Light composite flavor-singlet scalar in large N_f QCD

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(LatKMI Collaboration)

Refs. PRD86(2012)054506, PRD87(2013)094511,
PRL111(2013)162001, arXiv:1309.0711

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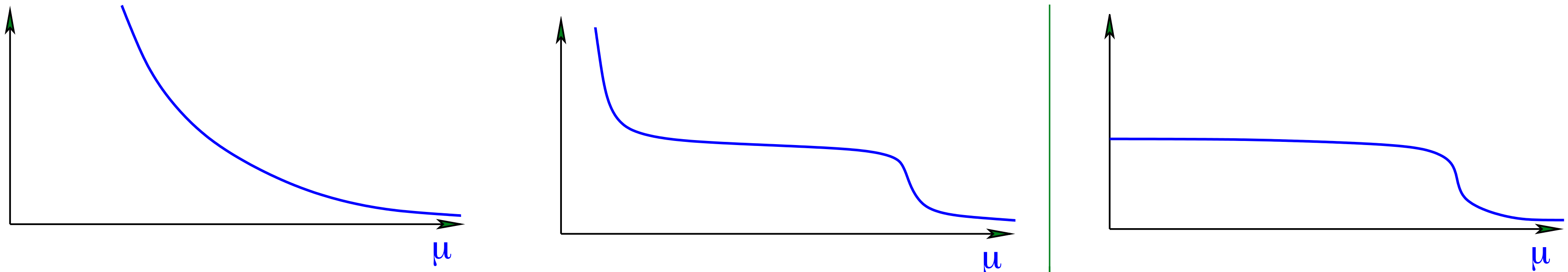
Walking technicolor

N_f massless fermions + SU(N_{TC}) gauge at $O(1)$ TeV

Model requirement:

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range



Small N_f | Middle N_f | Large $N_f \leq 16$

Chiral symmetry breaking ← phase boundary → Conformal

- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region

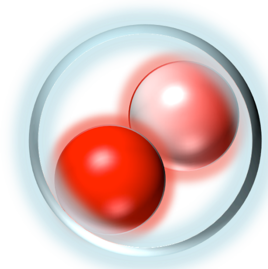
Walking technicolor

N_f massless fermions + SU(N_{TC}) gauge at $O(1)$ TeV

Model requirement:

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region

- Higgs $\approx$ Light composite scalar
  pNGB (technidilaton)
  of scale symmetry breaking



$$m_{\text{Higgs}}/v_{\text{EW}} \sim 0.5 = m_\sigma/(\sqrt{N_d}F)$$

F : decay constant, N_d : number of weak doublets

usual QCD $m_\sigma/F \sim$ 4–5

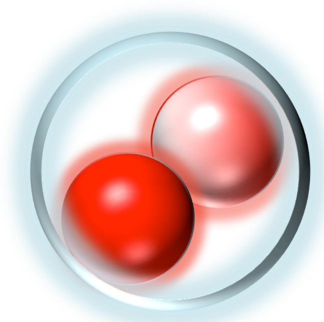
Conditions of walking technicolor

- Spontaneous chiral symmetry breaking
- Slow running (walking) coupling in wide scale range
- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region
- Light composite scalar

Question: Such a theory really exists?

Nonperturbative calculation is important.

$\rightarrow$ numerical calculation with lattice gauge theory



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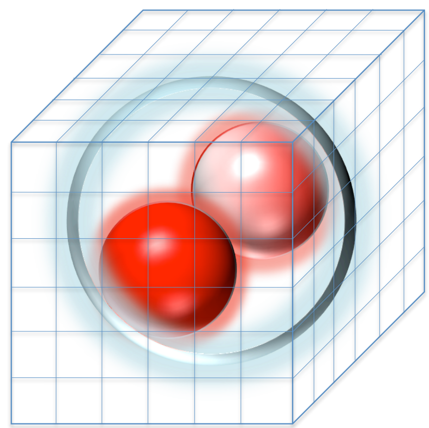


Recent studies in our project

Purpose in our project

Search for candidate of walking technicolor

Systematic investigation of N_f dependence

SU(3) gauge theory with $N_f = 0, 4, 8, 12, 16$ fermions

Common setup for all N_f: Improved staggered action (HISQ/Tree)

Cheaper calculation cost + small lattice systematic error

HISQ '07 HPQCD and UKQCD; HISQ/Tree '12 Bazakov *et al.*

Basic physical quantities: m_π, F_π, m_ρ, $\langle\overline{\psi}\psi\rangle$

$N_f = 12$: PRD86(2012)054506

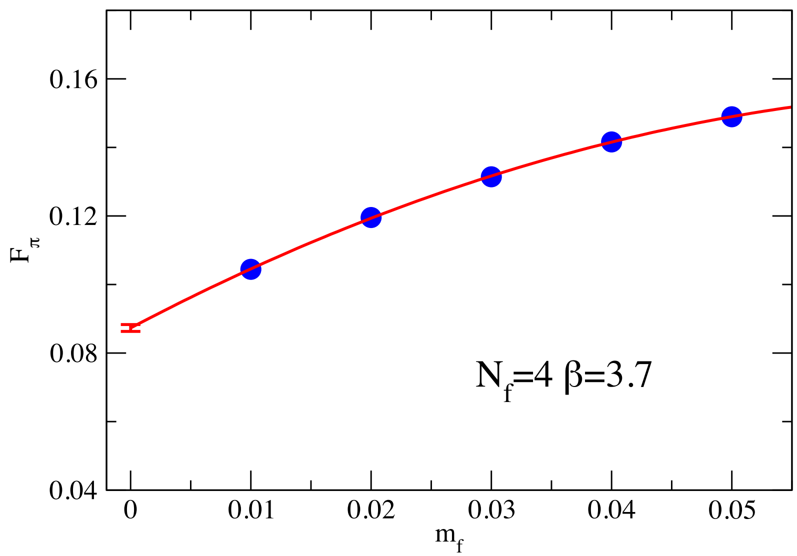
$N_f = 8$: PRD87(2013)094511

$N_f = 8$ may be candidate of walking theory

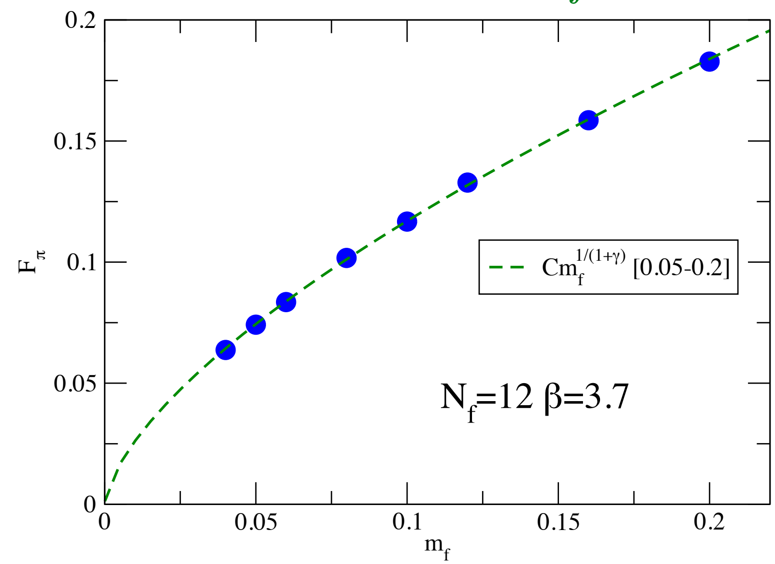
Recent study of LatKMI Collaboration

PRD86(2012)054506; PRD87(2013)094511

Chiral broken $F_\pi \to F \neq 0$



Conformal $F_\pi \to C m_f^{1/(1+\gamma)}$



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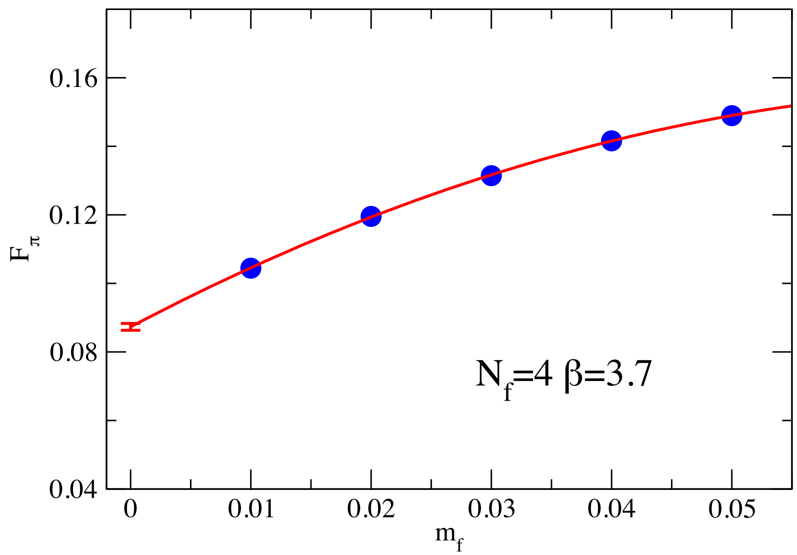

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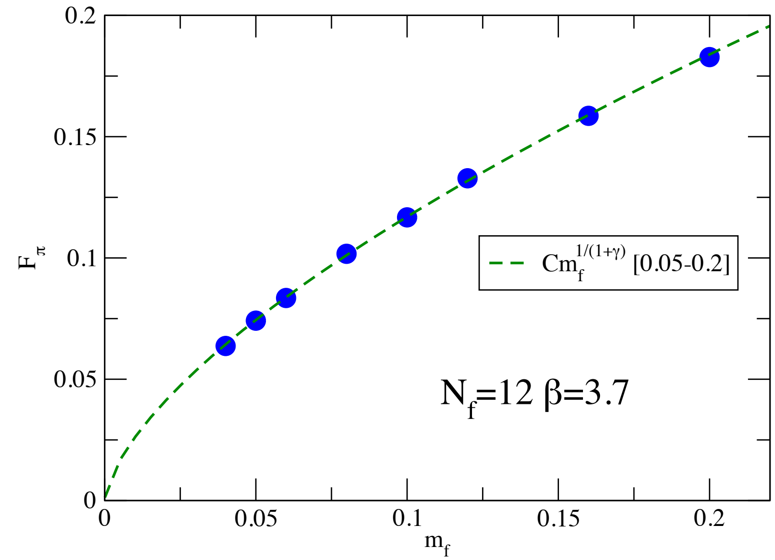

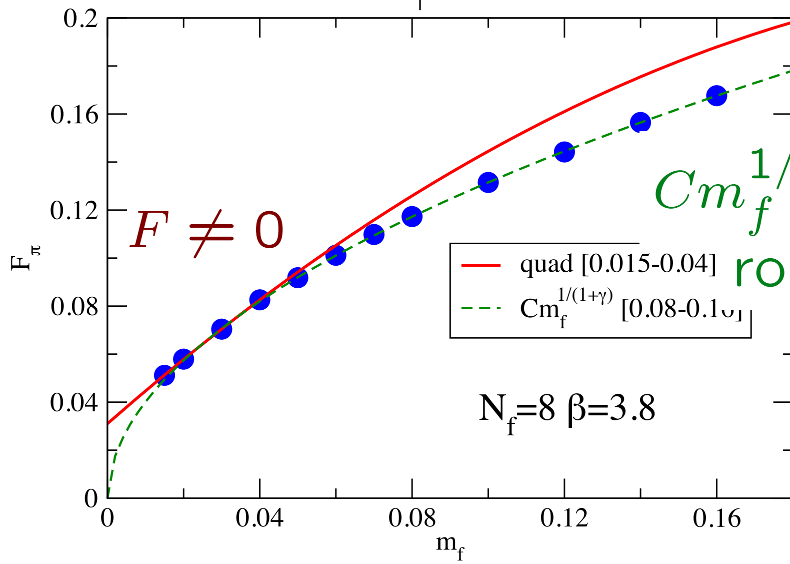

$C m_f^{1/(1+\gamma)}$

rough estimate of γ

Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

PRD86(2012)054506; PRD87(2013)094511

$N_f = 4$ QCD: Spontaneous chiral symmetry breaking

$N_f = 12$ QCD: Consistent with conformal phase

$N_f = 8$ QCD may be a candidate of Walking technicolor

- Spontaneous chiral symmetry breaking
  $F_\pi \neq 0$ and $F_\pi / m_\pi \to \infty$ towards $m_f \to 0$
- Slow running (walking) coupling in wide scale range
  Different behaviors of F_π in light and middle m_f
- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region
  $\gamma = 0.62$–0.97: Hyperscaling-like behavior in middle m_f
- Light composite scalar $\Leftarrow$ Important to check!

Next: Flavor-singlet scalar in (approximate) conformal theory

Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

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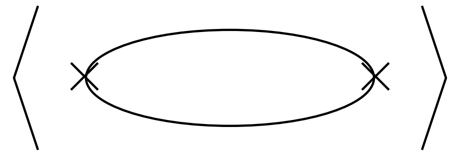
Next: Flavor-singlet scalar in (approximate) conformal theory

Composite flavor-singlet scalar in $N_f = 12$ and 8 QCD

Difficulty of flavor-singlet scalar meson

- Flavor non-singlet scalar meson $S_{NS}(t) = \sum_{\vec{x}} \overline{\psi}_a(\vec{x},t)\psi_b(\vec{x},t)$ $(a \neq b)$

$$\langle 0|S_{NS}(t)S^{\dagger}_{NS}(0)|0\rangle \quad =$$



$$= -C(t)$$

c.f. m_π, F_π from non-singlet pseudoscalar

$O(100)$ configurations $\times$ $O(1)$ $D^{-1}[U](x,y)$

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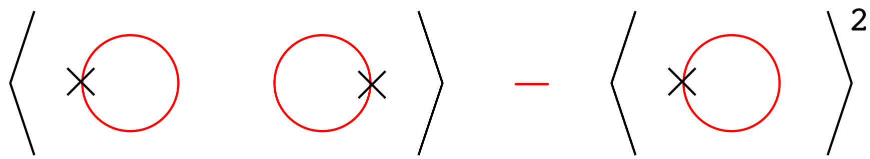
$$\langle 0|S(t)S^{\dagger}(0)|0\rangle \quad = -C(t) + (N_f/4)D(t) \text{ (disconnected)}$$

$$D(t) =$$



Much harder but essential for flavor-singlet

$O(10000)$ configurations $\times$ $O(100)$ $D^{-1}[U](x,x)$

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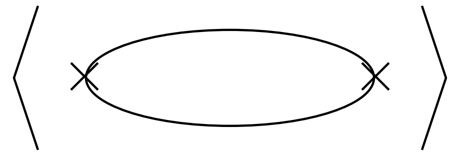

$= -C(t)$

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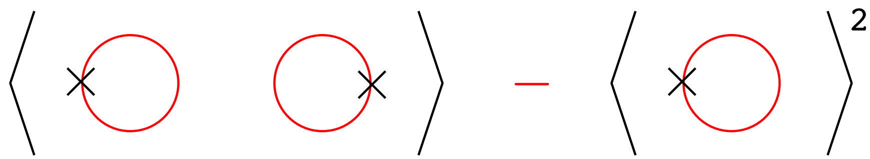


Much harder but essential for flavor-singlet

$O(10000)$ configurations $\times$ $O(10)$ $D^{-1}[U](x,x)$

using noise reduction method

'97 Venkataraman and Kilcup

Composite flavor-singlet scalar in $N_f = 12$ QCD

Purpose of $N_f = 12$ QCD calculation

Why $N_f = 12$

- Investigated by many groups

  '08,'09 Appelquist *et al.*, '10 Deuzeman *et al.*, '10,'12 Hasenfratz,
  '11 Fodor *et al.*, '11 Appelquist *et al.*, '11 DeGrand, '11 Ogawa *et al.*,
  '12 Lin *et al.*, '12,'13 Iwasaki *et al.*, '12,'13 Itou, '12 Jin and Mawhinney, and $\cdots$

