Strongly coupled gauge theories: In and out of the conformal window

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ArXiv:1301.1355,1310.1124 and in prep

Strongly coupled gauge-fermion systems

Attractive candidates for BSM phenomenology

- strongly coupled - need non-perturbative investigation
- gauge coupling is slowly walking (near marginal)

– Nearly conformal models are **very** different from QCD
$\rightarrow$ numerical methods from QCD are not always effective

- modified methods
  (finite size scaling with corrections)
- new approaches
  (running anomalous mass from spectral density)

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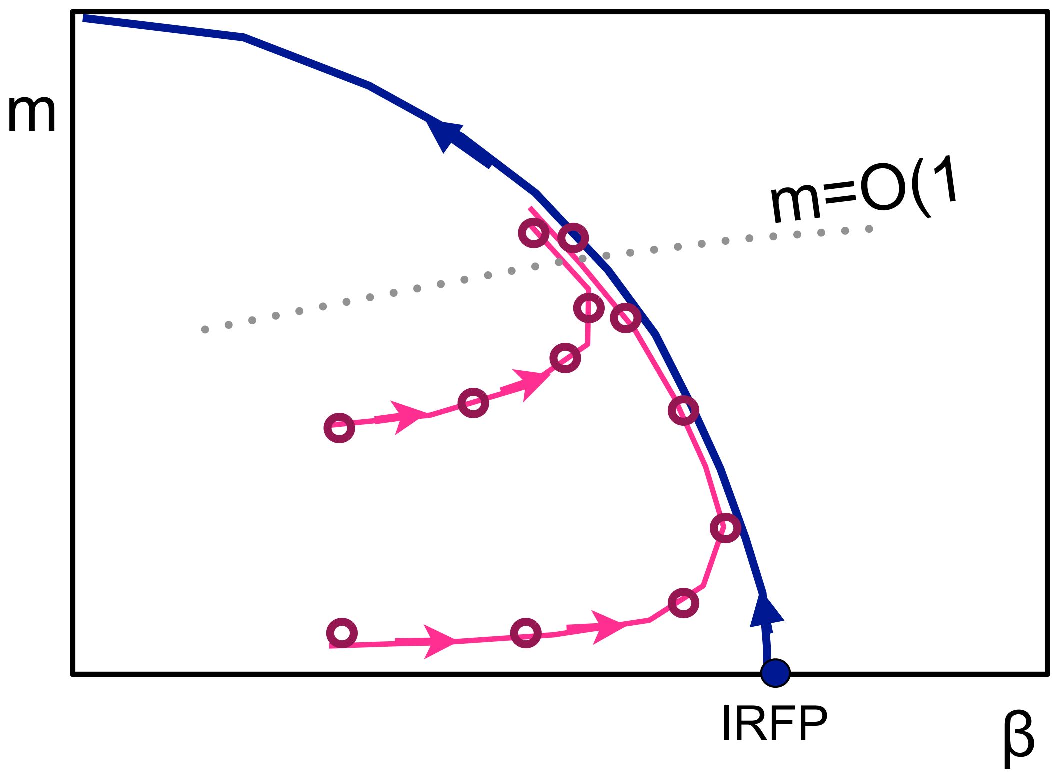
Universal scaling

Wilson RG



RG flow:
towards IRFP, away in m:

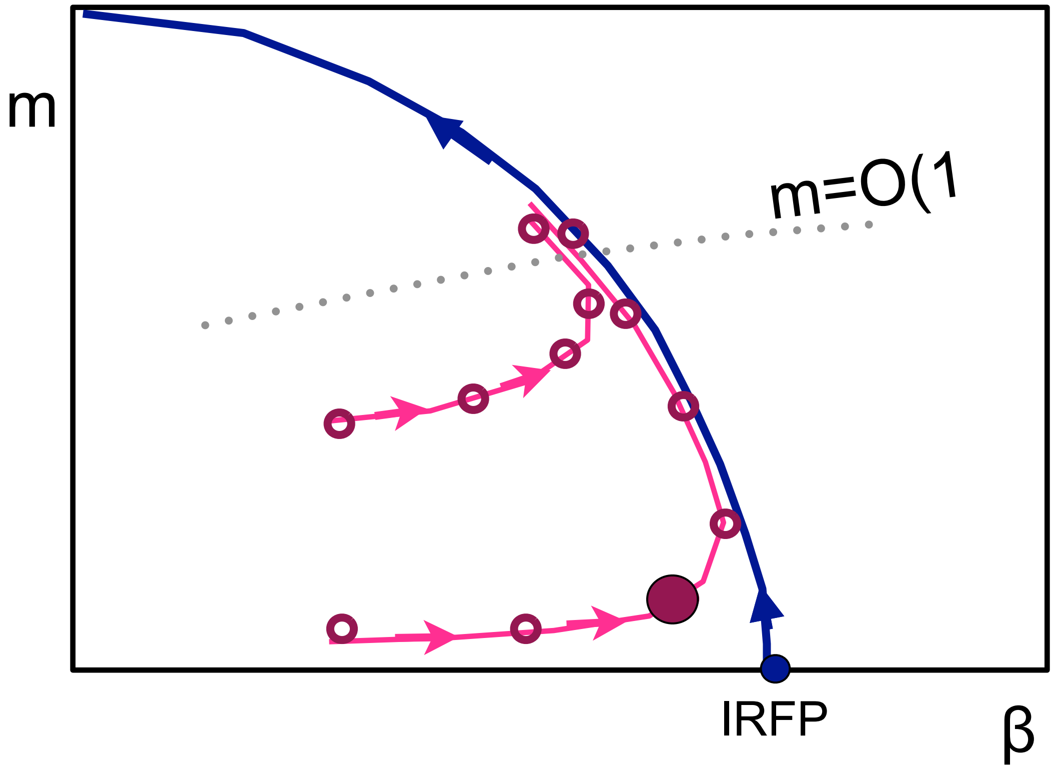
In n steps:
$m \to mb^{y} \to mb^{2y} \ldots \to mb^{ny}$
$L \to L/b \to L/b^{2} \ldots \to L/b^{n}$

but only as long as
$mb^{ny} < O(1)$ or
$L/b^{b} > O(1)$

Universal scaling behavior along the renormalized trajectory

NON - universal :

Wilson RG



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but only as long as
$mb^{ny} < O(1)$
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If m is large or L is small the flow does not RT:
no universal behavior → no scaling

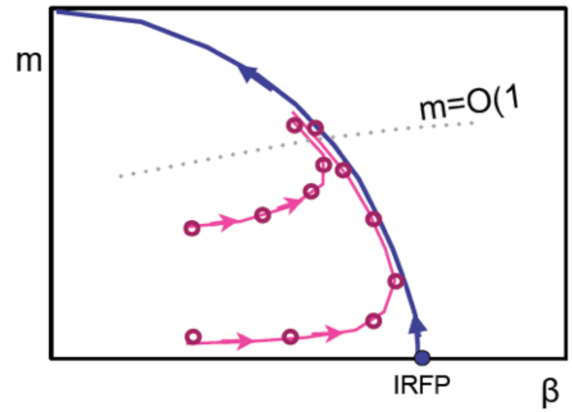
Finite size scaling - textbook case

Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

g_i with scaling dimensions $y_i < 0$.



