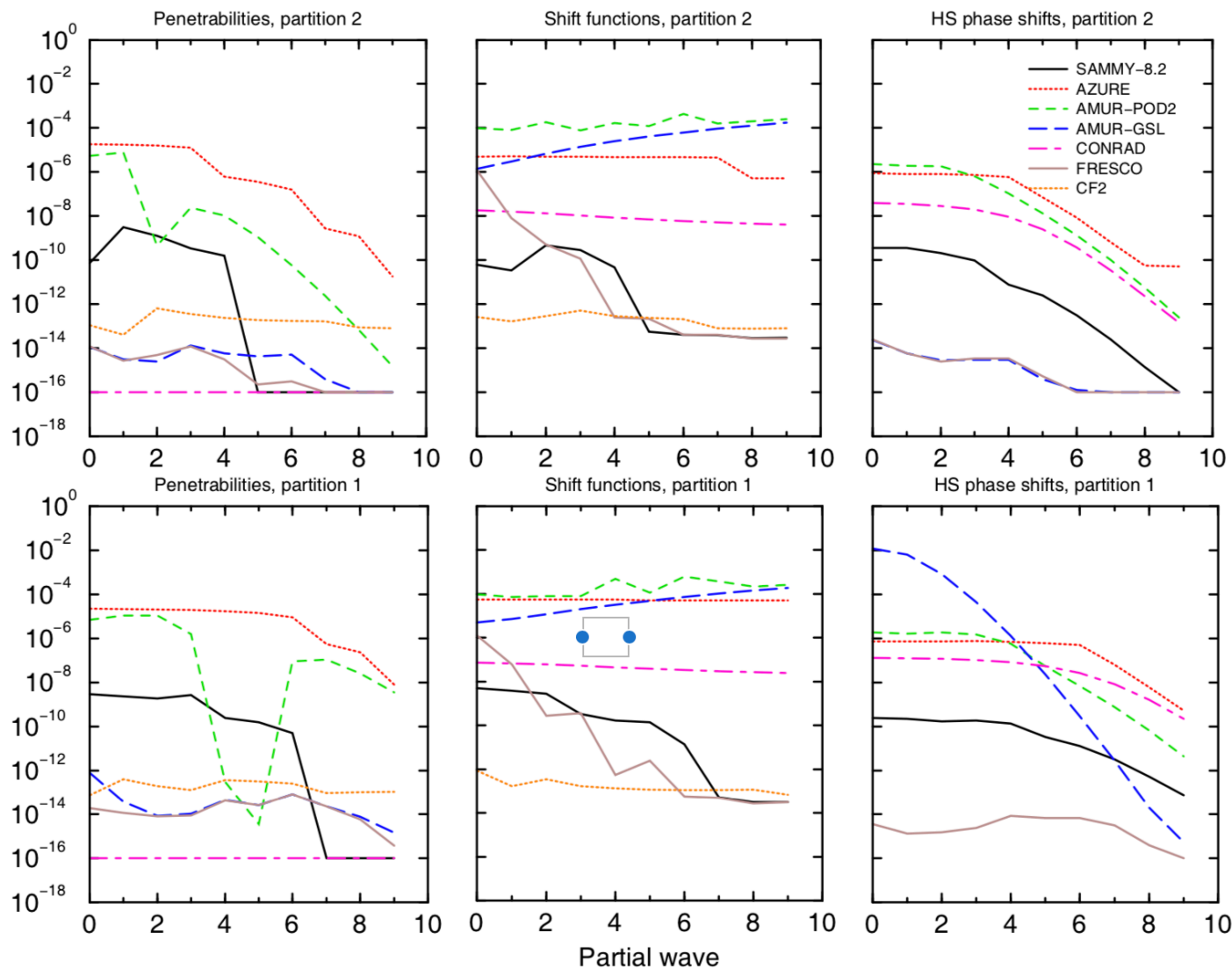
- AMUR: Japan - S. Kunieda
- AZURE2: Notre Dame – James de Boer
- EDA: LANL – Gerry Hale and Mark Paris
- FRESCOX: LLNL – Ian Thompson
- GECCOS: TUW – Thomas Srdinko
- SAMMY: ORNL – Marco Pigni and others
- CONRAD: CEA – Tamagno and Archier