In our work PRD86(2012)054506

consistent behavior with conformal phase

- A few studies of flavor-singlet scalar in conformal theory
  1. SU(2) Adjoint $N_f = 2$ glueball: '09 Del Debbio *et al.*
  2. SU(3) $N_f = 12$ meson: '12 Jin and Mawhinney

     c.f. SU(3) $N_f = 12$ meson: '13 LHC Collaboration

Purpouse of this work

Understand properties of flavor-singlet scalar in $N_f = 12$

regarded as pilot study of $N_f = 8$ theory

Flavor-singlet scalar in $N_f = 12$ QCD

PRL111(2013)162001

Simulation parameters

- $\beta = 4$ HISQ/Tree action
  calculation of m_σ
- Huge number of configurations
  measuring every 2 tarj.
- Four m_f on more than two volumes
- Noise reduction method with $N_r = 64$
- Local meson operator of $(1 \otimes 1)$

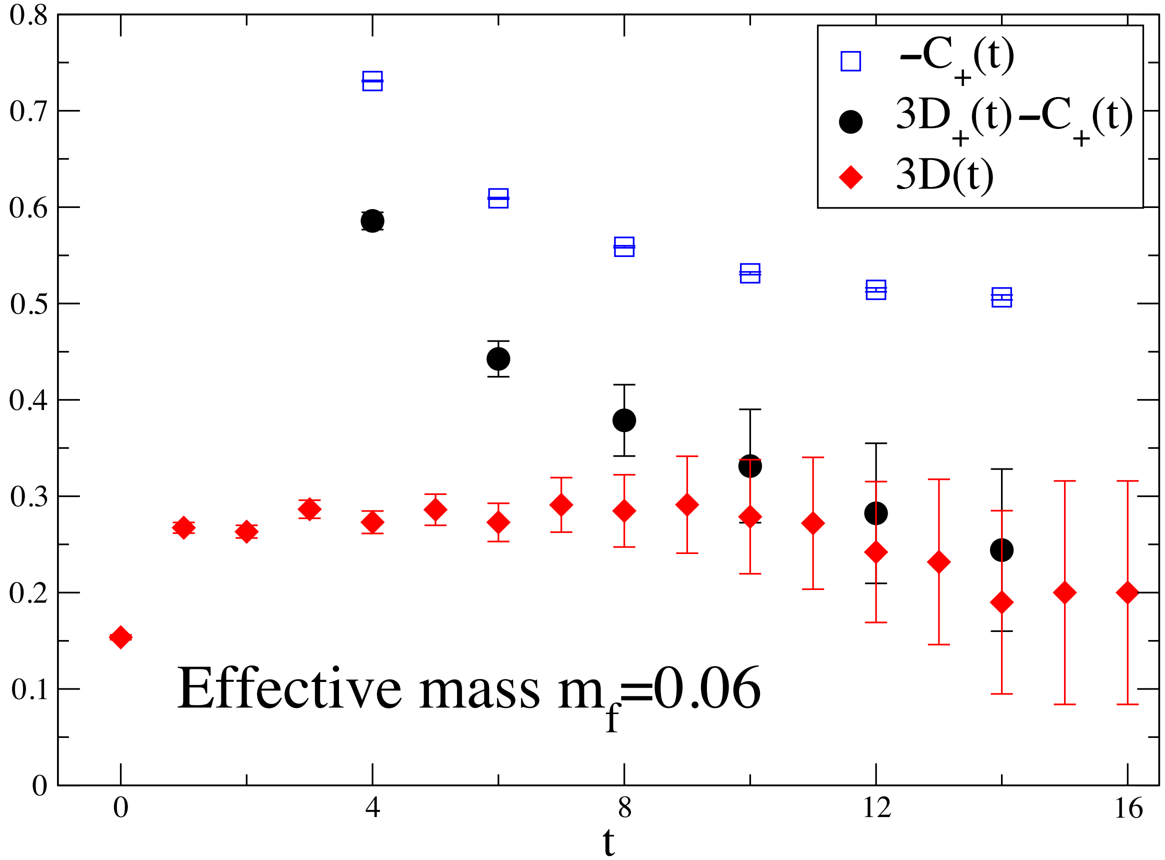
L, T	m_f	confs
24,32	0.05	11000
	0.06	14000
	0.08	15000
	0.10	9000
30,40	0.05	10000
	0.06	15000
	0.08	15000
	0.10	4000
36,48	0.05	5000
	0.06	6000

Machines: φ at KMI, CX400 at Kyushu Univ.

Effective mass in $N_f = 12$

PRL111(2013)162001

$m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$



Non-singlet scalar

a_0: $-C_+(t)$

Singlet scalar

σ: $3D_+(t) - C_+(t)$

$m_\sigma < m_{a_0}$

σ: $D(t)$

Consistent m_σ with smaller error

$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of m_σ from $D(t)$

Effective mass in $N_f = 12$

PRL111(2013)162001

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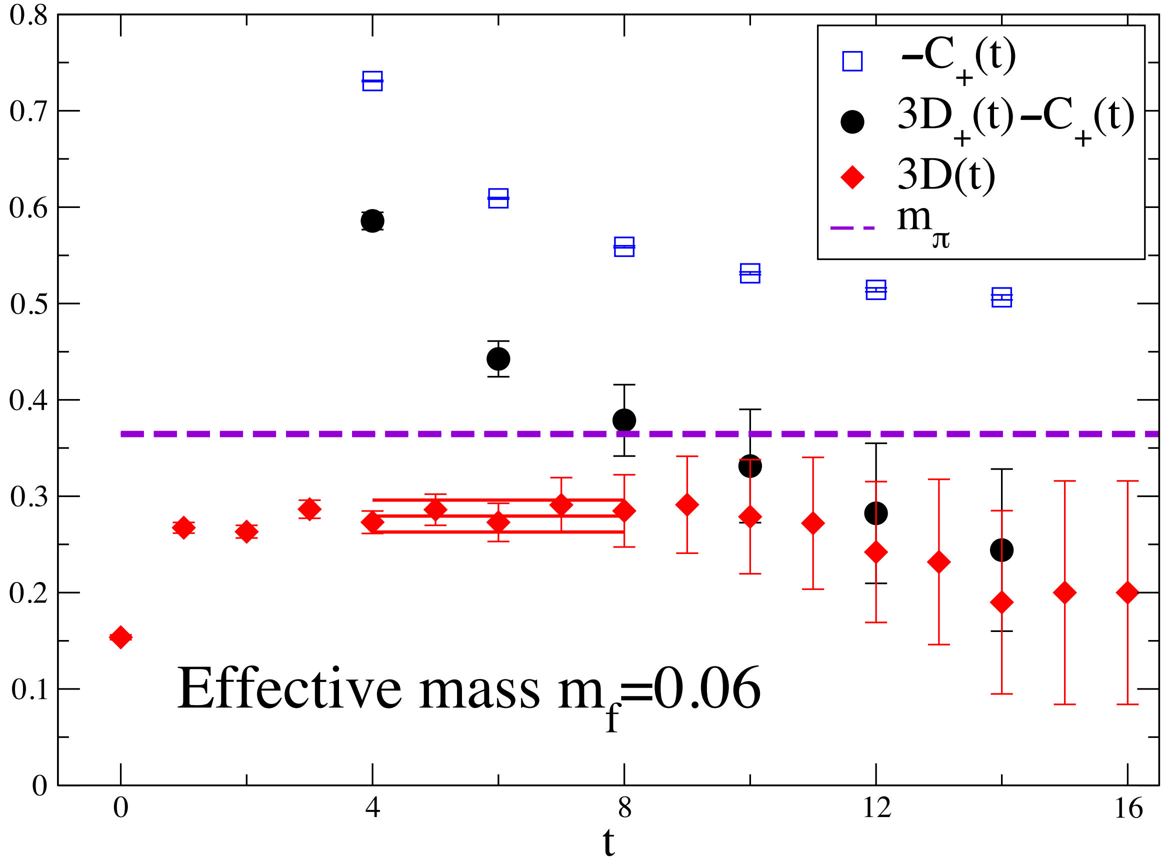

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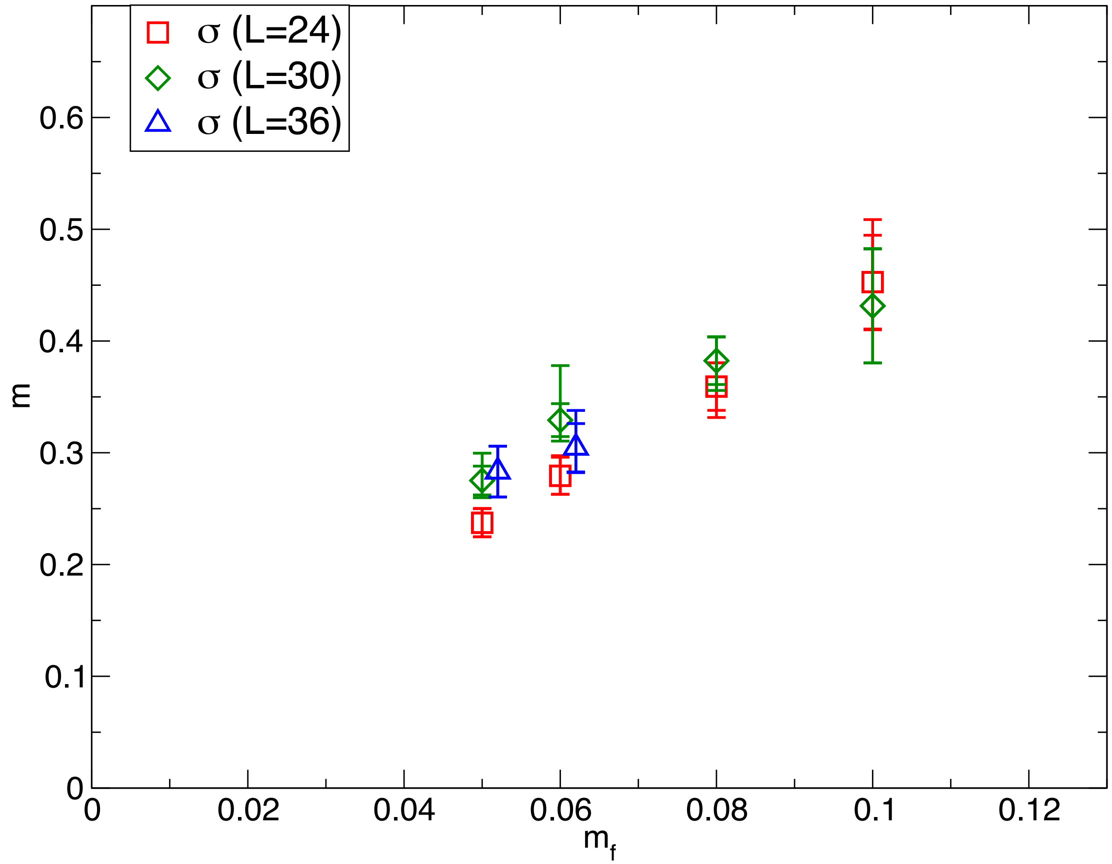
m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8



Reasonable signals with almost 10% statistical error

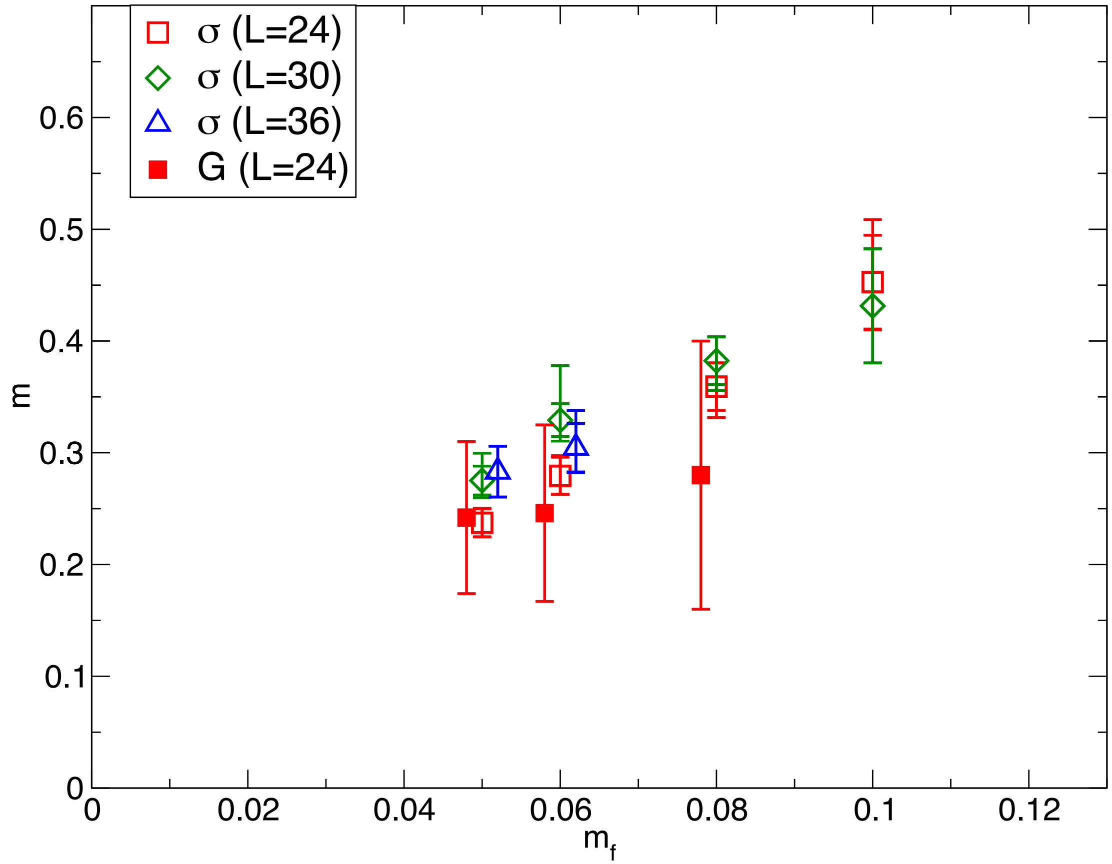
Systematic error from fit range dependence of m_σ

Finite volume effect under control $\leftarrow$ 2 larger volumes agree

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8

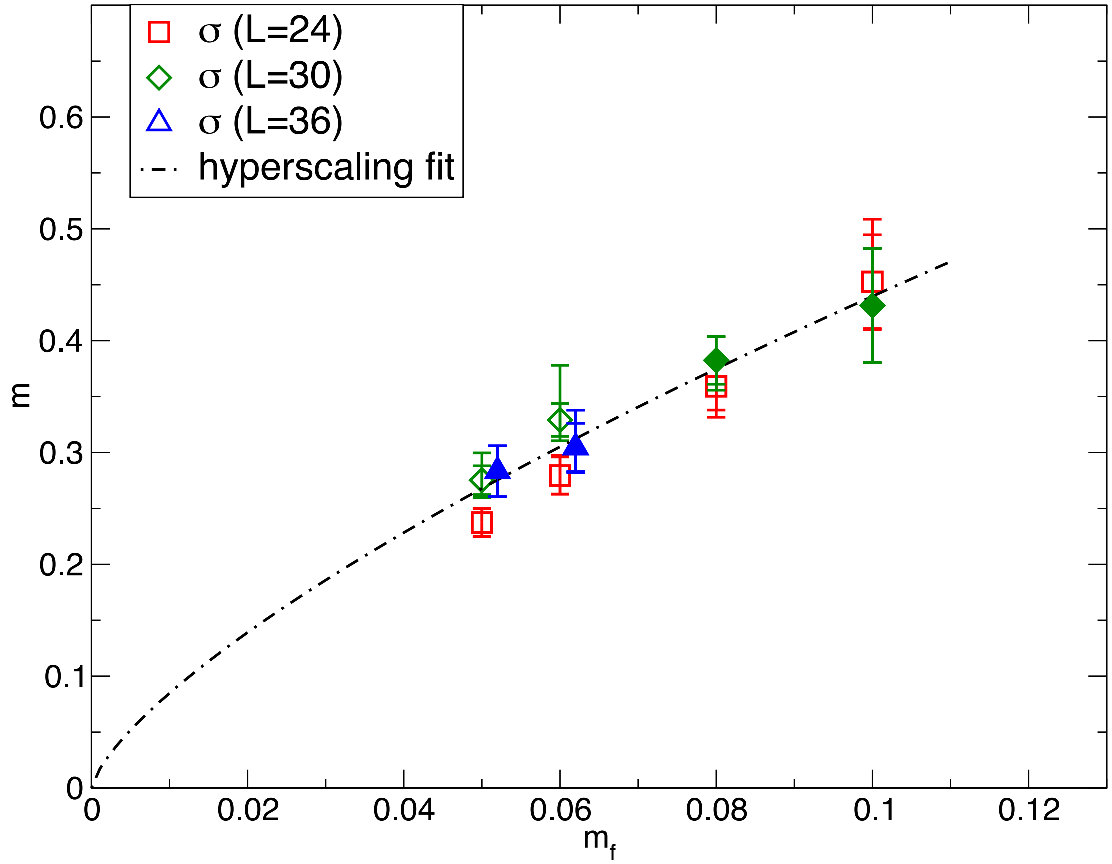


Consistent mass from glueball operator calculation

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8



Hyperscaling test with fixed γ using larget volume at each m_f

$$m_\sigma = C m_f^{1/(1+\gamma)}$$ with $\gamma = 0.414$ from hyperscaling of m_π

