



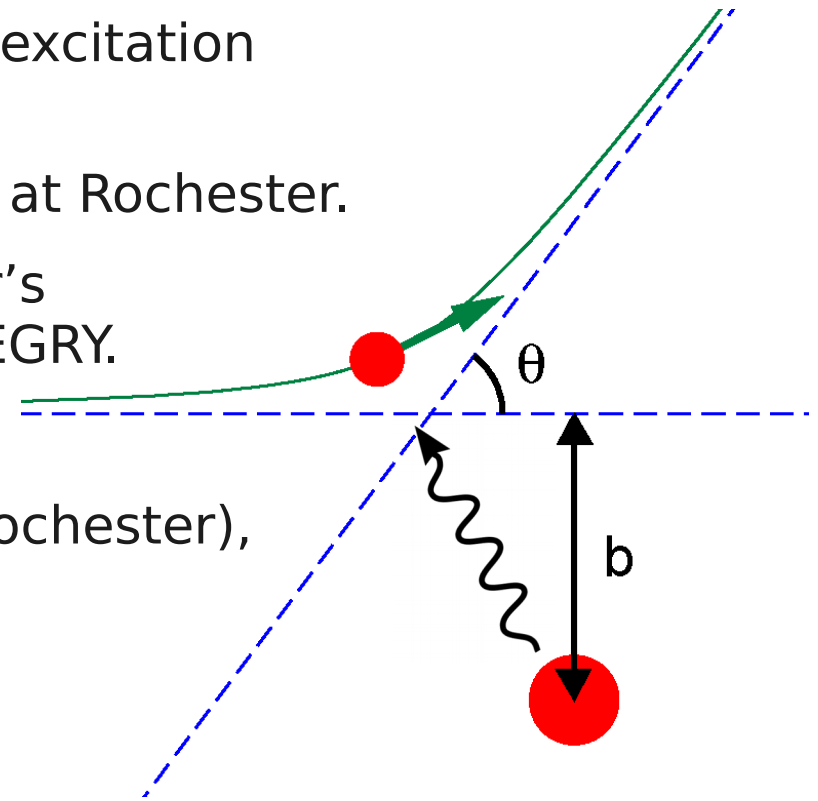
# GOSIA Results as ENSDF Data

Adam Hayes  
Nuclear Data Week  
2019

# What is Gosia?



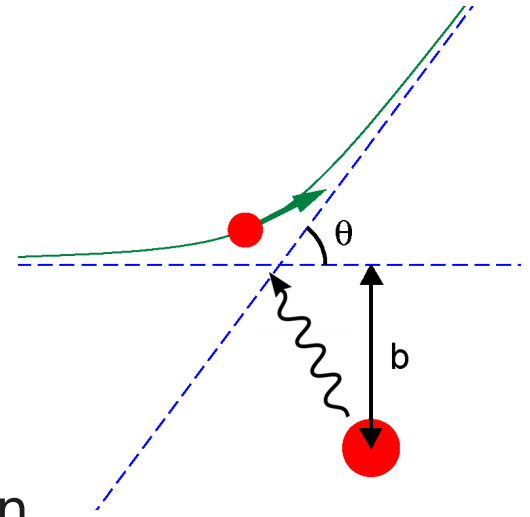
- Semi-classical, coupled-channels Coulomb excitation simulation and analysis code.
- Developed in 1980 by Czosnyka, Cline, Wu at Rochester.
- Some concepts from 1978 Winther, deBoer's COULEX & Rochester de-excitation code CEGRY.
- Maintained by Czosnyka 1980-2006.
- Gosia Steering Committee (2008): Cline (Rochester), Gaffney (CERN), Hayes (BNL), Napiorkowski (Warsaw), Warr (Cologne), Zielińska (Warsaw)
- Contributions: Hasselgren (Uppsala), Hayes, Ibbotson (Rochester), Kavka (Uppsala/Rochester), Kotlinski (Warsaw/Rochester), Srebrny (Warsaw), Vogt (Munchen/Rochester)
- <http://www.pas.rochester.edu/~cline/Gosia/index.html>



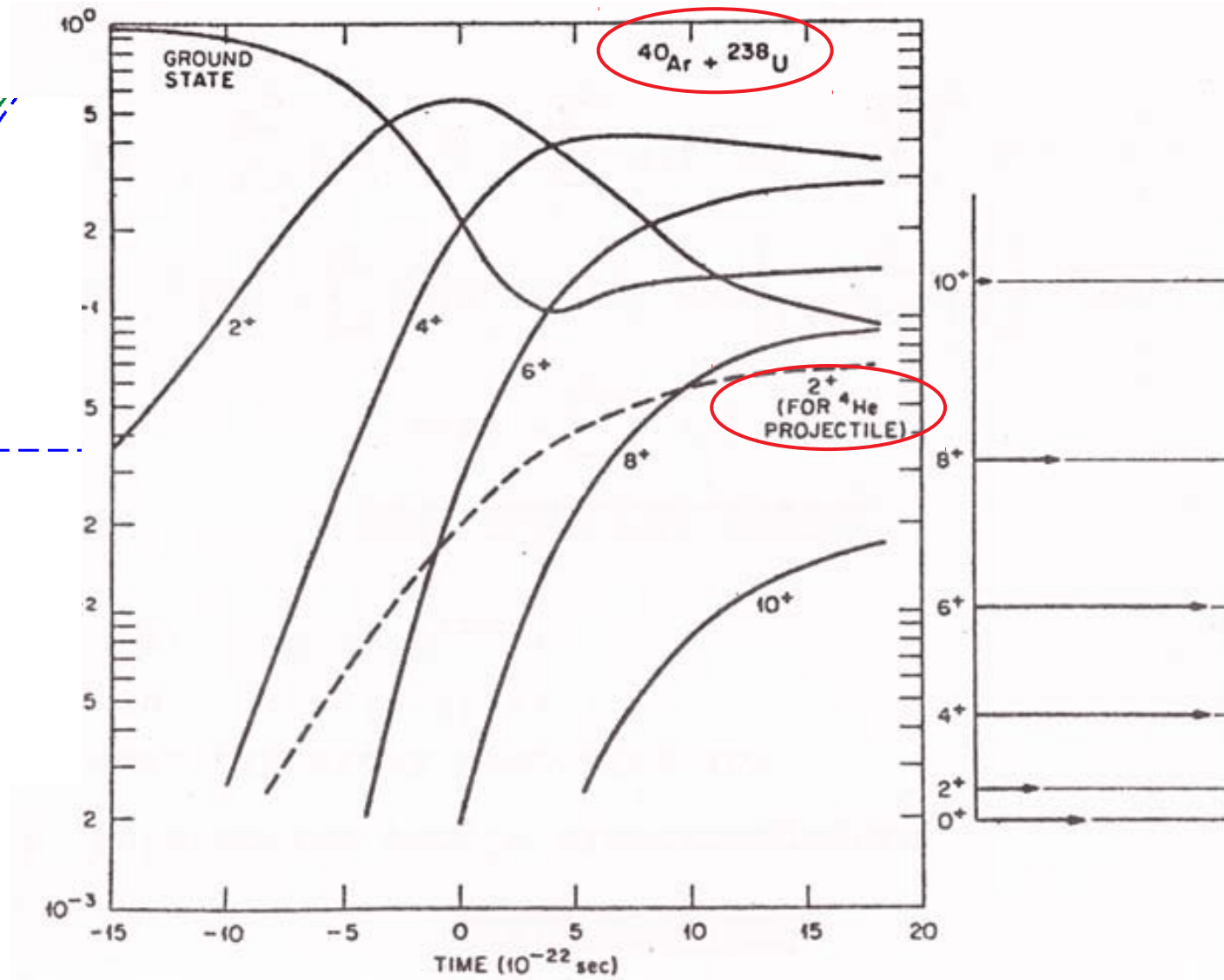
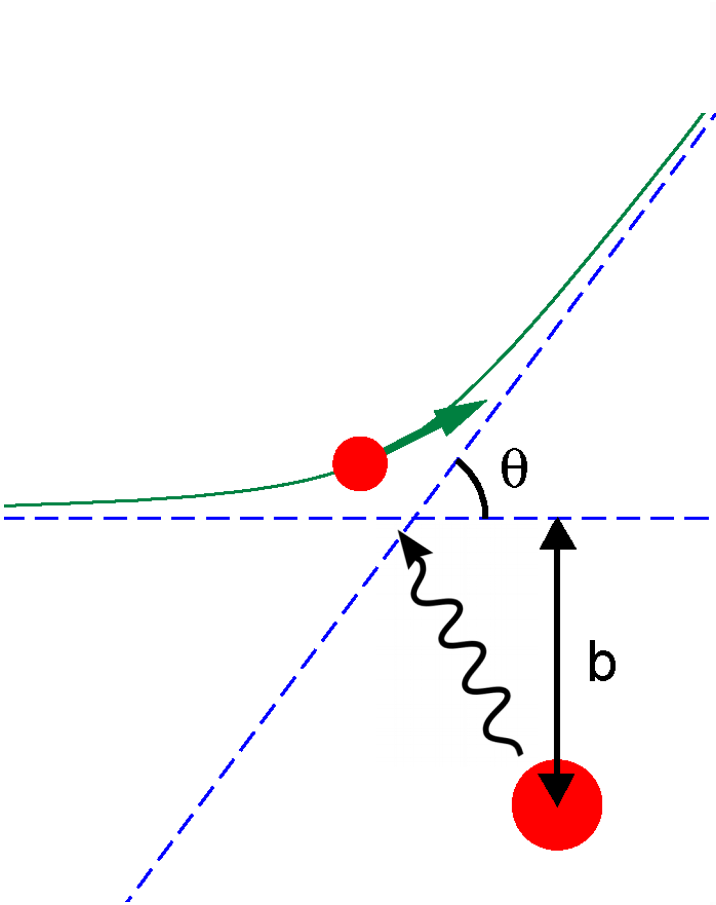
# Semi-classical



- Classical collision trajectory.
- Quantum-mechanical excitation & decay.
- Fully-quantal perturbation calculation not feasible for multi-step Coulex—calculated population of high-lying states sensitive to  $\sim 30^{\text{th}}$  order perturbation.
- Appropriate for “safe” Coulex—about  $< 80\%$  Coulomb barrier.
- Somewhat higher for heavy ions if small impact parameter scattering excluded.
- Sommerfeld parameter: 
$$\eta = \frac{a2\pi}{2\lambda}$$
 $\eta \gg 1 \rightarrow$  wave packet much smaller than interaction region of trajectory.