Renormalization group arguments in volume L^3 predict scaling of physical masses as

$$M_H L = f(Lm^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

as $$m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$$

$$M_H L = f(x), \quad x = Lm^{1/y_m}$$

–tune y_m until different volumes "collapse"

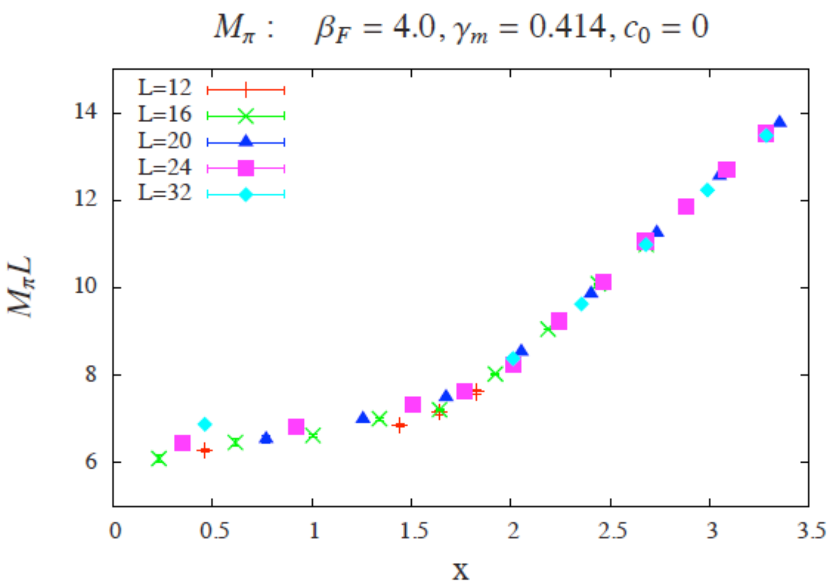
Finite size scaling N_f=12 (nHYP action)

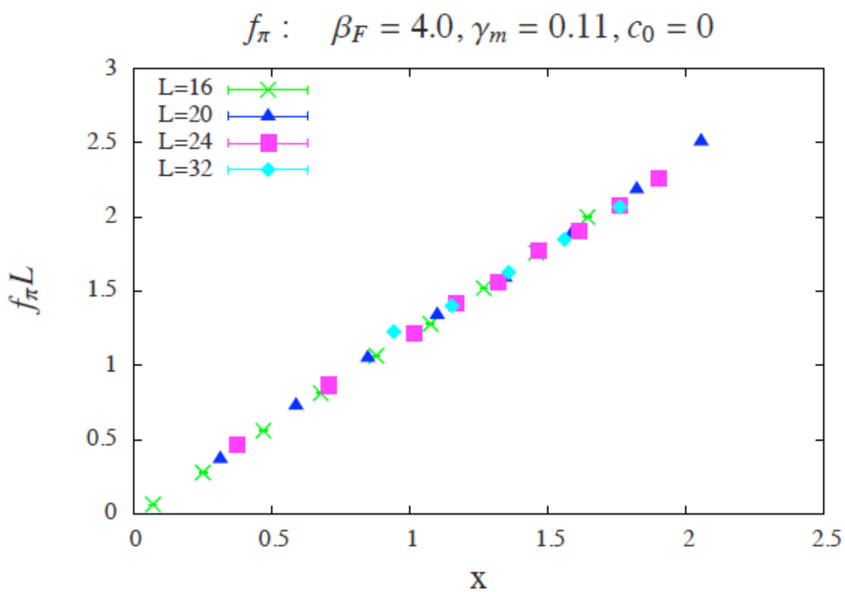
β= 4.0 (meson spectrum matches LatHiggs coll. β=2.2 closely)

– good curve collapse for larger $x = Lm^{1/y_m}$

– inconsistent exponents (see results from LHC, KMI as well)

– Not very good curve collapse at small x (small L)



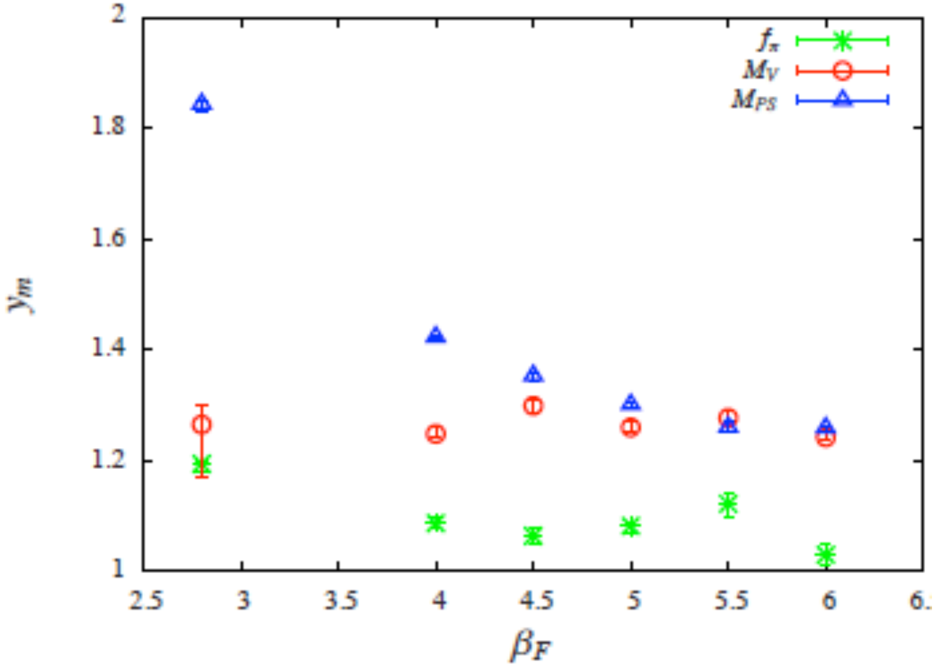


M_π : y_m=1.408(10)

f_π : y_m=1.11(5)

Scaling exponents

Result of “curve collapse” for pseudo-scalar, vector and f_π:

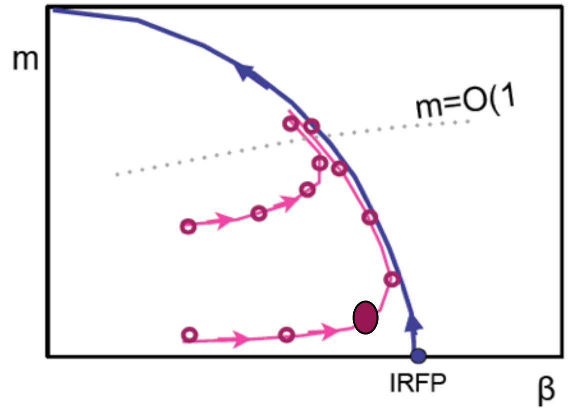


β=2.8 — 6.0
Volumes: 12^3, 16^3, 20^3, 24^3, 32^3
$N_T = 2\ N_S$
masses: 0.005 — 0.12
such that x= 0.2 - 5

25 - 35 data points at each β

y_m depends strongly on β and the operator considered

Finite size scaling with a **near-marginal operator**



Consider a FP with one relevant operator

$m \approx 0$ with scaling dimension $y_m > 0$

and irrelevant operators

g_i with scaling dimensions $y_i < 0$

g_0 (near) marginal, $y_0 \lesssim 0$

Renormalization group arguments in volume L^3 predict

$$M_H L = f(Lm^{1/y_m}, g_i m^{-y_i/y_m}) \quad \text{as} \quad m \approx 0$$

as $m \to 0, \quad L \to \infty: \quad g_i m^{-y_i/y_0} \to 0$

$$g_0 \to g_0 m^{\omega}, \quad \omega = -y_0 / y_m \gtrsim 0$$

$$M_H L = f(x, g_0 m^{\omega}), \quad x = Lm^{1/y_m}$$

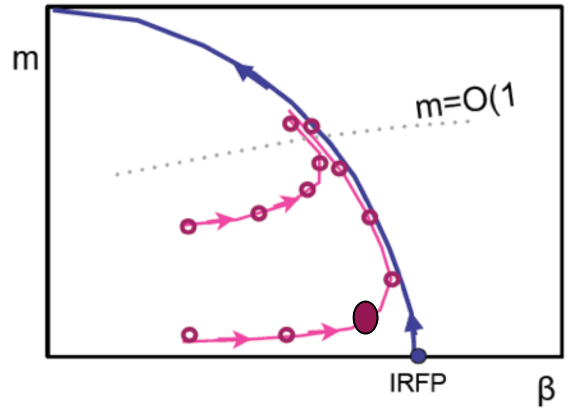
The scaling function depends on two variables now!

