Lattice calculation of proton decay matrix element

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collaborated with Y. Aoki, A. Soni (RBC/UKQCD collaboration)

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Plan

- Introduction
- Matrix element of proton decay
- Lattice calculation and new technique
- Preliminary results in high precision
- Summary

Proton decay: smoking gun of NP

- Baryon number violation in the SM

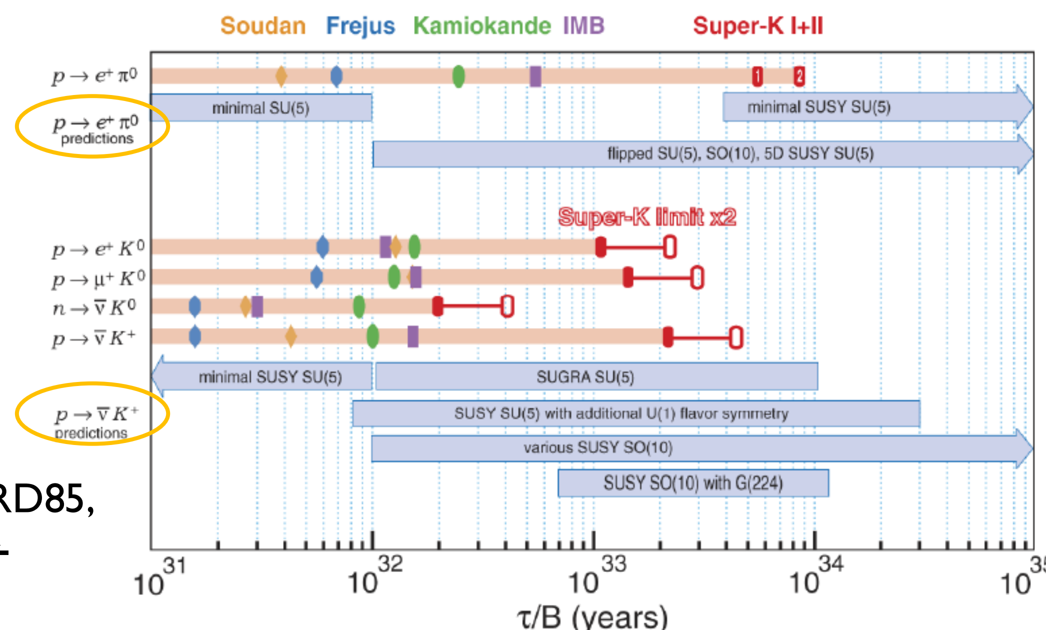
  via anomaly, B(and L) violation is very rare event ('tHooft 1976):

  $\Delta B = \Delta L = 2$: $\tau(d \to e^+ \nu_\mu) \sim 10^{120}$ years,

  $\Delta B = \Delta L = 3$: $\tau(^3He \to e^+ \nu_\tau \nu_\mu) \sim 10^{150}$ years

- baryons excess (not anti-baryon) in the universe
- (SUSY-) GUTs
  - Coupling unification
  - Proton decay
- Experiments
  - $\tau(pe^+\pi^0) > 8.2 \times 10^{33}$ years
  - $\tau(p\nu K^+) > 2.3 \times 10^{33}$ years

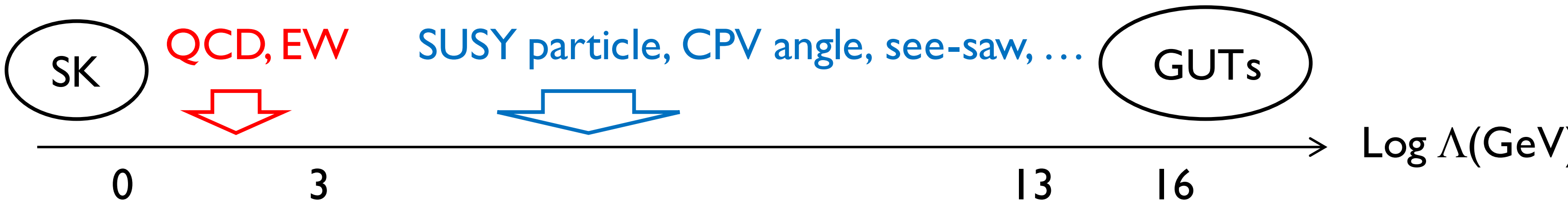
Nishino et al. (Super-Kamiokande), PRD85, 112001(2012), Kobayashi et al. (Super-Kamiokande), PRD72, 052007 (2005)