# Semi-classical Time-Evolution



# Typical Applications

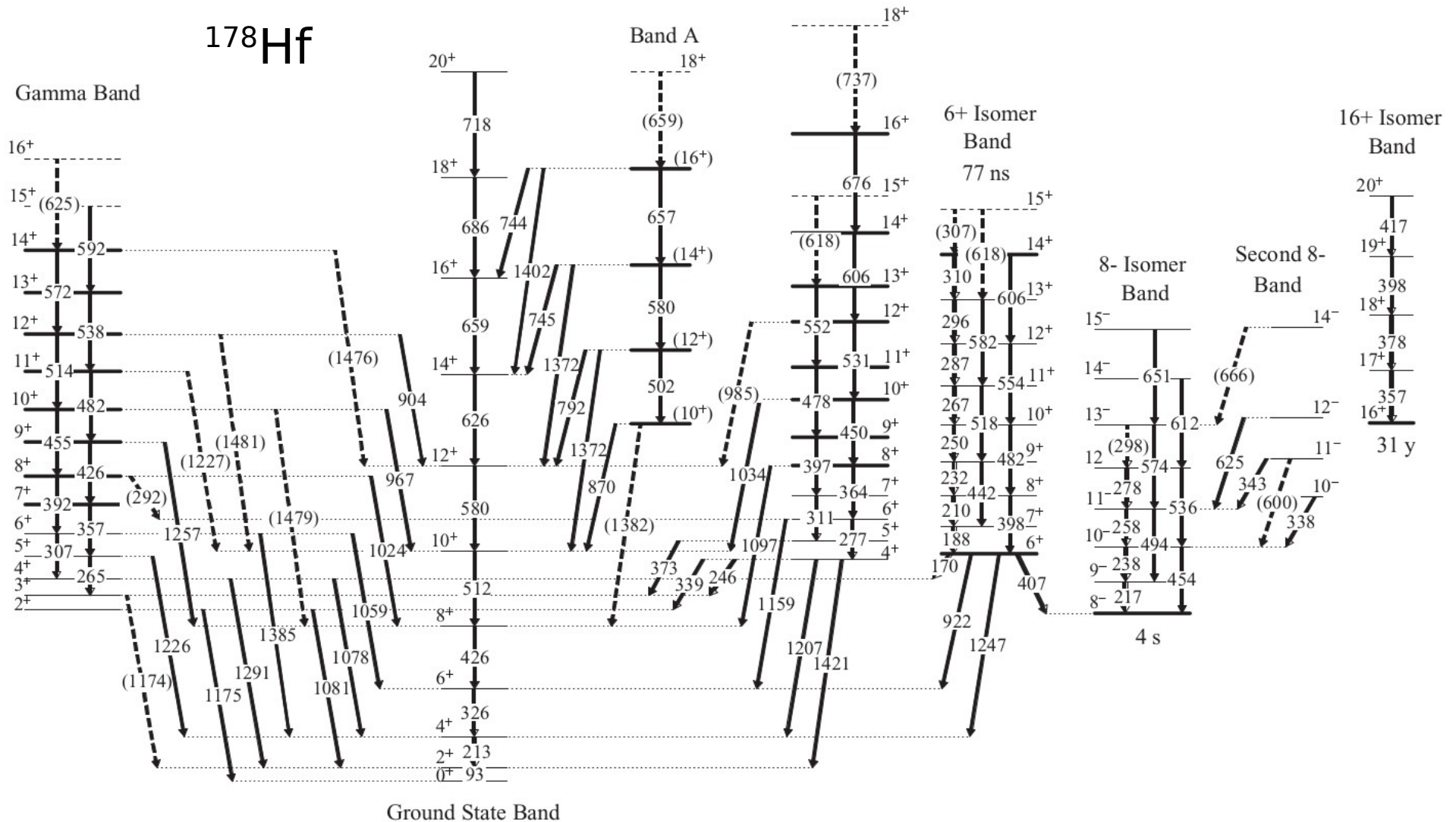


- Predominantly E2 & E3 matrix elements from excitation
- M1, E1... from decay
- Measure  $B(E2)$  and/or  $Q_s$  in  $\sim$ two-state system
- Many electric and magnetic matrix elements in strongly-collective system



## Strongly-deformed systems

# Collective Rotor: Intrinsic vs. Individual M.E.



# E2 Matrix Elements of Collective Bands



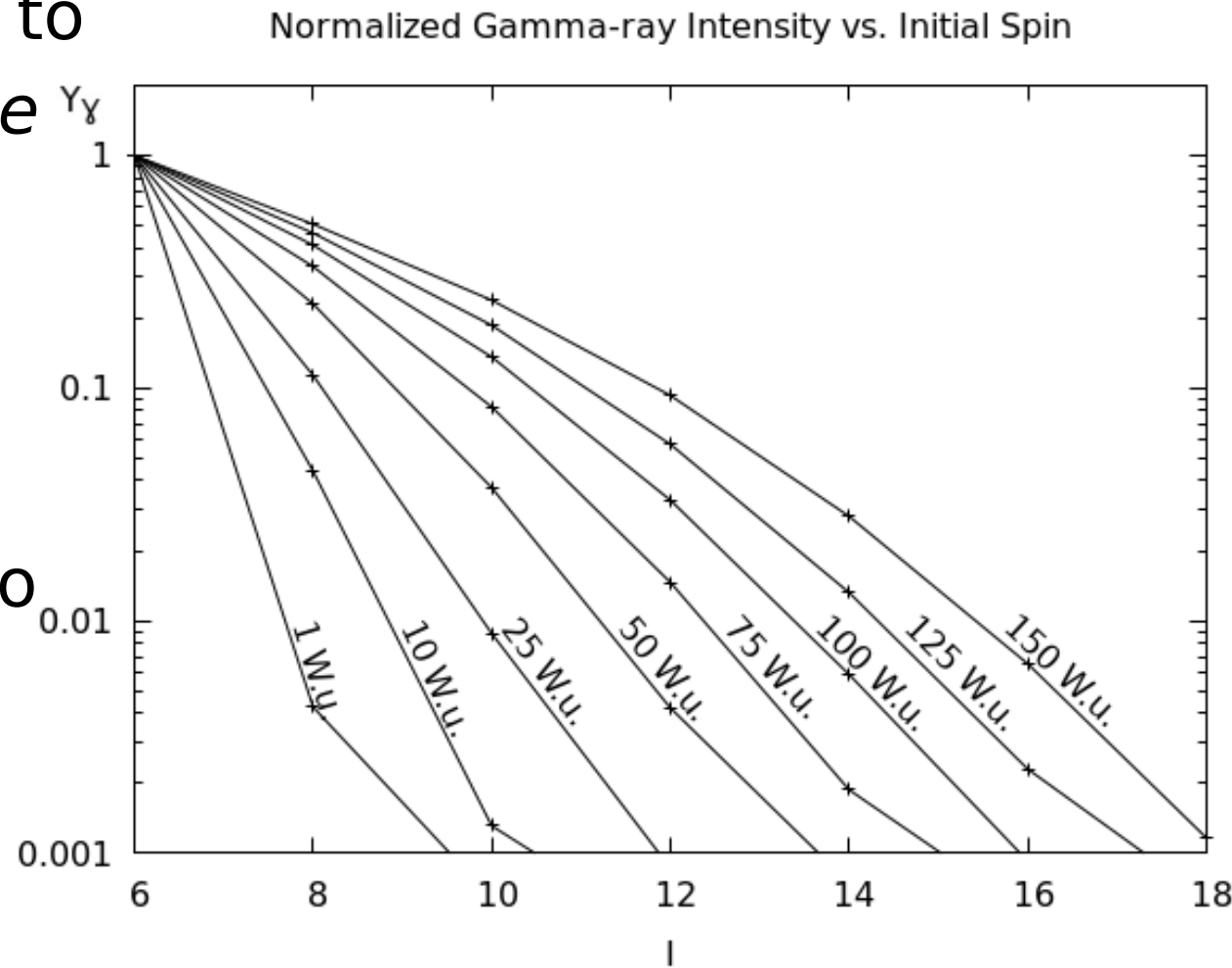
- Requires relative yield only.
- GSB is rigid rotor to good approximation.
  - 1) Measure  $Q_0$  assuming rigid rotor.
  - 2) Fit  $\langle I_i+2 || E2 || I_i \rangle$  for  $I^\pi > 6^+$  where observed yield is sensitive to  $Q_0$ .
  - 3) In reality, some iteration with fits to  $K^\pi = 2^+, 4^+$  required.



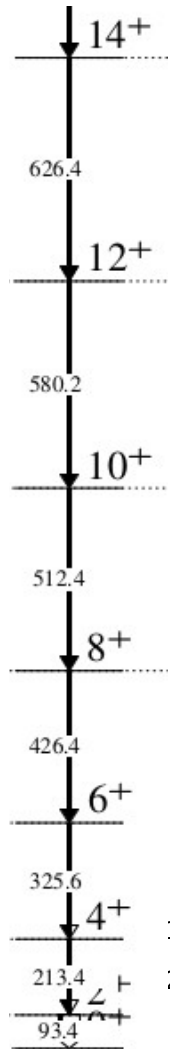
# Gamma Yield Sensitivity to Quadrupole Moment



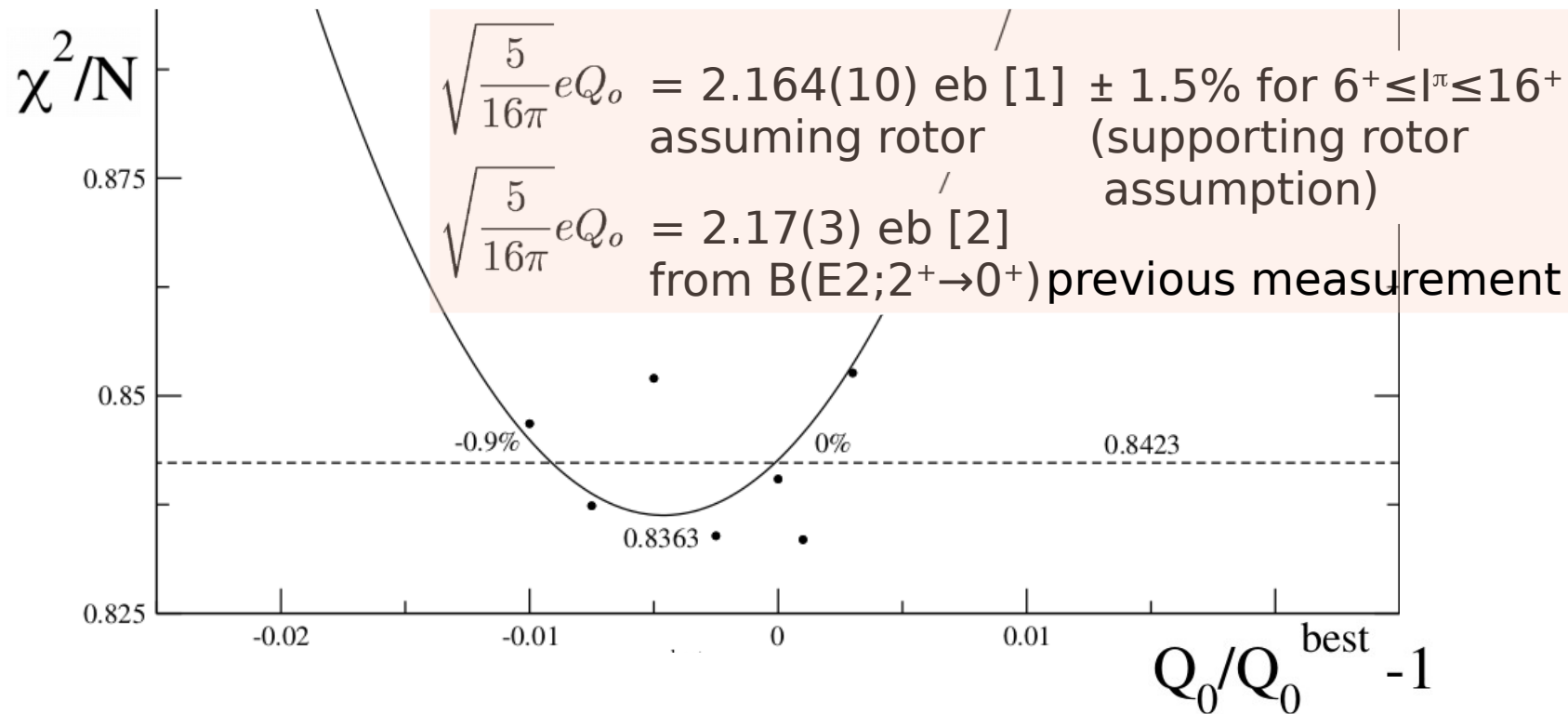
- Measured  $Q_0$  sensitive to rotational band *relative* gamma yield intensity
- No external normalization
- $Q_0$  typically sensitive to  $\leq 5\%$  level



# Example: Quadrupole moment of collective nucleus $^{178}\text{Hf}$



Rotor model fit of  $Q_0$  to measured  $\gamma$ -ray yield of GSB



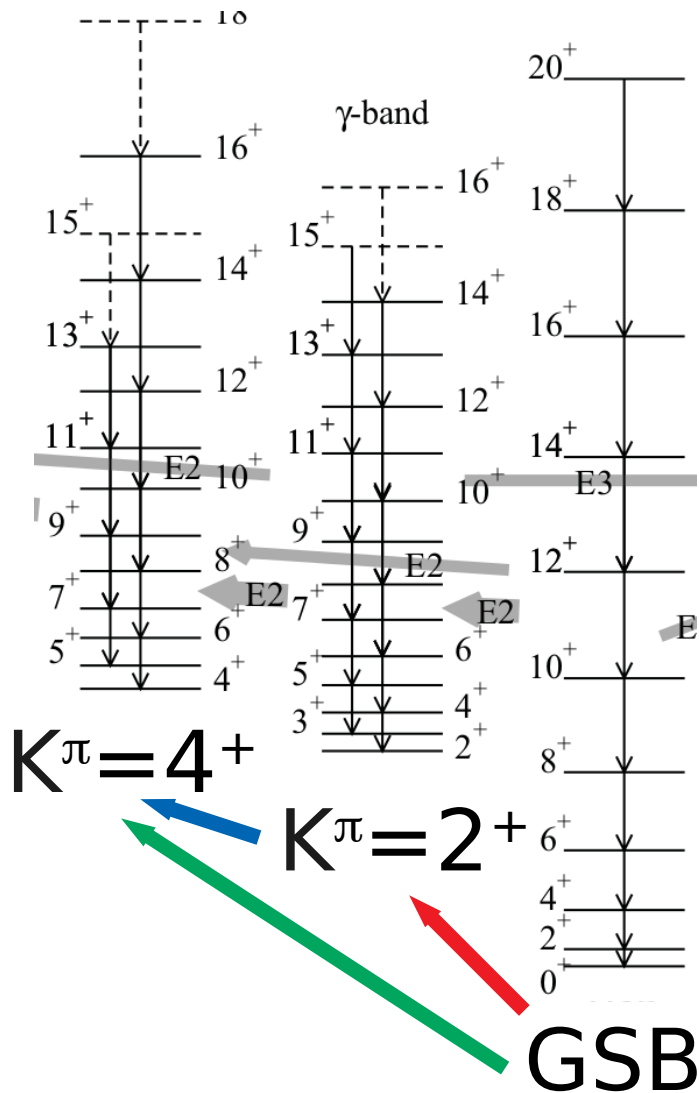
<sup>1</sup>Hayes et al., Phys. Rev. C **75**, 034308 (2007), Thesis (2005, unpublished)

