# Lattice calculation of the hadronic contributions to the muon anomalous magnetic moment

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*Lattice Meets Experiment: BSM*
*Brookhaven National Lab*
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# Collaborators

Past/On-going work on g-2 done in collaboration with

| HVP | HLbL |
|---|---|
| Christopher Aubin (Fordham U) | Saumitra Chowdhury (UConn) |
| Maarten Golterman (SFSU) | Masahi Hayakawa (Nagoya) |
| Santiago Peris (SFSU/Barcelona) | Taku Izubuchi (BNL/RBRC) |
| | Eigo Shintani (RBRC) |
| RBC/UKQCD | Norikazu Yamada (KEK) |

New work starting with RBC/UKQCD collab (Christ, Jin, ...)

# Outline

Motivation and Introduction

The hadronic vacuum polarization (HVP) contribution (O($\alpha^2$))

The hadronic light-by-light (HLbL) contribution (O($\alpha^3$))

Summary/Outlook

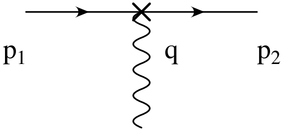
# New experiments + new theory = (?) new physics

muon anomaly $a_\mu$ provides important test of the SM

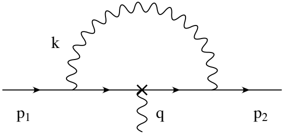
- BNL E821: $a_\mu^{\mathrm{exp}}$ accuracy is 0.54 ppm
- Fermilab E989, start is $\sim 3$ years away, goal is 0.14 ppm
- J-PARC E34
- $a_\mu(\mathrm{Expt})$-$a_\mu(\mathrm{SM}) = 287(63)(51)$ $(\times 10^{-11})$, or $\sim 3.6\sigma$
- If both central values stay the same,
  - E989 ($\sim 4\times$ smaller error) $\rightarrow\sim 5\sigma$
  - E989+new HLBL theory (models+lattice, 10%) $\rightarrow\sim 6\sigma$
  - E989+new HLBL +new HVP (50% reduction) $\rightarrow\sim 8\sigma$
- Big discrepancy! (New Physics $\sim 2\times$ Electroweak)
- Lattice calculations crucial

# The magnetic moment of the muon

In interacting quantum (field) theory $g$ gets corrections




$+$




$+ \ldots$

$$\gamma^\mu \;\; \rightarrow \;\; \Gamma^\mu(q) \; = \left(\gamma^\mu \, F_1(q^2) + \frac{i\,\sigma^{\mu\nu}\,q_\nu}{2m}\,F_2(q^2)\right)$$

which results from Lorentz and gauge invariance when the muon is <u>on-mass-shell</u>.

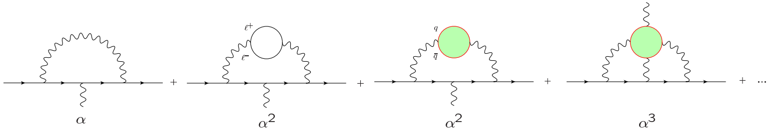
$$F_2(0) \;\; = \;\; \frac{g-2}{2} \equiv a_\mu \qquad\qquad (F_1(0) = 1)$$

(the anomalous magnetic moment, or anomaly)

# The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \dots$$




Corrections begin at $\mathcal{O}(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\dots$

hadronic contributions $\sim 6 \times 10^{-5}$ times smaller (leading error).