**Table 1.** Summary of R-matrix codes specifications.

Feature	EDA	AZURE2	AMUR	FRESCOX	SAMMY	CONRAD	GECCOS
R-matrix	Full	Full	Limited (for $\gamma$ 's)	Full	SLBW, MLBW, RM, Full <sup>a</sup>	SLBW, MLBW, RM, Full <sup>a</sup>	Full
Derivatives	Analytic	Numerical	Numerical	Numerical	Analytical ( $T = 0$ K) Numerical <sup>b</sup>	Analytic	Numerical
Reference frame	Lab/CM	In Lab, Out CM	Lab/CM	Lab/CM	Lab/CM	Lab/CM	Lab/CM
(Non)Relativistic kinematics	R + NR	NR	NR	R, NR	N-R	R, NR	NR
Channel Radii	Varied	Varied	Fitted (option)	Fixed	Varied	Varied	Fixed/Varied
Photons	In/Out	Out	Out	In/Out	Out	Out	
Observables: $E, \theta$ cross sections	All	All	All	All	All	All	All
Observables: polarization $T_{kq}$	All	No	Yes	All	No <sup>c</sup>	No	No (planned)
Inverse reactions	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Decay gammas	Post-processing	No	No	Post-processing	No	No	No
Isobaric reactions simultaneously	Yes	No	No	No	No	No	No
Doppler broadening	No	No	Yes	No	Yes	Yes	No
Resolution broadening	Yes	Yes	Yes	No	Yes	Yes	No
Normalization	Yes	Yes	Yes	yes	Yes	Yes	Yes
Background subtraction	No	No	Yes	No	Yes	Yes	No
Background R-matrix terms	E-dependent	Distant poles	Distant poles	Distant poles	Yes	No (planned)	Yes
Sample-size corrections	No	Yes	No	No	Yes	Yes	No
Close-geometry Q-corrections	No	Yes	No	No	Yes	No	No
Fitting procedure	LSQ	MINUIT2	KALMAN	MINUIT1	Bayesian (GLS)	Bayesian (GLS)	DAKOTA [41]
Multiple data sets	S	S	S	S	S	S	S
S: Simultaneously							
Uses data covariances	No	No	Yes	No	Yes	Yes	No <sup>d</sup>
Prior parameter covariances	Yes	No	Yes	No	Yes	Yes	No <sup>d</sup>
Data covariances (MF 32)	No	No	No	No	Yes	Yes	No
Brune parameter output	No (planned)	Yes	No	No	No (planned)	No (planned)	No (planned)
ENDF-6 format output	Yes	No	Yes	No	Yes	Yes	No <sup>d</sup>
ENDF-6 input	No	No	No	No	Yes	Yes	No
Code language	F77	C++	C++	F90	F77 <sup>e</sup>	C++	F03
Export controlled	Yes	No	Yes	No	RSICC <sup>f</sup>	Yes	No
Documentation	No	Yes	No	Yes	Online	No	No <sup>g</sup>
Parallelized	No	Yes	Yes	No	No	Yes	Yes
Interactive fitting	Yes	No	No	Yes	Yes	Yes	No
PPP modification			Unavailable fit	No	Experimental correction	Maximal iteration	