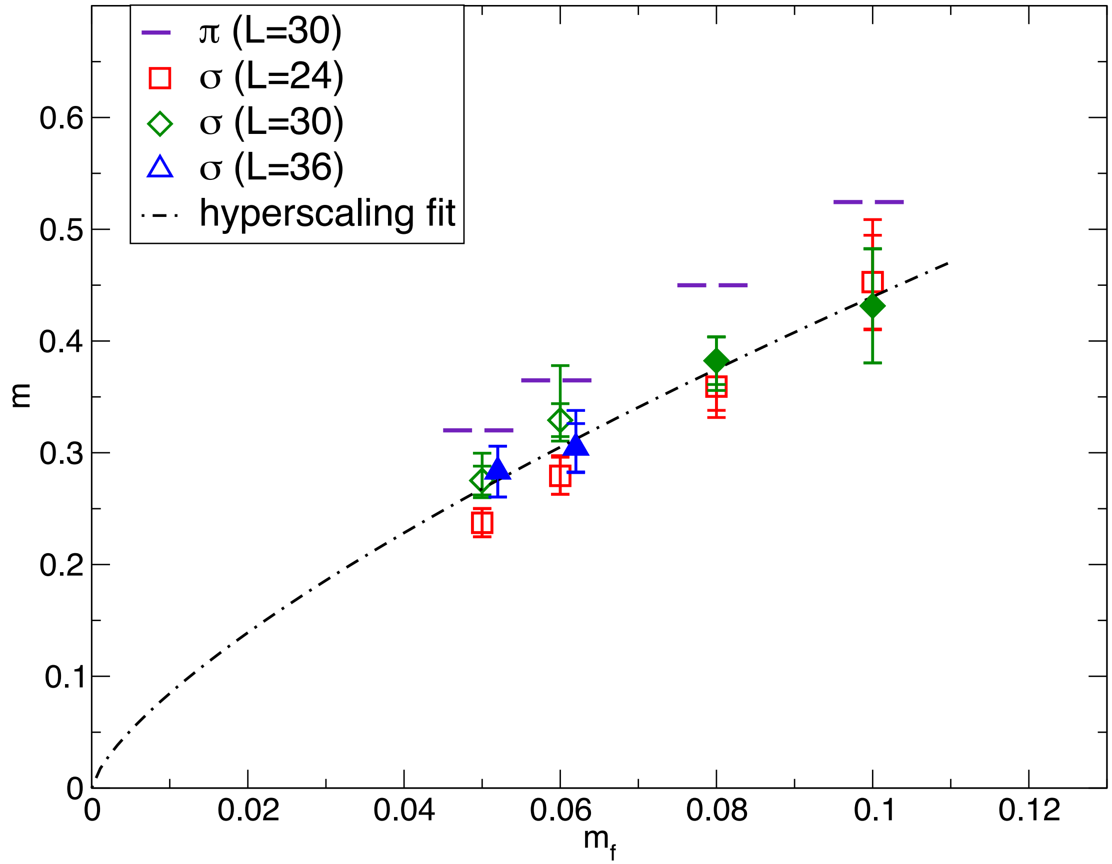
PRD86(2012)054506

Consistent hyperscaling as m_π

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8

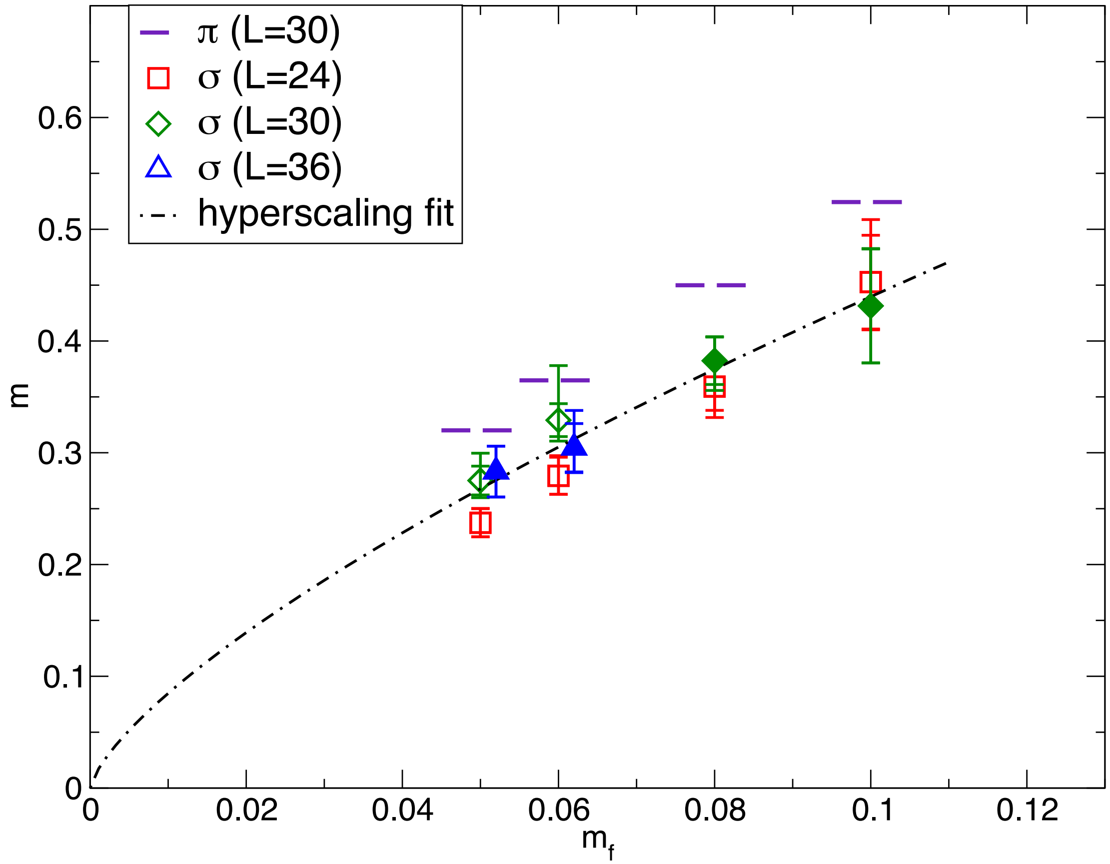


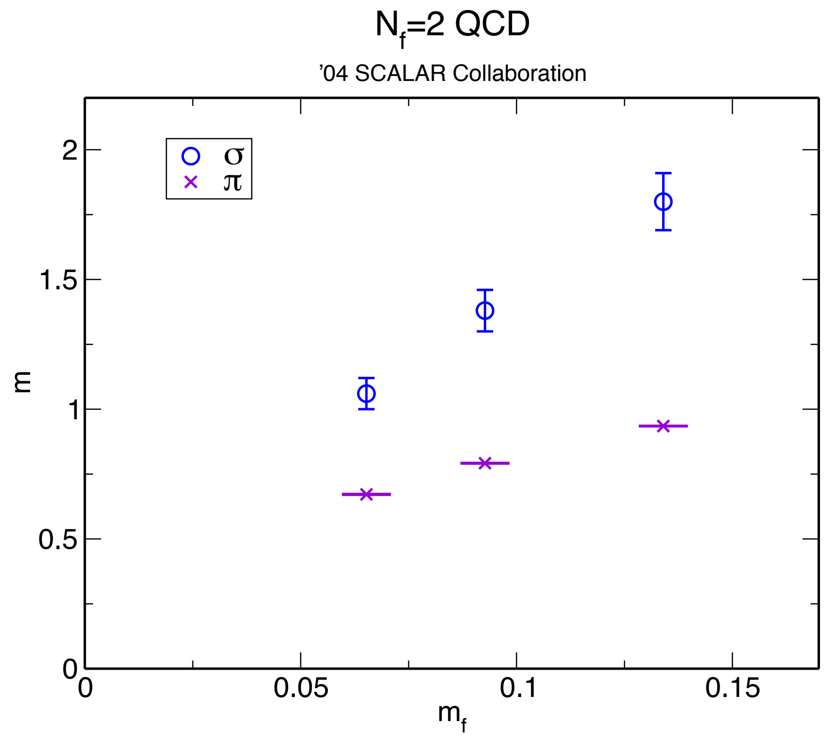
Lighter than π in all m_f

m_f dependence in $N_f = 12$

PRL111(2013)162001

m_σ from fit of $3D(t)$ with $t = 4$–8





Lighter than π in all m_f

Much different from usual QCD

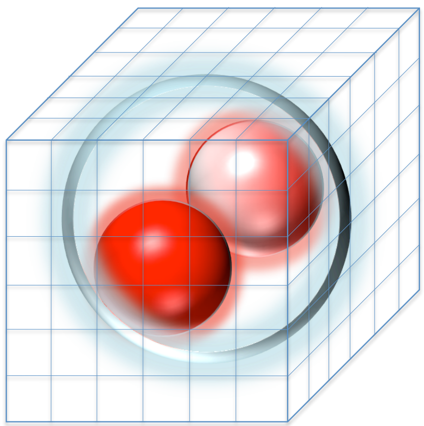
$\boxed{\text{Conformal symmetry may make } \sigma \text{ light}}$

Composite flavor-singlet scalar in $N_f = 8$ QCD

$N_f = 8$ QCD maybe candidate of walking theory; PRD87(2013)094511

If flavor-singlet scalar is light: $m_\sigma \sim F$ in $m_f = 0$

$\rightarrow$ Possibility of composite Higgs (technidilaton)



Flavor-singlet scalar in $N_f = 8$ QCD

report of preliminary results arXiv:1309.0711

Maybe candidate of walking theory; PRD87(2013)094511

Simulation parameters

- $\beta = 3.8$ HISQ/Tree action
  calculation of m_σ
- Huge number of configurations
  measuring every 2 tarj.
- Five m_f with three volumes
- Noise reduction method with $N_r = 64$
- Local meson operator of $(1 \otimes 1)$

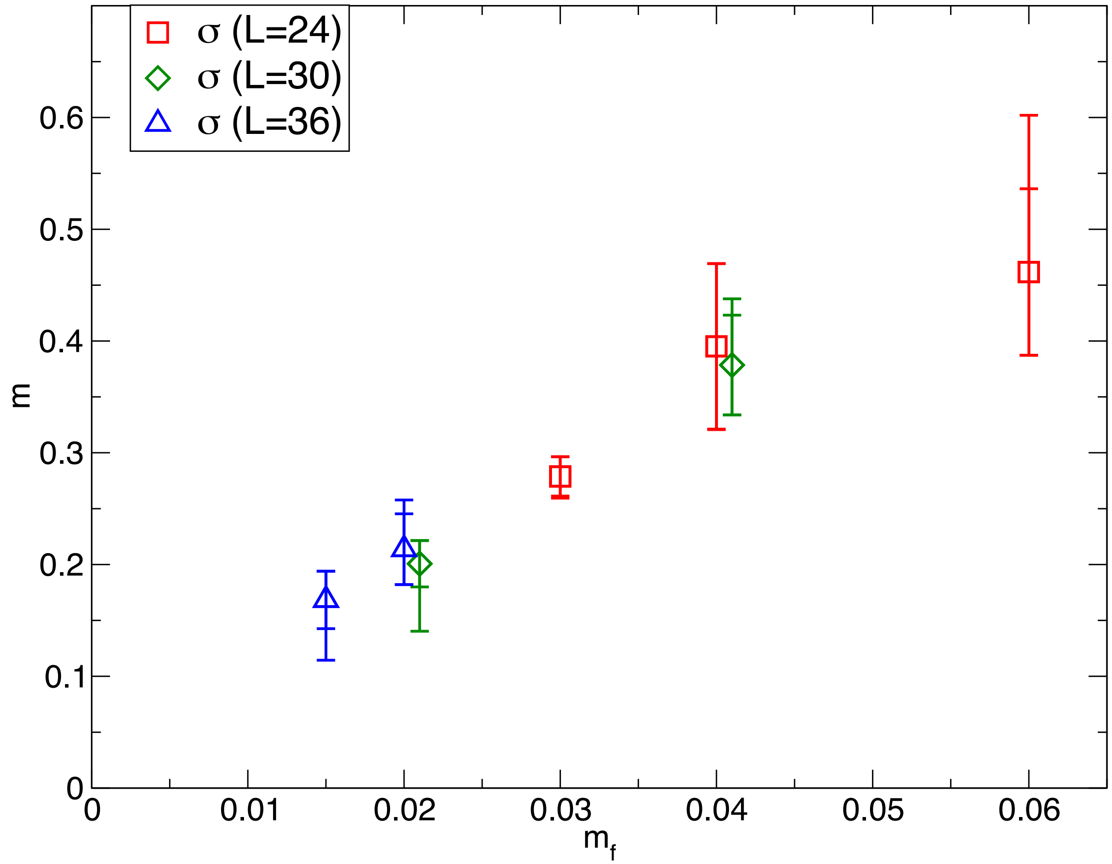
L, T	m_f	confs
24,32	0.03	27400
	0.04	5500
	0.06	18000
30,40	0.02	8000
	0.04	12600
36,48	0.02	2900
	0.015	2900

All results are preliminary.

Machines: φ at KMI, CX400 at Nagoya Univ.,
CX400 and HA8000 at Kyushu Univ.

m_f dependence in $N_f = 8$

m_σ from fit of $2D(t)$ with $t = 6$–11



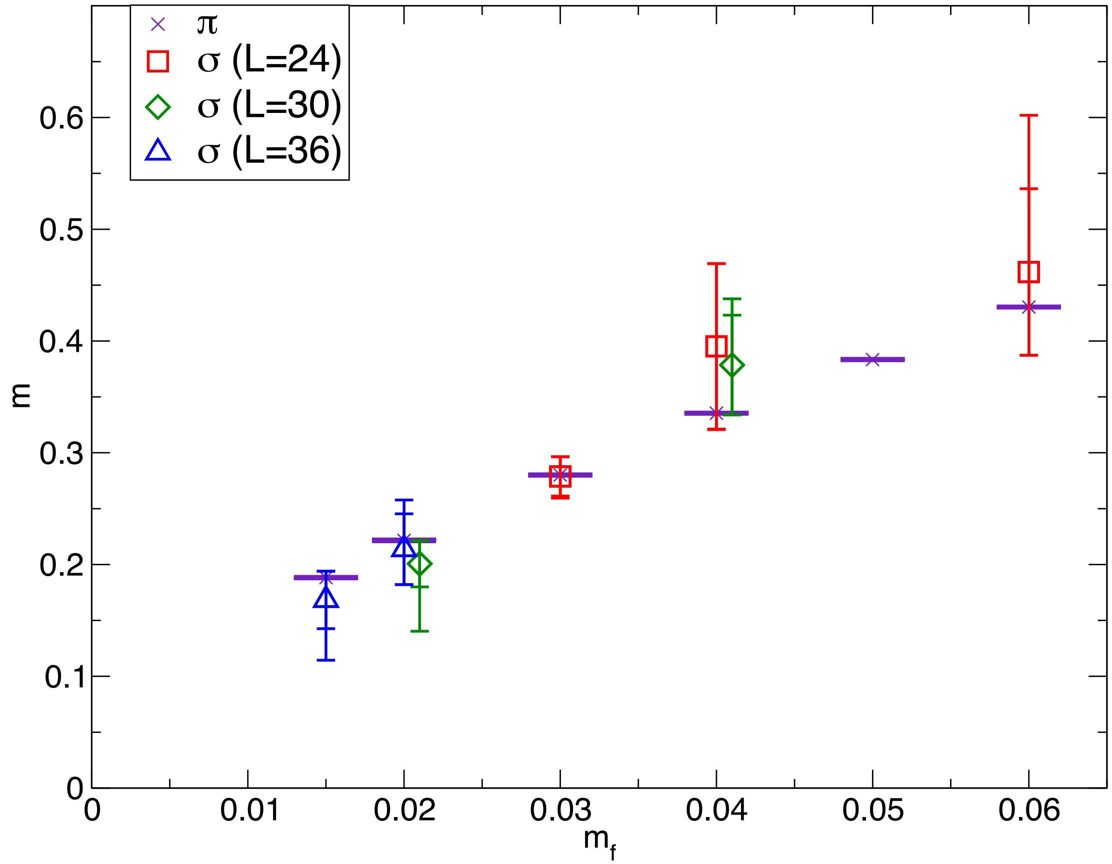
Reasonable signals with statistical error $< 20\%$

