Lattice calculation of the hadronic contributions to the muon anomalous magnetic moment

Tom Blum (UConn / RIKEN BNL Research Center)

Lattice Meets Experiment: BSM
Brookhaven National Lab
December 6, 2013
Motivation and Introduction
The hadronic vacuum polarization (HVP) contribution ($O(\alpha^2)$)
The hadronic light-by-light (HLbL) contribution ($O(\alpha^3)$)

Summary/Outlook

Collaborators

Past/On-going work on g-2 done in collaboration with

<table>
<thead>
<tr>
<th>HVP</th>
<th>HLbL</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christopher Aubin (Fordham U)</td>
<td>Saumitra Chowdhury (UConn)</td>
</tr>
<tr>
<td>Maarten Golterman (SFSU)</td>
<td>Masahi Hayakawa (Nagoya)</td>
</tr>
<tr>
<td>Santiago Peris (SFSU/Barcelona)</td>
<td>Taku Izubuchi (BNL/RBRC)</td>
</tr>
<tr>
<td>RBC/UKQCD</td>
<td>Eigo Shintani (RBRC)</td>
</tr>
<tr>
<td></td>
<td>Norikazu Yamada (KEK)</td>
</tr>
</tbody>
</table>

New work starting with RBC/UKQCD collab (Christ, Jin, ...)
Motivation and Introduction

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Summary/Outlook
muon anomaly $a_\mu$ provides important test of the SM

- BNL E821: $a_\mu^{\text{exp}}$ accuracy is 0.54 ppm
- Fermilab E989, start is $\sim$ 3 years away, goal is 0.14 ppm
- J-PARC E34
  - $a_\mu^{\text{Expt}}(\text{SM}) = 287(63)(51) \times 10^{-11}$, or $\sim 3.6\sigma$
  - If both central values stay the same,
    - E989 ($\sim 4 \times$ smaller error) $\rightarrow \sim 5\sigma$
    - E989+new HLbL theory (models+lattice, 10%) $\rightarrow \sim 6\sigma$
    - E989+new HLbL +new HVP (50% reduction) $\rightarrow \sim 8\sigma$
- Big discrepancy! (New Physics $\sim 2 \times$ Electroweak)
- Lattice calculations crucial
The magnetic moment of the muon

In interacting quantum (field) theory $g$ gets corrections

\[ \gamma^\mu \rightarrow \Gamma^\mu(q) = \left( \gamma^\mu F_1(q^2) + \frac{i \sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right) \]

which results from Lorentz and gauge invariance when the muon is on-mass-shell.

\[ F_2(0) = \frac{g - 2}{2} \equiv a_\mu \quad (F_1(0) = 1) \]

(the anomalous magnetic moment, or anomaly)
The magnetic moment of the muon

Compute these corrections order-by-order in perturbation theory by expanding $\Gamma^\mu(q^2)$ in QED coupling constant

$$\alpha = \frac{e^2}{4\pi} = \frac{1}{137} + \ldots$$

Corrections begin at $O(\alpha)$; Schwinger term $= \frac{\alpha}{2\pi} = 0.0011614\ldots$ hadronic contributions $\sim 6 \times 10^{-5}$ times smaller (leading error).
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Summary/Outlook
Hadronic vacuum polarization (HVP) ($\alpha^2$)

The blobs, which represent all possible intermediate hadronic states, are not calculable in perturbation theory, but can be calculated from

- dispersion relation + experimental cross-section for $e^+e^-$ (and $\tau$) → hadrons
  \[ a_\mu^{\text{had}(2)} = \frac{1}{4\pi^2} \int_{4m_{\pi}^2}^{\infty} ds \, K(s) \sigma_{\text{total}}(s) \]
- first principles using lattice QCD,
  \[ a_\mu^{(2)\text{had}} = \left( \frac{\alpha}{\pi} \right)^2 \int_0^{\infty} dQ^2 \, f(Q^2) \Pi(Q^2) \]  
  [Lautrup and de Rafael 1969, Blum 2002]
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Summary/Outlook

