Neutron-Antineutron Oscillation Matrix Elements on a Lattice

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Baryon Number Violation

- $\Delta B = 1$: Proton decay
- $\Delta B = 2$: Neutron-antineutron oscillation
  (neutron/antineutron oscillation through $\Delta B = 1$ is possible, but suppressed)

Which one (if any) is realized in nature?

Searches for $n \longrightarrow \bar{n}$ transitions

- In a nucleus

  stability of a nucleus w.r.t *(nn)* annihilation $$T_d = R\tau_{n\bar{n}}^2$$

  iron atoms $T_d(Fe) > 7.2 \cdot 10^{31}\ \text{year} \longrightarrow \tau_{n\bar{n}} > 1.4 \cdot 10^8\ \text{sec}$ [Soudan 2]

  oxygen atoms $T_d(O) > 1.77 \cdot 10^{32}\ \text{year} \longrightarrow \tau_{n\bar{n}} > 3.3 \cdot 10^8\ \text{sec}$ [Super-K]

- (Quasi) free neutrons

  probability of event $$P_{n\to\bar{n}} \sim \left(\frac{t}{\tau_{n\bar{n}}}\right)^2$$

  Reactor neutrons (Grenoble, 1990) $\tau_{n\bar{n}} > 0.86 \cdot 10^8 sec$

  Proposed experiments:

  - stored ultra-cold neutrons
  - (focused, vertical) cold neutron beams

Neutron <=> Antineutron Operators

Effective 6-quark operators *From Beyond (the Standard Model)* :
interaction with a massive Majorana lepton, unified theories, etc

[T.K.Kuo, S.T.Love, PRL45:93 (1980)]
[R.N.Mohapatra, R.E.Marshak, PRL44:1316 (1980)]

$$\mathcal{H}_{n\bar{n}} = \begin{pmatrix} E+V & \delta m \\ \delta m & E-V \end{pmatrix} \qquad \tau_{n\bar{n}} = (2\delta m)^{-1}$$

$$\mathcal{L}_{\text{eff}} = \sum_i \left[c_i \mathcal{O}_i^{6q} + \text{h.c.} \right] \qquad \delta m = -\langle \bar{n} | \int d^4x \, \mathcal{L}_{\text{eff}} | n \rangle = -\sum_i c_i \boxed{\langle \bar{n} | \, \mathcal{O}_i^{6\text{q}} \, | n \rangle}$$

Dimension-9 point-like operators suppressed by $(M_X)^{-5}$

Sensitivity of matter to BN-violating terms is determined by nuclear scale physics and non-perturbative QCD

Neutron <=> Antineutron Operators

Operators: pseudoscalar singlets w.r.t $SU(3)_c \otimes U(1)_{em} \left[\otimes SU(2)_L \right]$

$$\mathcal{O}_{1\,\chi_1\{\chi_2\chi_3\}} = T^s_{ijklmn} \left[u^{iT}_{\chi_1} C u^j_{\chi_1}\right] \left[d^{kT}_{\chi_2} C d^l_{\chi_2}\right] \left[d^{mT}_{\chi_3} C d^n_{\chi_3}\right]$$

$$\chi_{1,2,3} = R, L$$

$$\mathcal{O}_{2\,\{\chi_1\chi_2\}\chi_3} = T^s_{ijklmn} \left[u^{iT}_{\chi_1} C d^j_{\chi_1}\right] \left[u^{kT}_{\chi_2} C d^l_{\chi_2}\right] \left[d^{mT}_{\chi_3} C d^n_{\chi_3}\right]$$

$$\mathcal{O}_{3\,\{\chi_1\chi_2\}\chi_3} = T^a_{ijklmn} \left[u^{iT}_{\chi_1} C d^j_{\chi_1}\right] \left[u^{kT}_{\chi_2} C d^l_{\chi_2}\right] \left[d^{mT}_{\chi_3} C d^n_{\chi_3}\right]$$

Computed using MIT bag model

[S.Rao, R.Shrock, PLB116:238 (1982)]