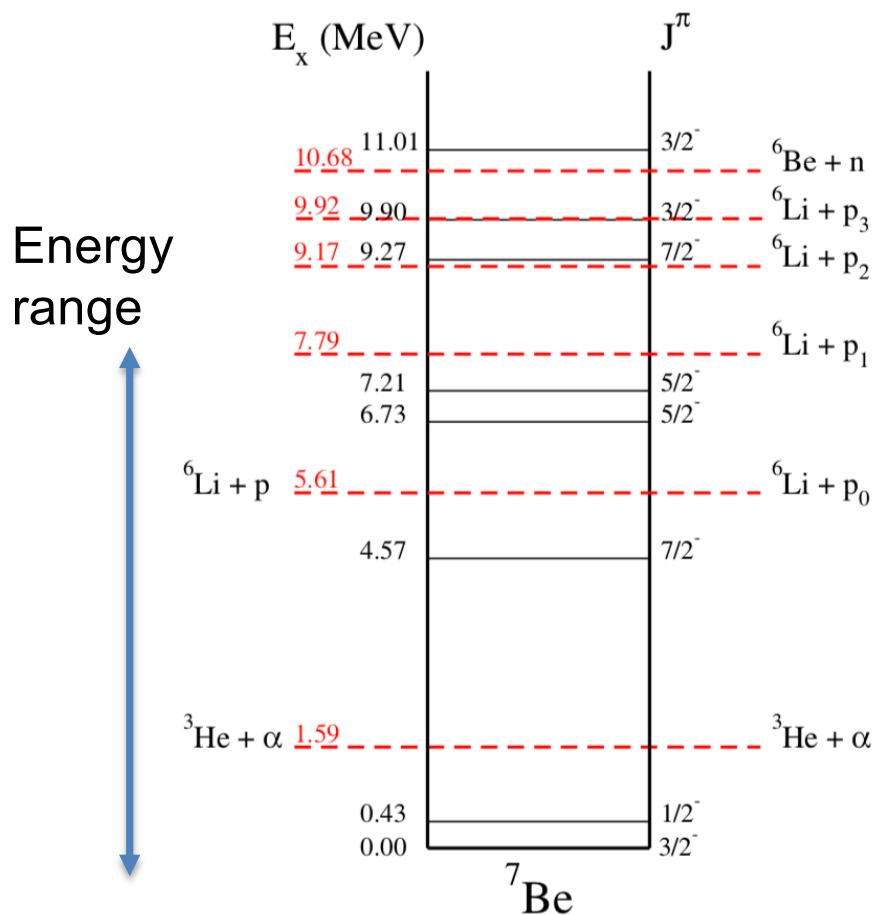
# Test the Coulomb Functions: $S$ , $P$ , $\phi$