$a_\mu$ (HVP) lattice results

<table>
<thead>
<tr>
<th>$a_\mu$</th>
<th>$N_f$</th>
<th>errors</th>
<th>action</th>
<th>group</th>
</tr>
</thead>
<tbody>
<tr>
<td>713(15)</td>
<td>2+1</td>
<td>stat.</td>
<td>Asqtad</td>
<td>Aubin, Blum (2006)</td>
</tr>
<tr>
<td>748(21)</td>
<td>2+1</td>
<td>stat.</td>
<td>Asqtad</td>
<td>Aubin, Blum (2006)</td>
</tr>
<tr>
<td>641(33)</td>
<td>2+1</td>
<td>stat., sys.</td>
<td>DWF</td>
<td>UKQCD (2011)</td>
</tr>
<tr>
<td>674(21)</td>
<td>2+1+1</td>
<td>stat., sys</td>
<td>TM</td>
<td>ETMC (2013)</td>
</tr>
<tr>
<td>572(16)</td>
<td>2</td>
<td>stat.</td>
<td>TM</td>
<td>ETMC (2011)</td>
</tr>
<tr>
<td>618(64)</td>
<td>2(+1)¹</td>
<td>stat., sys.</td>
<td>Wilson</td>
<td>Mainz (2011)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Exp.</th>
<th></th>
<th>$e^+e^-$</th>
<th>Davier, et al. (2011)</th>
</tr>
</thead>
<tbody>
<tr>
<td>692.3 (4.2)</td>
<td></td>
<td>$e^+e^-$</td>
<td>Davier, et al. (2011)</td>
</tr>
<tr>
<td>694.9 (4.3)</td>
<td></td>
<td>$e^+e^-$</td>
<td>Hagiwara, et al. (2012)</td>
</tr>
<tr>
<td>701.5 (4.7)</td>
<td></td>
<td>$e^+e^-+\tau$</td>
<td>Davier, et al. (2011)</td>
</tr>
</tbody>
</table>

¹strange quark is quenched
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**Summary/Outlook**

\(a_\mu(\text{HVP})\) integrand: low momentum region

Finite volume \(\rightarrow\) minimum finite momentum

Integral dominated by low \(Q^2 \sim m_\mu^2\) region. Stat. errors larger too

\(a_\mu(\text{HLO})\)

\(\mu\) integrand: low momentum region

Finite volume \(\rightarrow\) minimum finite momentum

Integral dominated by low \(Q^2 \sim m_\mu^2\) region. Stat. errors larger too

\(a_\mu(\text{HLO})\)

\(\mu\) integrand: low momentum region

Finite volume \(\rightarrow\) minimum finite momentum

Integral dominated by low \(Q^2 \sim m_\mu^2\) region. Stat. errors larger too

In progress...

**Integrand of** \(a_\mu^{\text{HLO}}/(4\alpha^2)\) compared with data

(MILC, \(a = 0.06\) fm , \(m_\pi = 220\) MeV)

⇒ need more data at low \(Q^2\) with smaller errors! In progress...

**ABGP** [Aubin, et al., arXive:1205:3695]

**UKQCD** [arXive:1107.1497]
a_\mu (HVP) Reducing statistical errors: All Mode Averaging

Use AMA, 1400 LM / 704 sources, 48^3 \times 144 (MILC), 20 configs, 2.6-20 \times error reduction for same cost!. RBC/UKQCD preliminary DWF results also show large error reduction (see Shintani, Lattice 2013).

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$a\mu(HVP)$ errors

Controlling errors at the 1% level

- $Q^2$ dependence
  - All mode averaging (AMA) (statistics) \cite{PhysRevD.88.094503}
  - Twisted BC’s or large box \cite{PhysRevD.88.074505}
  - Pade approximants for model independent fits \cite{PRD.86.054509}
  - avoid fit, analytic cont. (Ji and Jung, DESY+KEK, Mainz)

- physical quark masses / large boxes
- disconnected diagrams / isospin breaking
- charm contribution

Will give confidence that dispersive calculation is right

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Lattice calculation of the hadronic contributions to the muon a
RBC/UKQCD calculation of the HVP

- physical u,d,s quarks and quenched c
- large volume: \(48 \times 0.114 = 5.47\) fm box (2\(\times\) in t dir) \((q_{\text{min}} = 0.113\) GeV)
- Use AMA+random Z2 noise sources
- twisted b.c. for valence quarks for \(q^2 = 0\)
- eventually 0.086 fm ensemble as well
- Disconnected quark loop diagrams (Hyung-Jin Kim, others)
- Calculation starting on FNAL bc cluster
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HLbL ($\alpha^3$)

Blobs: all possible hadronic states

Model estimates put this $O(\alpha^3)$ contribution at about $(10-12) \times 10^{-10}$ with a 25-40% uncertainty

No dispersion relation *a’la* vacuum polarization

**Lattice regulator**: model independent, approximations systematically improvable

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HLbL: QCD+QED on the lattice

\[
\left\langle \begin{array}{c}
\text{quark} \\
\text{QCD+QED}
\end{array} \right\rangle = \left\langle \begin{array}{c}
\text{quark} \\
\text{QCD+QED}
\end{array} \right\rangle + 3 \times \left\langle \begin{array}{c}
\text{quark} \\
\text{QCD+QED}
\end{array} \right\rangle + \cdots.
\]