arXiv:1205.2671v1

Motivation

- To increase the confidence level of bound of proton lifetime
  - account non-perturbative ingredients from GUT scale to QCD ($< \Lambda_{QCD}$)



Decay rate is contributed from squared of matrix element

$$\Gamma_{p\to\pi^0 e^+} = \frac{m_p}{32\pi}\left[1-\left(\frac{m_e}{m_p}\right)^2\right]^2 \left|\sum_i C_i W_0^i(p\to\pi^0)\right|^2$$

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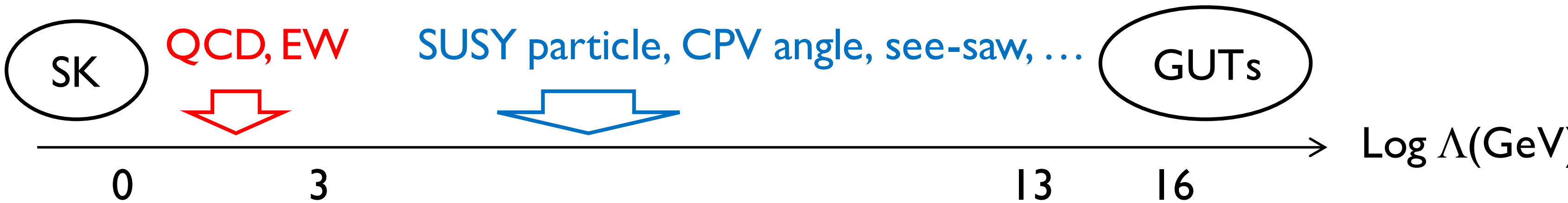

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- To remove the theoretical uncertainties
  - The QCD effect in the matrix element is one of the main uncertainties.
  - Most GUTs predictions have been based on BChPT, and there are also unknown LECs and higher order effect.

LECs α (also β) are also estimated from lattice QCD.

Y. Aoki et al. (RBC-UKQCD), PRD78, 054505 (2008)

Lattice QCD is able to determine W_0 without relying on BChPT !

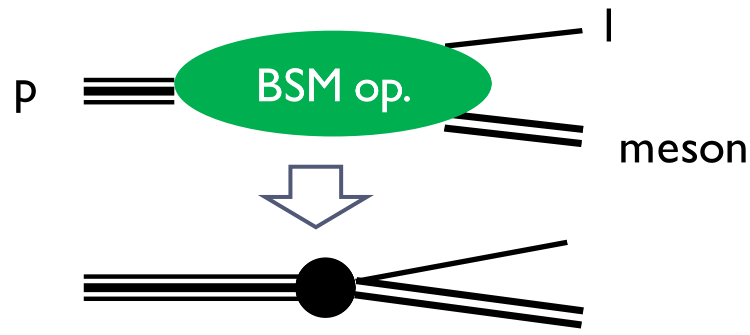
BV effective operators at low-energy

- Dimension-6 operator

$$\mathcal{L}_{\rm GUT} \simeq \mathcal{L}_{\rm SM} + \sum_i C_i(\mu) O_i(\mu)/\Lambda^2_{\rm GUT}$$

"i" labels chirality (Γ) and flavor (q,l)

C_i : Wilson coefficients depending on type of GUT models.