Symmetry reduces # of operators: 24 -> 14
(of which only 4 are $SU(2)_L$ symmetric)

$$3\mathcal{O}_{3\,\{\chi\chi\}\chi'} = \mathcal{O}_{2\,\{\chi\chi\}\chi'} - \mathcal{O}_{1\,\chi\{\chi\chi'\}}$$

[W.Caswell, J.Milutinovic, G.Senjanovic, PLB122:373 (1983)]

On a lattice:
7 independent quantities
(R<-->L symmetry)

$$\mathcal{O}_{1\,\chi\{\chi\chi\}} \quad \mathcal{O}_{1\,\chi\{\chi\eta\}} \quad \mathcal{O}_{1\,\chi\{\eta\eta\}}$$

$$\mathcal{O}_{2\,\{\chi\chi\}\chi} \quad \mathcal{O}_{2\,\{\chi\chi\}\eta} \quad \mathcal{O}_{2\,\{\chi\eta\}\chi}$$

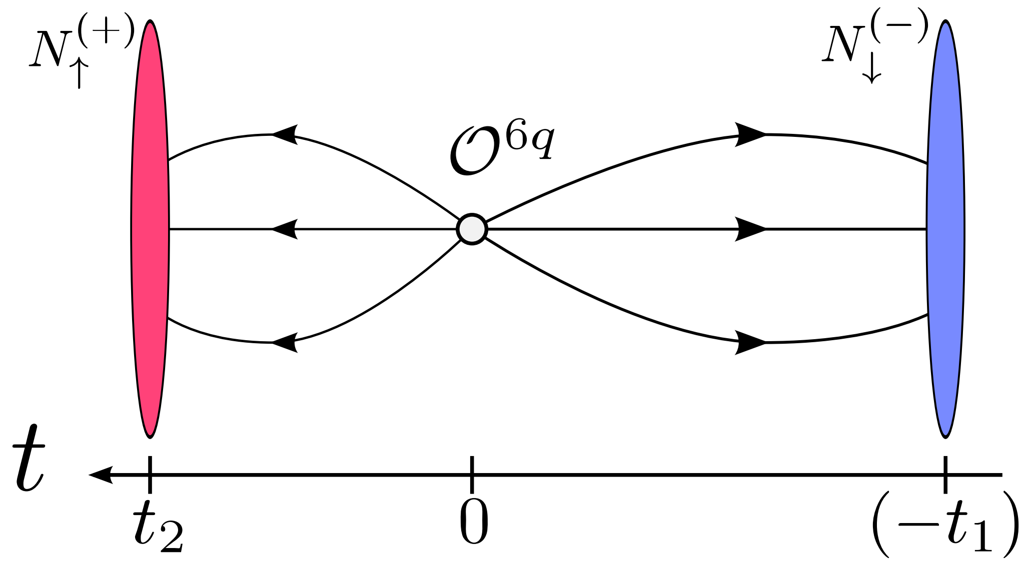
$$\mathcal{O}_{3\,\{\chi\eta\}\chi}$$

$$(\chi, \eta = R, L, \quad \chi \neq \eta)$$

SU(2)$_V$ and parity further reduce independent $\langle\bar{n}|\mathcal{O}^{6q}_i|n\rangle$ to 6 [B. Tiburzi]