Corrections to finite size scaling

Physical masses scale as

$$M_H = L^{-1} f(x, g_0 m^{\omega}), \quad \omega = -y_0 / y_m$$

$f(x, g_0 m^{\omega})$ is analytic both in x and g_0.



If the $g_0 m^{\omega}$ corrections are small, expand

$$LM_H = F(x)\left(1 + g_0 m^{\omega} G(x)\right)$$

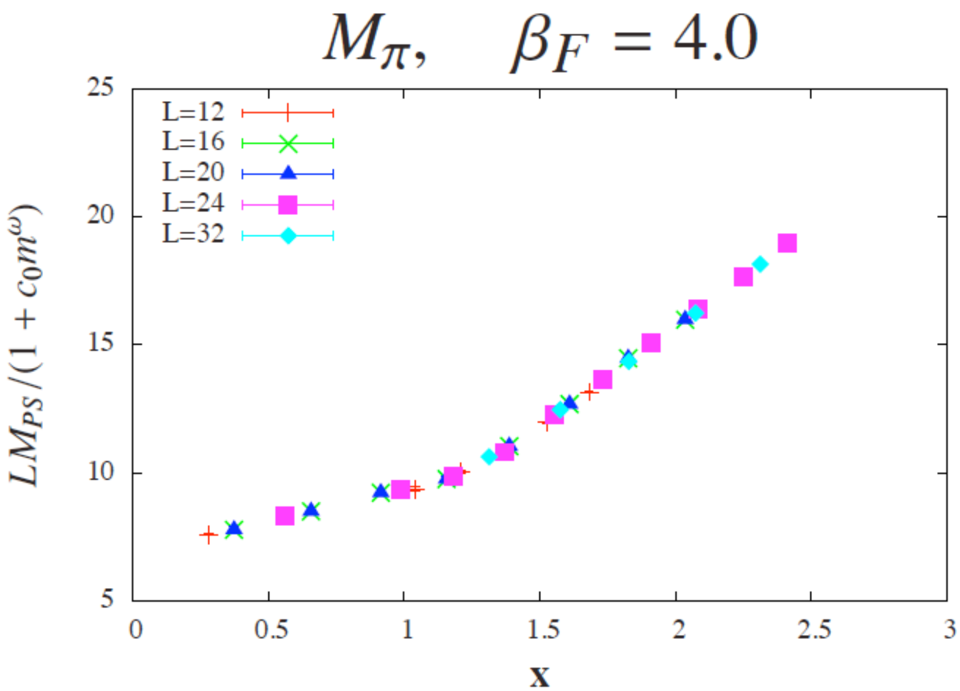
– F(0), G(0) are finite constants
– as $L \to \infty$: $M_H \propto m^{1/y_m} \to F(x) \propto x$, $G(x) = \text{const}$

Approximate $G(x) = c$ (should be checked) $\to \dfrac{LM_H}{1 + c\, g_0 m^{\omega}} = F(x)$

Need minimization in y_m, ω, and $c_0 = c g_0$

Scaling test **with** corrections

Curve collapse: 2 parameters, y_m and c_0; y_0=-0.36 fixed (2-loop PT)



Fit:
two quadratic polynomials
one at $x < x_0$, one at $x > x_0$,
separation point x_0 free
(here x_0 = 1.36)

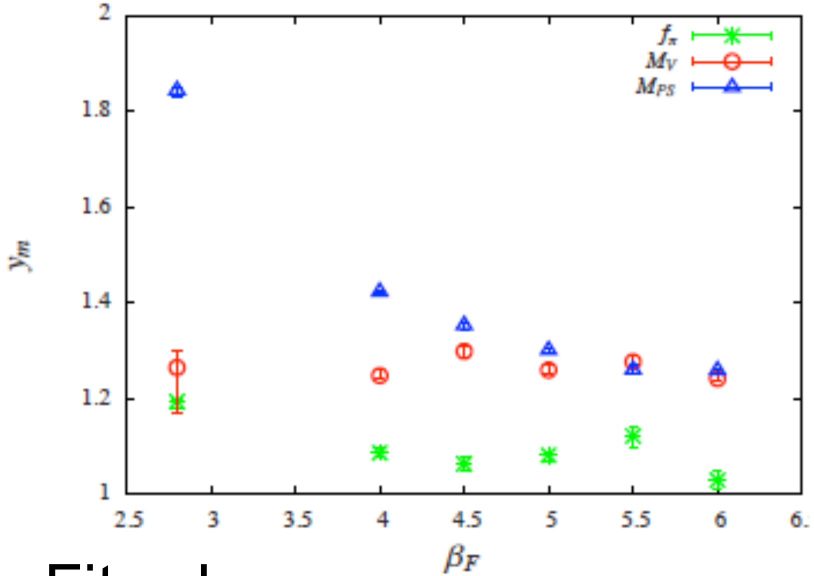
Consistent curve collapse both at small and large $x = Lm^{1/y_m}$

y_m=1.23(2) , c_0 = -0.67 - $\chi^2/\mathrm{dof} = 1.2$ (from 3.3)

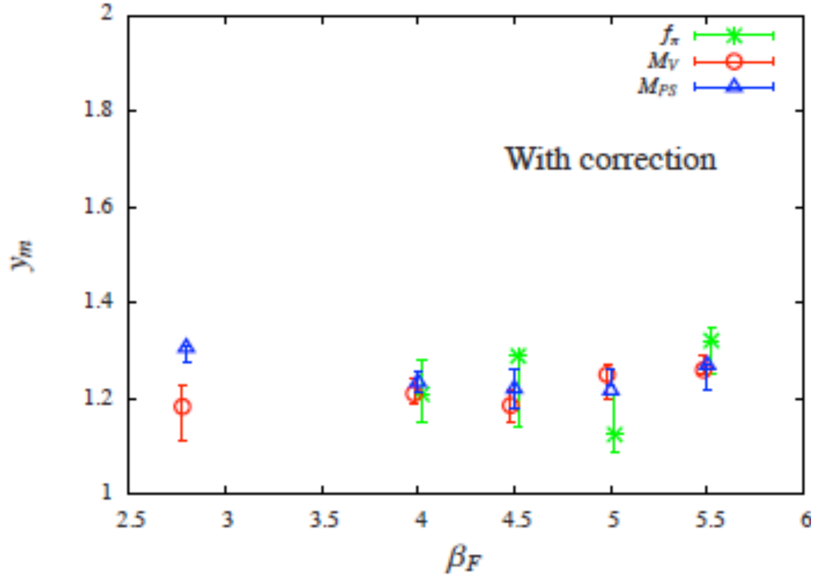
Scaling exponent **with** corrections

Include all data M_π L , M_V L , f_π L points

Leading operator only



With correction



Fits show

- good curve collapse
- consistent scaling exponent y_m=1.22(2)
- can we constrain the fit parameters better?