Systematic error from fit range dependence of m_σ

Finite volume effect seems under control

m_f dependence in $N_f = 8$

m_σ from fit of $2D(t)$ with $t = 6$–11



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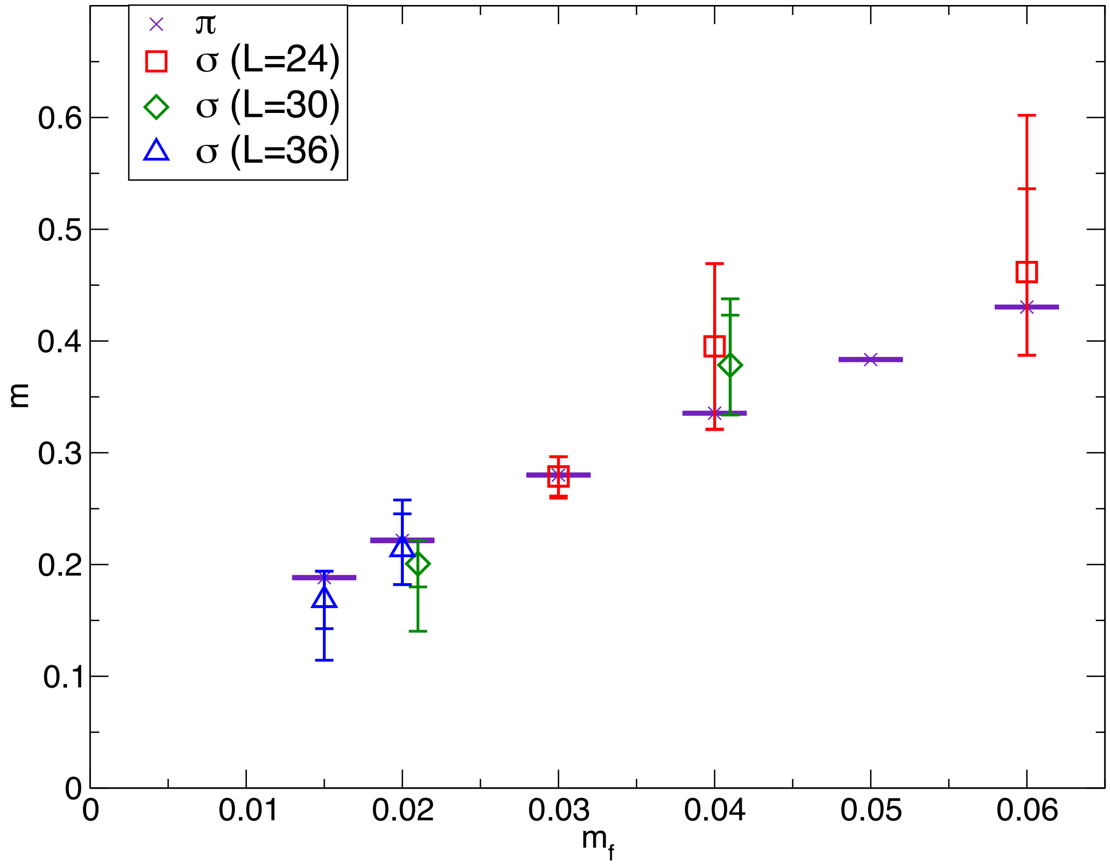
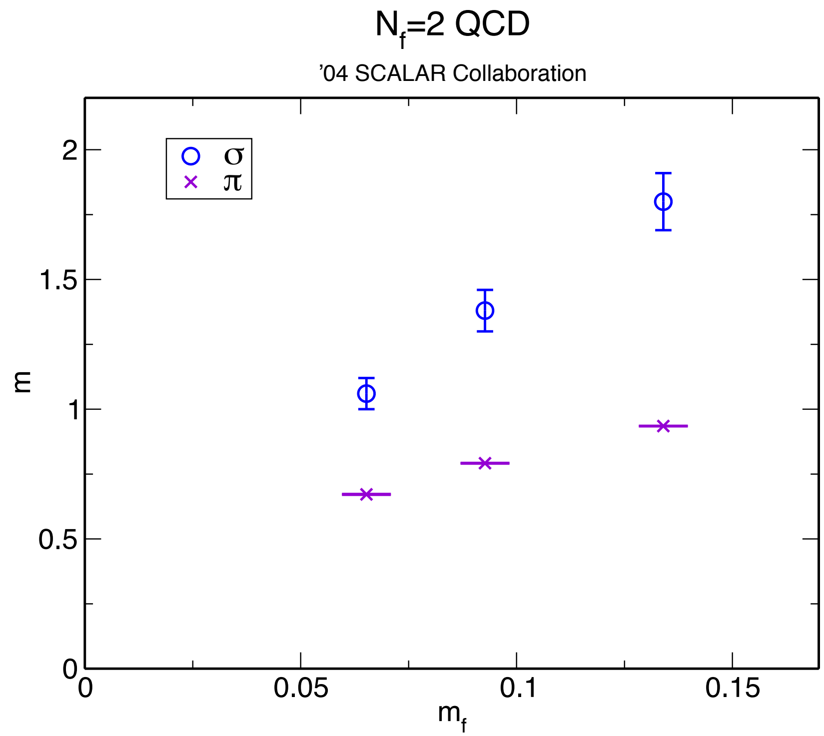
Systematic error from fit range dependence of m_σ

$m_\sigma \sim m_\pi$ in all m_f

m_f dependence in $N_f = 8$

m_σ from fit of $2D(t)$ with $t = 6$–11



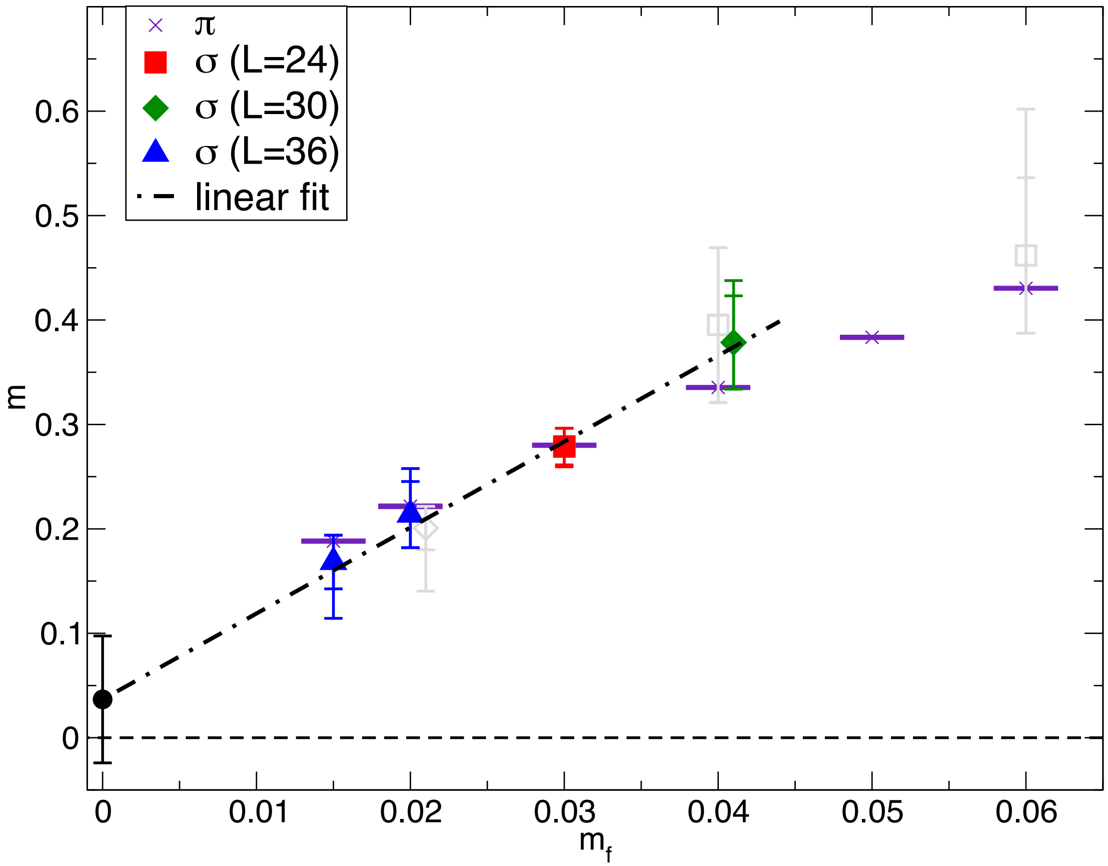


Reasonable signals with statistical error $< 20\%$

Systematic error from fit range dependence of m_σ

$m_\sigma \sim m_\pi$ in all m_f, much different from $N_f = 2$ QCD

Chiral extrapolation (1) in $N_f = 8$



$$m_\sigma = m_0 + Am_f\text{: } m_0 = 0.028(53) \rightarrow \frac{m_\sigma}{F} = 1.7(2.7)$$

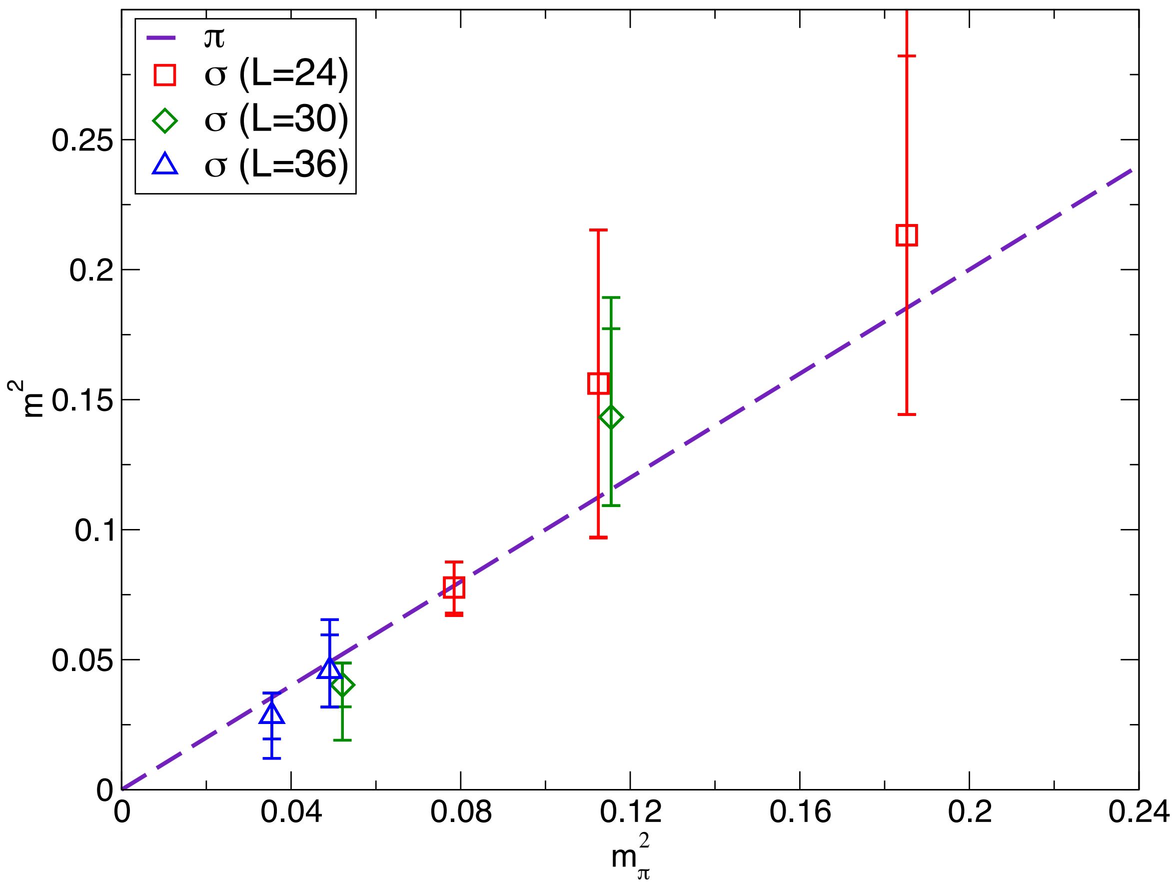
$F = 0.0219(7)$ PRD87(2013)094511

c.f.) 1 family model: $m_{\text{Higgs}} = 210(340)$ GeV

Chiral extrapolation (2) in $N_f = 8$

ChPT with scale symmetry breaking '13 Matsuzaki and Yamawaki

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$



$m_\sigma \sim m_\pi \to C \sim 1$

Chiral extrapolation (2) in $N_f = 8$

ChPT with scale symmetry breaking '13 Matsuzaki and Yamawaki

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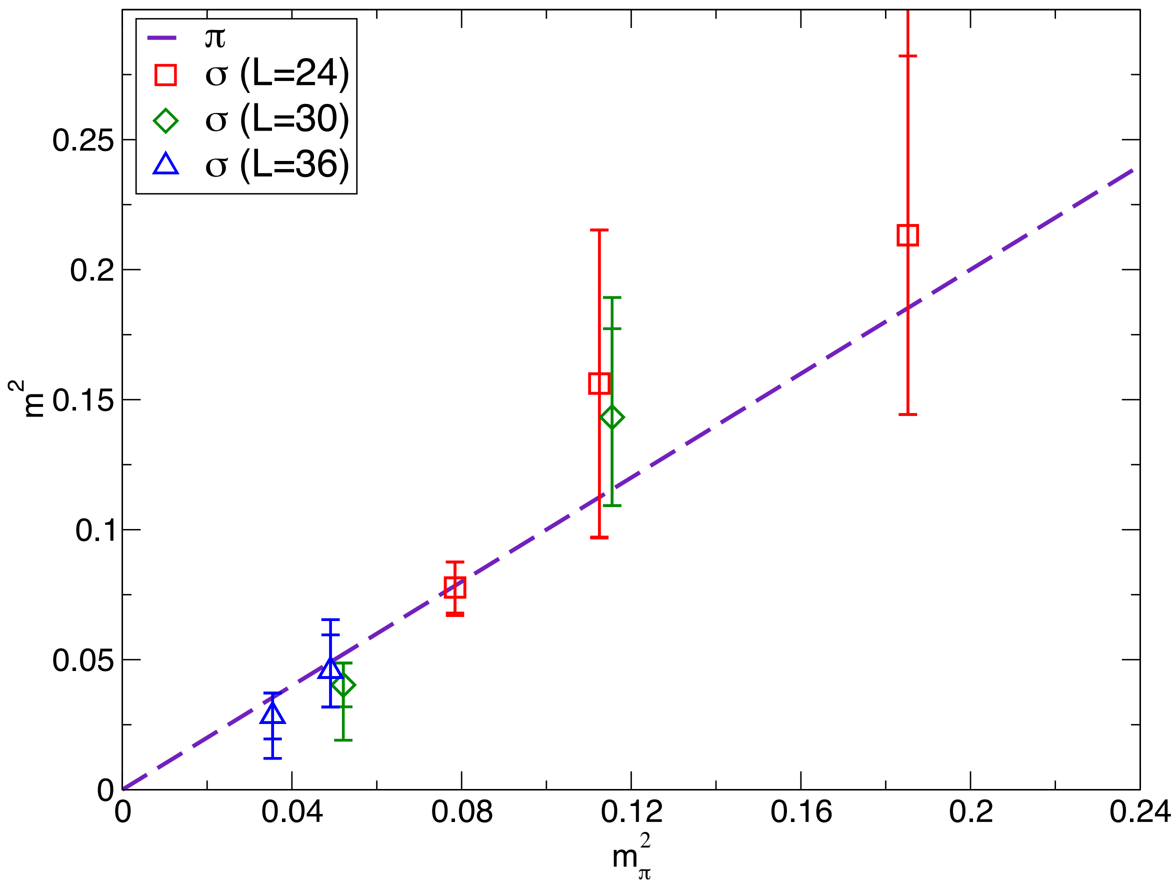