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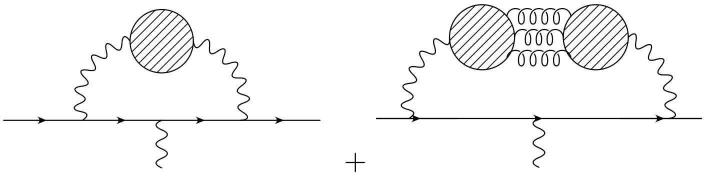
# Hadronic vacuum polarization (HVP) ($\alpha^2$)



The blobs, which represent all possible intermediate hadronic states, are not calculable in perturbation theory, but can be calculated from

- dispersion relation + experimental cross-section for $e^+e^-(\text{and } \tau) \to$ hadrons $a_\mu^{\text{had}(2)} = \frac{1}{4\pi^2}\int_{4m_\pi^2}^{\infty} \mathrm{d}s\, K(s)\sigma_{\text{total}}(s)$
- first principles using lattice QCD, $a_\mu^{(2)\text{had}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2\, f(Q^2)\,\Pi(Q^2)$ [Lautrup and de Rafael 1969, Blum 2002]

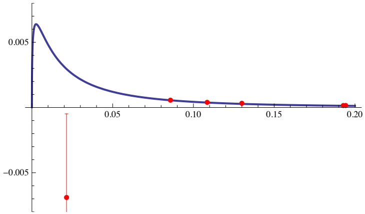
# $a_\mu$(HVP) lattice results

| $a_\mu$ | $N_f$ | errors | action | group |
|---|---|---|---|---|
| 713(15) | 2+1 | stat. | Asqtad | Aubin, Blum (2006) |
| 748(21) | 2+1 | stat. | Asqtad | Aubin, Blum (2006) |
| 641(33)(32) | 2+1 | stat., sys. | DWF | UKQCD (2011) |
| 674(21)(18) | 2+1+1 | stat., sys | TM | ETMC (2013) |
| 572(16) | 2 | stat. | TM | ETMC (2011) |
| 618(64) | 2(+1)[1] | stat., sys. | Wilson | Mainz (2011) |
| Exp. | | | | |
| 692.3 (4.2) | | | $e^+e^-$ | Davier, *et al.* (2011) |
| 694.9 (4.3) | | | $e^+e^-$ | Hagiwara, *et al.* (2012) |
| 701.5 (4.7) | | | $e^+e^-+\tau$ | Davier, *et al.* (2011) |

[1] strange quark is quenched

# $a_\mu$(HVP) integrand: low momentum region

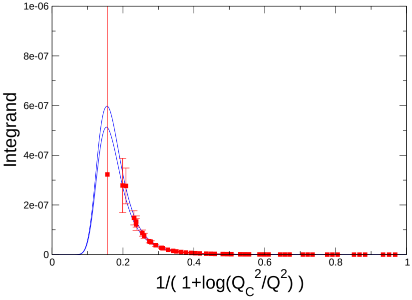
Finite volume $\rightarrow$ minimum finite momentum
Integral dominated by low $Q^2 \sim m_\mu^2$ region. Stat. errors larger too




Integrand of $a_\mu^{\rm HLO}/(4\alpha^2)$ compared with data
(MILC, $a = 0.06$ fm , $m_\pi = 220$ MeV)

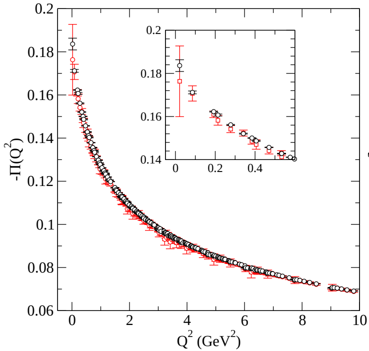
$\Rightarrow$ need more data at low $Q^2$ with smaller errors! In progress...

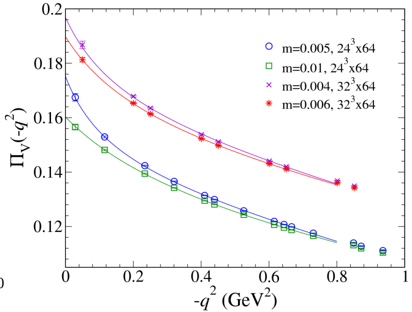
ABGP [Aubin, *et al.*, arXive:1205:3695]




UKQCD [arXive:1107.1497]

# $a_\mu$(HVP) Reducing statistical errors: All Mode Averaging




Use AMA, 1400 LM / 704 sources, $48^3 \times 144$ (MILC), 20 configs, 2.6-20 $\times$ error reduction for same cost!. RBC/UKQCD preliminary DWF results also show large error reduction (see Shintani, Lattice 2013).