Scaling exponent **with** corrections

Include all data $M_\pi L$, $M_V L$, $f_\pi L$ points

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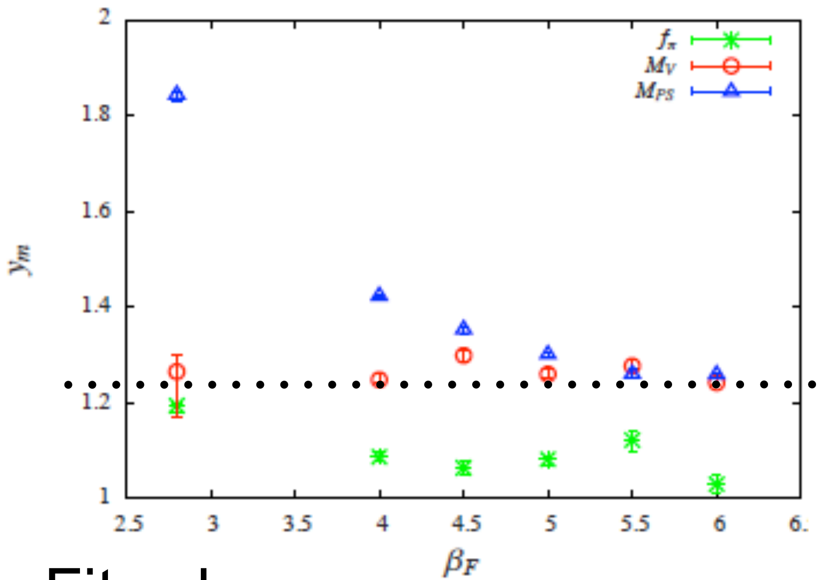

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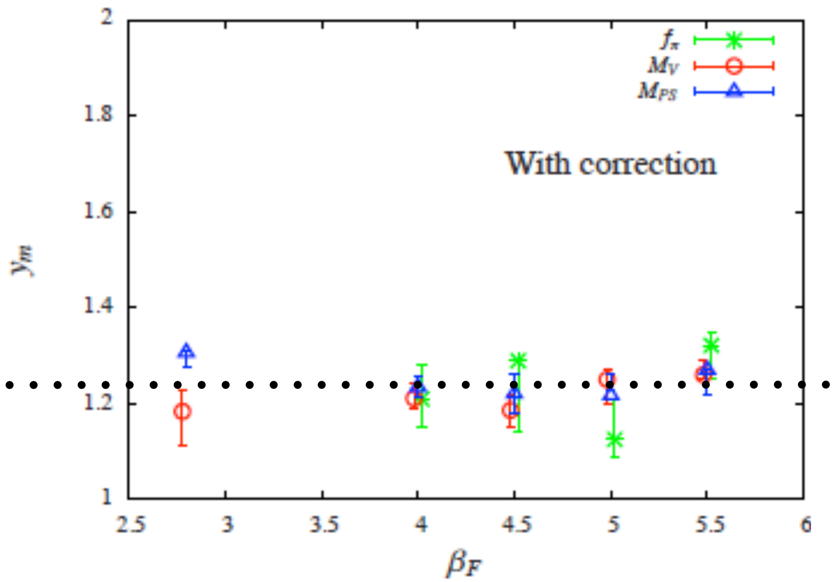

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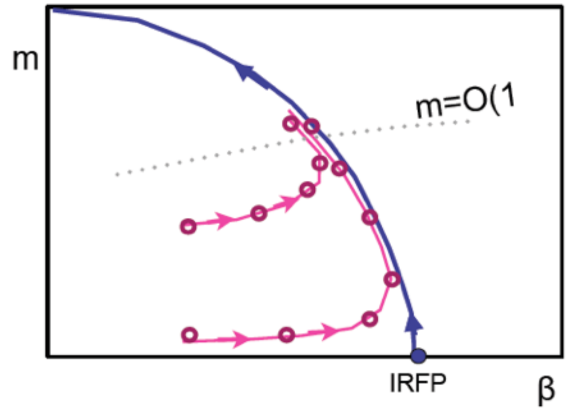
Combining data sets:



If the gauge coupling is irrelevant,
the scaling function F(x)

$$\frac{LM_H}{1+c\,g_0 m^{\omega}} = F(x)$$

is unique, independent of

- gauge coupling β
- lattice action (nHYP or stout or HISQ or Wilson or DW ...)

Combine different data sets

- we need to rescale the bare fermion mass $m(\beta) \to s\, m(\beta)$
- remnant scaling violations could be different for different sets
  $\to$ most noticeable at small x (or L)

Combining data sets:

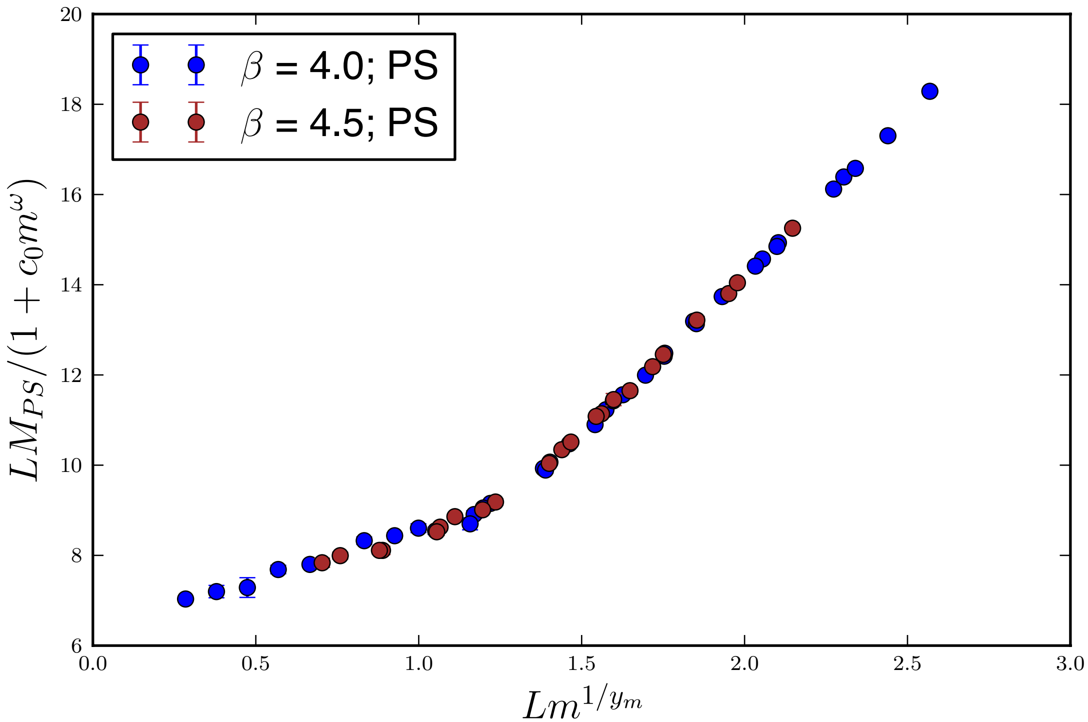
Fit with :

- common y_m, y_0,
- $F(x)$ depends on the operator only
- mass rescale factor depends on β
- correction term c_0 depends on β, operator

Combining gauge couplings:

pion at β=4.0,4.5 (all available volumes):

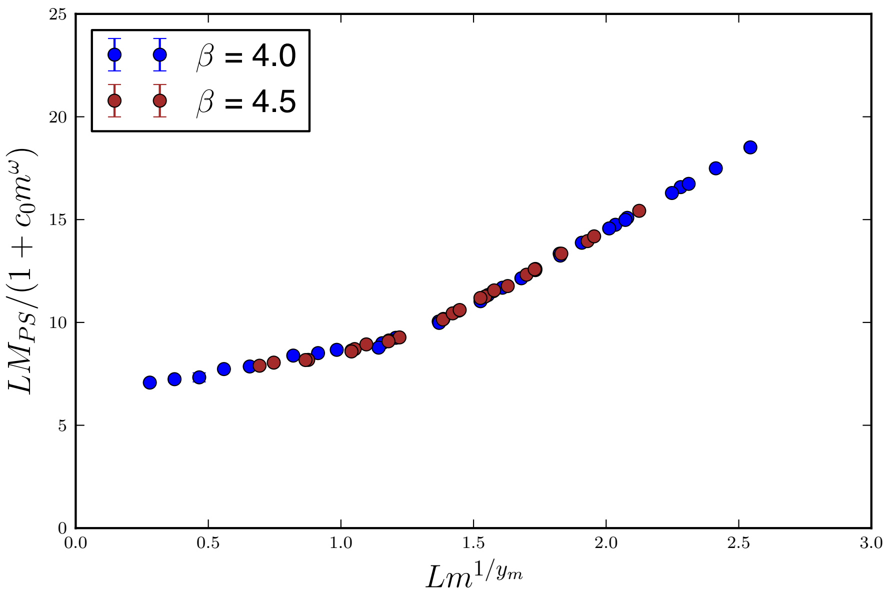
y_m=1.23[2], y_0=-0.47[6] ; χ^2 /dof =1.2 [60]