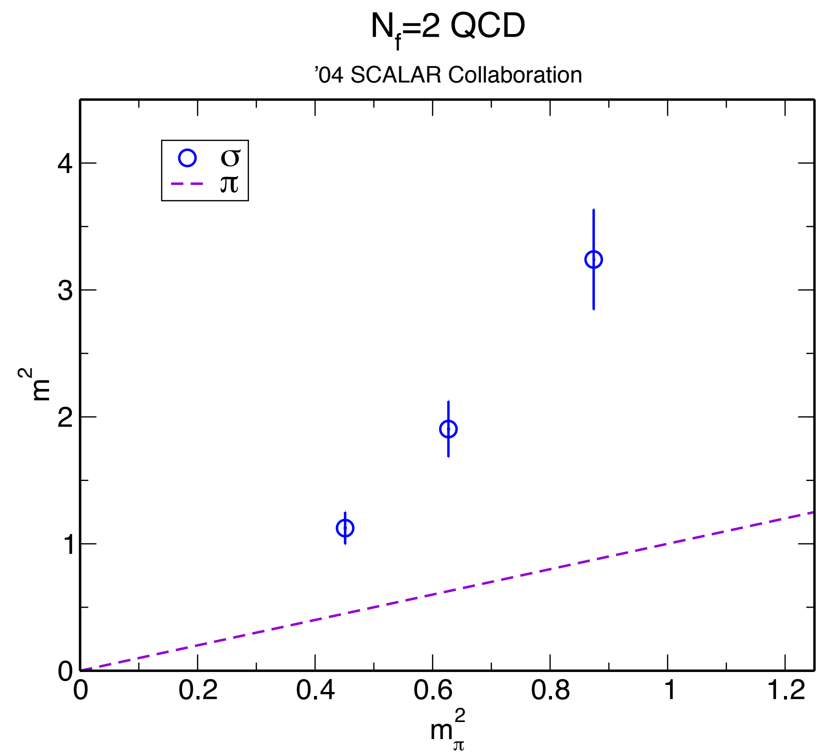

$m_\sigma \sim m_\pi \to C \sim 1$: different from $N_f = 2$ QCD

Chiral extrapolation (2) in $N_f = 8$

ChPT with scale symmetry breaking   '13 Matsuzaki and Yamawaki

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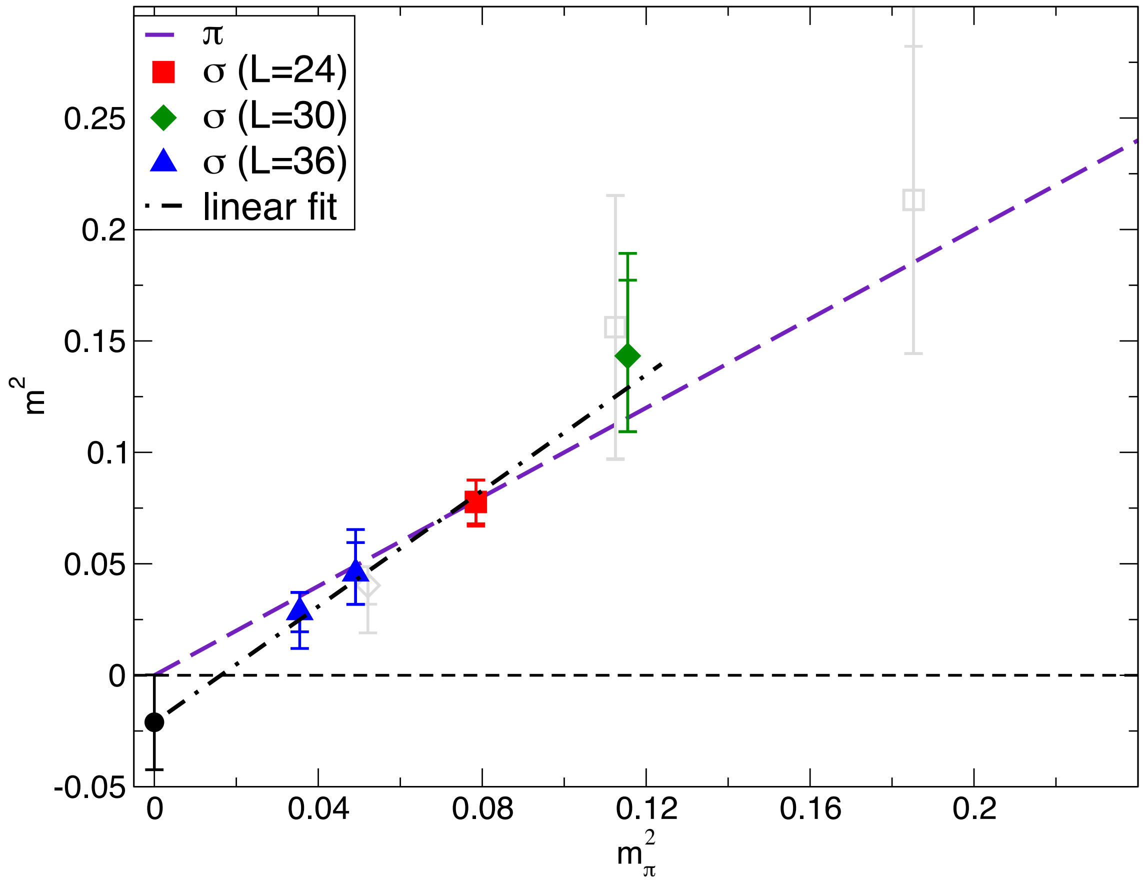


$m_0^2 < 0$: data not in $m_\sigma > m_\pi$ region

Need data at smaller m_f where $m_\sigma > m_\pi$ as in usual QCD

Summary

Flavor-singlet scalar is important in walking technicolor theory.
Difficult due to huge noise in lattice simulation
$\Rightarrow$ Noise reduction method and large N_{conf} $O(10000)$

Results of $N_f = 12$ QCD (consistent with conformal phase)
- $m_\sigma < m_\pi$; much different from small N_f QCD
- Conformal symmetry may make σ light

Results of $N_f = 8$ QCD (maybe candidate of walking technicolor)
- $m_\sigma \sim m_\pi$; much different from small N_f QCD
- Might be reflection of approximate conformal symmetry
- Need more data at smaller m_f for chiral extrapolation
  Linear chiral extrapolation : $m_\sigma/F = 1.7(2.7)$

Possibility of light composite Higgs $m_{\text{Higgs}} \sim v_{EW}$
(technidilaton)

Future work:

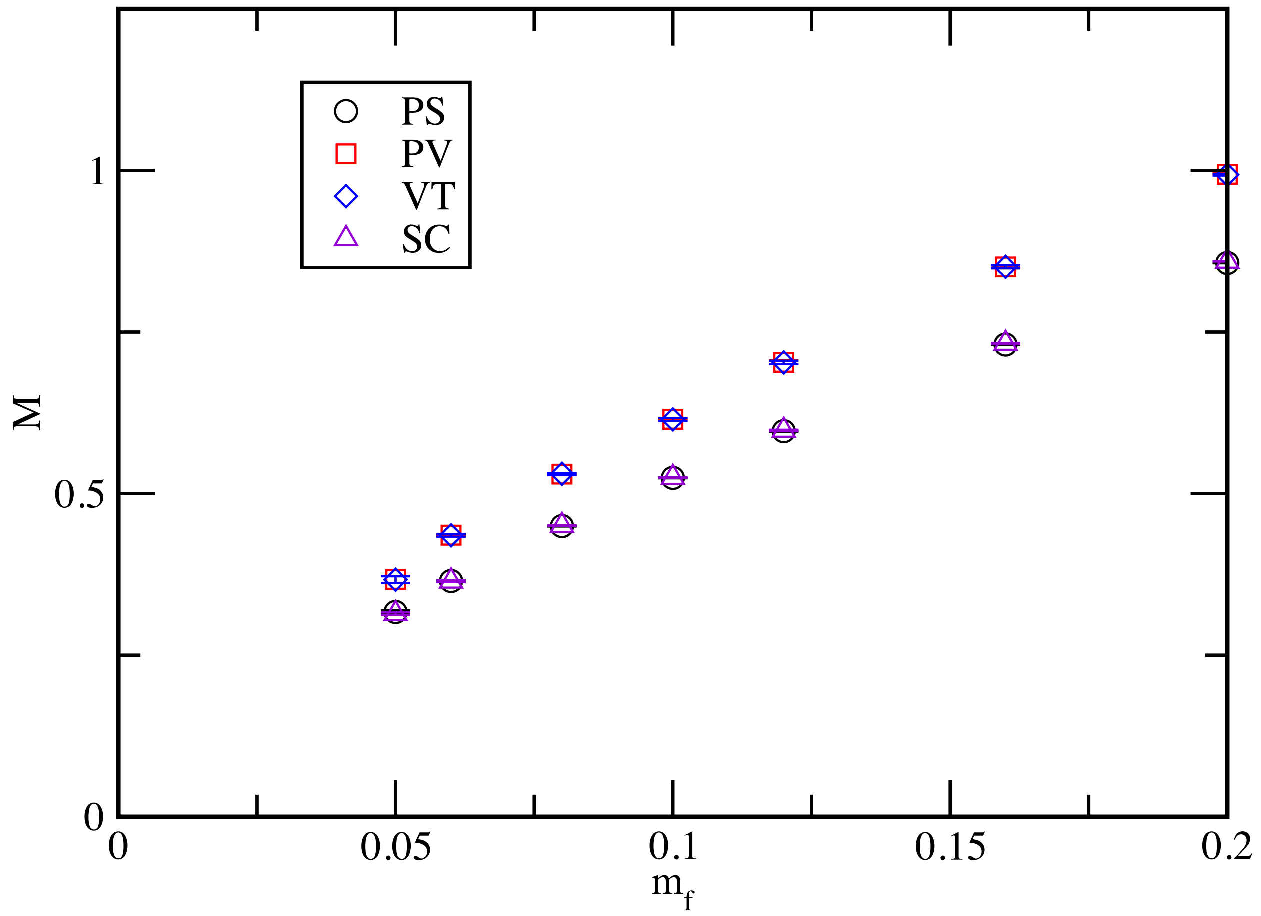
Smaller m_f data for more reliable chiral extrapolation

Back up

$N_f = 12$ taste symmetry breaking effect

LatKMI; PRD86(2012)054506

0^-: PS, SC; 1^-: PV, VT



Small taste symmetry breaking in meson masses

States in $D(t)$

$A_H(t) = A_H \exp(-M_H t)$

Connected part

$$-C(t) = A_{a_0}(t) + (-1)^t A_{\pi_{\mathsf{SC}}}(t)$$

Connected + disconnected

$$N_f D(t) - C(t) = A_\sigma(t) + (-1)^t A_{\pi_{\overline{\mathsf{SC}}}}(t)$$

$$\xrightarrow{\text{taste symmetric limit}} \pi_{\overline{\mathsf{SC}}} = \pi_{\mathsf{SC}} = \pi_{\mathsf{PS}}$$

$\pi_{\overline{\mathsf{SC}}}$: Species-singlet but taste-non-singlet 0^-

η in PRD76:094504(2007)

disconnected part

$$N_f D(t) = A_\sigma(t) - A_{a_0}(t) + (-1)^t \left(A_{\pi_{\overline{\mathsf{SC}}}}(t) - A_{\pi_{\mathsf{SC}}}(t)\right)$$

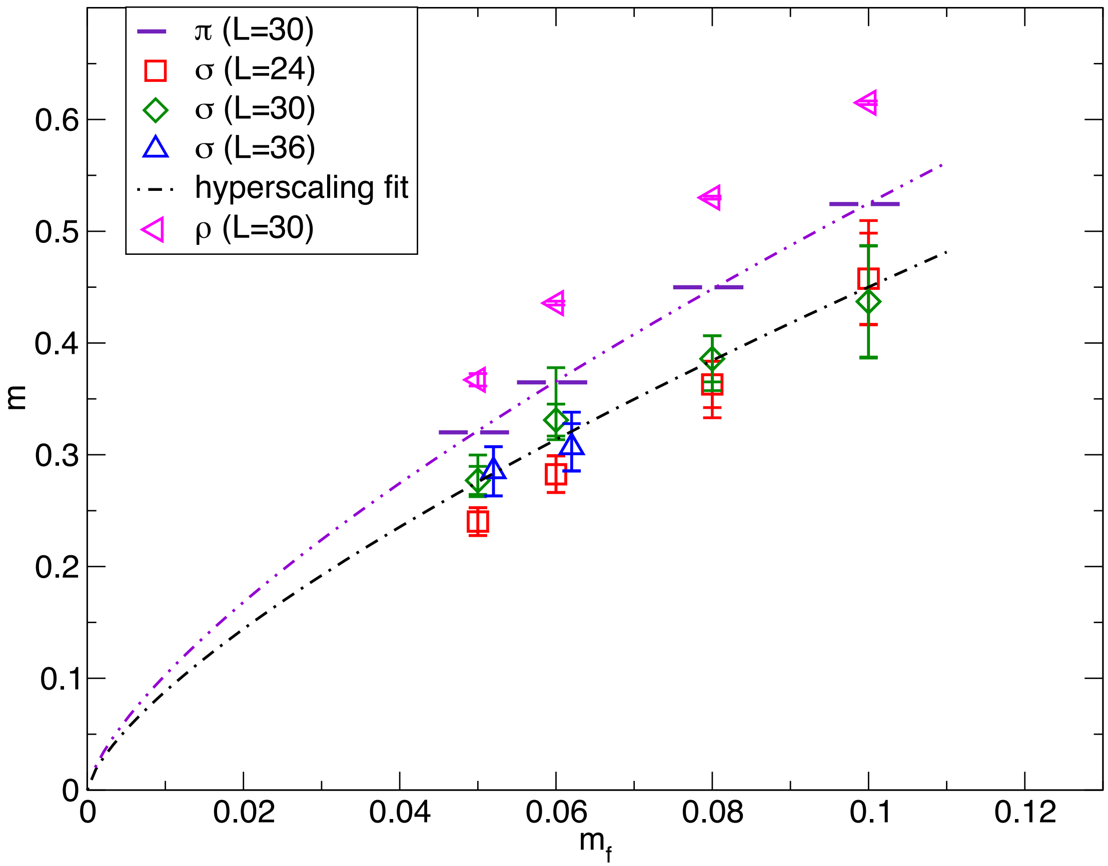
$$\xrightarrow{\text{taste symmetric limit}} A_\sigma(t) - A_{a_0}(t)$$

$\Rightarrow$ small oscillation in $D(t)$ if good taste symmetry

m_f dependence of m_σ in $N_f = 12$

arXiv:1305.6006

m_σ from fit of $D(t)$ with $t = 4$–8



Lighter than π in all m_f

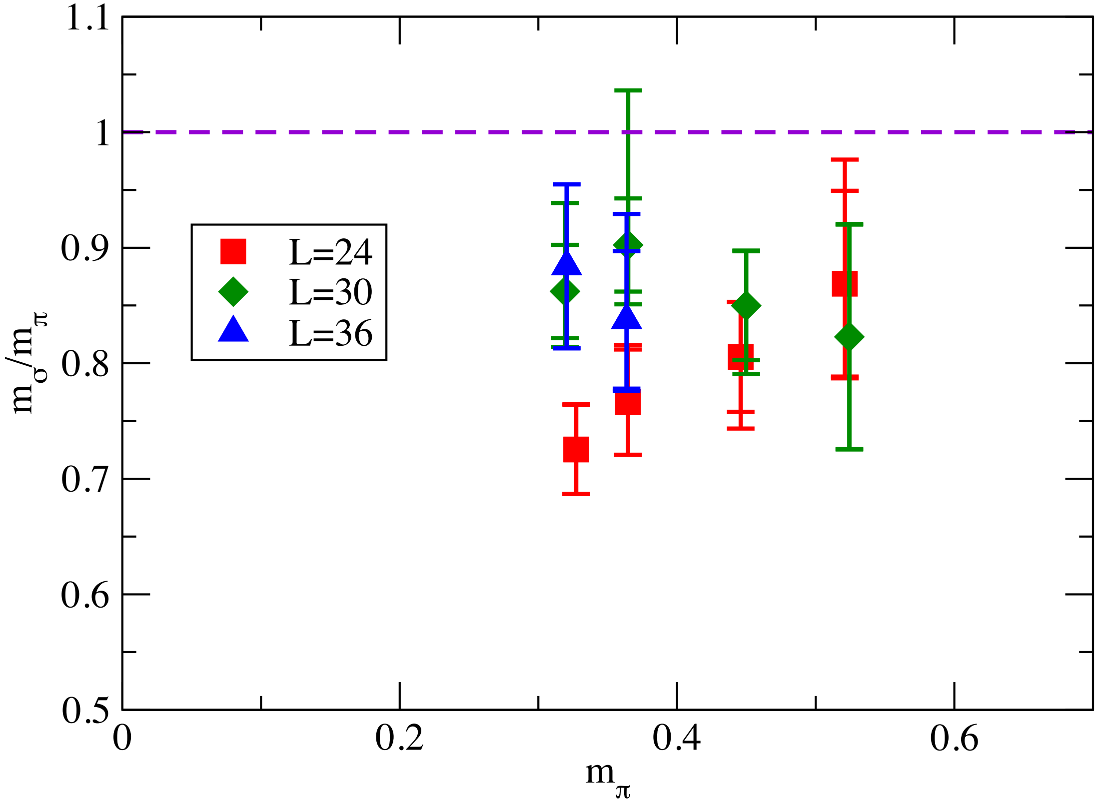
Much different from usual QCD

Conformal symmetry may make σ light

m_f dependence of m_σ/m_π

arXiv:1305.6006

m_σ from fit of $D(t)$ with $t = 4$–8



Discussion

Why flavor-singlet scalar calculation is possible?