Lattice Calculation

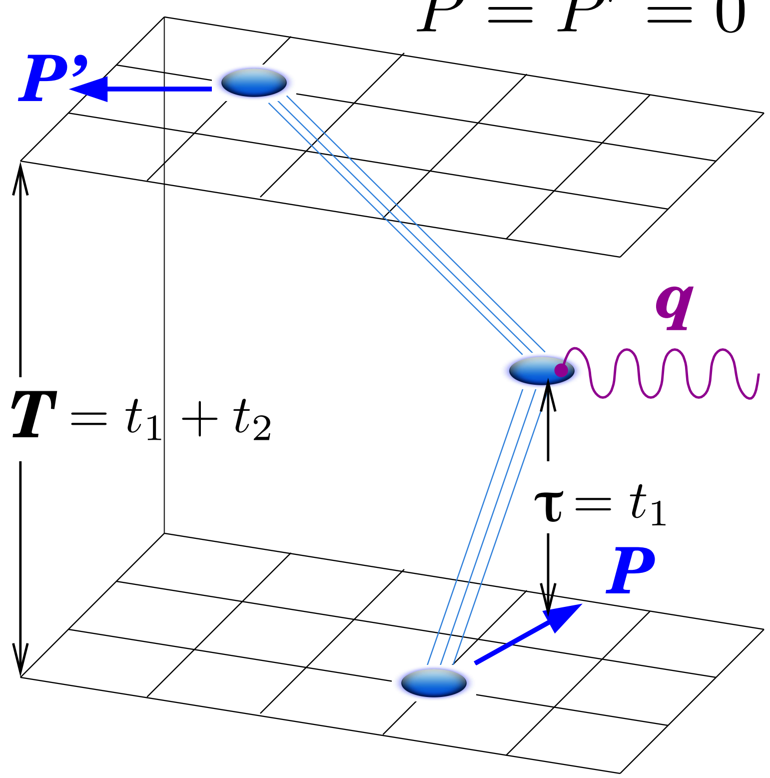
$$\langle N_\uparrow^{(+)}(t_2)\,\mathcal{O}^{6q}(0)\,N_\downarrow^{(-)}(-t_1)\rangle \underset{t_1,\,t_2,\,t_1+t_2\to\infty}{\sim} e^{-M_n(t_2+t_1)}\langle n_\uparrow|\mathcal{O}^{6q}|\overline{n}_\uparrow\rangle$$



No quark-disconnected contractions!

Single propagator $\longrightarrow\quad \forall\quad t_1,\,t_2$

$\vec{P} = \vec{P}' = 0$



$$\langle n|\mathcal{O}|\bar{n}\rangle\Big|_{\rm lat} = \langle n|\mathcal{O}|\bar{n}\rangle + O\left(e^{-\Delta E_{\rm exc}t_1},\, e^{-\Delta E_{\rm exc}t_2},\, e^{-\Delta E_{\rm exc}(t_1+t_2)}\right)$$

Complete set of correlators for sophisticated exc.state analysis:

- Exponential fits
- Variational (GPoF)

Preliminary Results

- 20^3x256 anisotropic lattice, clover-improved Wilson fermions
- pion mass *390 MeV*
- lattice spacing *a = 0.123 fm*

[Hadron Spectrum collaboration]