Comparisons with COULCCv36



Differences of codes with  $S$ ,  $P$ , and  $\phi$  with values from COULCC

# Test Case: ${}^3\text{He}+{}^4\text{He} // p + {}^6\text{Li}$ .



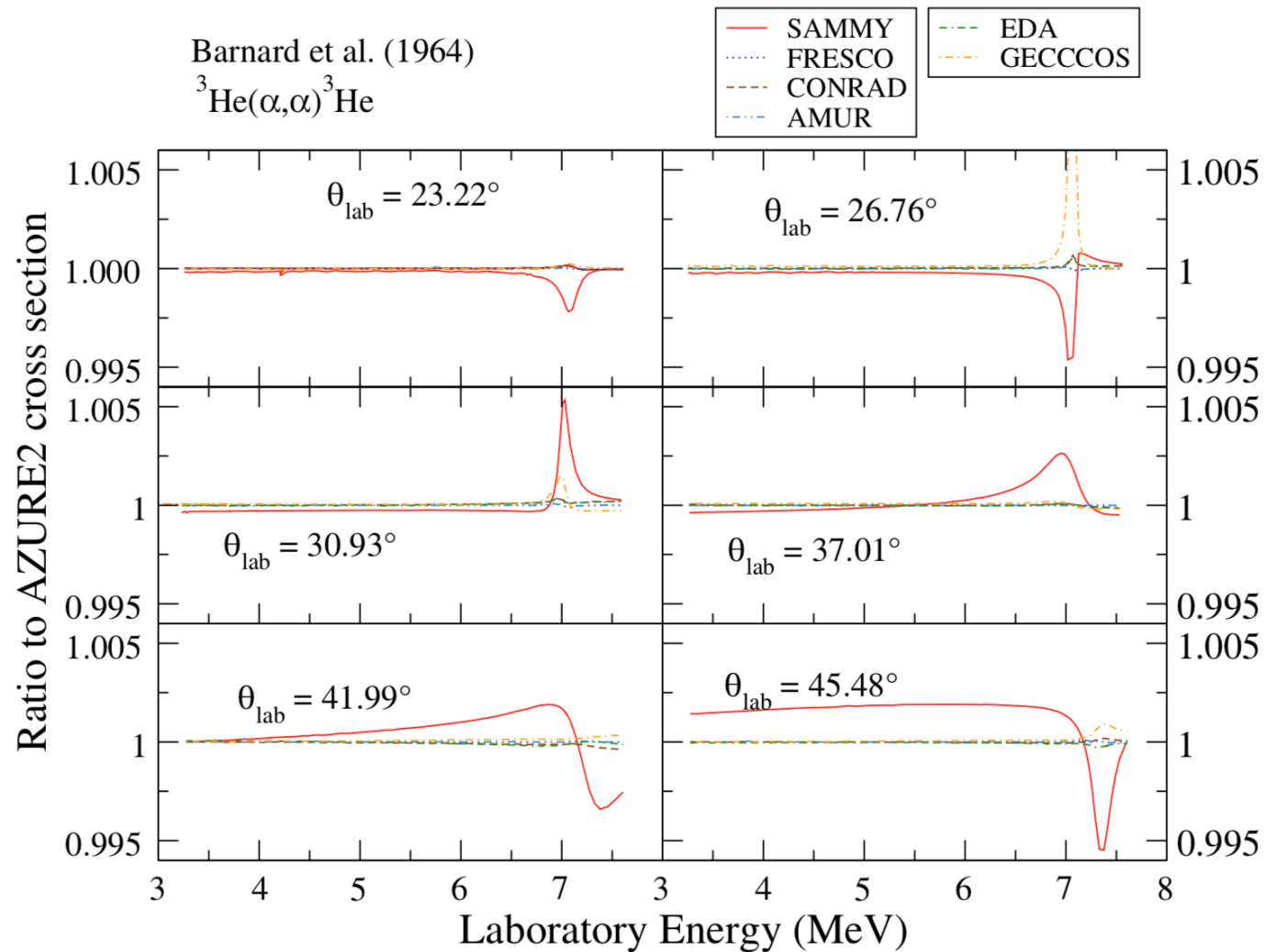
**Fig. 1.** Level diagram of  ${}^7\text{Be}$ . The low mass nucleus has only two bound states, the ground state and the level at  $E_x = 0.429$  MeV.

## R-matrix parameters

**Table 3.** R-matrix parameters in the  $B = -\ell$  basis. Pole energies in the center-of-mass frame of the elastic channel. Reduced width amplitudes are given in  $\text{MeV}^{1/2}$ .