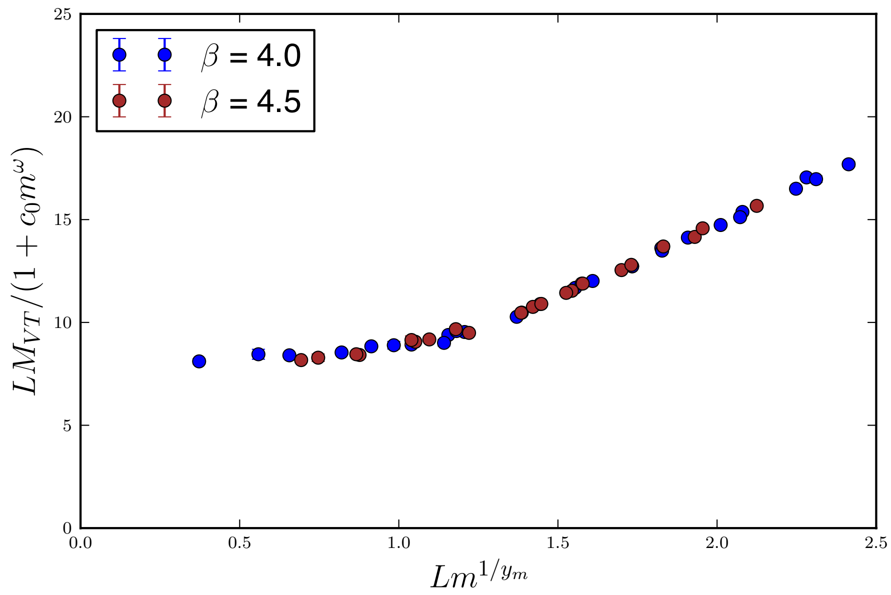


Combining gauge couplings AND operators

pion and vector at β=4.0,4.5 (new fit!)

y_m=1.22[2], y_0=-0.50[5] ; χ^2 /dof =1.4 [108]

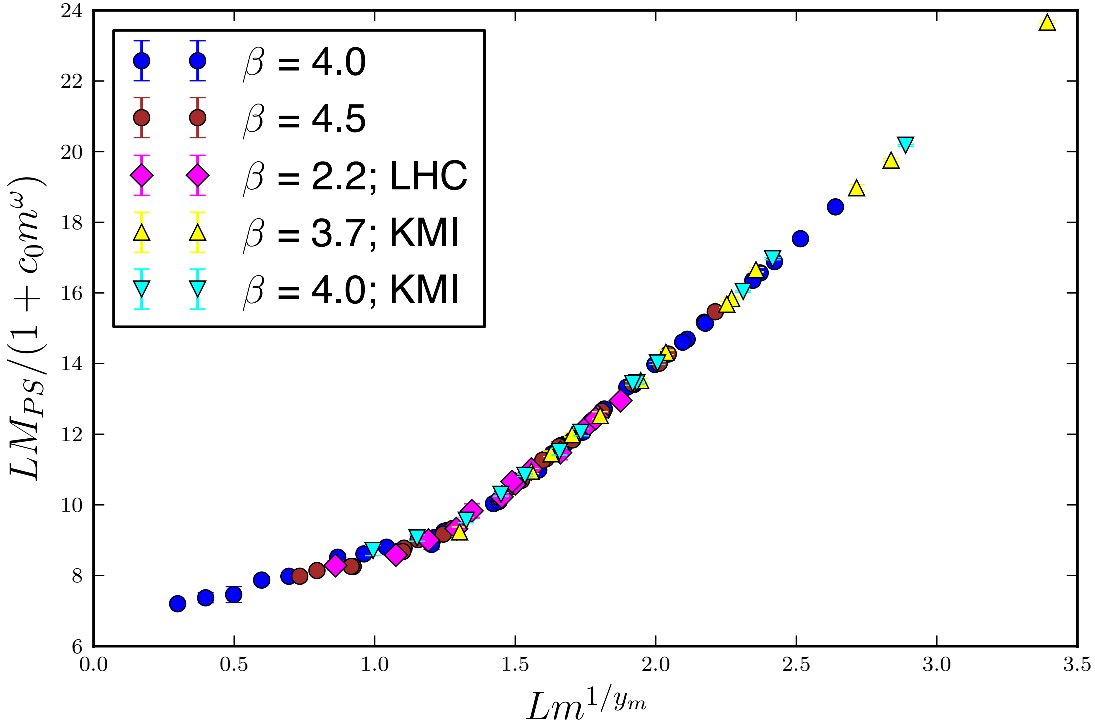




Combining gauge couplings AND actions

pion at β=4.0,4.5, LHC, KMI :

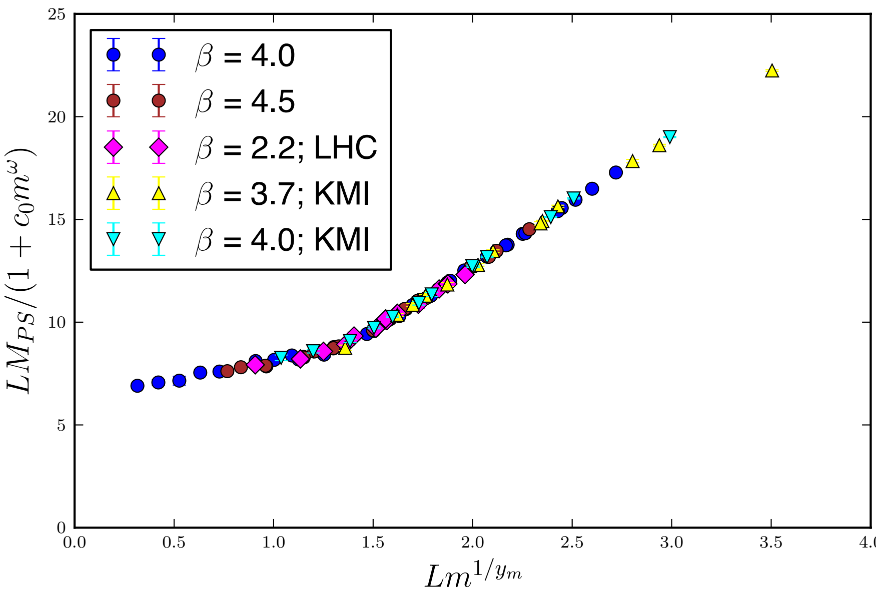
y_m=1.27[1], y_0=-0.43[5] ; χ^2 /dof =1.8 [99]

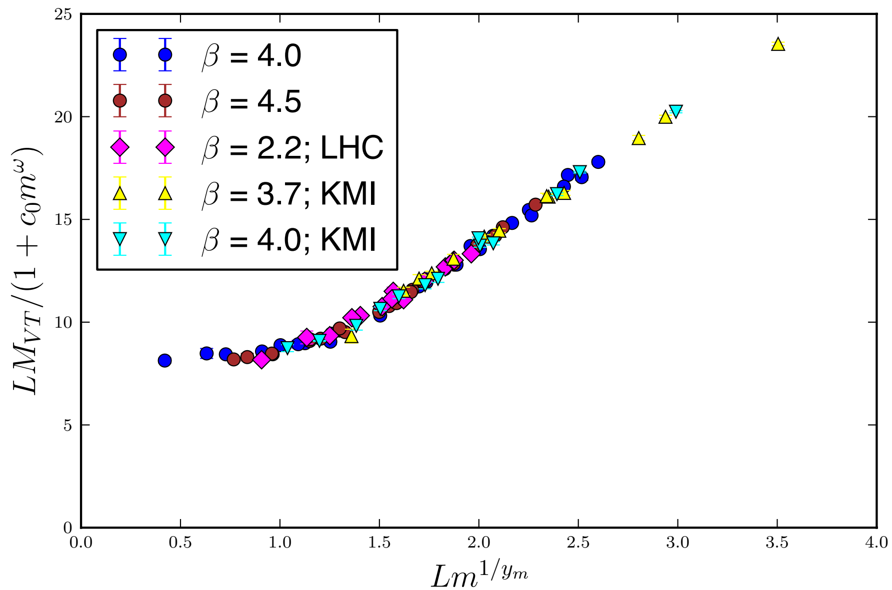


Combining gauge couplings AND actions AND operators

pion and vector at β=4.0,4.5, LHC, KMI :

y_m=1.27[1], y_0=-0.51[5] ; χ^2 /dof =2.7 [188]