<sup>2</sup>From  $B(E2; 2^+ \rightarrow 0^+) = 159(5) \text{ W.u.}$

E. Brown, Nuclear Data Sheet **54**, 199 (1988)

Citing R.M. Ronningen et al., Phys. Rev. C **15**, 1671 (1977)

# Collective Rotor: Intrinsic vs. Individual M.E.

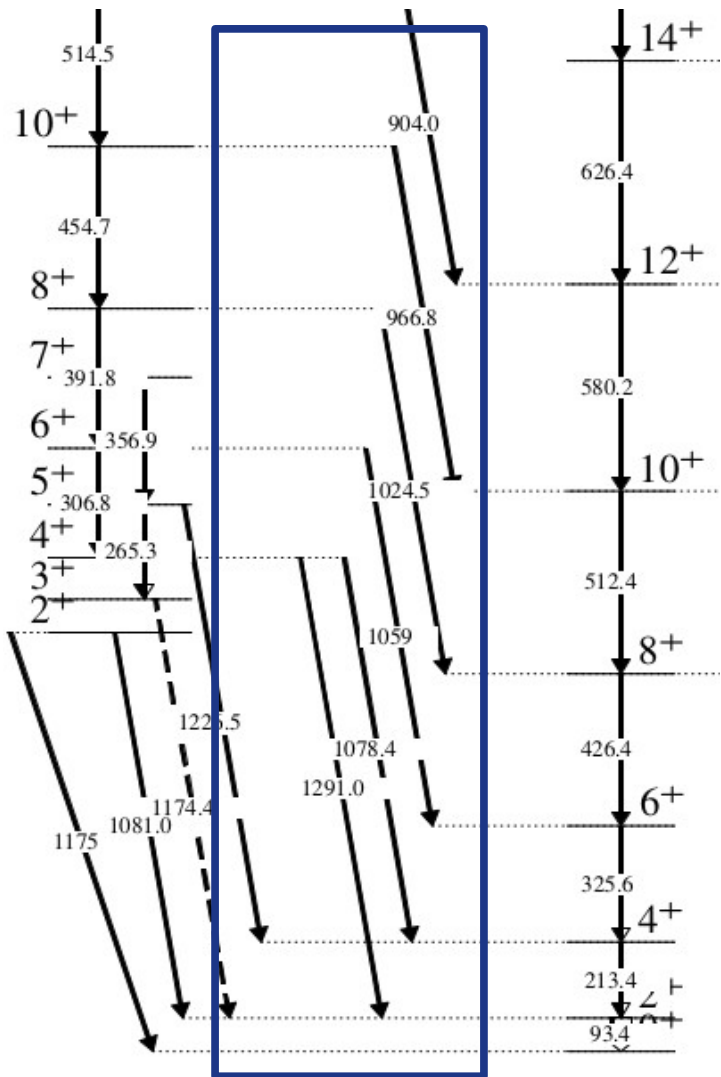


The intrinsic matrix elements  $\langle K_f | \mathcal{M}\lambda | K_i \rangle = m_0 + \Delta I^2 m_1$

	$m$	$\nu$	$m_0$	$\sim 5\% \text{ error}$	Comment
$\langle 2^+   E2   0^+ \rangle$	0	0	0.266(12) eb	-0.00347(15) eb	<sup>a</sup> $m = 0.252(11)$
$\langle 4^+   E2   2^+ \rangle$	0	0	0.447(19) eb		
$\langle 4^+   E2   0^+ \rangle$	2	2	$9.1 \times 10^{-4}$ eb	$-1.47 \times 10^{-5}$ eb	$\pm 6\%$
$\langle 4^+   M1   0^+ \rangle$	3	3	$6.3 \times 10^{-5} \mu_N$	$-9.5 \times 10^{-7} \mu_N$	$\pm 30\%$
$\langle 6^+   E2   4^+ \rangle$	0	0	0.094(3) eb		
$\langle 6^+   E2   2^+ \rangle$	2	2	0.00116(10) eb		
$\langle 6^+   E2   0^+ \rangle$	4	4	$1.57 \times 10^{-6}$ eb	$-2.10 \times 10^{-8}$ eb	$\pm 3.5\%$
$\langle 6^+   M2   8^- \rangle$	0	0	0.102(9) $\mu_N b^{1/2}$		
$\langle 8^-   E3   2^+ \rangle$	3	3	$0.36_{-0.07}^{+0}$ eb <sup>3/2</sup>		<sup>b</sup> Alaga rule
$\langle 8^-   E3   0^+ \rangle$	5	5	$0.37_{-0.01}^{+0.07}$ eb <sup>3/2</sup>		<sup>b</sup> Alaga rule

Hayes et al., Phys. Rev. C 75, 034308 (2007)

# Collective Rotor: Intrinsic vs. Individual M.E.



Quotable

GSB  $\rightarrow$   $\gamma$ -band  
 $K$ -allowed  $\langle \gamma | E2 | \text{GSB} \rangle = 0.266(12)$   
 $-3.47(15) \times 10^{-3} \{I_\gamma(I_\gamma + 1) - I_0(I_0 + 1)\}$  (eb)

$I_i$	$I_f$	$m$ (eb)	$B(E2) \times 10^3$ ( $e^2b^2$ )	$I_i$	$I_f$	$m$ (eb)	$B(E2) \times 10^3$ ( $e^2b^2$ )
0	2	0.347	120.	10	10	0.489	11.1
2	2	0.449	40.3	10	11	-0.683	42.8
2	3	-0.548	60.1	10	12	0.212	52.4
2	4	0.318	20.2	10	11	-0.683	22.2
4	2	0.119	1.57	10	12	0.212	2.14
4	3	-0.415	19.1	12	10	0.595	14.2
4	4	0.668	49.6	12	11	-1.10	48.8

Informational

Table 9.5: Reduced matrix elements  $m = \langle I_f, K = 2^+ || E2 || I_i, K = 0^+ \rangle$  and  $B(E2) = B(E2; \text{GSB} \rightarrow \gamma)$  values for the  $K$ -allowed E2 transitions.  $\langle \gamma | E2 | \text{GSB} \rangle = 0.252(11)$  eb to first order in  $\Delta I^2$ .

# Typical Applications

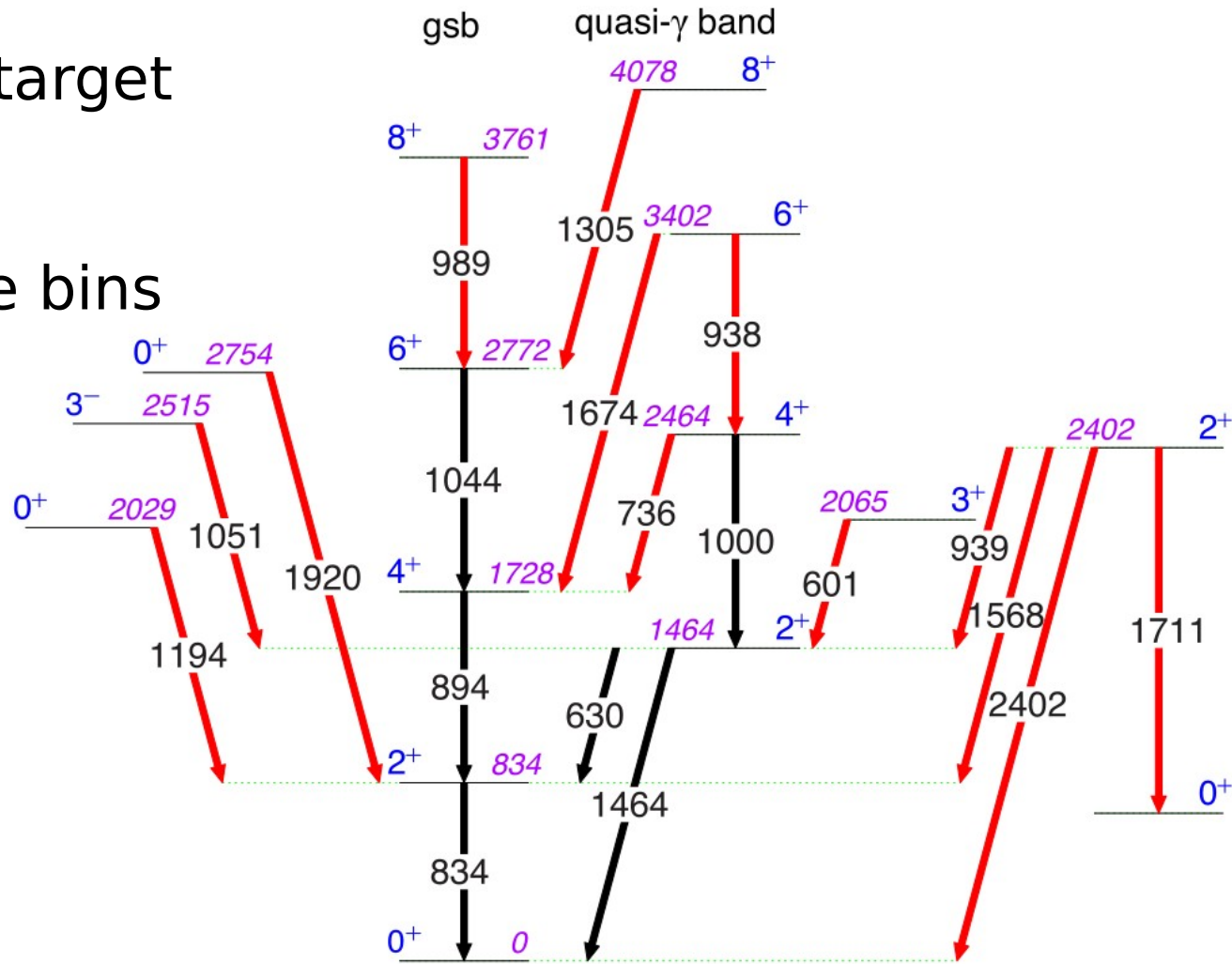


Weaker  $B(E2)$ , several-state problems

# $^{72}\text{Ge}$ Coulex



- $^{72}\text{Ge}$  on  $^{208}\text{Pb}$  0.5 mg target at 301 MeV
- 7  $10^\circ$  scattering angle bins from  $30^\circ$  to  $165^\circ$
- E2 couplings of primary interest



Ayangeakaa et al., PLB **754**, 254 (2016)



# $^{72}\text{Ge}$ Coulex



Reduced  $E2$  matrix elements for transitions of  $^{72}\text{Ge}$ , deduced from the present work, in comparison with previous measurements.

- Results very similar to previous measurements
- What is a reasonable expectation of the errors?