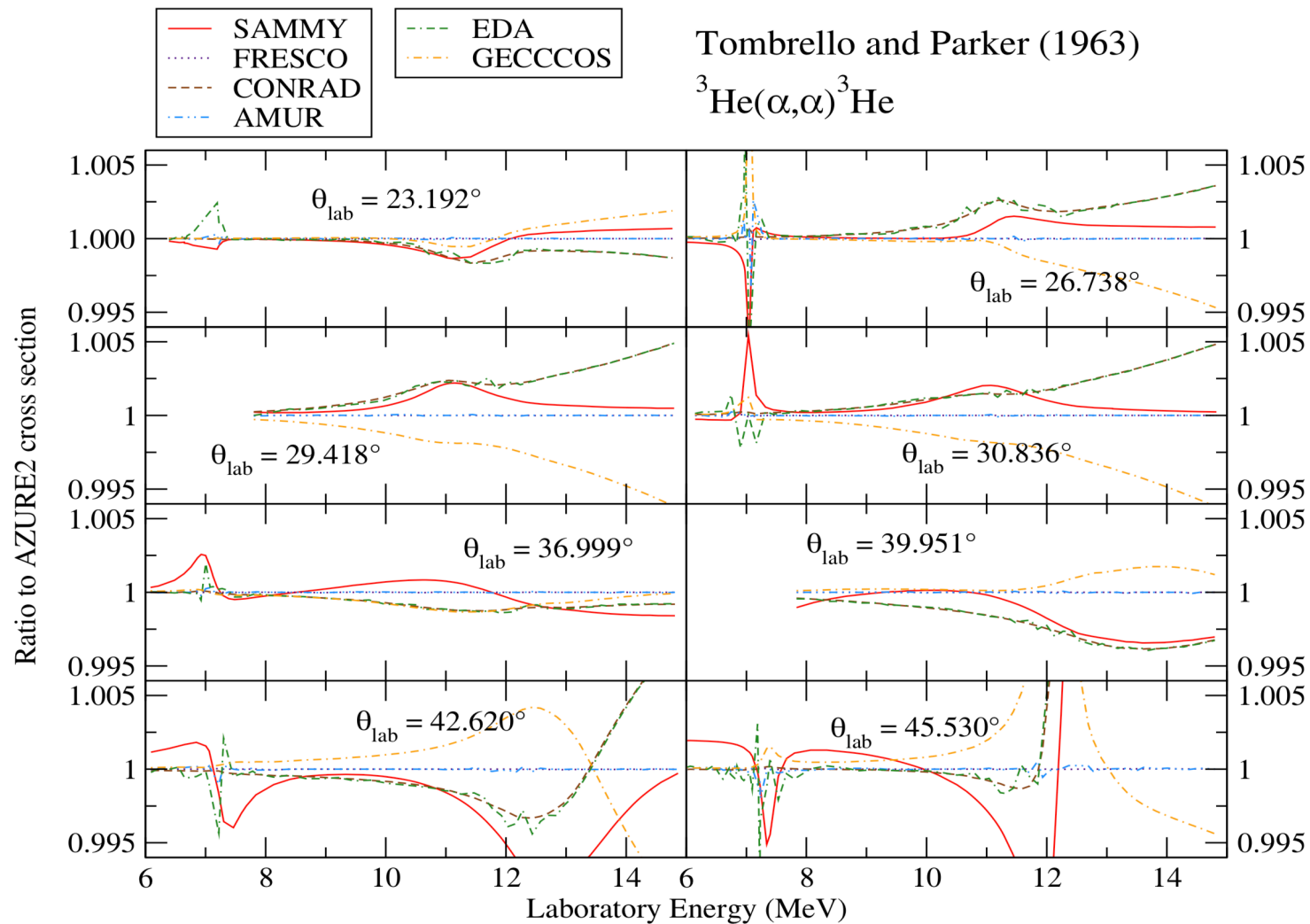
$J^\pi = 1.5^-$				
$E$	${}^1\text{H}+{}^6\text{Li}$	${}^1\text{H}+{}^6\text{Li}$	${}^1\text{H}+{}^6\text{Li}$	${}^4\text{He}+{}^3\text{He}$
(MeV)	$\ell S: 1, 1/2$	$\ell S: 1, 3/2$	$\ell S: 3, 3/2$	$\ell S: 1, 1/2$
-1.586097	-1.34077	-0.41816	0.00000	1.05725
$J^\pi = 2.5^-$				
$E$	${}^1\text{H}+{}^6\text{Li}$	${}^1\text{H}+{}^6\text{Li}$	${}^4\text{He}+{}^3\text{He}$	
(MeV)	$\ell S: 1, 3/2$	$\ell S: 3, 1/2$	$\ell S: 3, 1/2$	
5.746671	0.94880	0.00000	0.18770	
7.088367	-0.34947	0.00000	1.18381	
$J^\pi = 3.5^-$				
$E$	${}^1\text{H}+{}^6\text{Li}$	${}^1\text{H}+{}^6\text{Li}$	${}^1\text{H}+{}^6\text{Li}$	${}^4\text{He}+{}^3\text{He}$
(MeV)	$\ell S: 3, 1/2$	$\ell S: 3, 3/2$	$\ell S: 5, 3/2$	$\ell S: 3, 1/2$
3.483949	0.00000	0.00000	0.00000	0.79362