[AMA method: Blum, Izubuchi, Shintani, Phys. Rev. D 88, 094503 (2013)]

# $a_\mu(\mathrm{HVP})$ errors

Controlling errors at the 1% level

- $Q^2$ dependence
  - All mode averaging (AMA) (statistics) Phys. Rev. D 88, 094503 (2013)
  - Twisted BC's or large box Mainz; Aubin etal, Phys. Rev. D 88, 074505 (2013)
  - Pade approximants for model independent fits PRD 86 054509 (2012)
  - avoid fit, analytic cont. (Ji and Jung, DESY+KEK, Mainz)
- physical quark masses / large boxes
- disconnected diagrams / isospin breaking
- charm contribution

Will give confidence that dispersive calculation is right

# RBC/UKQCD calculation of the HVP

- physical u,d,s quarks and quenched c
- large volume: $48 * 0.114 = 5.47$ fm box ($2\times$ in t dir) ($q_{\mathrm{min}} = 0.113$ GeV)
- Use AMA+random Z2 noise sources
- twisted b.c. for valence quarks for $q^2 = 0$
- eventually 0.086 fm ensemble as well
- Disconnected quark loop diagrams (Hyung-Jin Kim, others)
- Calculation starting on FNAL bc cluster

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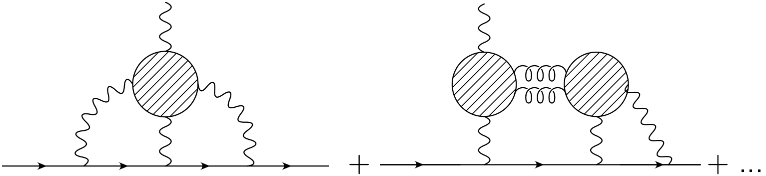
# HLbL ($\alpha^3$)



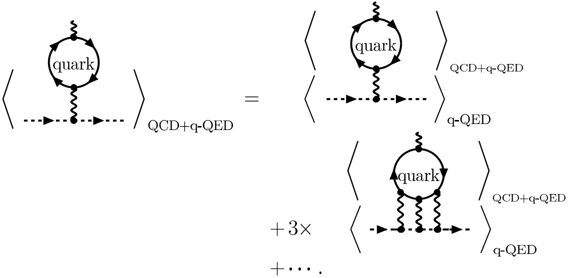
Blobs: all possible hadronic states

Model estimates put this $\mathcal{O}(\alpha^3)$ contribution at about $(10-12)\times 10^{-10}$ with a 25-40% uncertainty

No dispersion relation *a'la* vacuum polarization

Lattice regulator: model independent, approximations systematically improvable

# HLbL: QCD+QED on the lattice




Average over combined gluon *and* photon gauge configurations

Quarks coupled to gluons and photons, muon coupled to photons

Correlation function and subtraction highly correlated

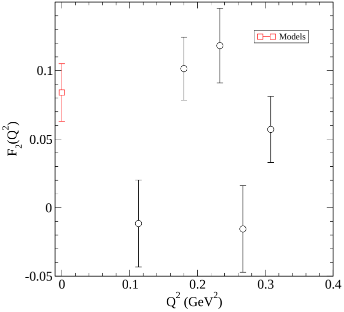
[Hayakawa, *et al.* hep-lat/0509016; Chowdhury *et al.* (2008); Chowdhury Ph. D. thesis (2009)]