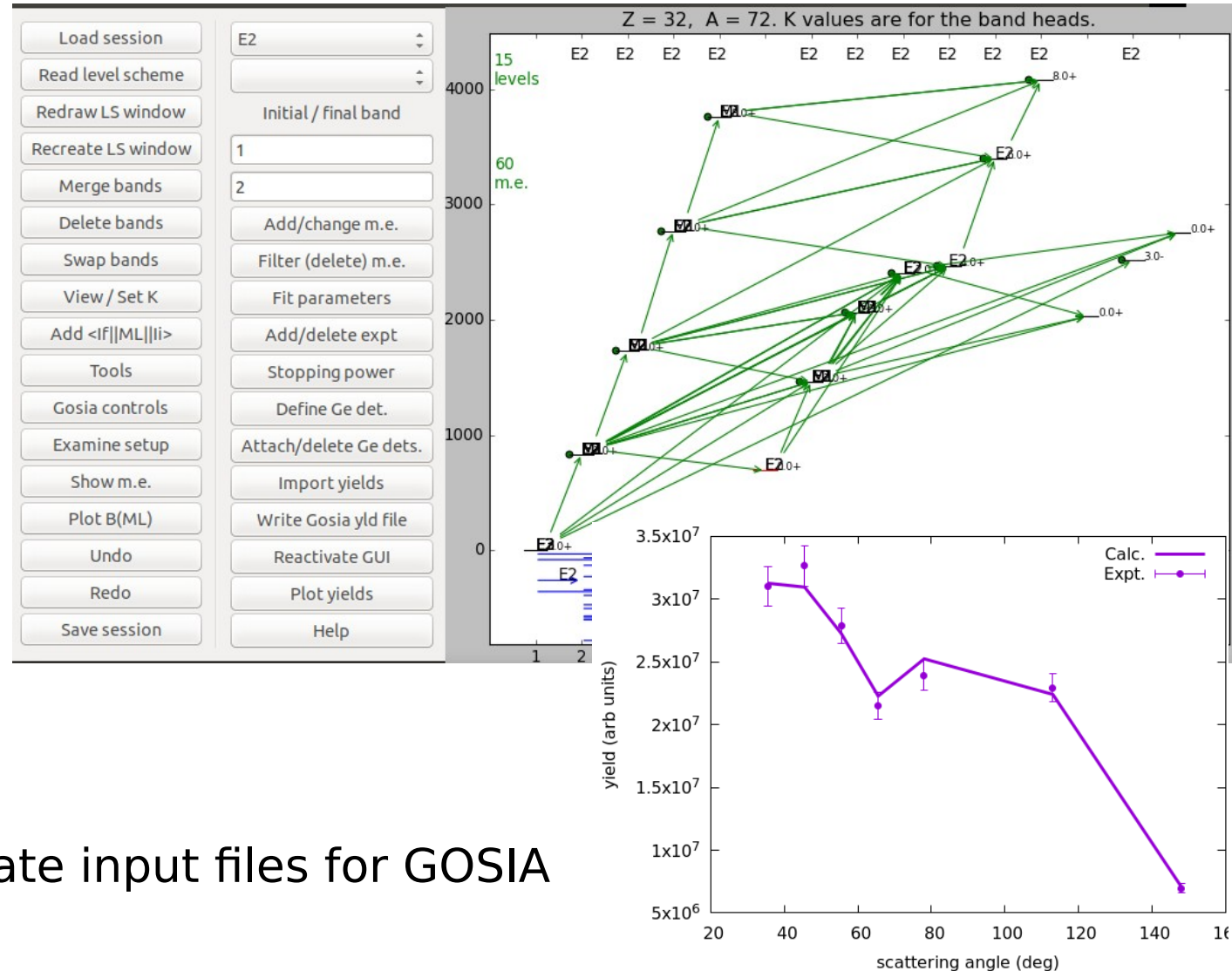
$I_i^\pi \rightarrow I_f^\pi$	$\langle I_i    \mathcal{M}(E2)    I_f \rangle$ (eb)		
	This work	Ref. [34]	Refs. [40,41]
$0_1^+ \rightarrow 2_1^+$	0.457(4)	0.46(1)	0.457(7)
$2_1^+ \rightarrow 4_1^+$	0.90(2)	0.89(4)	0.76(4)
$4_1^+ \rightarrow 6_1^+$	$1.11^{+0.04}_{-0.05}$	1.2(3)	
$6_1^+ \rightarrow 8_1^+$	$1.1^{+0.2}_{-1.6}$		
$2_1^+ \rightarrow 2_1^+$	$-0.16^{+0.07}_{-0.02}$	$-0.16^{+0.10}_{-0.07}$	-0.17(8)
$4_1^+ \rightarrow 4_1^+$	$-0.14^{+0.09}_{-0.04}$	-0.01(1)	
$6_1^+ \rightarrow 6_1^+$	$-0.20^{+0.08}_{-0.25}$	-0.1(5)	
$2_1^+ \rightarrow 0_2^+$	$0.35^{+0.01}_{-0.02}$	0.36(4)	0.45(2)
$4_1^+ \rightarrow 2_2^+$	$-0.06^{+0.03}_{-0.04}$	-0.08(5)	
$6_1^+ \rightarrow 4_2^+$	$0.28^{+0.10}_{-0.05}$	< 0.4	

Ayangeakaa et al., PLB **754**, 254 (2016)

# Aside: RACHEL UI for Gosia



- “Semi-GUI”
- Developed in 2005, updates in progress for Python3, Qt...
- Simulation
- Experiment planning
- **Design experiments for analysis**
- Data analysis
- Plots of results
- Run in RACHEL / generate input files for GOSIA



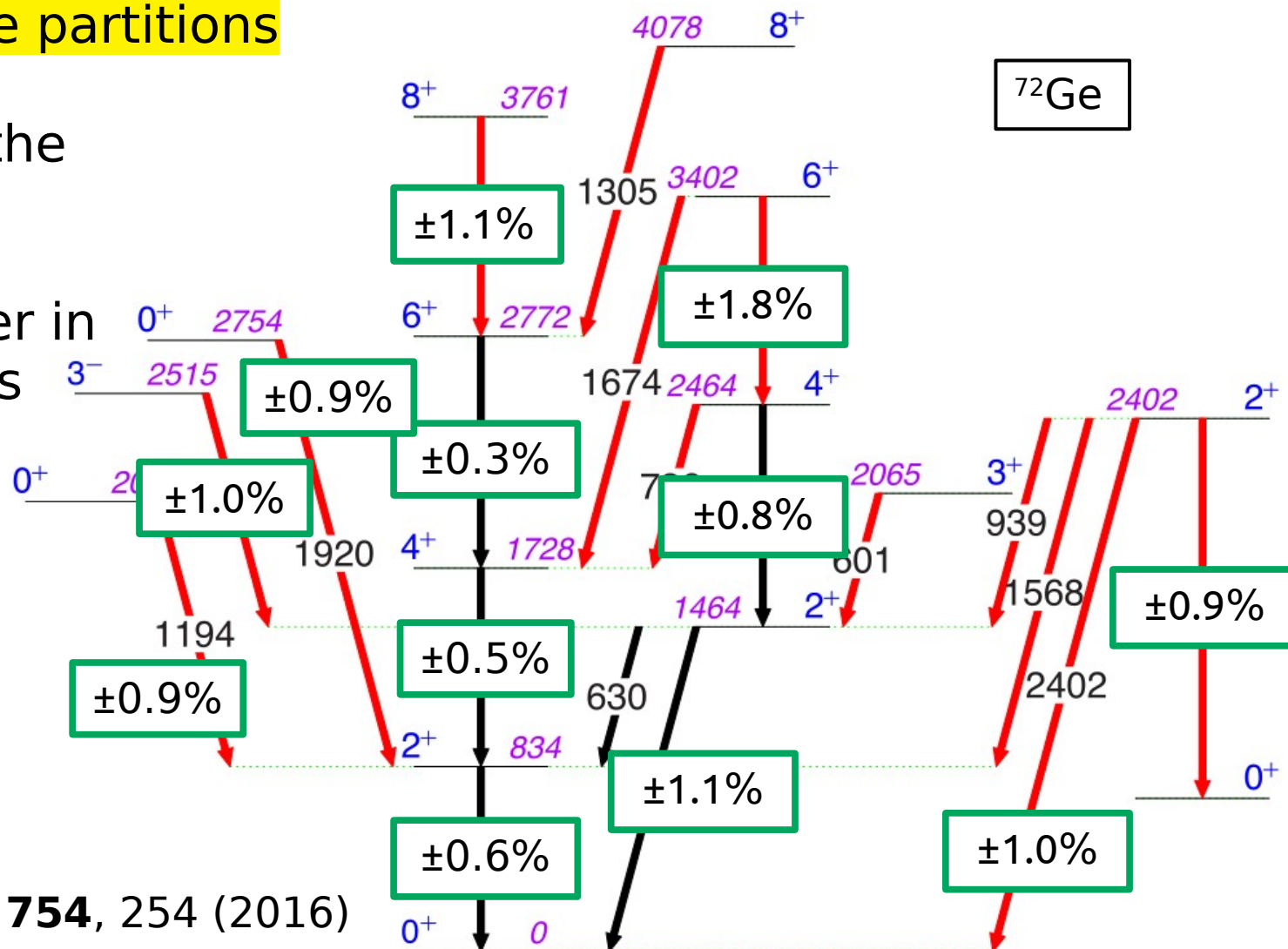


# Simulation: Statistical (only) Error with ~5% Error in Yields



<sup>72</sup>Ge

- 7 scattering angle partitions
- 5% error bars in the simulated data
- No random scatter in data → no conflicts

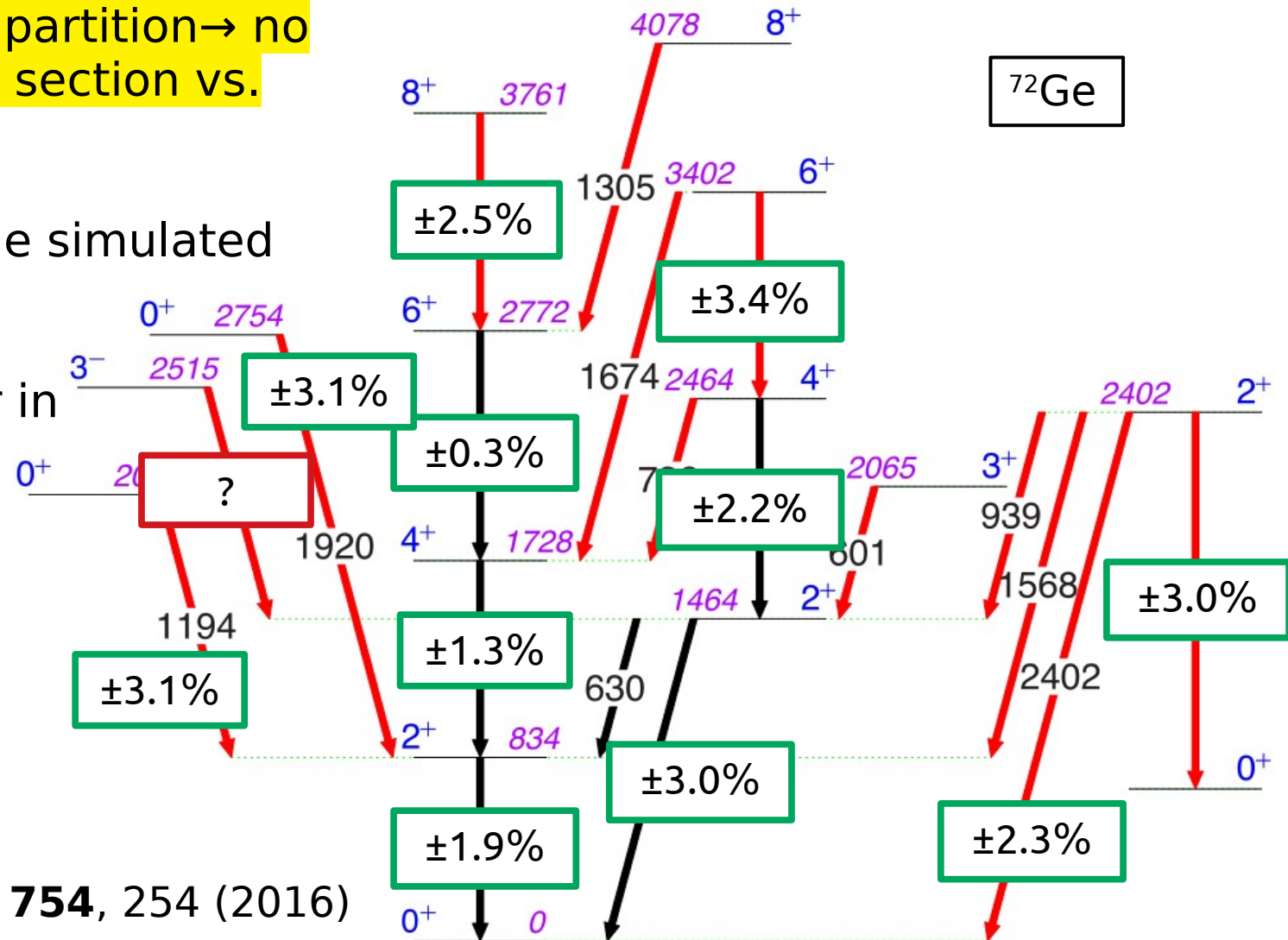


Ayangeakaa et al., PLB **754**, 254 (2016)