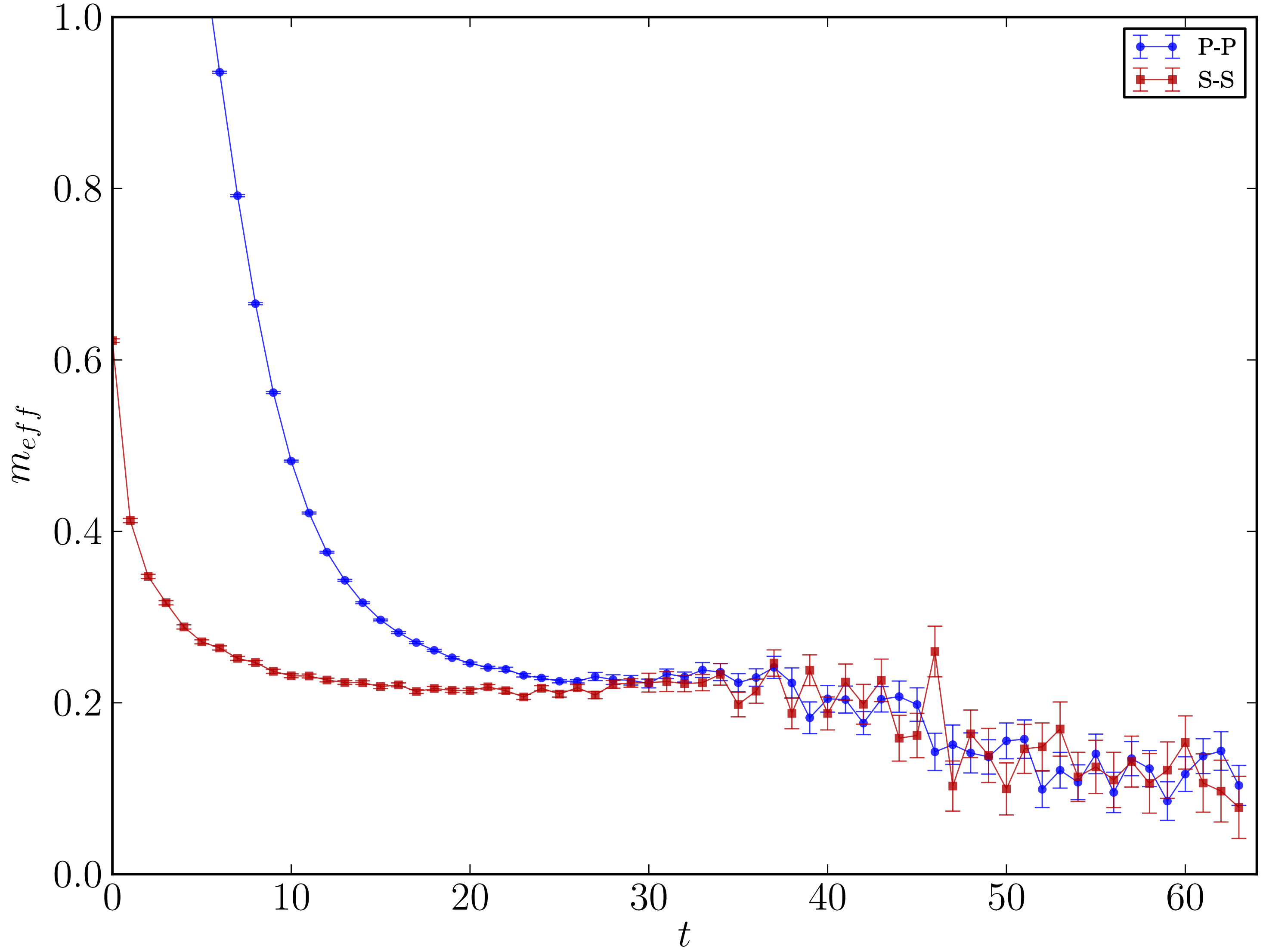
- no renormalization yet, only *bare* lattice quantities
- 886 gauge configurations x 9 samples per lattice

PRELIMINARY ANALYSIS !

Simple ratios to inspect signal for matrix elements