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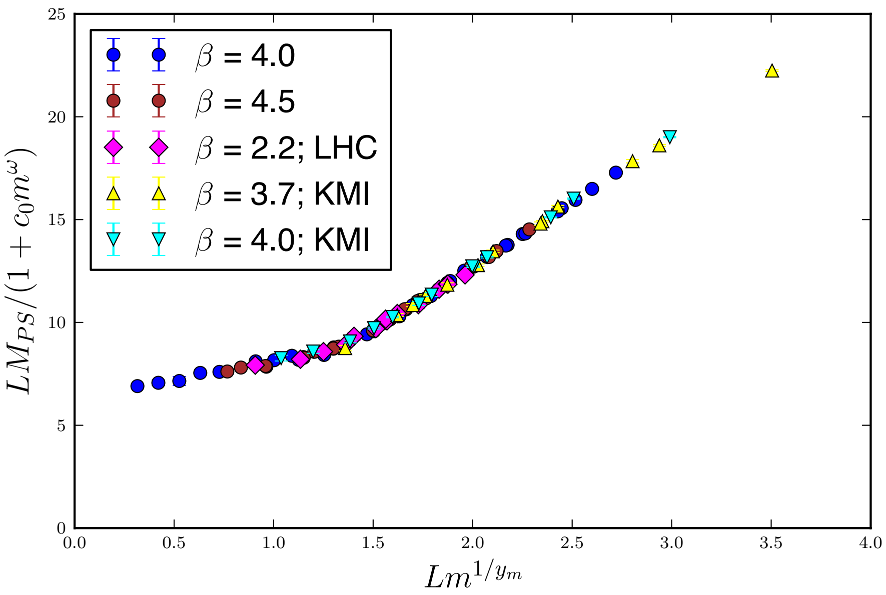
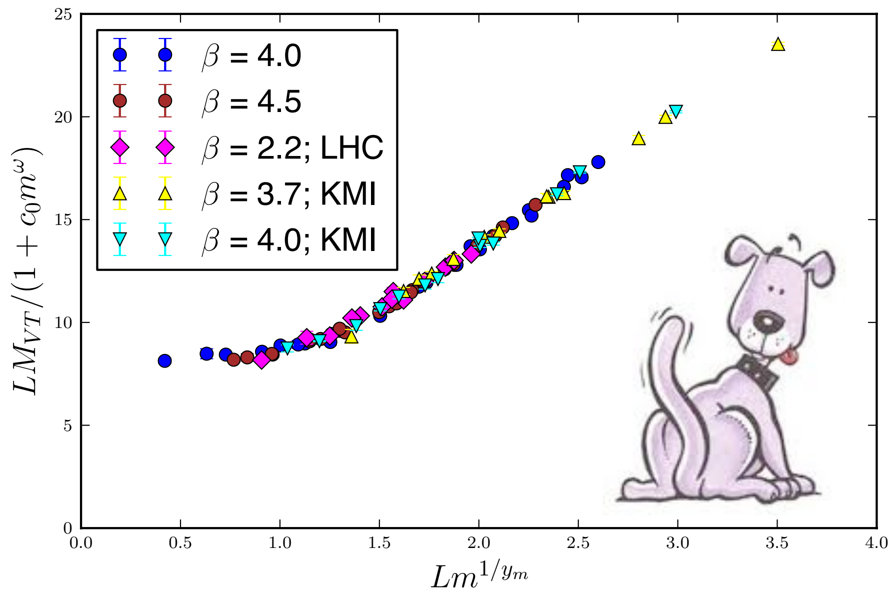
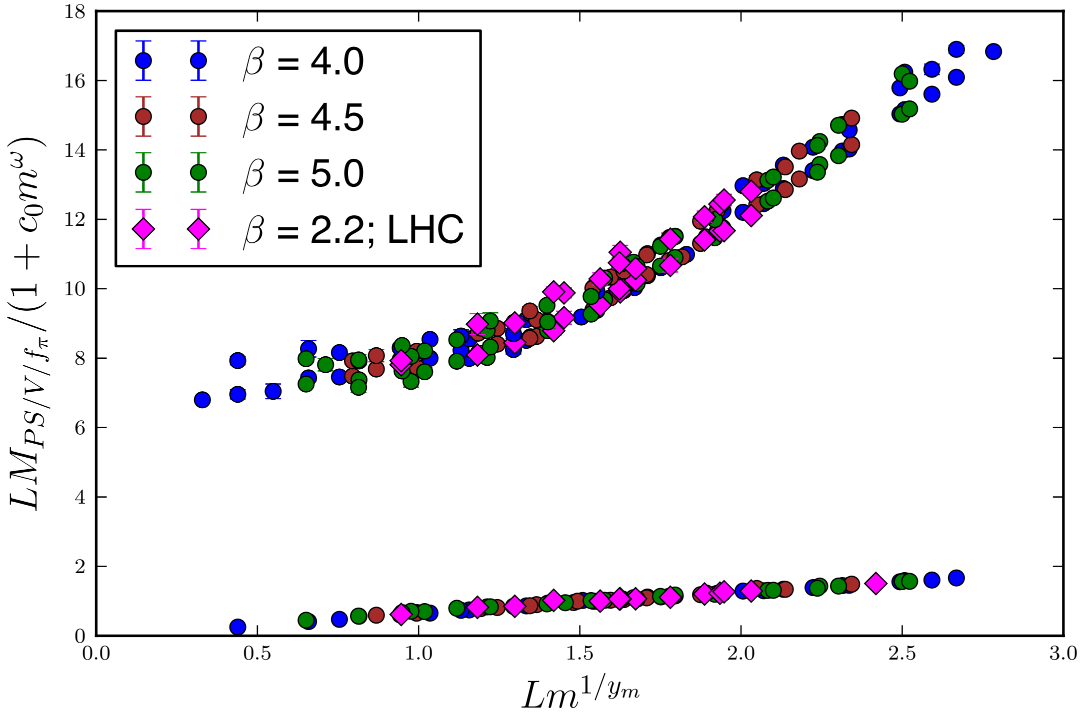




Combining gauge couplings AND actions AND operators

pion, vector and f_π at β=4.0, 4.5, 5.0, LHC :

y_m=1.28[1], y_0=-0.56[3] ; χ^2 /dof =3.2 [286]



Full disclosure : f_π is worst in the fit, especially when including KMI data

Consistency:

Fit 30-300 points with 10 - 20 parameters ...