Average over combined gluon and photon gauge configurations
Quarks coupled to gluons and photons, muon coupled to photons
Correlation function and subtraction highly correlated

[Hayakawa, et al. hep-lat/0509016; Chowdhury et al. (2008); Chowdhury Ph. D. thesis (2009)]
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Summary/Outlook

$a_\mu(\text{HLbL})$ in 2+1 flavor lattice QCD+QED

- Lattice size, $24^3 ((2.7 \text{ fm})^3)$
- Pion mass, $m_\pi = 329$ MeV
- Muon mass (190 MeV)
- $0.11 \lesssim Q^2 \lesssim 0.31 \text{ GeV}^2$
- Use **All Mode Averaging** (AMA)
  - $6^3 (5^3)$ point sources/configuration = 216 (125)
  - AMA approximation: “sloppy CG”, $r_{\text{stop}} = 10^{-4}$

[Blum, Hayakawa, and Izubuchi (arXiv:1301.2607)]

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Signal emerging in the model ballpark

model value/error is “Glasgow Consensus”

(arXiv:0901.0306 [hep-ph])

$m_\pi = 329$ MeV

Stat. error only

Low points: fewer combinations in average. Insufficient statistics?
Check of subtraction (using heavier quark and muon masses)

- Change charge to $e = 0.84, 1.19$
- HLbL amplitude ($\sim e^4$) changes by $\sim 0.5$ and 2 ✓
- While unsubtracted amplitude stays the same ✓
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Summary/Outlook

\(a_\mu(\text{HLbL})\) “Disconnected” diagrams

not calculated yet (not suppressed)

Omission due to use of quenched QED, i.e., sea quarks not electrically charged. Two possibilities,

2. dynamical QED(+QCD) in HMC

Use same non-perturbative method as for quenched QED
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**$a_\mu^{(HLbL)}$** Disconnected quark loop diagrams

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Lattice calculation of the hadronic contributions to the muon anomalous magnetic moment
$a_\mu(\text{HLbL})$ Disconnected quark loop diagrams in our non-perturbative method
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**Summary/Outlook**

\(a_\mu(\text{HLbL})\) Disconnected quark loop diagrams in our non-perturbative method

Diagrams in non-perturbative method have various “multiplicities”

<table>
<thead>
<tr>
<th>Diagram</th>
<th>(\mathcal{M}<em>C + \mathcal{M}</em>{C'})</th>
<th>(\mathcal{M}_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>LBL(4)</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>LBL(1,3)</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>LBL(2,2)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>LBL(3,1)</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>LBL(1,1,2)</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>LBL(2,1,1)</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>LBL(1,1,1,1)</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

But, physical linear combination, \(\mathcal{M}_C + \mathcal{M}_{C'} + \mathcal{M}_D\) has overall factor of 3
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$a_\mu(\text{HLbL})$ Errors

Need to address

- statistics
- $q^2 \to 0$ extrapolation
- excited states/“around the world” effects
- Finite volume
- $m_q \to m_{q,\text{phys}}$
- $m_\mu \to m_{\mu,\text{phys}}$
- $a \to 0$
- QED renormalization
- ...

Even 20-30% total error, if solid, is very interesting

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Summary/Outlook
Dark photon: $U(1)'$ extension(s) of SM ("dark charge")

- Explanation for astrophysical obs. of excess positrons (PAMELA, INTEGRAL,...). Contributes to $a_\mu$ (Pospelov 2008)
  - $\gamma' - \gamma$ Mixing couples SM, Dark sectors
  - Like LO Schwinger term
  - $m = 10 - 1000$ MeV
  - coupling $\epsilon^2 = 10^{-8} - 10^{-2}$
  - Pospelov (2008): explains $g - 2$ discrepancy
  - Assumes $\gamma' \rightarrow e^+ e^-$
  - Search at Mainz, RHIC, Jlab, ...

Plot courtesy Bill Marciano
Summary/Outlook

- Important testing ground for new physics
- Hadronic contributions dominate theory error
- Demanding, but straightforward calculations
- Great interest in HVP in lattice community
- First HLbL lattice calculation encouraging
- Expected precision (next 3-5 years)
  - E989 (J-PARC 34?): 0.14 PPM (3-4 better than E821)
  - SM theory, HVP: 0.3% (factor of 2 exp, lattice?)
  - SM theory, HLbL 10-20% (?)
  - Same central values, $a_\mu$ discrepancy $\rightarrow 5-8 \sigma$
Acknowledgments

- This research is supported in part by the US DOE
- Computational resources provided by the RIKEN BNL Research Center and USQCD Collaboration
- Lattice computations done on
  - QCDOC at BNL
  - Ds cluster at FNAL
  - q-series clusters at JLab