- Nice noise reduction method
- Huge N_{conf}
- Small $m_\sigma \rightarrow$ slow exponential damp of correlator
- Small taste symmetry breaking $\leftarrow$ improved action, large N_f, etc.

Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

PRD86(2012)054506; PRD87(2013)094511

N_f increasing : chiral broken $\rightarrow$ walking $\rightarrow$ conformal

Signal of phase

- Chiral broken phase

  Simulations at $m_f \neq 0$

$$m_f \rightarrow 0\text{: } m_\pi \rightarrow 0 \text{ and } F_\pi \neq 0 \Rightarrow \frac{F_\pi}{m_\pi} \xrightarrow{m_\pi \rightarrow 0} \infty$$

- Conformal phase

Simulations at $m_f \neq 0$: scale invariance breaking $\rightarrow$ confinement phase

Hyperscaling with anomalous dimension γ^* at small m_f

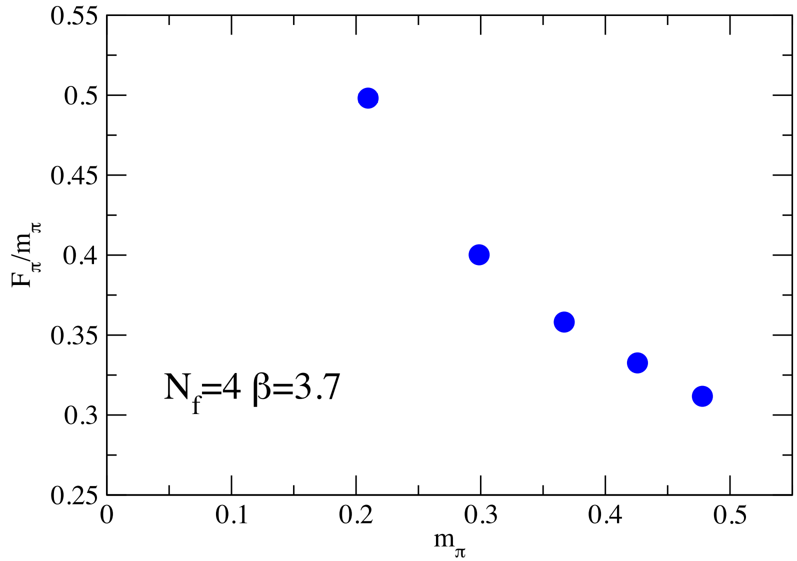
$$\begin{aligned} m_H &= C_H \ m_f^{1/(1+\gamma^*)} \\ F_\pi &= C_F \ m_f^{1/(1+\gamma^*)} \end{aligned} \Rightarrow \frac{F_\pi}{m_\pi} \xrightarrow{m_\pi \rightarrow 0} \text{constant}$$

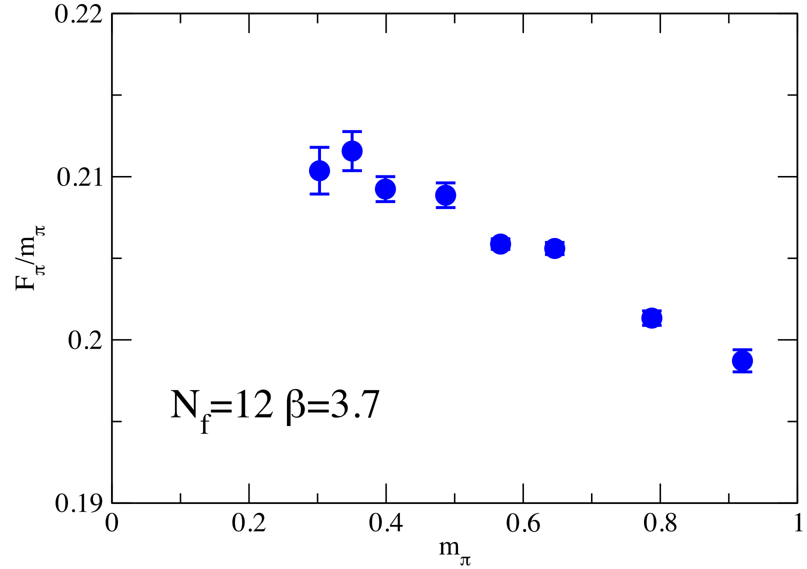
Different $m_f(m_\pi)$ dependence in two phases

Recent study of LatKMI Collaboration

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511

$F_\pi/m_\pi \to \infty$ $F_\pi/m_\pi \to$ constant

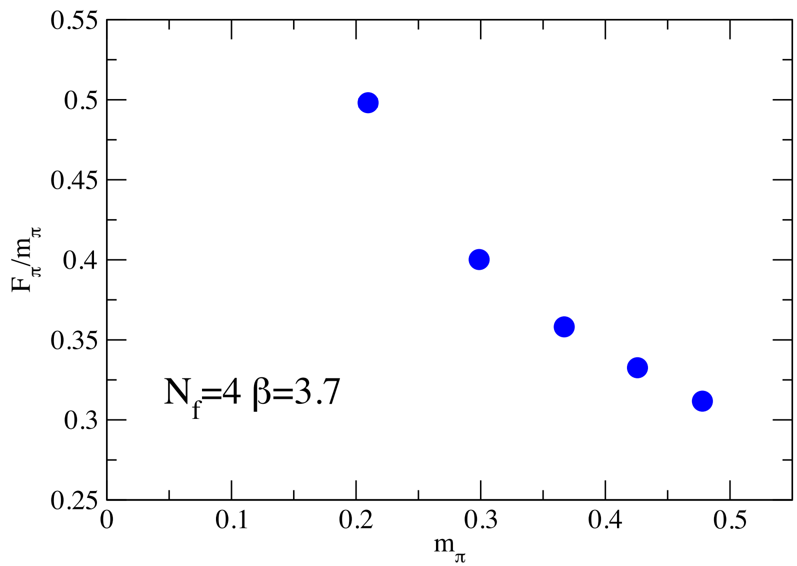




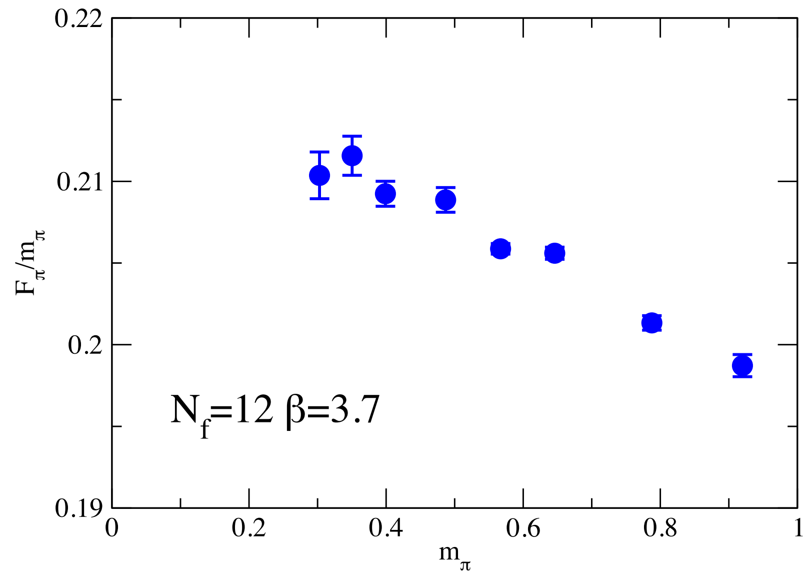
Recent study of LatKMI Collaboration

$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511

Chiral broken



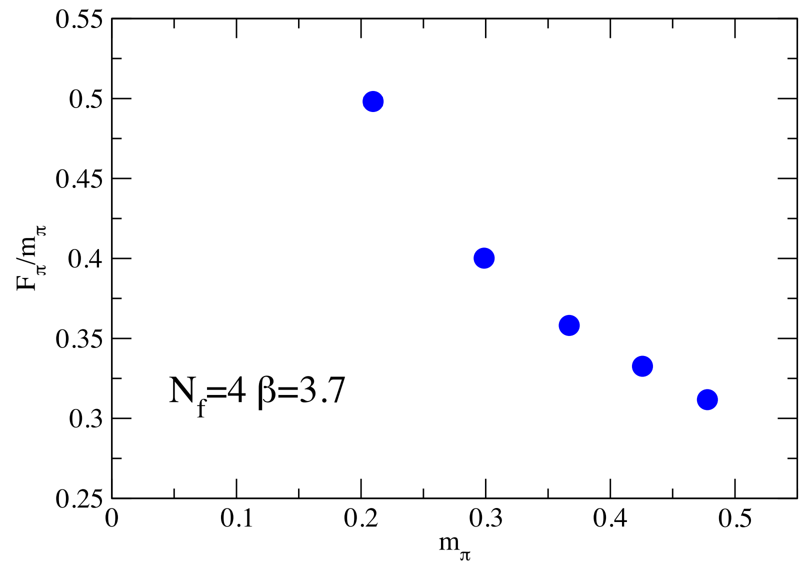
Conformal

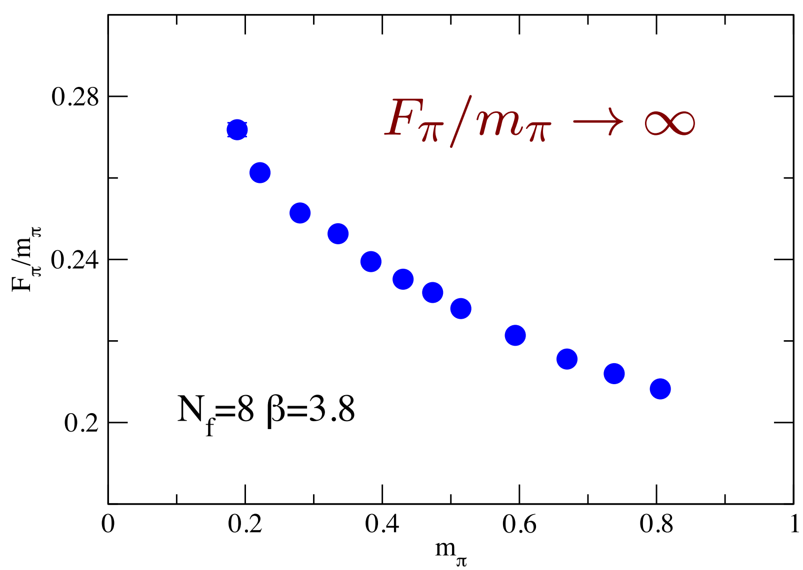


Recent study of LatKMI Collaboration

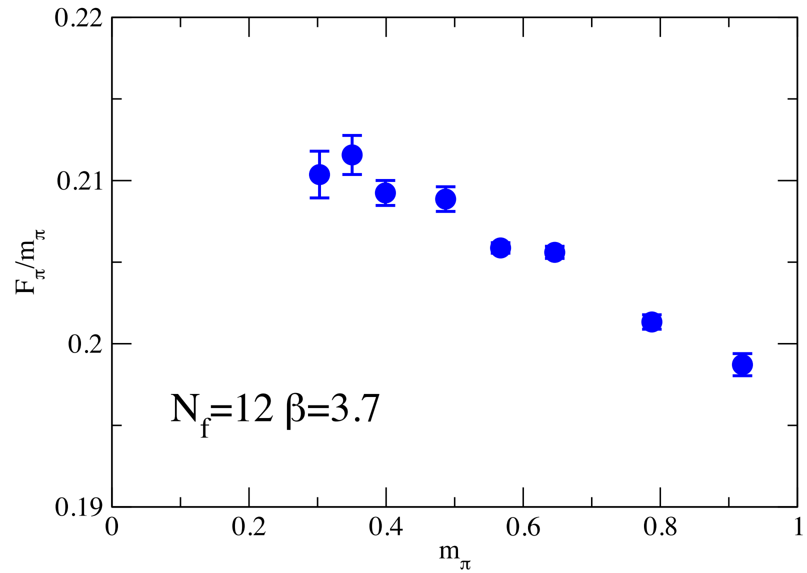
$N_f = 12$: PRD86(2012)054506; $N_f = 8$: PRD87(2013)094511

Chiral broken





Conformal



Recent study of LatKMI Collaboration

Search for candidate of walking technicolor

PRD86(2012)054506; PRD87(2013)094511

$N_f = 4$ QCD: Spontaneous chiral symmetry breaking

$N_f = 12$ QCD: Consistent with conformal phase

$N_f = 8$ QCD may be a candidate of Walking technicolor

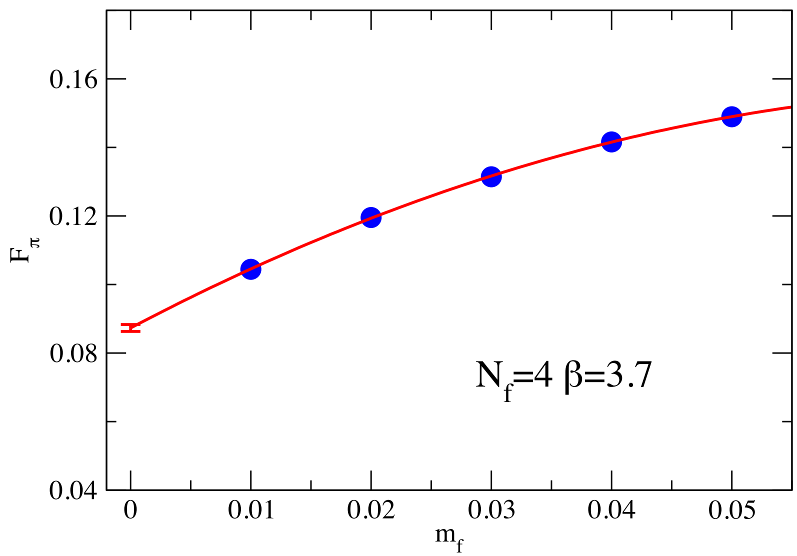
- Spontaneous chiral symmetry breaking
  $F_\pi \neq 0$ and $F_\pi/m_\pi \to \infty$ towards $m_f \to 0$
- Slow running (walking) coupling in wide scale range
  Different behaviors of F_π in light and middle m_f
- Large anomalous mass dimension $\gamma^* \sim 1$ in walking region
  $\gamma = 0.62$–0.97: Hyperscaling-like behavior in middle m_f
- Light composite scalar $\Leftarrow$ Important to check!