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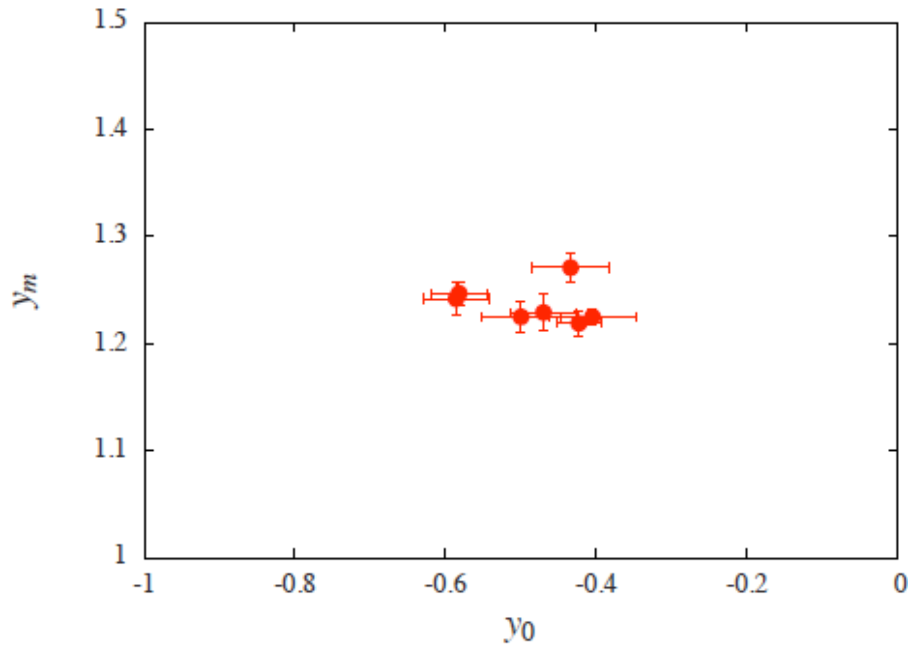
"With four parameters I can fit an elephant, and with five I can make him wiggle his trunk."

John von Neumann

Consistency:

Fit 30-300 points with 10 - 20 parameters ...

yet y_m, y_0, are consistent



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Fits combining different data sets, operators, predict y_m=1.24[2] with □$\chi^2/\mathrm{dof} \approx 1 - 3$

FSS summary, N_f=12

FSS fits with corrections that takes the walking gauge coupling into account give consistent results:

- good curve collapse, consistent exponents at each gauge coupling
- combined fit of many β values with common scaling function has χ^2 close to individual fits
- even different actions can be combined

Message from FSS

The gauge coupling of strongly coupled conformal systems are expected to run slowly ("walking")

→ scaling is strongly influenced by this near-marginal coupling

This is universal in every walking system!

- In finite size scaling analysis the marginal coupling can be accounted for
- Its effect should be considered in every other approach

Dirac operator spectral density and mode number

The **mode number** $\nu(\lambda) = V\int_{-\lambda}^{\lambda} \rho(\omega) d\omega \propto V\lambda^{\alpha+1}$ is RG invariant

(Giusti,Luscher)

→ α is related to the anomalous dimension

(Zwicky,DelDebbio;Patella)

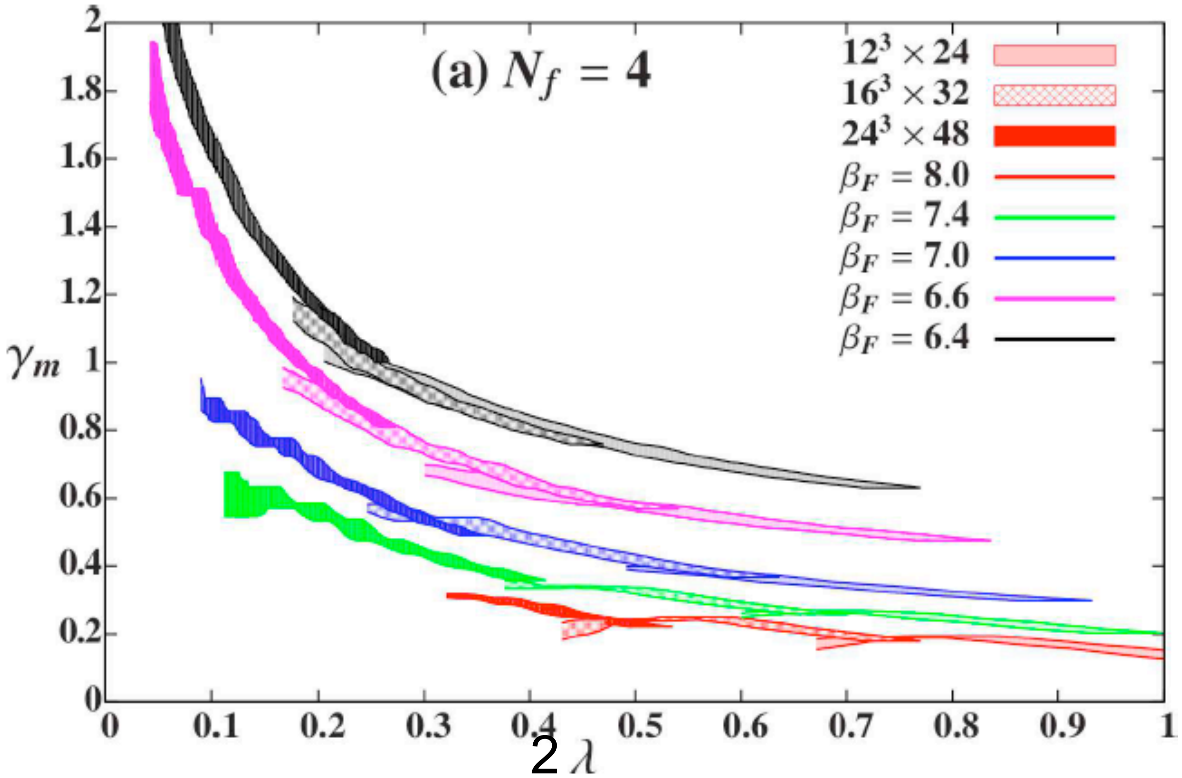
$$\frac{4}{1+\alpha} = y_m = 1 + \gamma_m$$

λ is an energy scale → α(λ) predicts a scale dependent (running) anomalous dimension

$$\gamma_m(\lambda \to 0) = \gamma_m^* \qquad \gamma_m(\lambda = \mathcal{O}(1)) = \gamma_0 g^2 + \ldots$$

N_f =4 : chirally broken

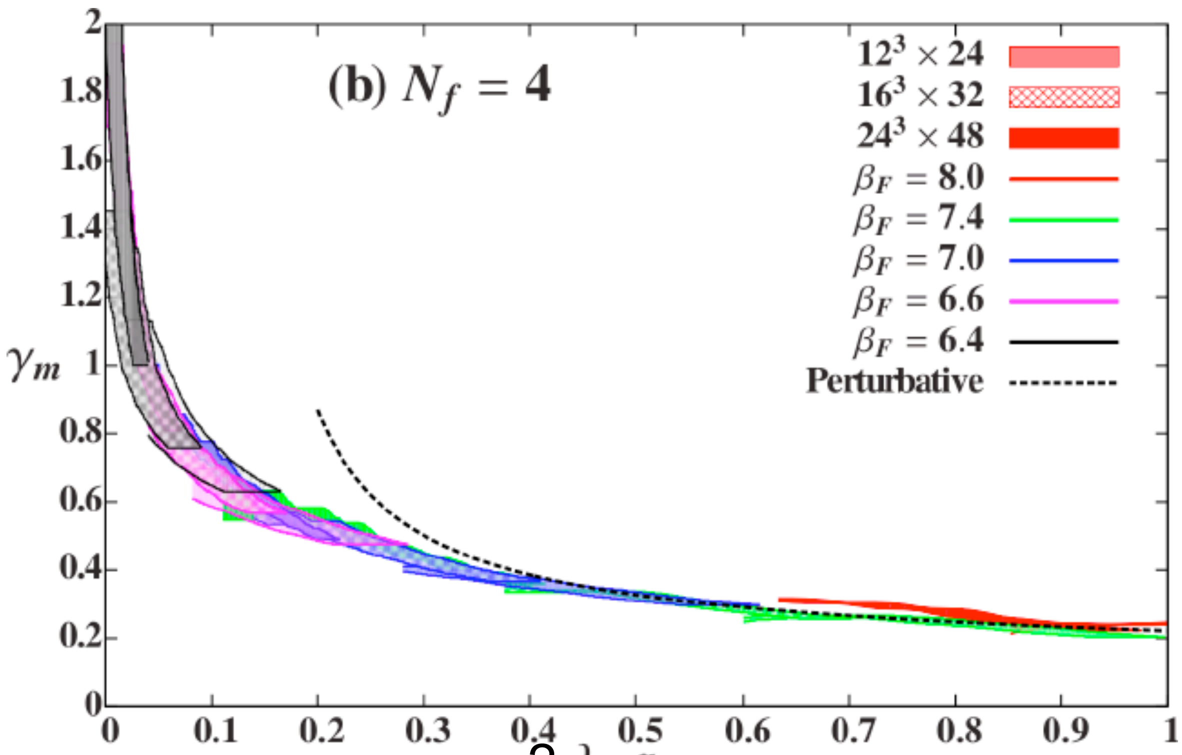
Broken chiral symmetry in IR, asymptotic freedom in UV