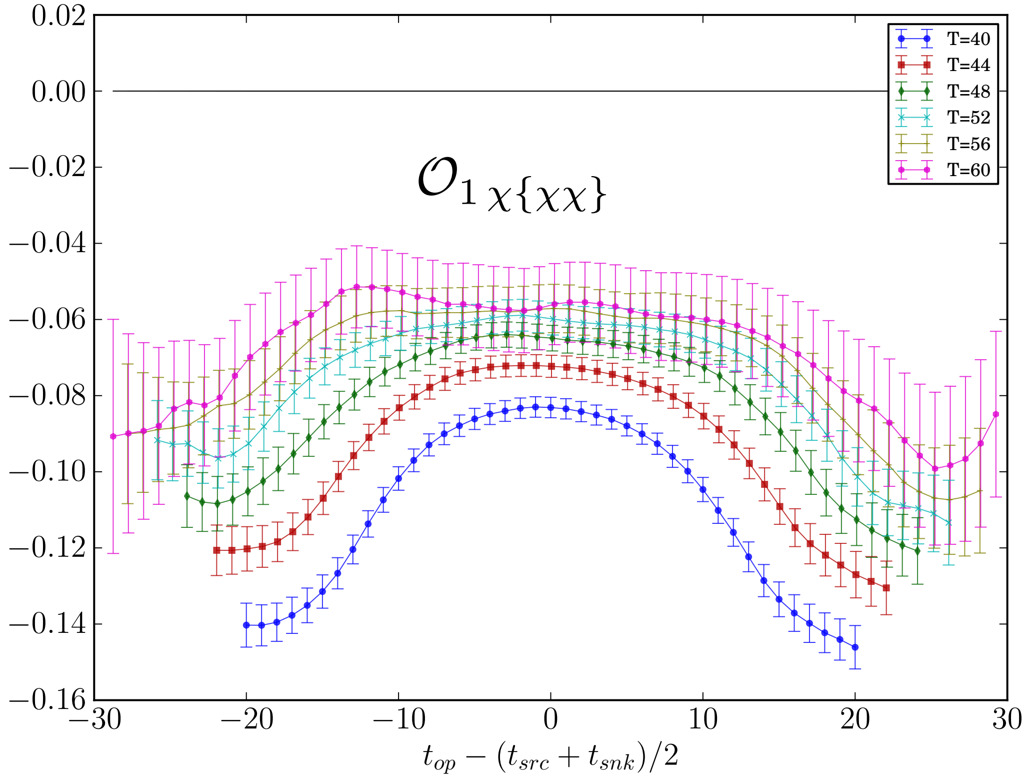
$$\langle n|\mathcal{O}|\bar{n}\rangle \sim \frac{C_{n\mathcal{O}\bar{n}}(t_2, 0, -t_1)}{\sqrt{C_{nn}(t_2, 0) C_{\bar{n}\bar{n}}(0, -t_1)}}$$