# Simulation: Statistical (only) Error with ~5% Error in Yields



- 1 scattering angle partition → no sensitivity to cross section vs. impact parameter
- 5% error bars in the simulated data
- No random scatter in data → no conflicts



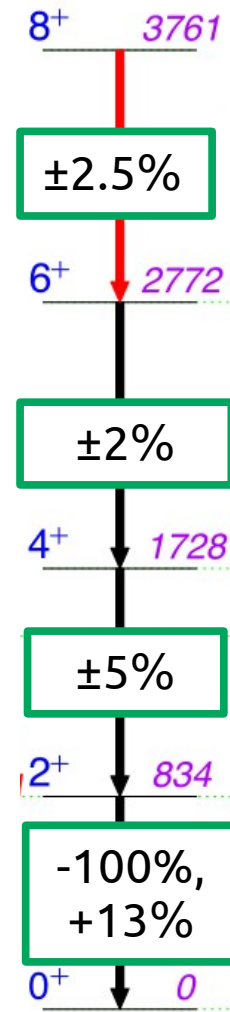
Ayangeakaa et al., PLB **754**, 254 (2016)

# Simulation: Statistical (only) Error with ~5% Error in Yields



- 1 scattering angle partition → no sensitivity to cross section vs. impact parameter
- Ground sequence only
- 5% error bars in the simulated data
- No random scatter in data → no conflicts
- NOTE: strongly deformed case would have no sensitivity to  $2 \rightarrow 0$

$^{72}\text{Ge}$

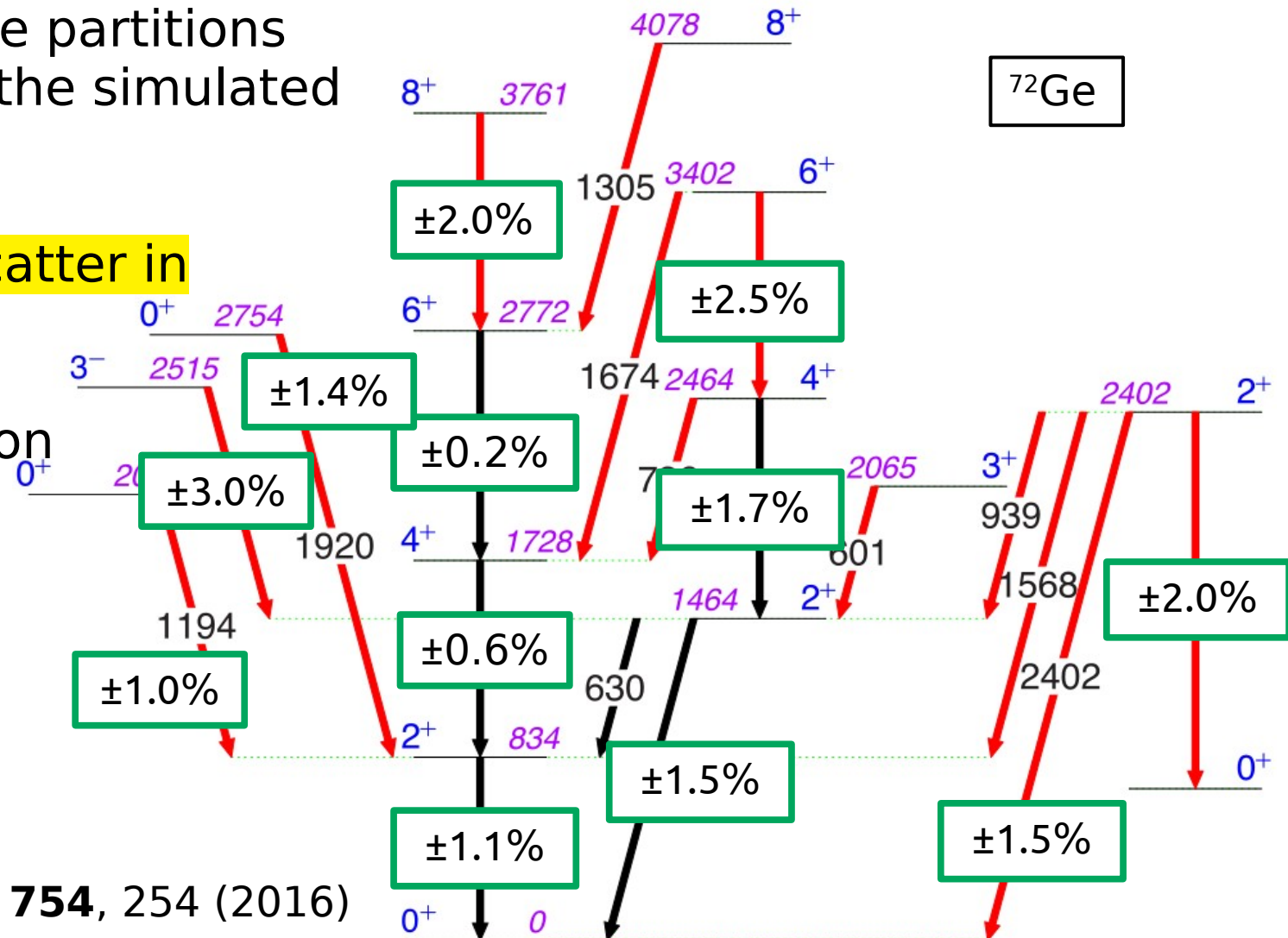


Ayangeakaa et al., PLB **754**, 254 (2016)

# Simulation: Statistical (only) Error with ~5% Error in Yields



- 7 scattering angle partitions  
5% error bars in the simulated data
- **WITH** random scatter in data → conflicts
- Gives an indication of the best-case sensitivity in the measurement

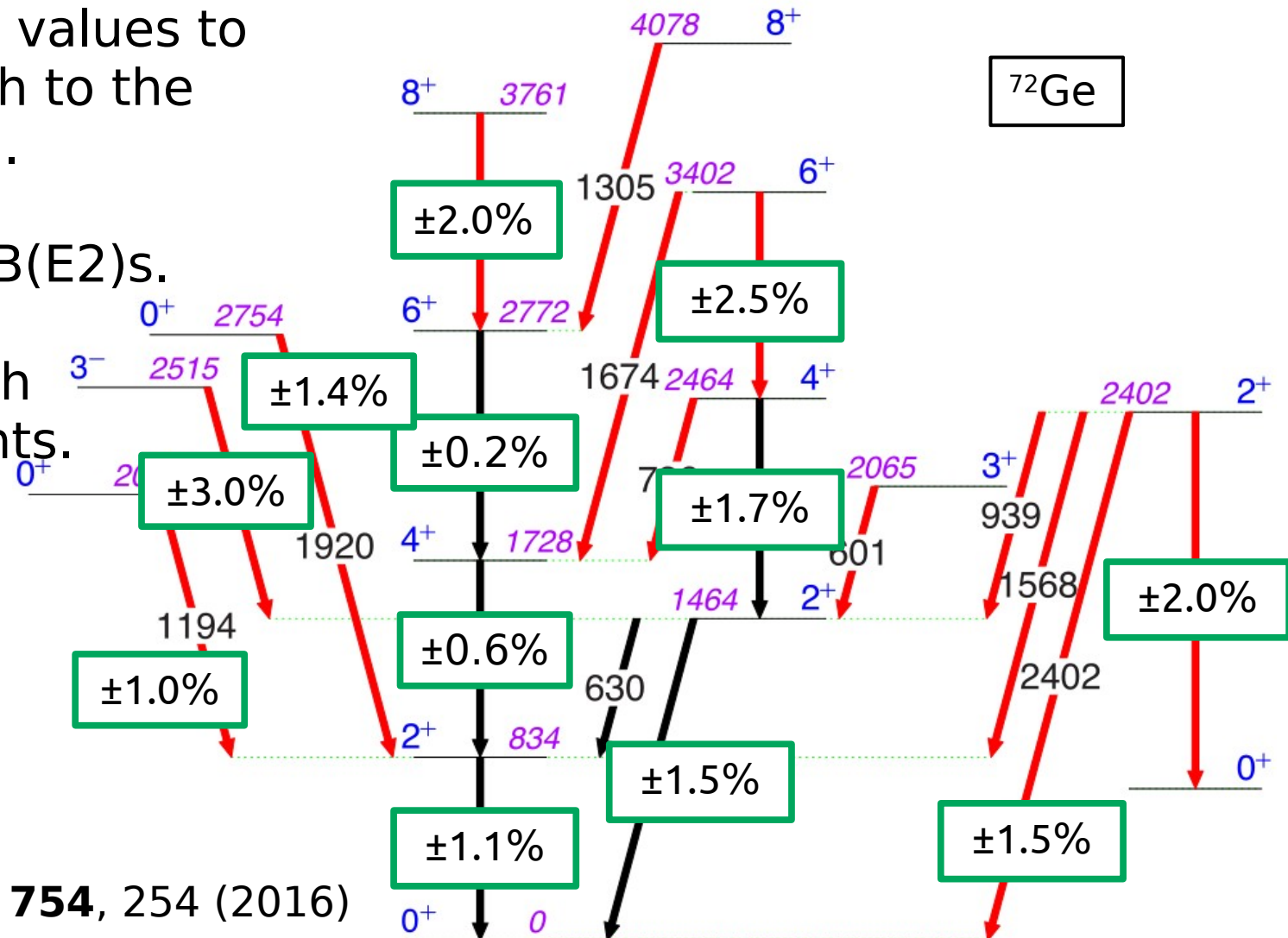


Ayangeakaa et al., PLB **754**, 254 (2016)

# Simulation: Statistical (only) Error with ~5% Error in Yields



- Use known B(E2) values to narrow the search to the correct minimum.
- Remove known B(E2)s.
- Repeat the search without constraints.



Ayangeakaa et al., PLB **754**, 254 (2016)



# Typical Applications

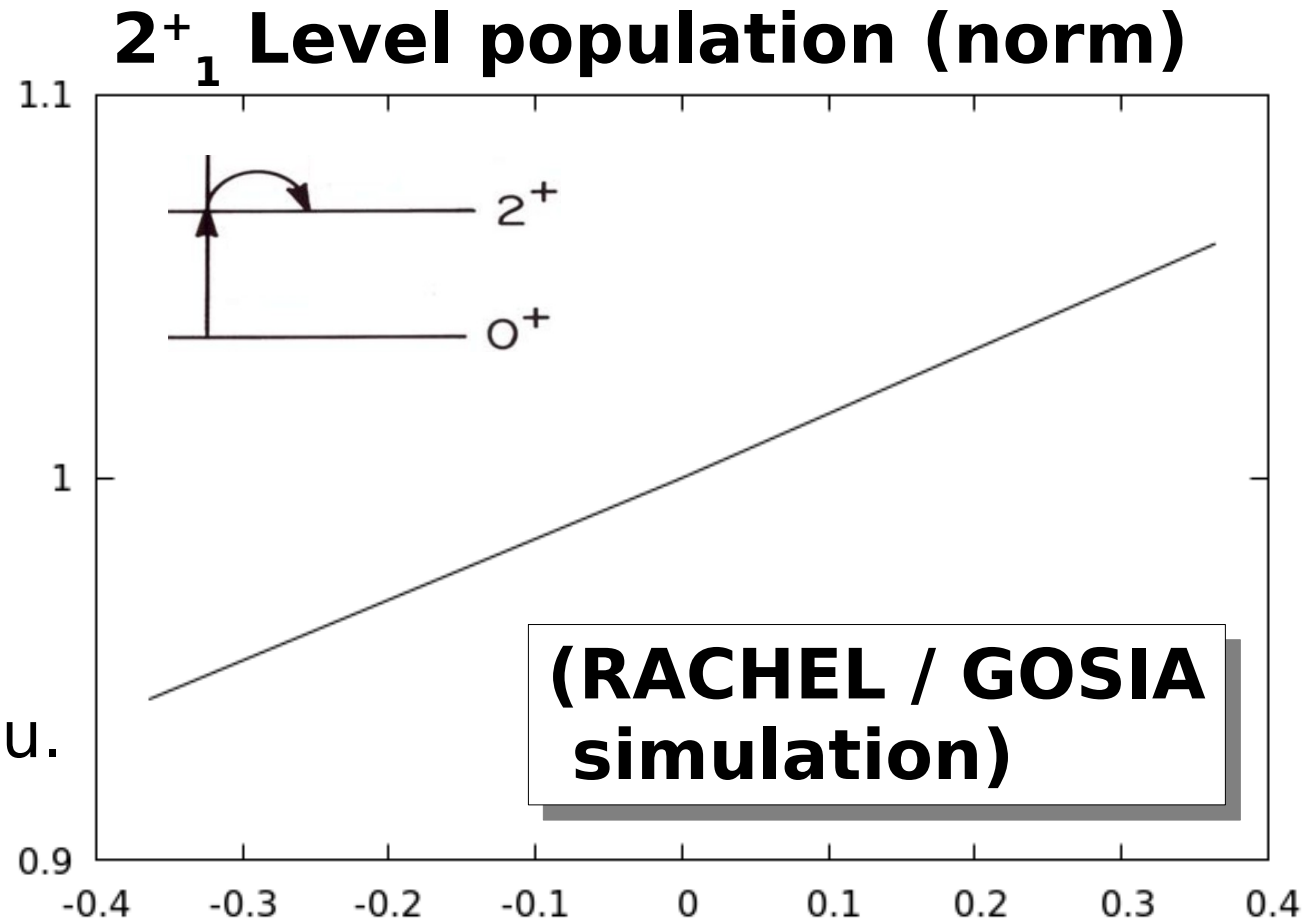


(Approximately) Two-state Problems

# Reorientation Effects



- Coulomb excitation of  $^{60}\text{Ni}$  beam by  $^{16}\text{O}$  beam at safe energy of 30MeV
- Predicted population of  $2^+_{1}$  excited state
- Known  $B(E2; 2^+_{1} \rightarrow 0^+_{1}) = 13 \text{ W.u.}$
- Constructive / destructive interference