Rescaling: N_f =4

The dimension of λ is carried by the lattice spacing: $\lambda_{lat} = \lambda_p a$
Rescale to a common physical scale:

$$\lambda_\beta \to \lambda_\beta \left(\frac{a_{7.4}}{a_\beta}\right)^{1+\gamma_m(\lambda_\beta)}$$



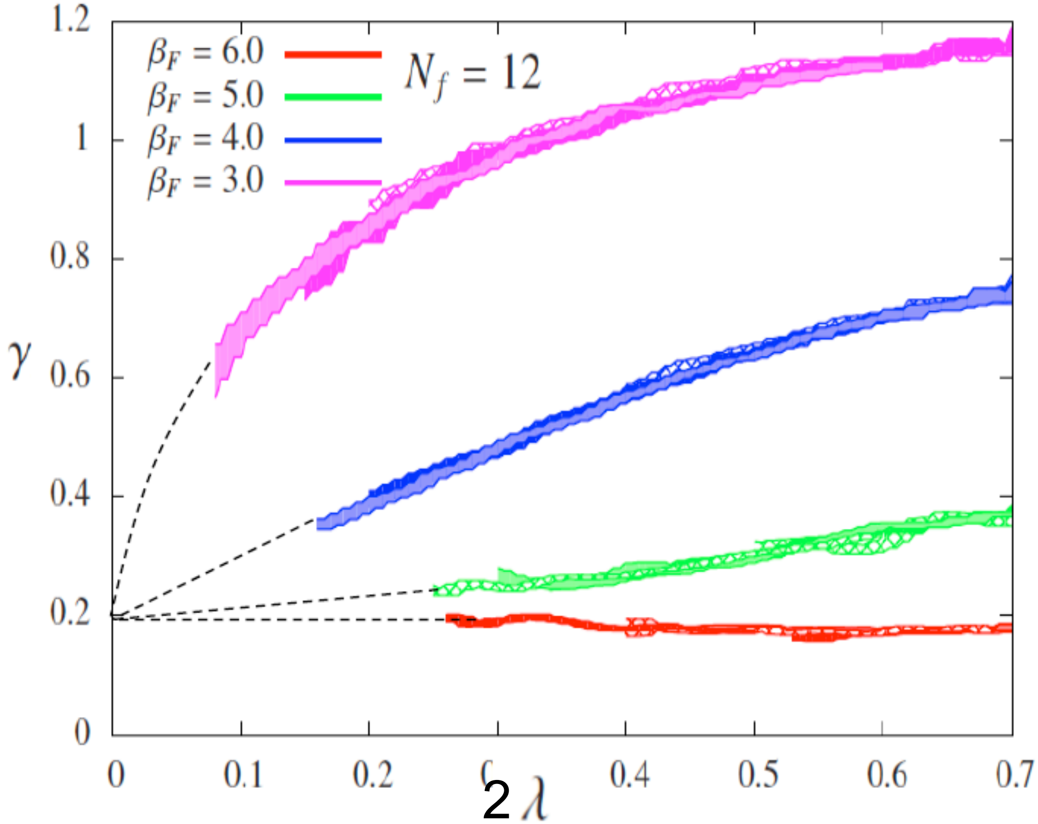
$2\,\lambda \cdot a_{7.4}$

Universal curve covering almost 2 orders of magnitude in energy!

Perturbative: functional form from 1-loop PT, relative scale is fitted

Most of these data were obtained on deconfined (small) volumes at m=0!

N_f =12 : controversial system



β=3.0, 4.0, 5.0, 6.0

- There is no sign of asymptotic freedom behavior for β<6.0,

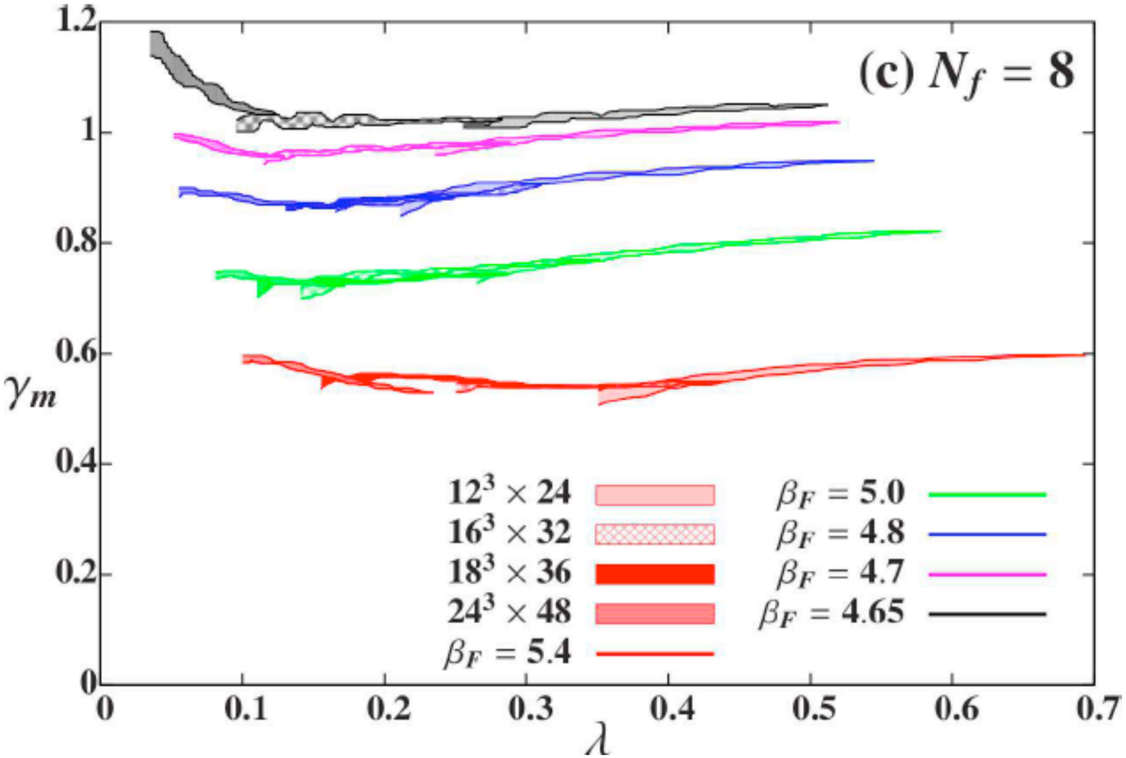
γ_m grows towards UV

- Not possible to rescale different β's to a single universal curve

Looks as if there was an IRFP between β=5.0 -6.0

N_f =8

Expected to be chirally broken - looks like walking!



-No asymptotic free scaling
-No rescale of different couplings

-When $\gamma_m \sim 1$ in the UV, the S^4b lattice phase develops

Dirac operator eigenvalue spectrum and spectral density

Unique & promising method !

- Can distinguish strong and weak coupling region of conformal /chirally broken systems

Predictions:

N_f=4 : scaling & anomalous dimension

N_f=12 : looks conformal

N_f=8 : could be walking with large anomalous dimension!

Conclusion

The gauge coupling of strongly coupled conformal systems are expected to run slowly ("walking")

$\rightarrow$ scaling is strongly influenced by this near-marginal coupling

This is universal in every walking system!

- Dirac spectral density shows this walking
- In finite size scaling analysis the marginal coupling can be accounted for
- Its effect should be considered in every other approach