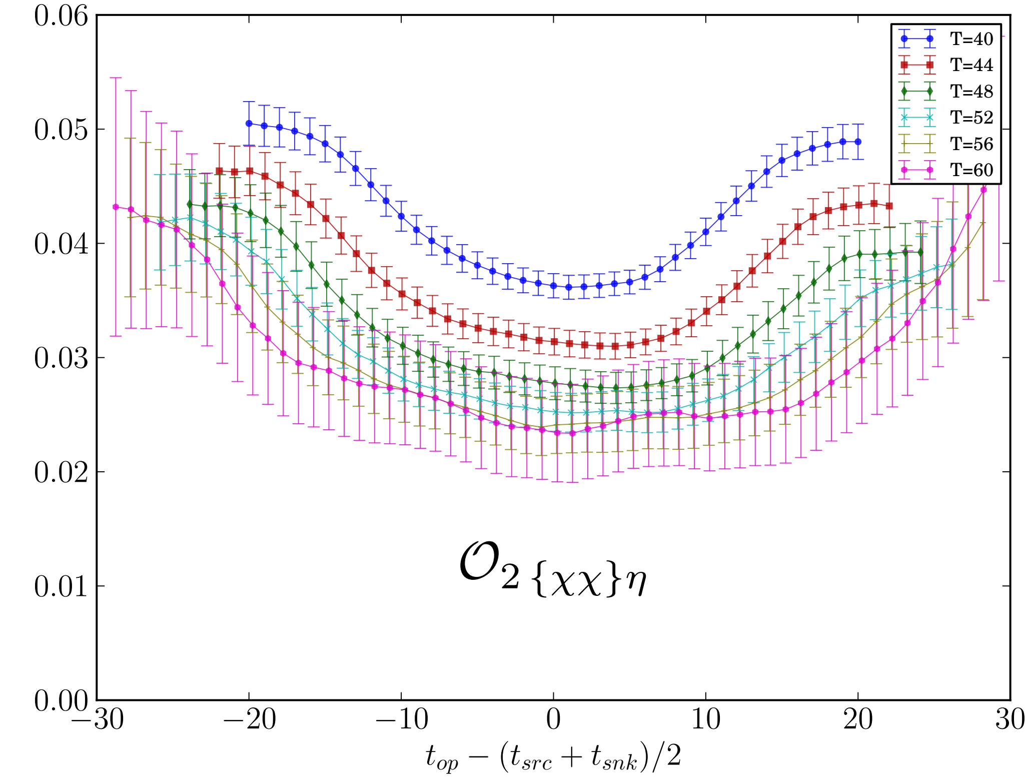
Effective Mass: Gauging Excited States

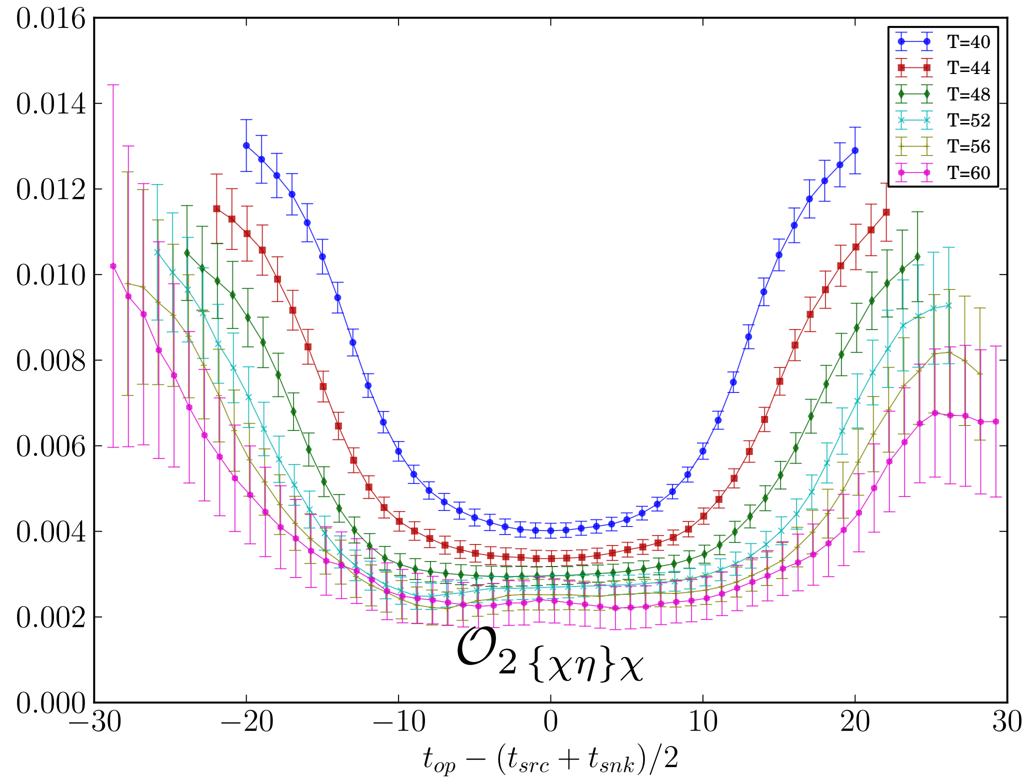