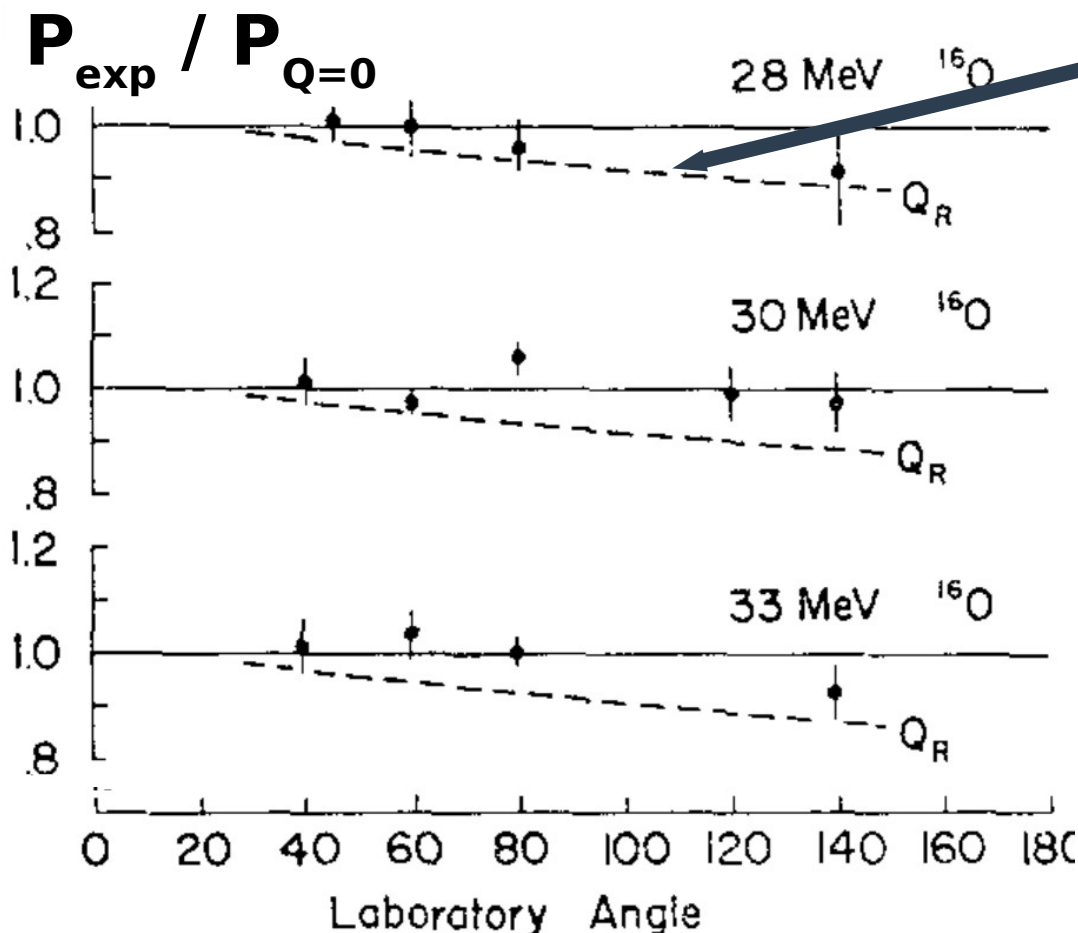


$$Q_{2+} = \langle 2^+_{1} || E2 || 2^+_{1} \rangle$$

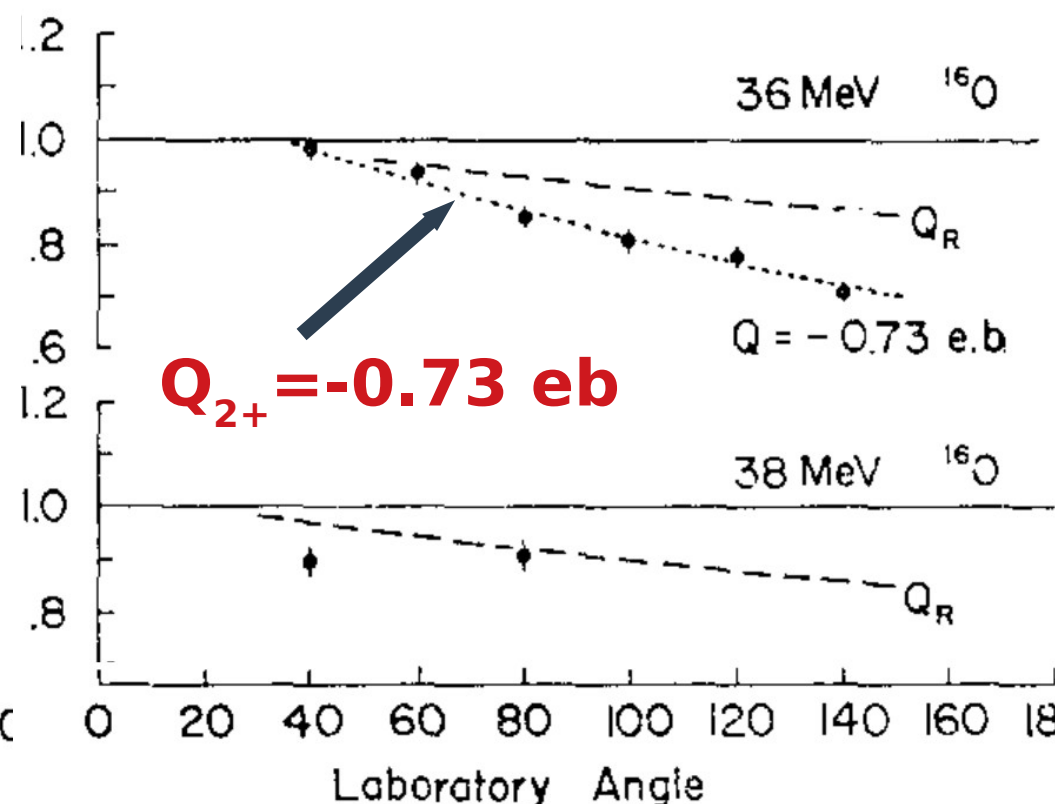
# Reorientation Effects: Coulomb-Nuclear Interference



Population  $P_{\text{exp}}/P_{Q=0}$  of  $2^+$  state vs. scattering angle (spectrograph)  
Cline et al., Nucl Phys A 133, 445 (1969)



**Assuming rotor**



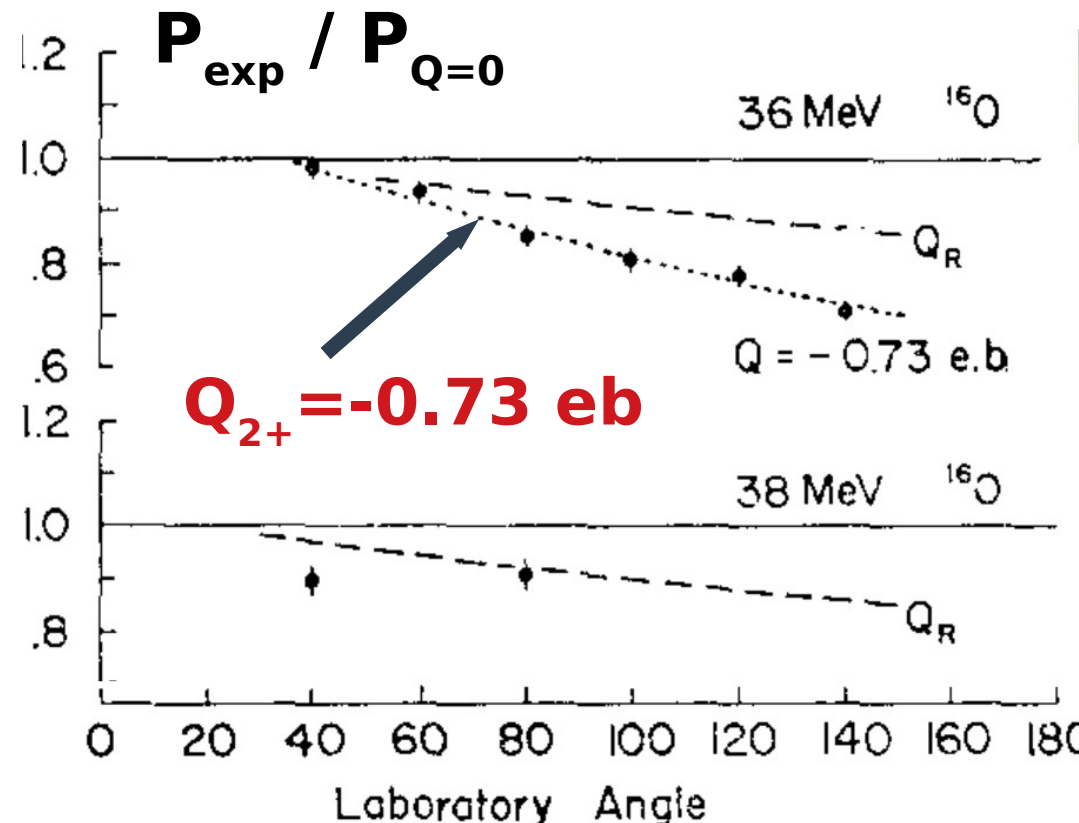


# Reorientation Effects: Systematic Error



Population  $P_{\text{exp}}/P_{Q=0}$  of  $2^+$  state vs. scattering angle (spectrograph)  
Cline et al., Nucl Phys A 133, 445 (1969)

- Requires  $E_{\text{beam}} \leq 30 \text{ MeV}$
- Equivalently, surface separation of  $r=1.25\text{fm} (A_t^{1/3} + A_p^{1/3}) \geq 5 \text{ fm}$
- Coulex is not “safe” for high energy by limiting scattering angle!
- Static moment is the first thing to go.