# $a_\mu$(HLbL) in 2+1 flavor lattice QCD+QED

- Lattice size, $24^3$ ($(2.7\ \text{fm})^3$)
- Pion mass, $m_\pi = 329$ MeV
- Muon mass (190 MeV)
- $0.11 \lesssim Q^2 \lesssim 0.31\ \text{GeV}^2$
- Use All Mode Averaging (AMA)
  - $6^3$ ($5^3$) point sources/configuration $= 216$ (125)
  - AMA approximation: "sloppy CG", $r_{\text{stop}} = 10^{-4}$

[Blum, Hayakawa, and Izubuchi (arXiv:1301.2607)]

# $a_\mu$(HLbL) in 2+1f lattice QCD+QED (PRELIMINARY)




[Blum, Hayakawa, and Izubuchi (Lattice 2013)]

- Signal emerging in the model ballpark
- model value/error is "Glasgow Consensus"

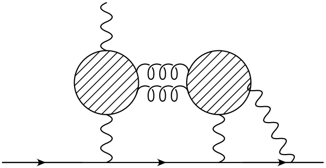
  (arXiv:0901.0306 [hep-ph])
- $m_\pi = 329$ MeV
- Stat. error only
- Low points: fewer combinations in average. Insufficient statistics?

# $a_\mu(\mathrm{HLbL})$ in 2+1f lattice QCD+QED (PRELIMINARY)

Check of subtraction (using heavier quark and muon masses)

- Change charge to $e = 0.84$, $1.19$
- HLbL amplitude ($\sim e^4$) changes by $\sim 0.5$ and 2 ✓
- while unsubtracted amplitude stays the same ✓

# $a_\mu(\mathrm{HLbL})$ "Disconnected" diagrams



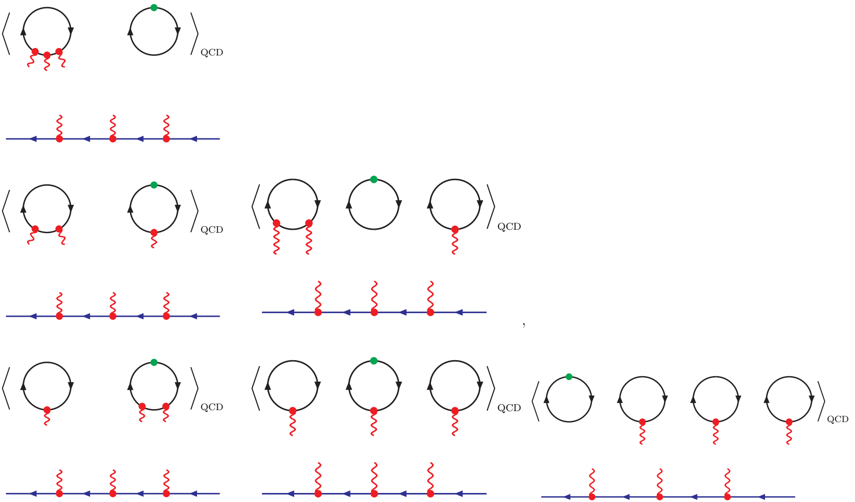
not calculated yet (not suppressed)

Omission due to use of quenched QED, i.e., sea quarks not electrically charged. Two possibilities,

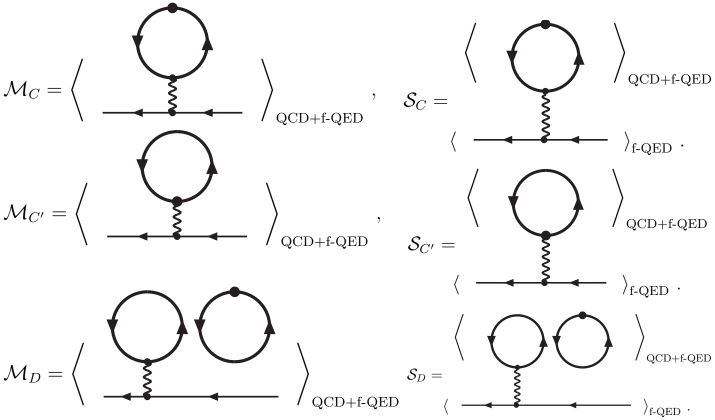
1. Re-weight in $\alpha$ (T. Ishikawa, *et al.*, Phys.Rev.Lett. 109 (2012) 072002 ) or
2. dynamical QED(+QCD) in HMC