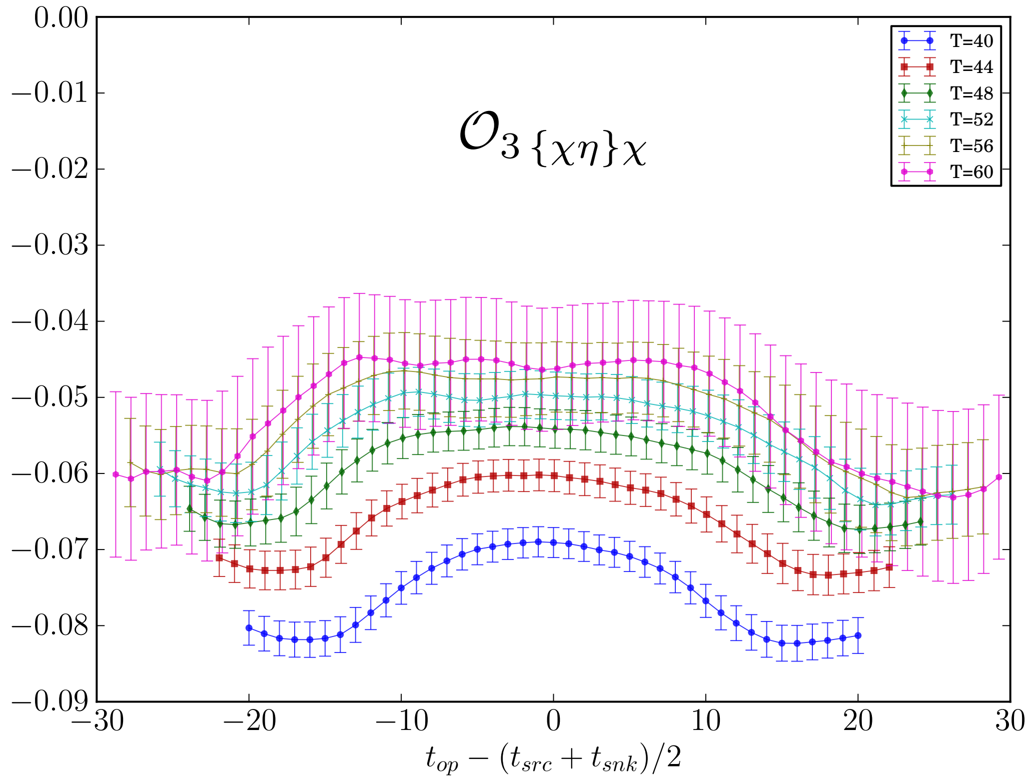
$$m_{eff}(t) = \log \frac{C_2(t)}{C_2(t+1)}$$