Next: Flavor-singlet scalar in (approximate) conformal theory

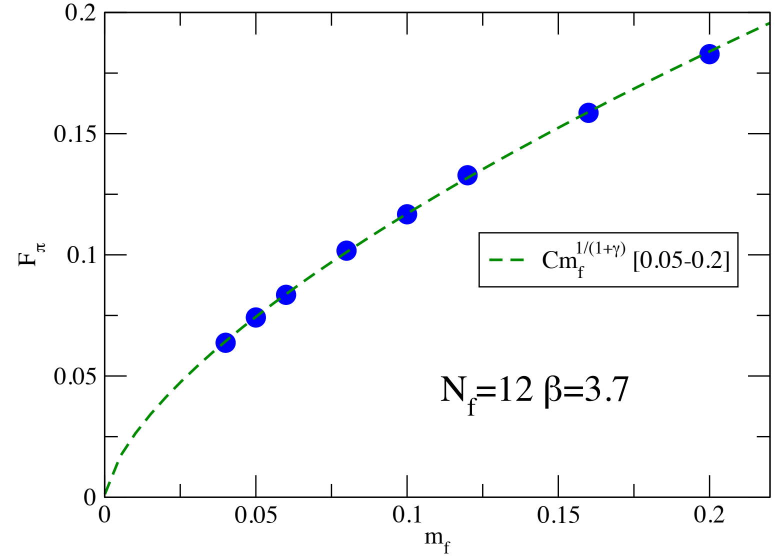
Recent study of LatKMI Collaboration

PRD86(2012)054506; PRD87(2013)094511

Chiral broken $F_\pi \to F \neq 0$



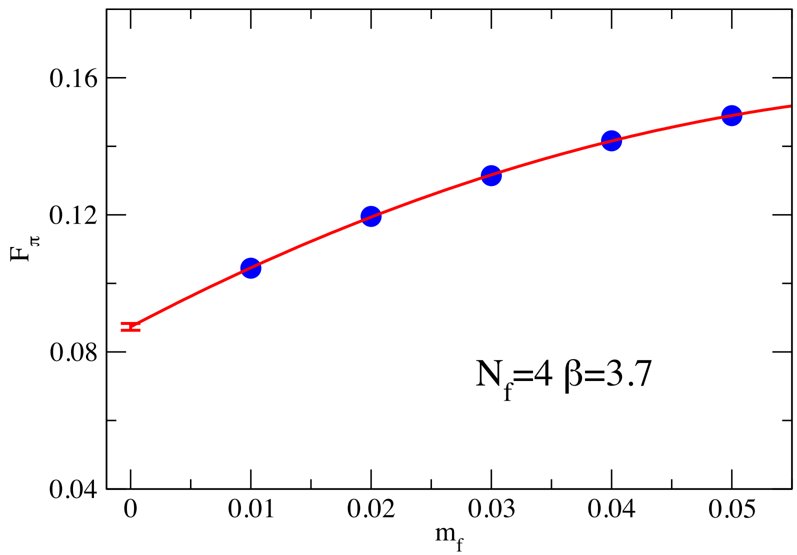
Conformal $F_\pi \to C m_f^{1/(1+\gamma)}$



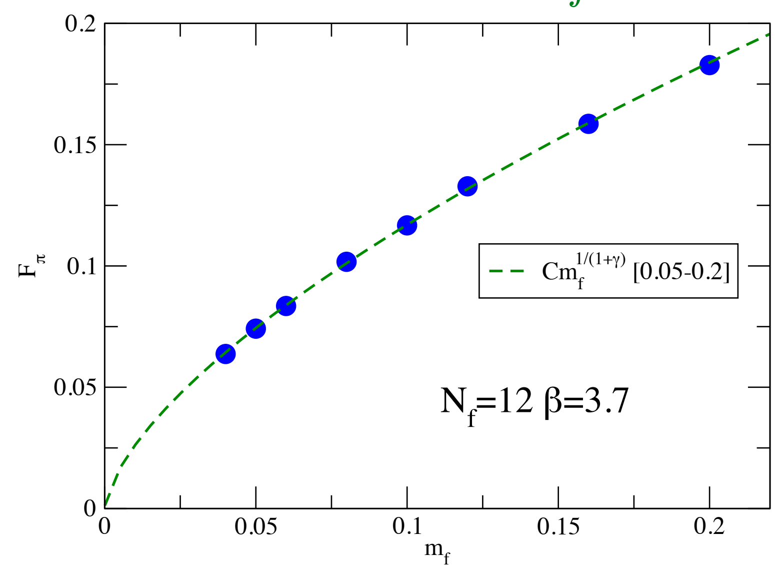
Recent study of LatKMI Collaboration

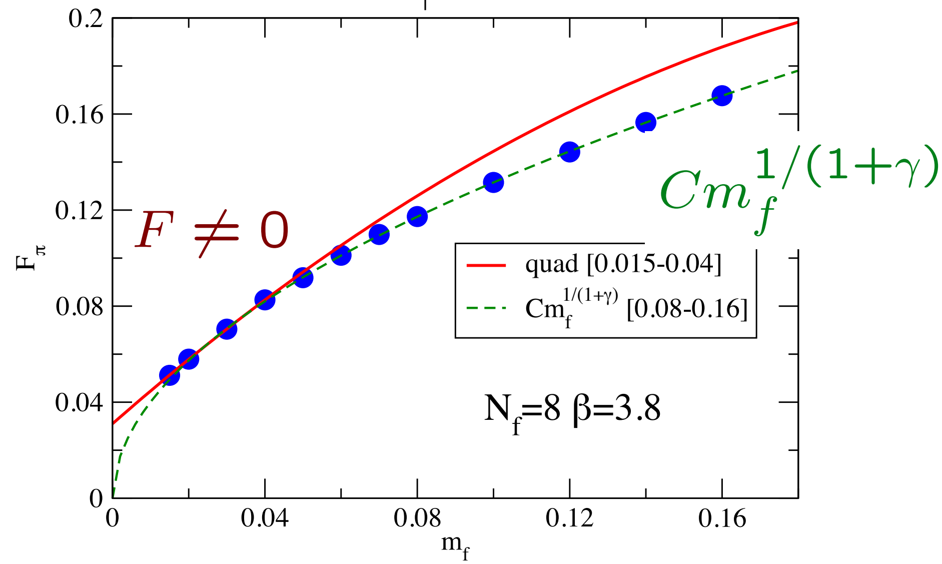
PRD86(2012)054506; PRD87(2013)094511

Chiral broken $F_\pi \to F \neq 0$



Conformal $F_\pi \to C m_f^{1/(1+\gamma)}$

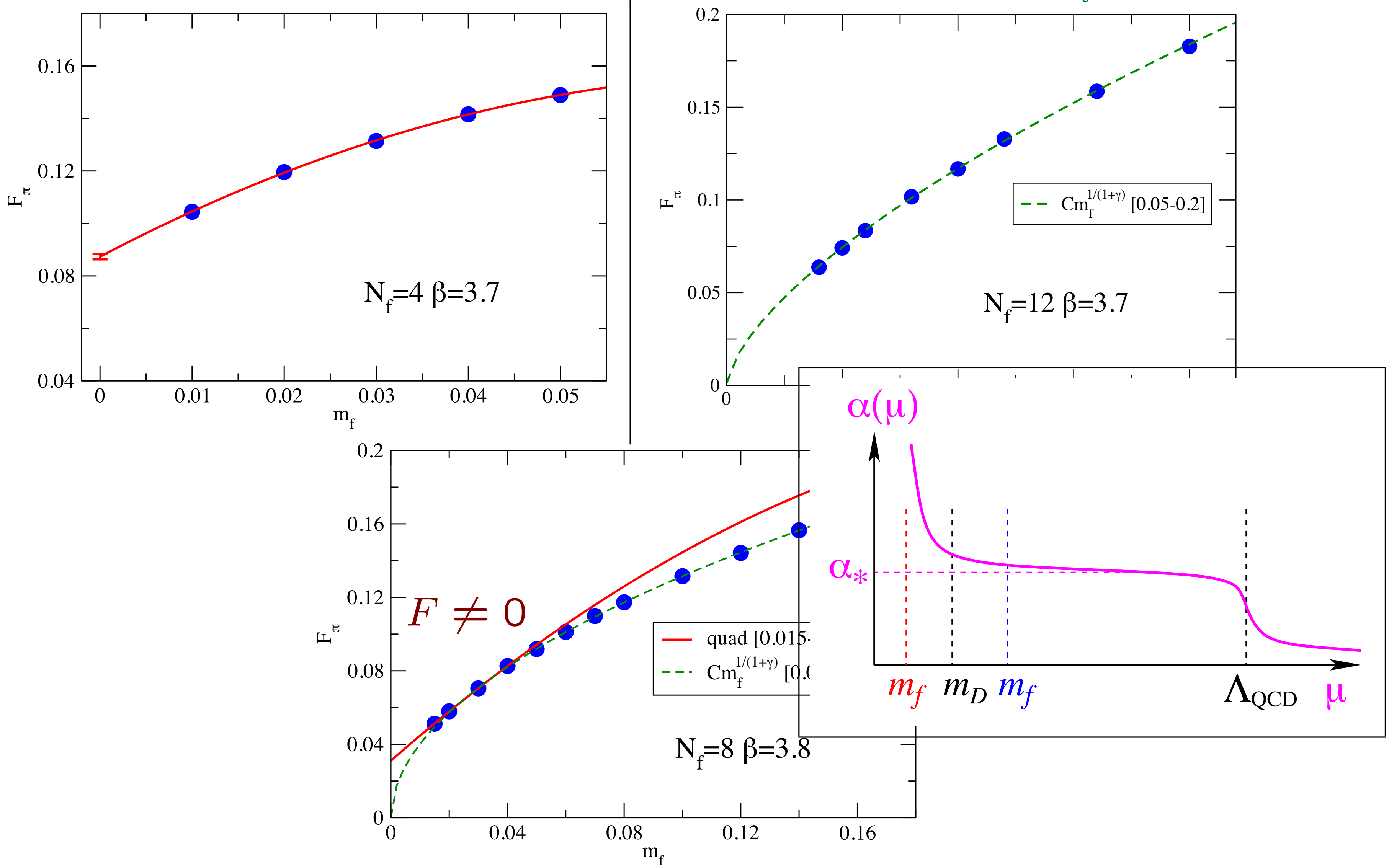




Recent study of LatKMI Collaboration

PRD86(2012)054506; PRD87(2013)094511

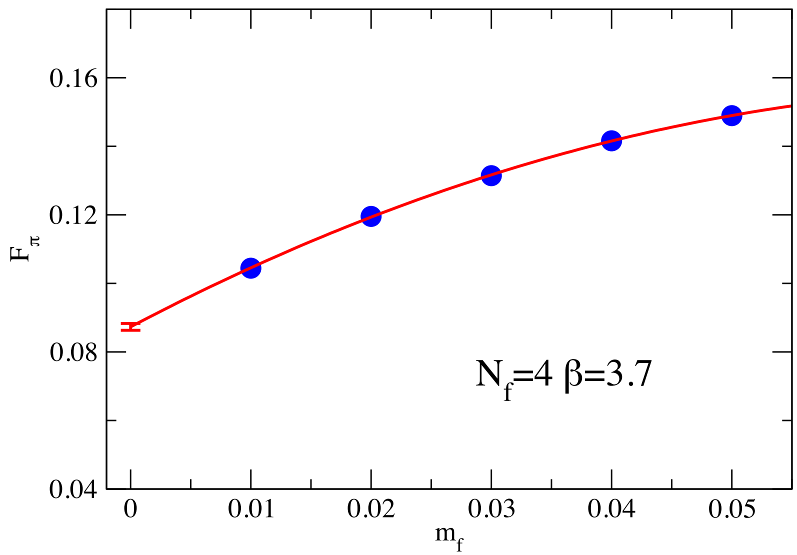
Chiral broken $F_\pi \to F \neq 0$ | Conformal $F_\pi \to C m_f^{1/(1+\gamma)}$