Use same non-perturbative method as for quenched QED

# $a_\mu$(HLbL) Disconnected quark loop diagrams



# $a_\mu$(HLbL) Disconnected quark loop diagrams in our non-perturbative method



# $a_\mu(\mathrm{HLbL})$ Disconnected quark loop diagrams in our non-perturbative method

Diagrams in non-perturbative method have various "multiplicities"

| | $\mathcal{M}_C + \mathcal{M}_{C'}$ | $\mathcal{M}_D$ |
|---|---|---|
| LBL(4) | 3 | 0 |
| LBL(1,3) | 0 | 3 |
| LBL(2,2) | 1 | 2 |
| LBL(3,1) | 2 | 1 |
| LBL(1,1,2) | 0 | 3 |
| LBL(2,1,1) | 1 | 2 |
| LBL(1,1,1,1) | 0 | 3 |

But, physical linear combination,
$\mathcal{M}_C + \mathcal{M}_{C'} + \mathcal{M}_D$
has overall factor of 3

# $a_\mu(\mathrm{HLbL})$ Errors

Need to address

- statistics
- $q^2 \to 0$ extrapolation
- excited states/"around the world" effects
- Finite volume
- $m_q \to m_{q,\,\mathrm{phys}}$
- $m_\mu \to m_{\mu,\,\mathrm{phys}}$
- $a \to 0$
- QED renormalization
- ...

Even 20-30% total error, if solid, is very interesting

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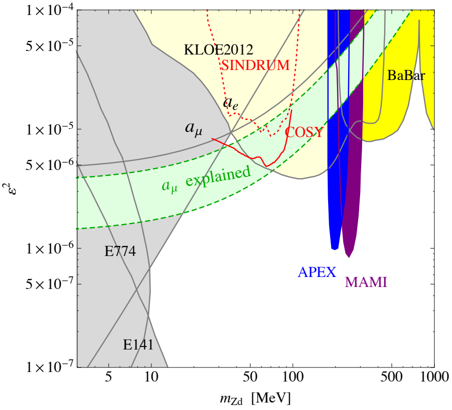
# Dark photon: $U(1)'$ extension(s) of SM ("dark charge")

- Explanation for astrophysical obs. of excess positrons (PAMELA, INTEGRAL,...). Contributes to $a_\mu$ (Pospelov 2008)




- $\gamma' - \gamma$ Mixing couples SM, Dark sectors
- Like LO Schwinger term
- $m = 10 - 1000$ MeV
- coupling $\epsilon^2 = 10^{-8} - 10^{-2}$
- Pospelov (2008): explains $g-2$ discrepancy
- Assumes $\gamma' \to e^+e^-$
- Search at Mainz, RHIC, Jlab, ..

Plot courtesy Bill Marciano

# Summary/Outlook

- Important testing ground for new physics
- Hadronic contributions dominate theory error
- Demanding, but straightforward calculations
- Great interest in HVP in lattice community
- First HLbL lattice calculation encouraging
- Expected precision (next 3-5 years)
  - E989 (J-PARC 34?): 0.14 PPM (3-4 better than E821)
  - SM theory, HVP: 0.3% (factor of 2 exp, lattice?)
  - SM theory, HLbL 10-20% (?)
  - Same central values, $a_\mu$ discrepancy $\rightarrow$ 5-8 $\sigma$

# Acknowledgments


- This research is supported in part by the US DOE
- Computational resources provided by the RIKEN BNL Research Center and USQCD Collaboration
- Lattice computations done on
  - QCDOC at BNL
  - Ds cluster at FNAL
  - q-series clusters at JLab