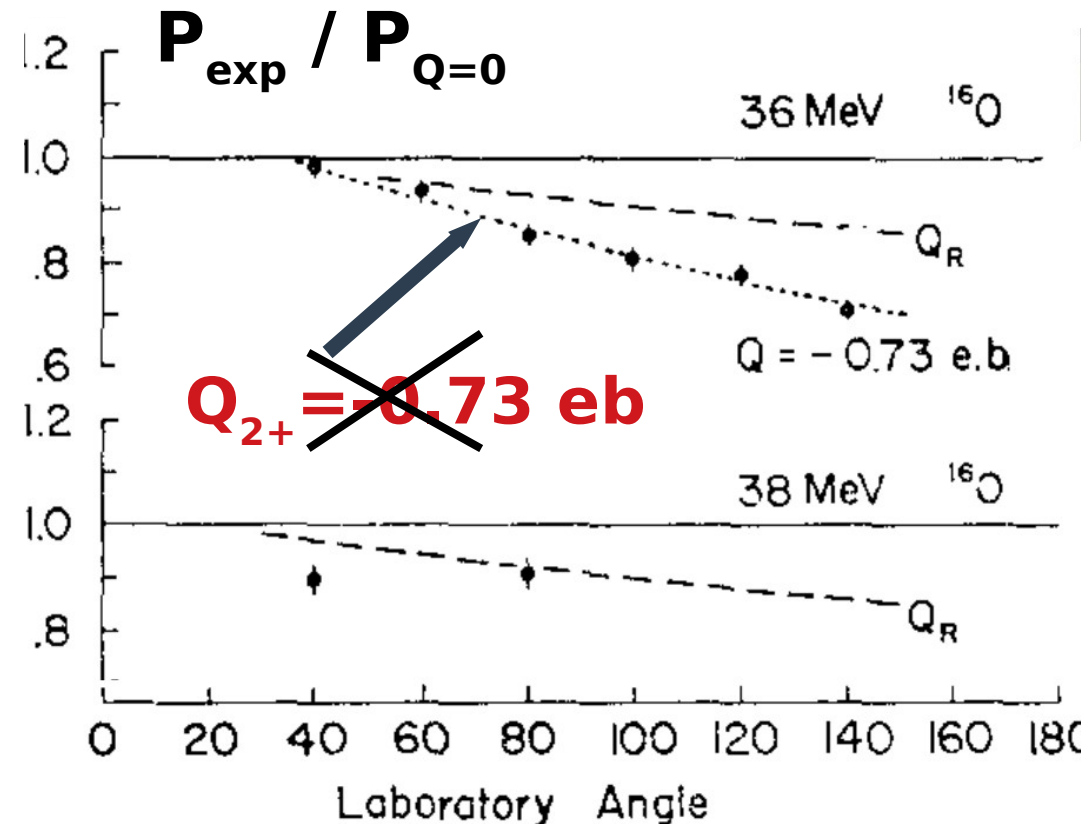


# Reorientation Effects: Systematic Error

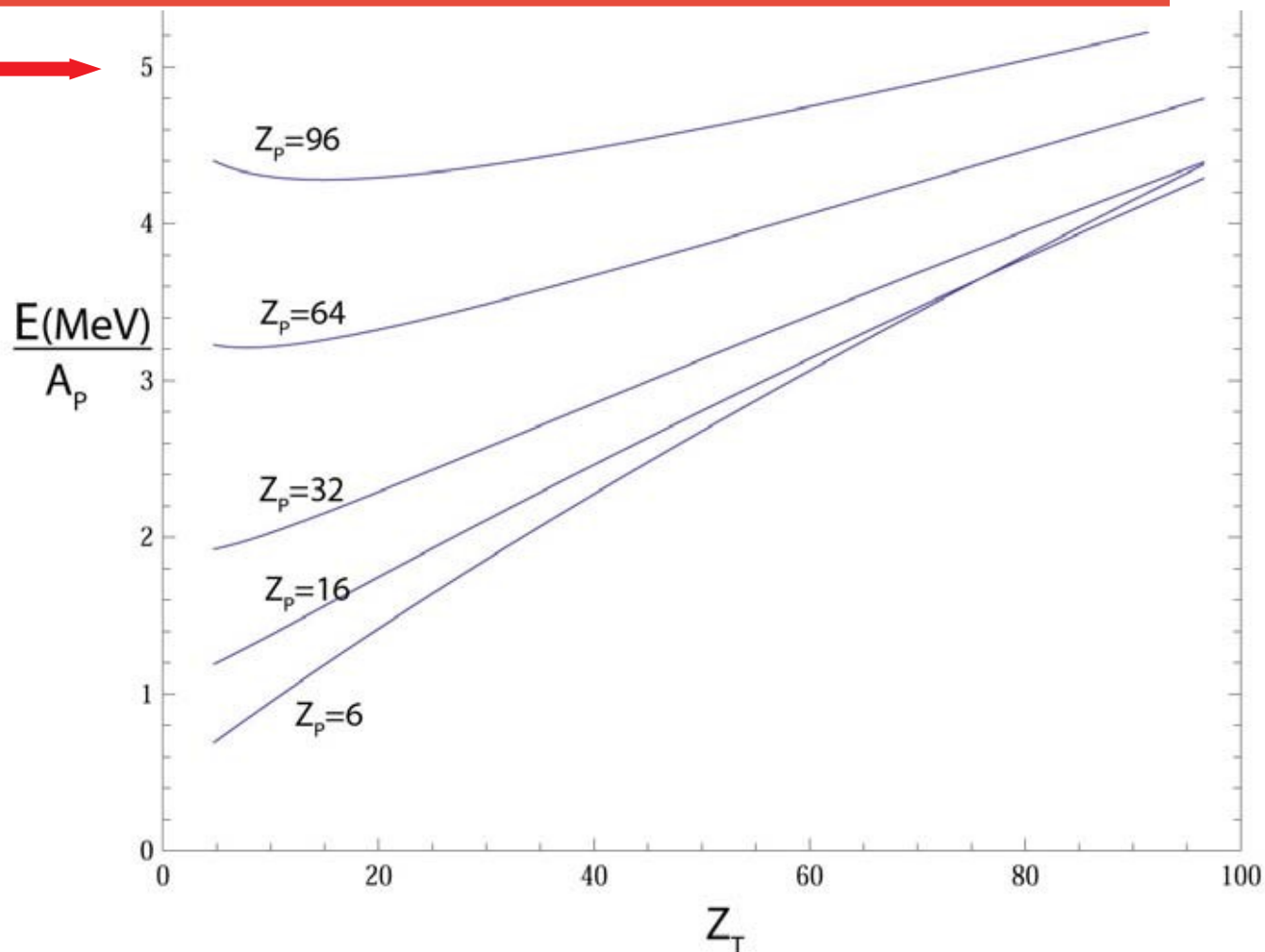


Population  $P_{\text{exp}}/P_{Q=0}$  of  $2^+$  state vs. scattering angle (spectrograph)  
Cline et al., Nucl Phys A 133, 445 (1969)

- $E_{\text{beam}} \leq 30 \text{ MeV}$
- $B(E2; 0^+ \rightarrow 2^+) = 0.0917(18) e^2 b^2$
- $Q_{2^+} = 0.00(8) \text{ eb}$



# Semi-classical



Maximum safe bombarding energy per nucleon as a function of target  $Z$ .  
(Gosia manual)

# Gosia2



- ~1 strongly populated state; can't self-normalize
- Normalization to Rutherford is difficult experimentally (as opposed to older spectrograph data).

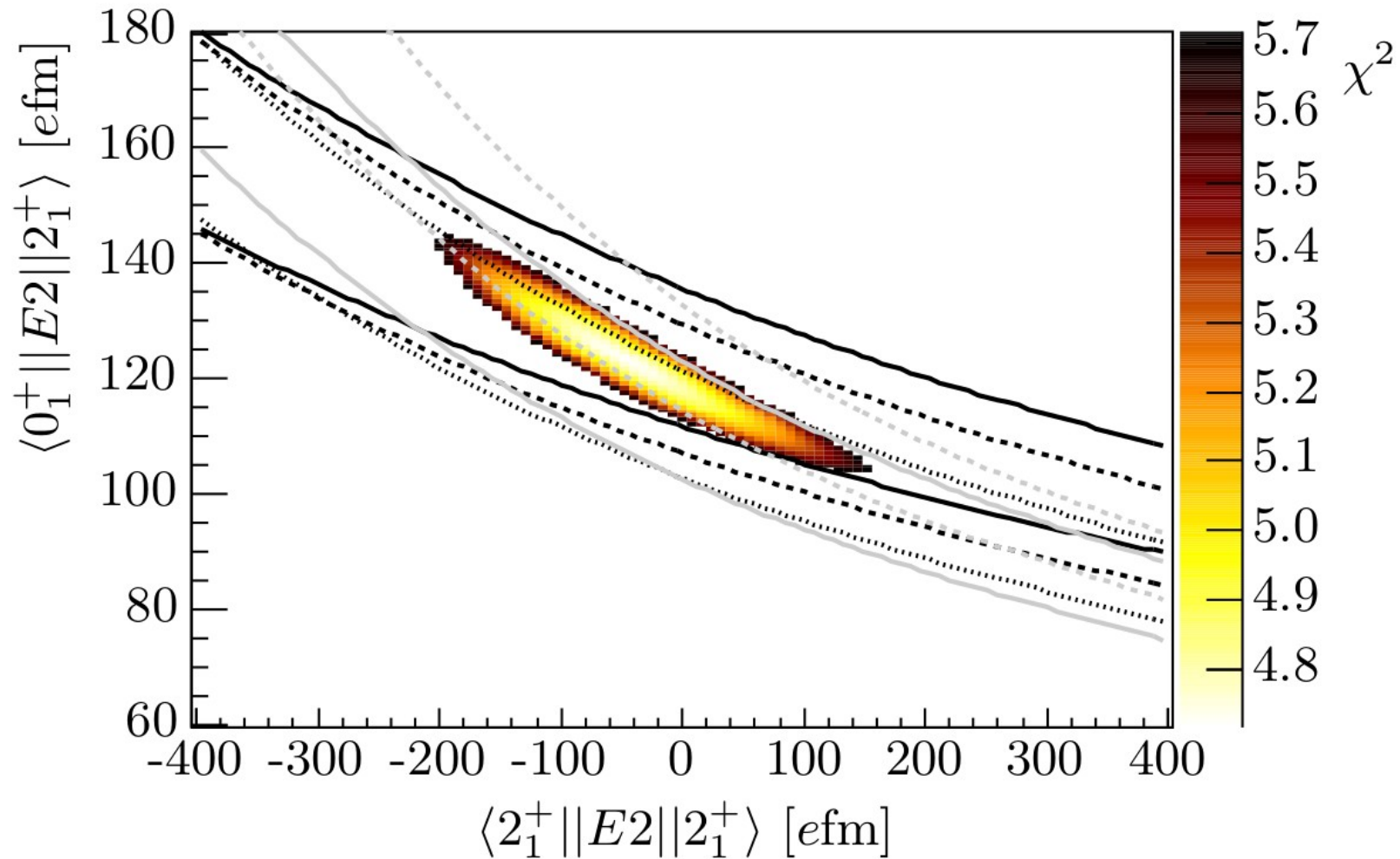
- Make use of mutual target / projectile excitation.

$$\frac{N_p}{N_t} = \frac{b_p \epsilon_\gamma(E_p) \sigma_p}{b_t \epsilon_\gamma(E_t) \sigma_t}$$

- Deduce transition probability (usually B[E2]) from known transition probability in collision partner.

→ Independent measurement of **quantity of interest**, but does require input of **previous measurements for collision partner**.

- **The static moment and B(E2) both affect population. Accuracy and realistic uncertainty require correlated error calculation.**

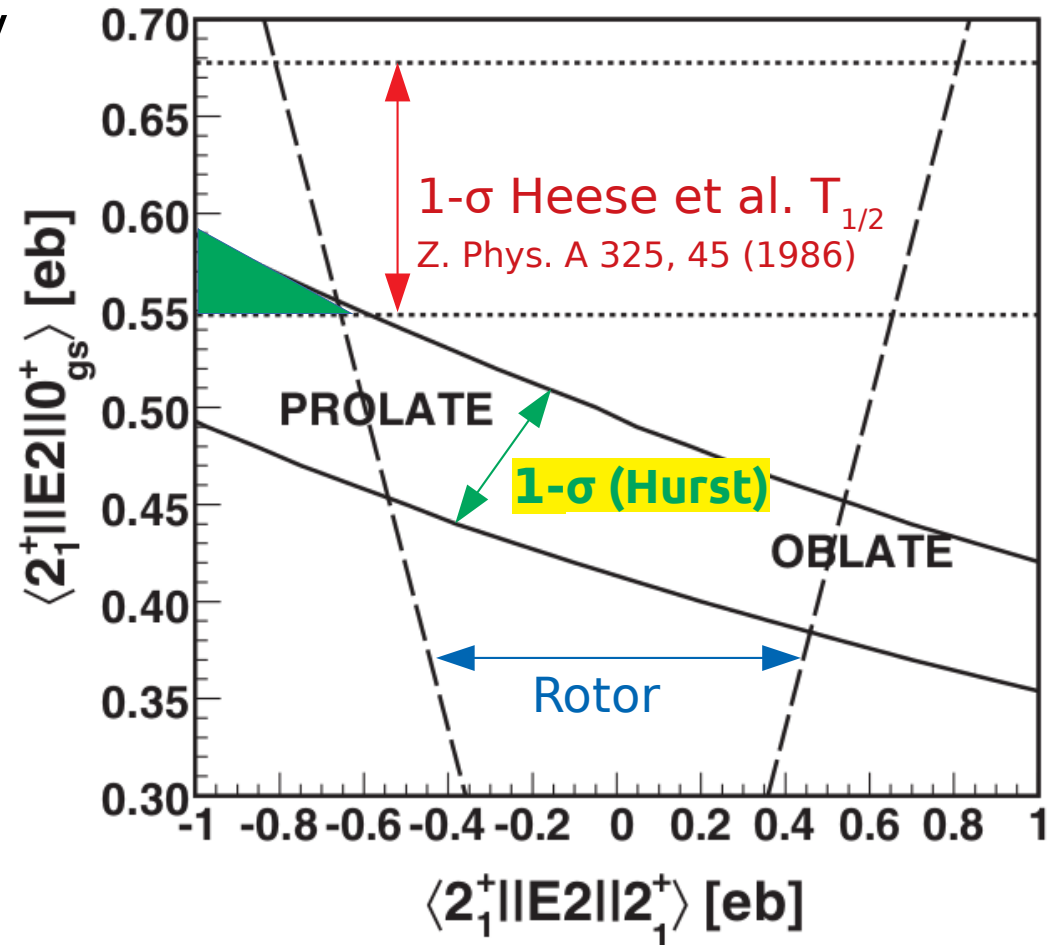


M. Zielińska et al.: Eur. Phys. J. A (2016) 52: 99

# $^{70}\text{Se}$ : Oblate or Prolate?



- Hurst et al.:  $^{70}\text{Se}$  on  $^{104}\text{Pd}$  @ 206MeV (>7fm separation)
  - <2% population of states other than  $2^+_1$
- Gosia2: norm proj to targ  $\gamma$ -yield
- Fit  $B(E2; 0^+ \rightarrow 2^+_1)$ ,  $\langle 2^+_1 || E2 || 2^+_1 \rangle$
- Requires accurate data for collision partner  $^{104}\text{Pd}$ 
  - Luontama et al.  $^{104}\text{Pd}$  (p,2n), (p,p'), Coulex (1986)
- Consistency with  $T_{1/2}$  meas requires  $\langle 2^+_1 || E2 || 2^+_1 \rangle$  less than -0.5eb
  - consistent with **prolate** deformation