# Comparison of calculations to AZURE2 results for the ${}^3\text{He}(\alpha,\alpha){}^3\text{He}$



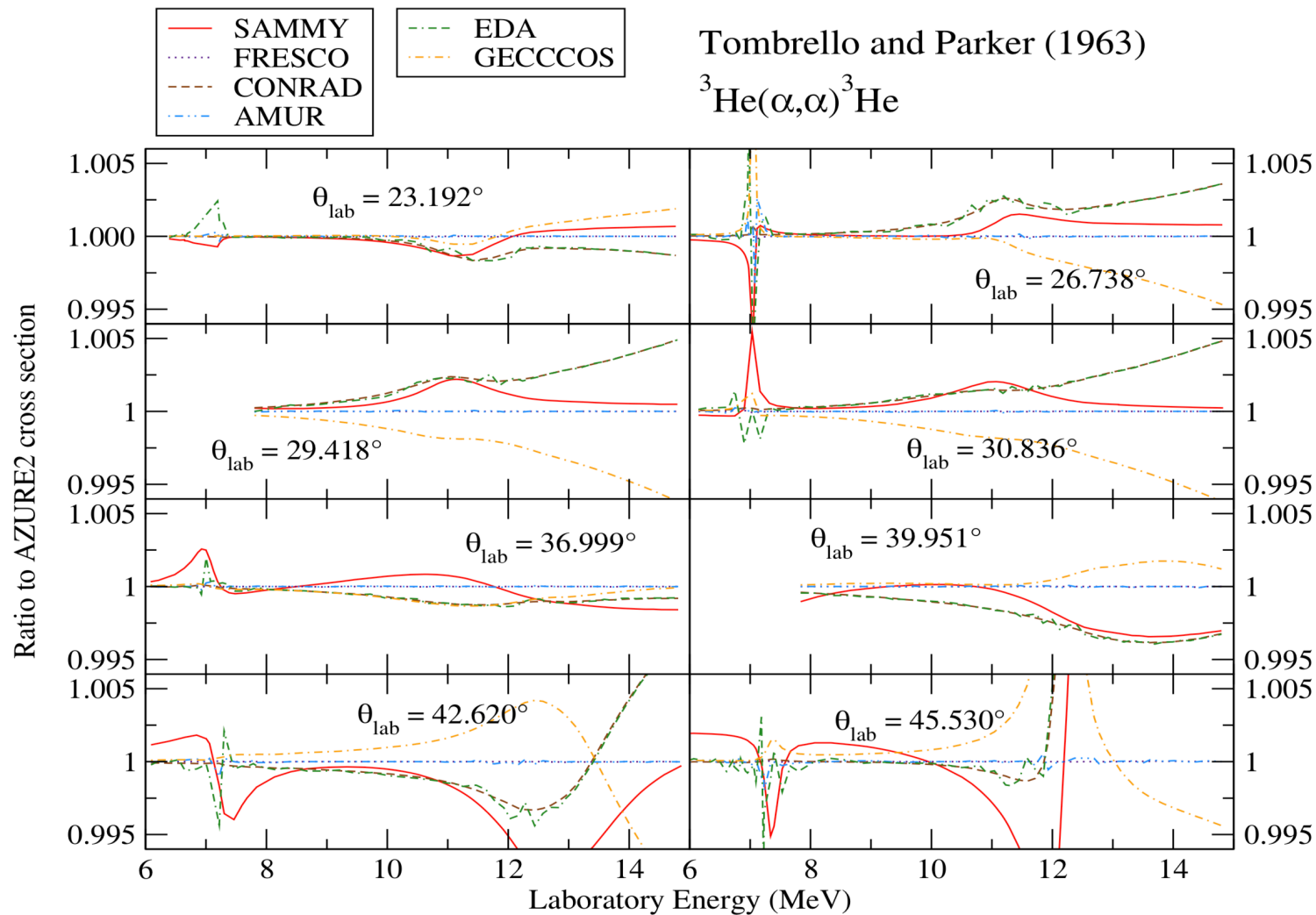
Lower  
incident  
energies

# Comparison of calculations to AZURE2 for ${}^3\text{He}(\alpha,\alpha){}^3\text{He}$



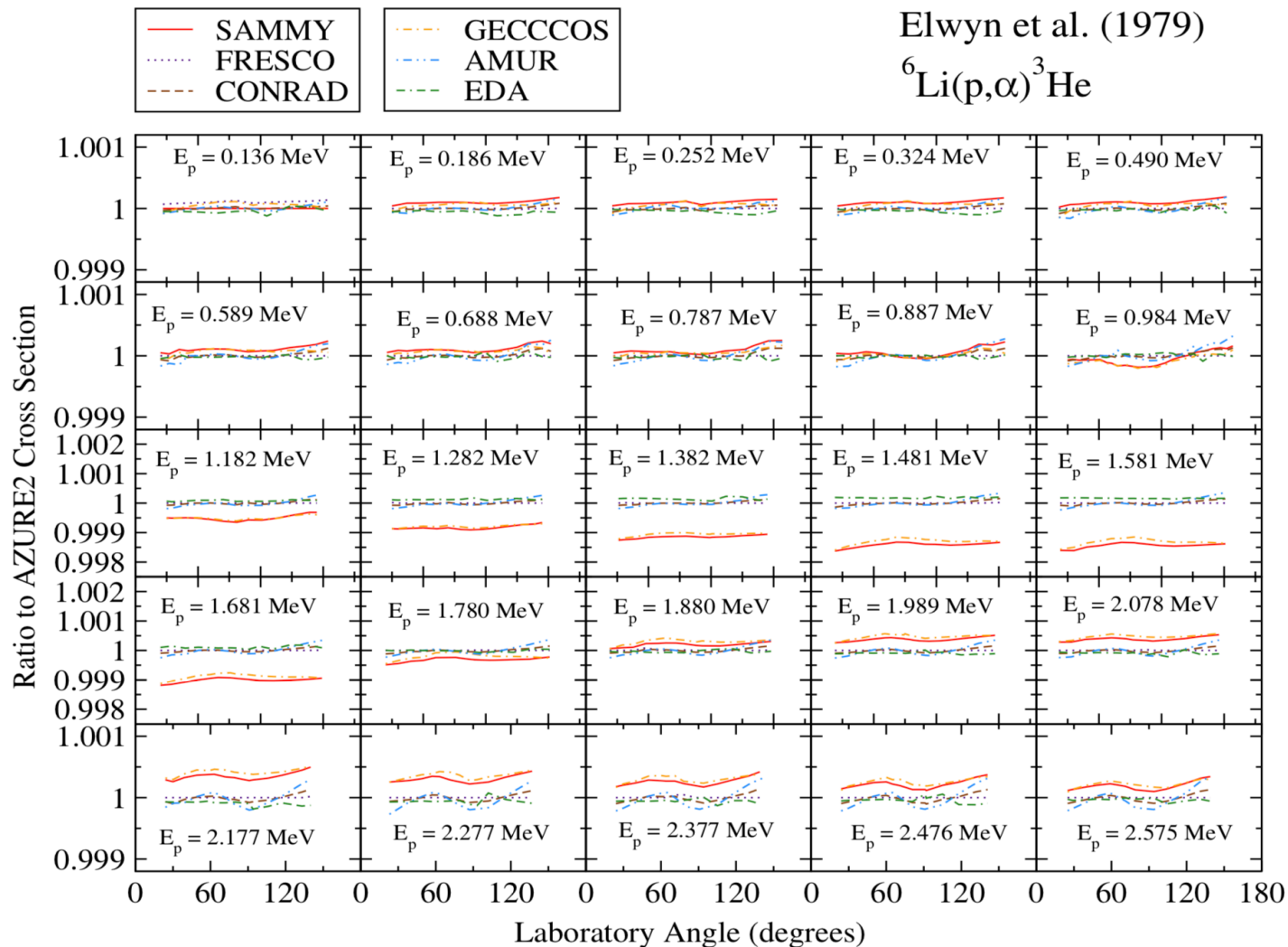


# Comparison of calculations to AZURE2 for ${}^6\text{Li}(p,p){}^6\text{Li}$



Higher incident energies

# Comparison of calculations to AZURE2 for ${}^6\text{Li}(p, \alpha){}^3\text{He}$



Low  
incident  
energies

# Conclusions

- A very useful exercise!!
- Used LLNL code FERDINAND.py translate parameter files between all the code inputs.
- We thought this would be all easy, but no -
  - Several bugs were found in several codes.
- On the whole, all the codes agreed to within 0.1–0.3% which is far below the uncertainties of the existing experimental data in this low-energy region.