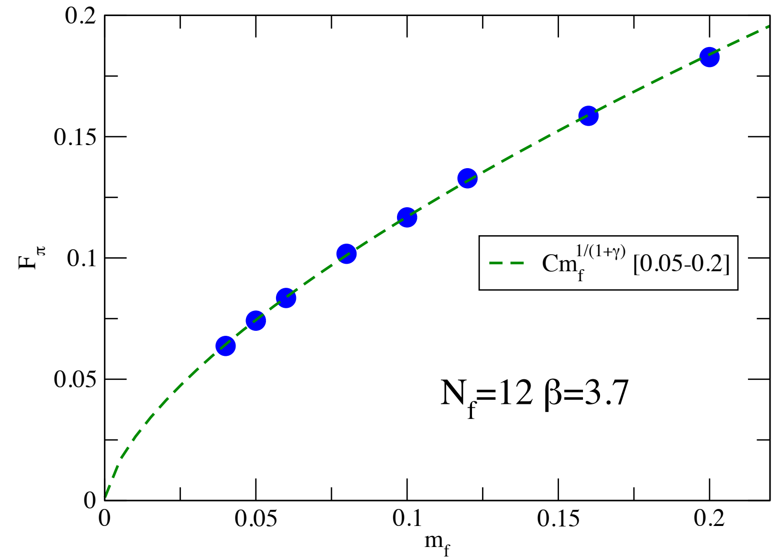
Recent study of LatKMI Collaboration

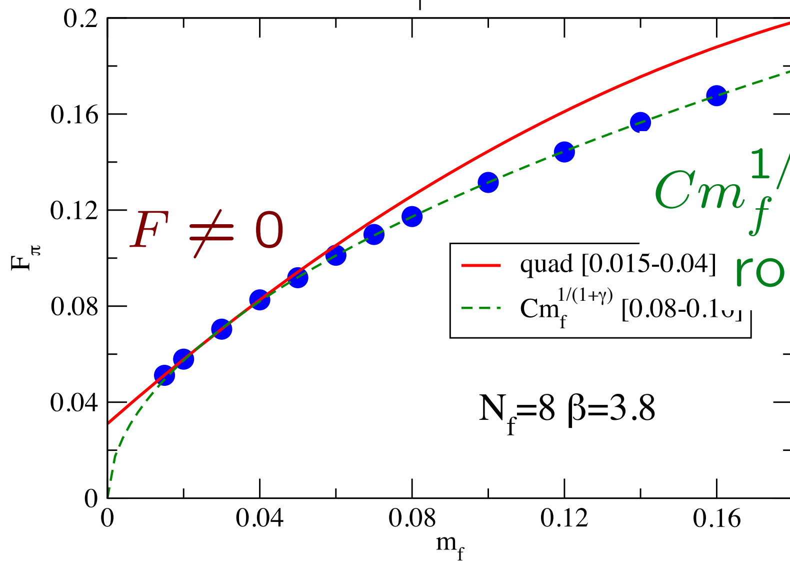
PRD86(2012)054506; PRD87(2013)094511

Chiral broken $F_\pi \to F \neq 0$



Conformal $F_\pi \to C m_f^{1/(1+\gamma)}$





$F \neq 0$

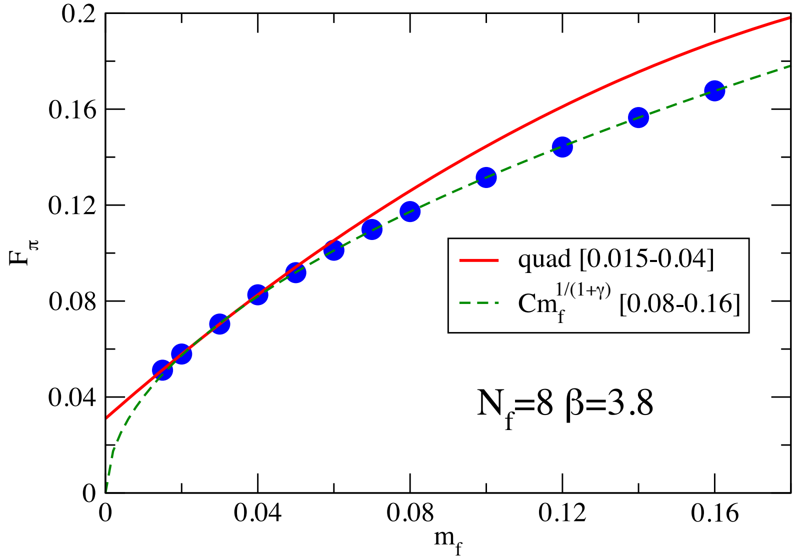
$C m_f^{1/(1+\gamma)}$

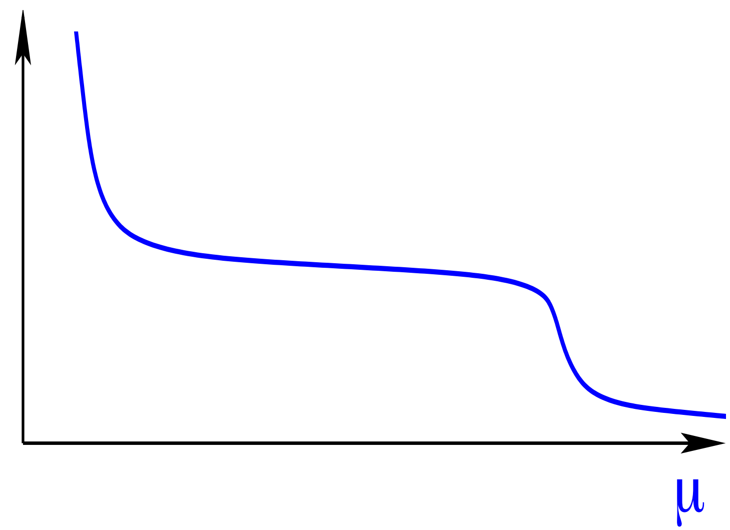
rough estimate of γ

Recent study of LatKMI Collaboration

PRD86(2012)054506; PRD87(2013)094511

Possible explanation through walking coupling



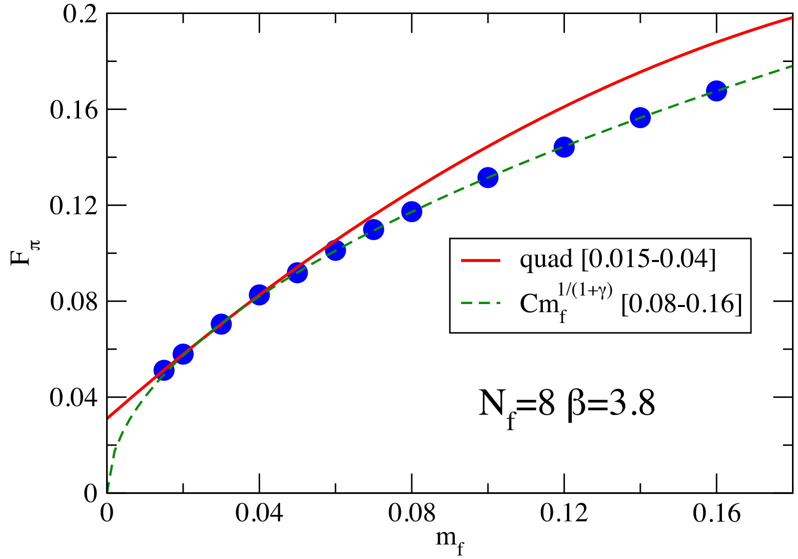


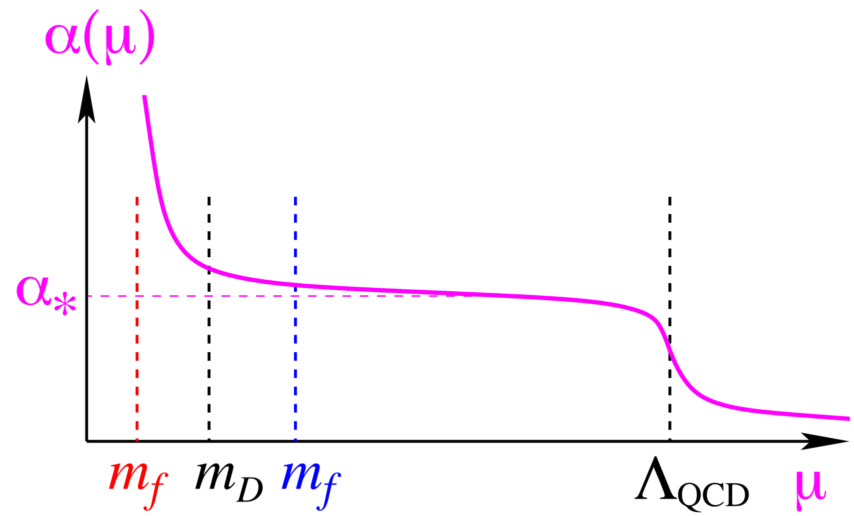
If $N_f = 8$ QCD has walking coupling ...

Recent study of LatKMI Collaboration

PRD86(2012)054506; PRD87(2013)094511

Possible explanation through walking coupling





m_f is regarded as IR scale cutoff of system.

Large $m_f \gg m_D$

Confine system at m_f

Not care spontaneous chiral symmetry breaking

$\rightarrow$ same as conformal system with large m_f

Small $m_f \lesssim m_D$

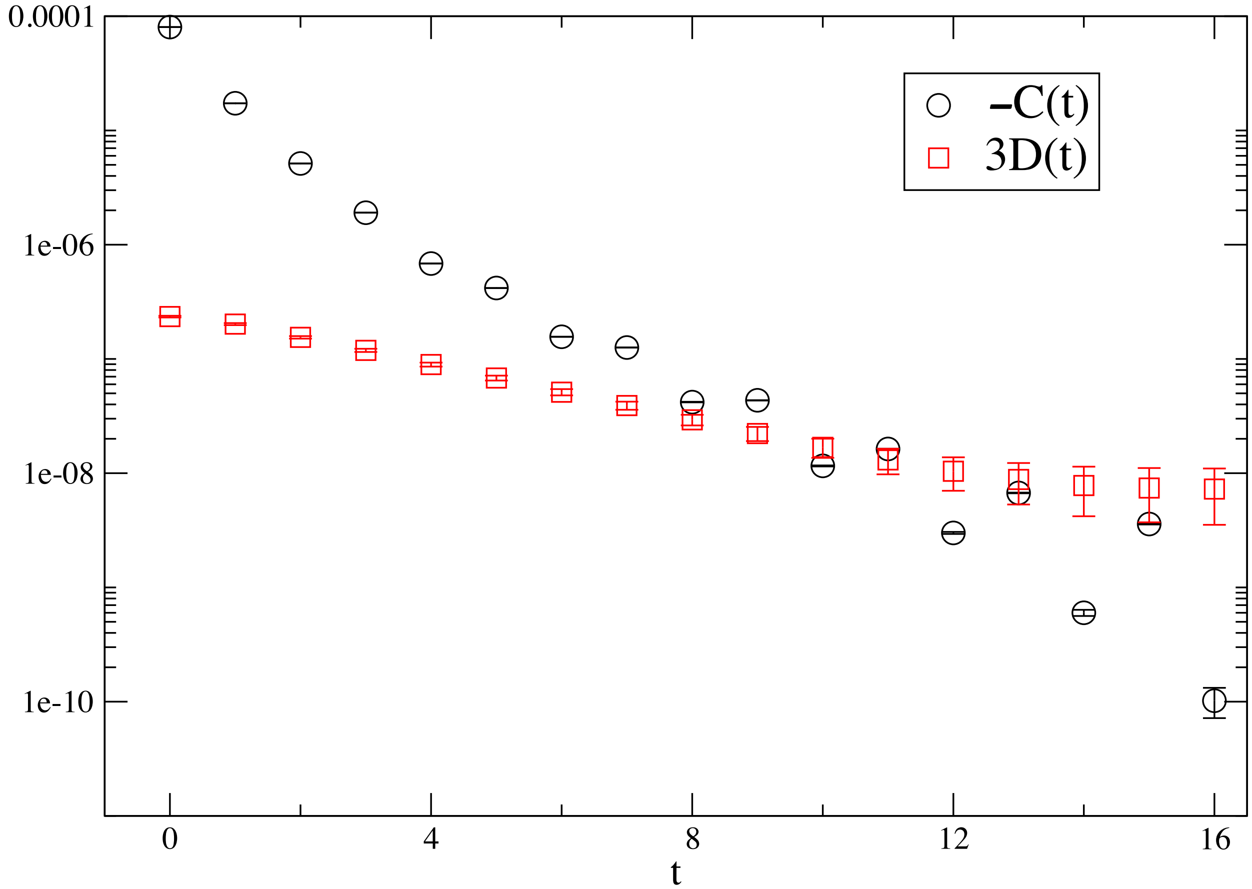
Contain spontaneous chiral symmetry breaking effect

Dual nature maybe signal of walking coupling

Correlators in $N_f = 12$

PRL111(2013)162001

$m_f = 0.06, L = 24$ with $N_{\text{conf}} = 14000$



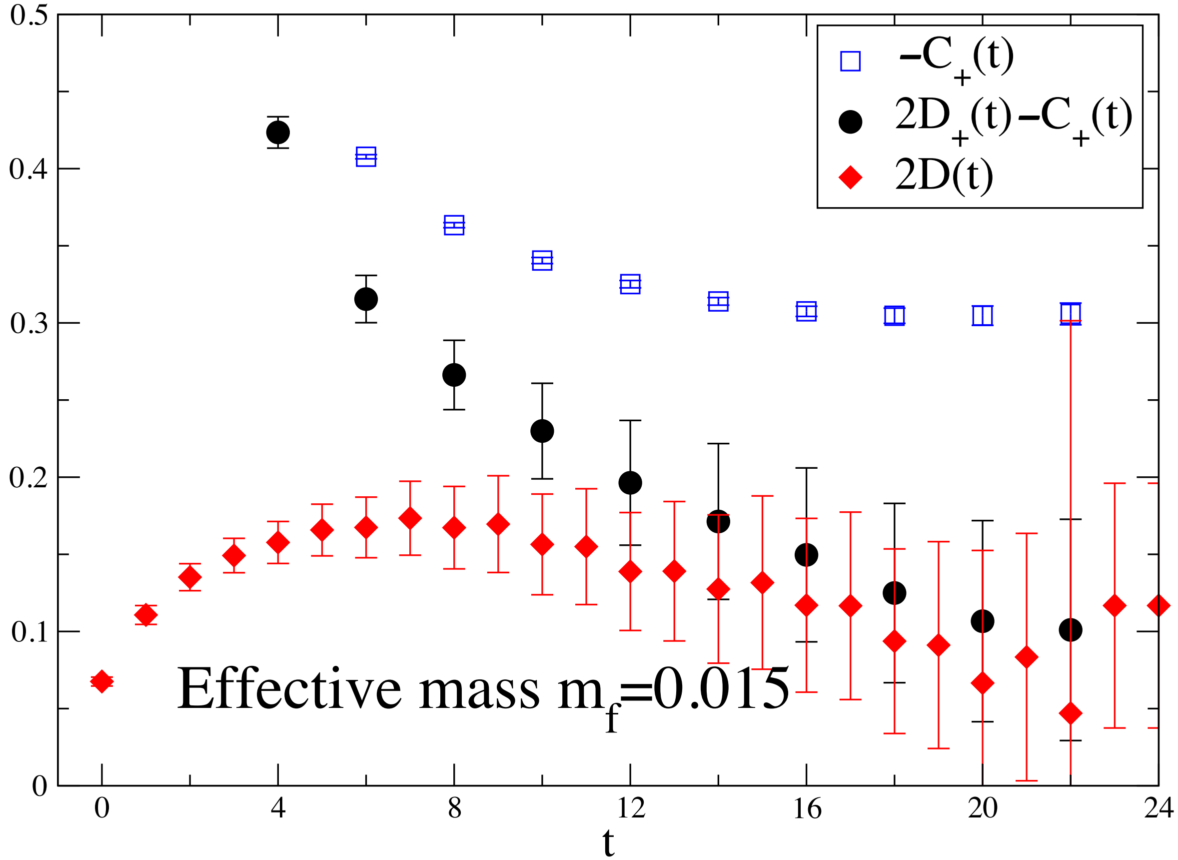
$-C(t)$ oscillates, but $D(t)$ does not

cancellation: species-singlet and non-singlet π_{SC} in $D(t)$

thanks to small taste symmetry breaking; PRD86(2012)054506

Effective mass in $N_f = 8$

$m_f = 0.015, L = 36$ with $N_{\text{conf}} = 2600$



Non-singlet scalar

a_0: $-C_+(t)$

Singlet scalar

σ: $2D_+(t) - C_+(t)$

$m_\sigma < m_{a_0}$

σ: $2D(t)$

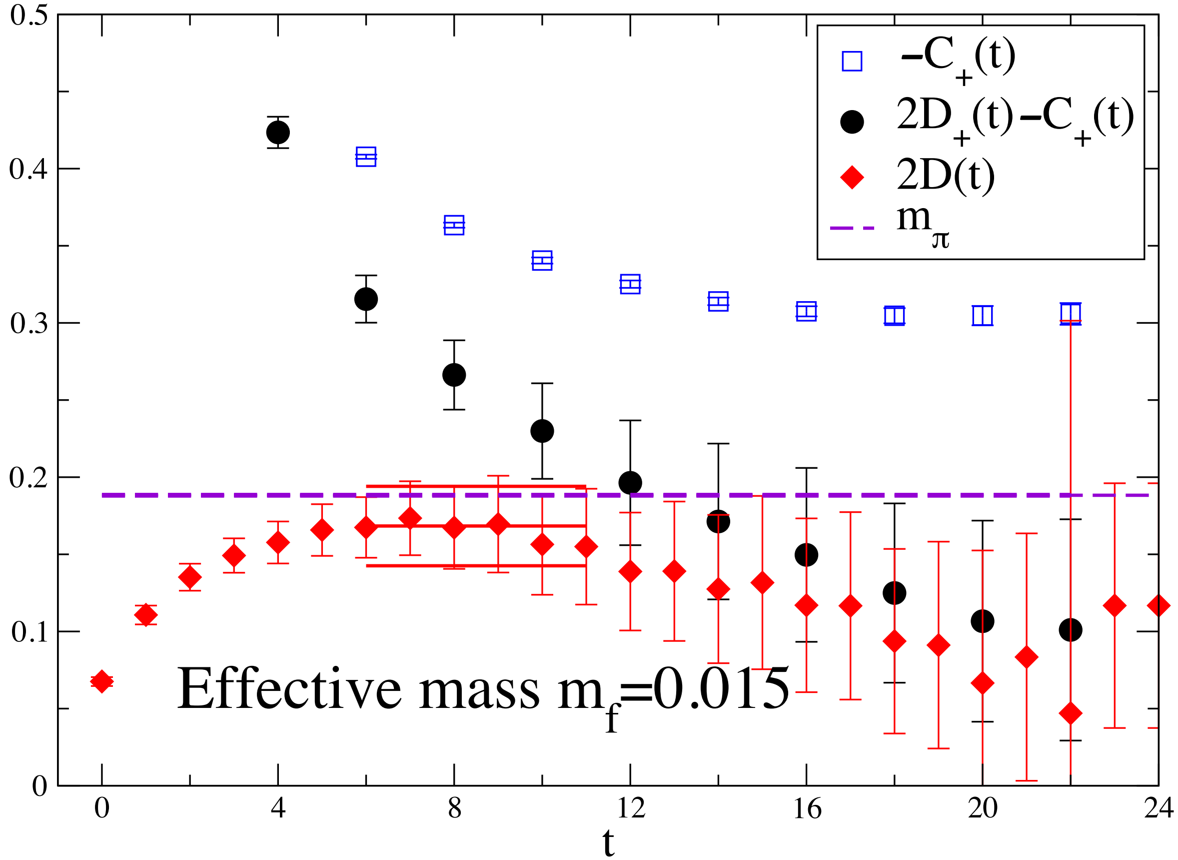
Consistent m_σ with smaller error

$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of m_σ from $2D(t)$

Effective mass in $N_f = 8$

$m_f = 0.015, L = 36$ with $N_{\text{conf}} = 2600$



Non-singlet scalar

a_0: $-C_+(t)$

Singlet scalar

σ: $2D_+(t) - C_+(t)$

$m_\sigma < m_{a_0}$

σ: $2D(t)$

Consistent m_σ

with smaller error

$m_\sigma \sim m_\pi$

$$X_+(t) = 2X(t) + X(t+1) + X(t-1)$$

Good signal of m_σ from $2D(t)$

fit range dependence $\rightarrow$ systematic error

m_f dependence of m_σ in $N_f = 8$

ChPT with spontaneous scale symmetry breaking

'13 Matsuzaki and Yamawaki

$$m_\sigma^2 = m_0^2 + C \cdot m_\pi^2 + (\text{chiral log of } m_\pi)$$

$$C = \frac{(3-\gamma_m)(1+\gamma_m)}{4} \cdot \frac{2N_f F_\pi^2}{F_\sigma^2}$$

F_σ: decay constant of σ

Valid in small m_π region $\to$ $m_\sigma > m_\pi$

c.f.)

- $\gamma_m \sim 1$
- 1 family model $F_\pi = v_{EW}/2$

  $C \lesssim 1$ from phenomenology and Holographic estimate