Bare Matrix Elements on a Lattice

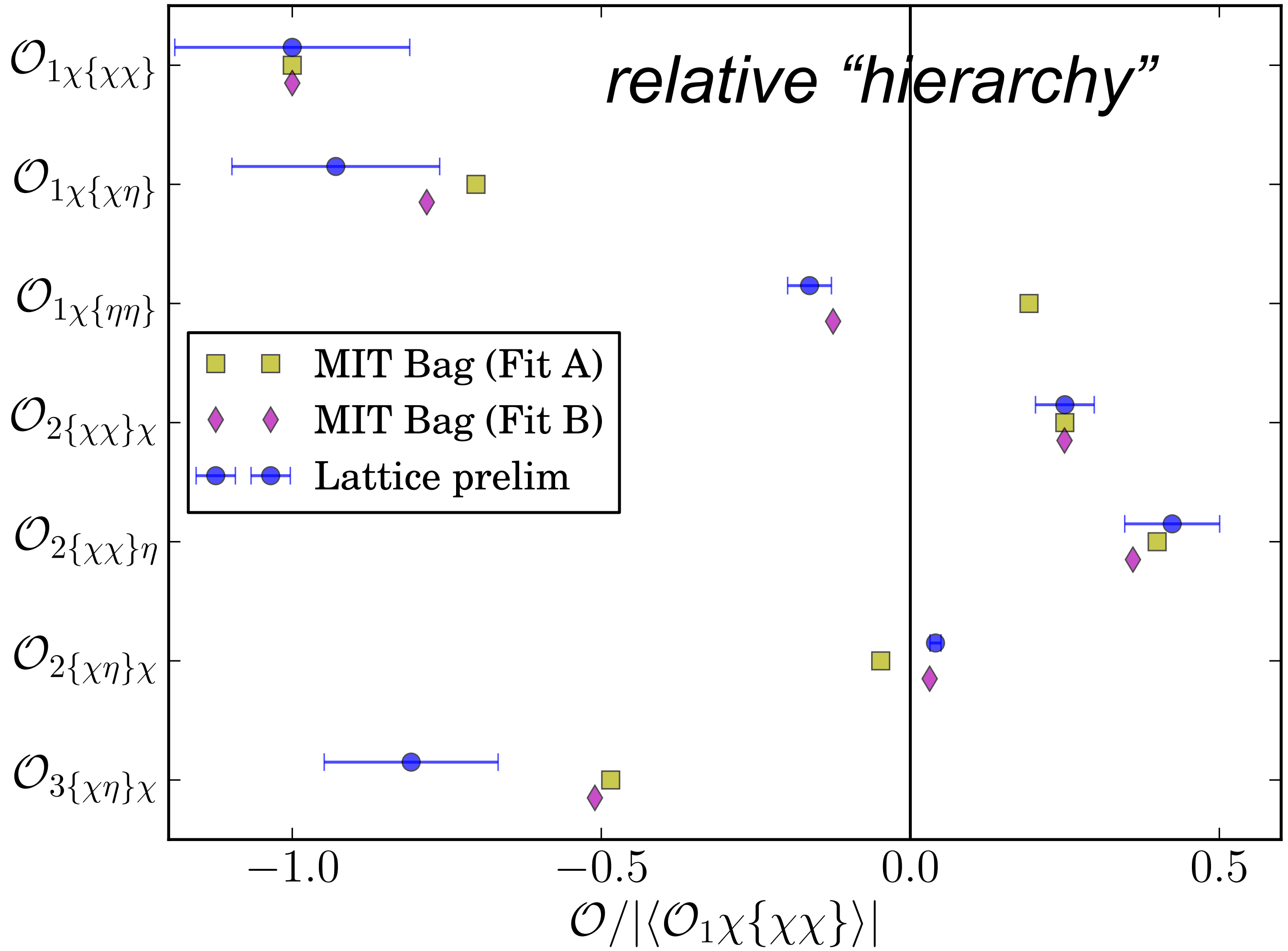








Comparison to MIT Bag Model Calculation



(values normalized by $\mathcal{O}_{1\,\chi\{\chi\chi\}}$)

MIT Bag Model calc.: [S.Rao, R.Shrock, PLB116:238 (1982)]

- Operators have different anomalous dimensions
- Some operators mix
- Chiral symmetry breaking effects?

Next Step: Renormalization

Non-perturbative lattice renormalization: bare quantities $\longrightarrow$ MOM-scheme

Perturbative 1-loop scaling is known [W.Caswell *et al* PLB122:373 (1983)]

- need higher loop calculations?
- matching coefficients for conversion to MSbar?

In the chiral limit, mixing between operators is restricted by flavor *SU(2)$_{L,R}$* symmetry:

$\mathcal{O}^1_{L(LL)} + 4\mathcal{O}^2_{(LL)L}$	$(\mathbf{3},-1)_L$	$\otimes$	$(\mathbf{0},0)_R$	$(\alpha_S/4\pi)(-12)$
$\mathcal{O}^2_{(LL)L} - \mathcal{O}^1_{L(LL)}$	$(\mathbf{1},-1)_L$	$\otimes$	$(\mathbf{0},0)_R$	$(\alpha_S/4\pi)(-2)$
$\mathcal{O}^2_{(LL)R} - \mathcal{O}^1_{L(LR)}$	$(\mathbf{0},0)_L$	$\otimes$	$(\mathbf{1},-1)_R$	0
$\mathcal{O}^3_{(LR)L}$	$(\mathbf{1},-1)_L$	$\otimes$	$(\mathbf{0},0)_R$	$(\alpha_S/4\pi)(+2)$
$\mathcal{O}^1_{L(LR)} + 2\mathcal{O}^2_{(LL)R}$	$(\mathbf{2},0)_L$	$\otimes$	$(\mathbf{1},-1)_R$	$(\alpha_S/4\pi)(-6)$
$\mathcal{O}^2_{(LR)L}$	$(\mathbf{2},-1)_L$	$\otimes$	$(\mathbf{1},0)_R$	$(\alpha_S/4\pi)(-6)$
$\mathcal{O}^1_{R(LL)}$	$(\mathbf{2},-2)_L$	$\otimes$	$(\mathbf{1},+1)_R$	$(\alpha_S/4\pi)(-6)$

(+ $L \leftrightarrow R$ counterparts)

Lattice: chiral symmetry is broken, renormalize $(L \pm R)$ multiplets instead

or switch to chiral (domain wall) fermions

Summary & Outlook

- ✦ Clear lattice signal for non-zero $\langle n|\mathcal{O}^{6\mathrm{q}}|\bar{n}\rangle$ with modest statistics
- ✦ Qualitative agreement with the MIT Bag Model is reassuring
- ✦ Expect only moderate renormalization and mixing effects

Outlook

- ✦ Non-perturbative renormalization
- ✦ Improve analysis to extract ground state M.E.
- ✦ Additional volumes, pion masses, possibly with chiral quarks