Note: Common mistake is to fit  $B(E2)$  without including correlations with  $\langle 2^+_1 || E2 || 2^+_1 \rangle$  in error calculation.

A.M. Hurst et al., PRL 98, 072501 (2007).



# Validation of the Method in $^{70}\text{Se}$ Experiment Using $^{74}\text{Se}$



- Measured gamma yields for Coulex of  $^{74}\text{Se}$  on  $^{104}\text{Pd}$
- Combined with  $\langle 2^+ || E2 || 2^+ \rangle = -0.36(7)$  eb (19% error) from Lecomte PRC 18, 2801 (1978).
- Adopted  $B(E2) = 0.387(8)$  e<sup>2</sup>b<sup>2</sup> (2% err)
- Hurst et al. obtained  $B(E2; 0^+ \rightarrow 2^+) = 0.36(2)$ e<sup>2</sup>b<sup>2</sup> (5.5% err)

A.M. Hurst *et al.*, PRL 98, 072501 (2007).

# Conclusions



- Very precise measurements are possible using Coulex and GOSIA
- Partitioning of data is very important (i.e. scattering angle)
- Collective strength / many populated states → relative gamma yields give absolute measurements
- Two-state problems require
  - Normalization to collision partner yields
  - Known  $B(E2)$ ,  $\langle 2+ || E2 || 2+ \rangle$  of collision partner
- Other problems lie somewhere in the middle
- Safe Coulex usually better than high statistics
- Inverse kinematics—don't get me started...
- Include all matrix elements in correlated error calculations
- Plan and simulate analysis before submitting proposals!





END



END



END



**END**

# Terminology



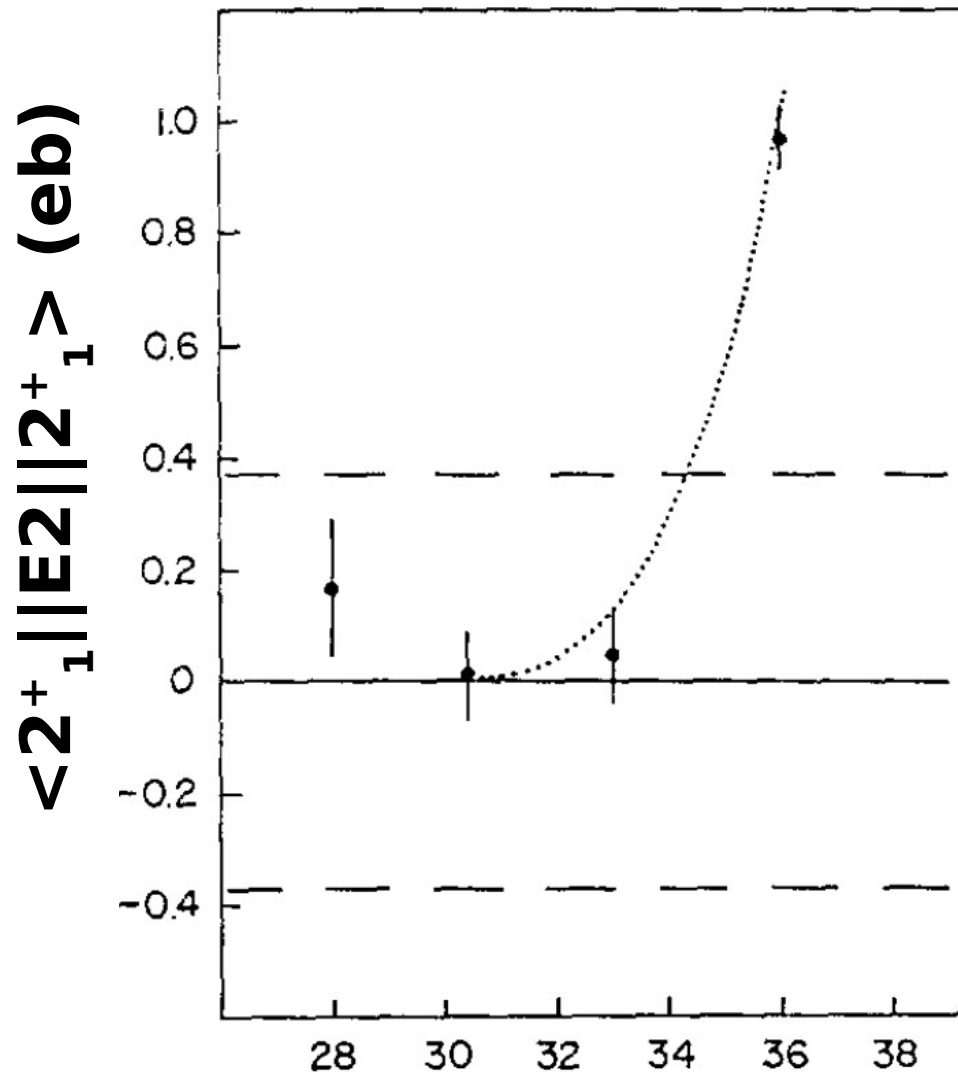
Yield	Gamma-ray intensity following Coulex
EM multipole operator matrix element	$\langle I_s M_s   M(\lambda, \mu)   I_f M_f \rangle = (-1)^{I_s - M_s} \begin{pmatrix} I_s & \lambda & I_f \\ -M_s & \mu & M_f \end{pmatrix} \langle I_s    M(\lambda)    I_f \rangle$
Reduced matrix element	$\langle I_s    M(\lambda)    I_f \rangle$
Quadrupole moment (in-band transitions)	$\langle I_f K    E2    I_i K \rangle = (2I_i + 1)^{1/2} \langle I_i K 20   I_f K \rangle \sqrt{\frac{5}{16\pi}} eQ_o$
"Intrinsic" matrix element (inter-band transitions)	$\langle K_f I_f    \mathcal{M}(\lambda)    K_i I_i \rangle = \sqrt{2I_i + 1} \langle I_i 0 \lambda K_f   I_f K_f \rangle \times \langle K_f   \mathcal{M}(\lambda, \mu = K_f)   K_i = 0 \rangle \begin{cases} \sqrt{2} & K_f \neq 0 \\ 1 & K_f = 0 \end{cases}$
Reduced transition probability	$B(E(M)\lambda; I_i K_i \rightarrow I_f K_f) \equiv \frac{ \langle I_f K_f    E(M)(\lambda, \mu)    I_i K_i \rangle ^2}{2I_i + 1}$

# Terminology



GSB	Ground-state rotational Band
Static moment	$\langle I    M(\lambda)    I \rangle$
“Safe” Coulex	Collision energy low enough that Coulomb-nuclear interference is negligible. Rule of thumb: $E_{\text{beam}} \leq 80\% E_{\text{barrier}}$
Sommerfeld parameter?	
Adiabaticity?	
Eccentricity?	

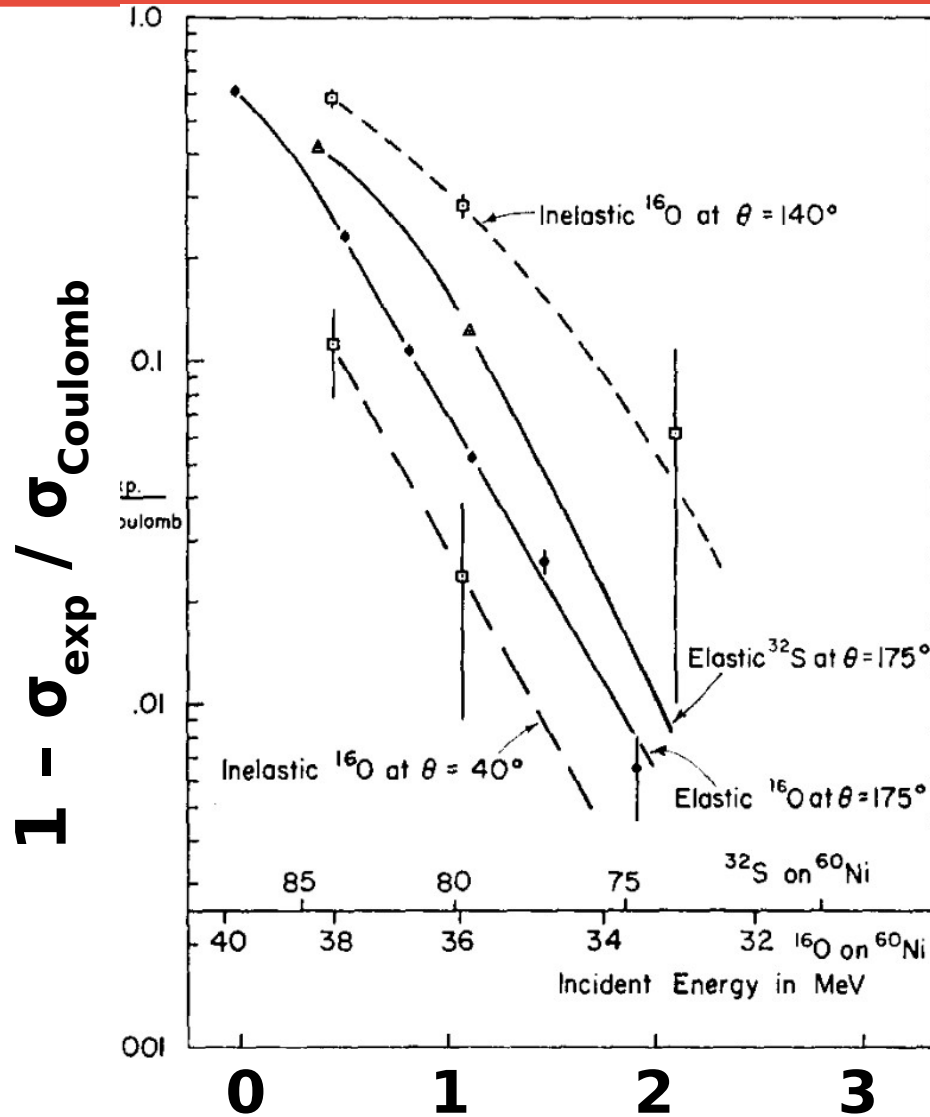
# Reorientation Effects: Systematic Error



- Requires  $E_{\text{beam}} \leq 30$  MeV
- Equivalently, surface separation of  $r=1.25\text{fm} (A_t^{1/3} + A_p^{1/3}) \geq 5$  fm
- Coulex is not “safe” for high energy by limiting scattering angle!
- Static moment is the first thing to go.

$^{16}\text{O}$  Incident Energy (MeV)

# Reorientation Effect



- Requires  $E_{\text{beam}} \leq 30$  MeV
- Equivalently, surface separation of  $r = 1.25 \text{ fm} (A_t^{1/3} + A_p^{1/3}) \geq 5$  fm
- Coulex is not “safe” for high energy by limiting scattering angle!
- Static moment is the first thing to go.

**Surface separation (fm)**