$C_i(\mu)O_i(\mu)$

- B violating operators

$$\begin{aligned}
\mathcal{O}^1_{abcd} &= (D^i_a, U^j_b)_R (q^{k\,\alpha}_c, l^\beta_d)_L \varepsilon^{ijk}\varepsilon^{\alpha\beta}, && :(q,q)_R\,(q,l)_L \\
\mathcal{O}^2_{abcd} &= (q^{i\,\alpha}_a, q^{j\,\beta}_b)_L (U^k_c, l_d)_R \varepsilon^{ijk}\varepsilon^{\alpha\beta}, && :(q,q)_L\,(q,l)_R \\
\tilde{\mathcal{O}}^4_{abcd} &= (q^{i\,\alpha}_a, q^{j\,\beta}_b)_L (q^{k\,\gamma}_c, l^\delta_d)_L \varepsilon^{ijk}\varepsilon^{\alpha\delta}\varepsilon^{\beta\gamma}, && :(q,q)_L\,(q,l)_L \\
\mathcal{O}^5_{abcd} &= (D^i_a, U^j_b)_R (U^k_c, l_d)_R \varepsilon^{ijk} && :(q,q)_R\,(q,l)_R
\end{aligned}$$

a,b,c,d : generation, α,β,γ,δ: SU(2) indices, i,j,k: color indices

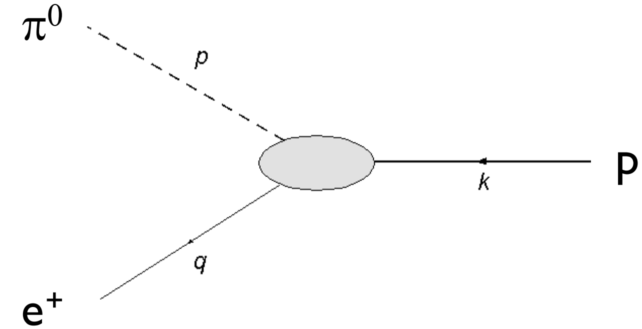
Weinberg, PRL43, 1566 (1979), Wilczek and Zee, PRL43, 1571 (1979)

2. Matrix element of proton decay

Hadronic effect in proton decay

- Kinematics in decaying into PS meson and lepton

$$\langle \pi^0 e^+ | p \rangle_{\rm GUT} = \sum_{i=\Gamma,\Gamma'} C_i \langle \pi^0 e^+ | (ud)_\Gamma (ul)_{\Gamma'} | p \rangle_{\rm SM}$$
$$= \sum_{i=\Gamma,\Gamma'} C_i \langle \pi^0 | (ud)_\Gamma u_{\Gamma'} | p \rangle \bar{v}^c_{e^+}$$



$$\langle \pi^0(\vec{p}) | (ud)_\Gamma (ul)_{\Gamma'} | p(\vec{k}, s) \rangle = \bar{v}^c_{e^+}(\vec{q}) P_{\Gamma'} \left[W_0^{\Gamma\Gamma'}(q^2) + \frac{m_{e^+}}{m_p} W_1^{\Gamma\Gamma'}(q^2) \right] u_p(\vec{k}, s)$$
$$= \bar{v}^c_{e^+}(\vec{q}) P_{\Gamma'} u_p(\vec{k}, s) W_0^{\Gamma\Gamma'}(0) + \mathcal{O}(m_l/m_N)$$

Aoki et al. (JLQCD), PRD62, 014506 (2000); Aoki et al.(RBC), PRD75, 014507 (2007)

W_0 at physical point ($q^2 = m_l^2 \simeq 0$) is relevant to proton decay matrix element.

How to obtain W_0 from lattice QCD

- The "indirect" method
  - Measurements of low-energy constant

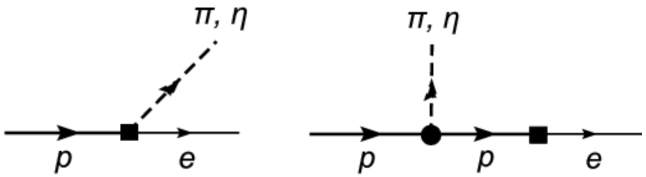
in BChPT at LO:

$$W_0^{LR}(p \to \pi^0) \simeq \alpha(1+D+F)/\sqrt{2}f_0,$$

where D and F is given by experiment, and α is LECs given by 2-pt function:

$$\langle 0|((ud)_R u_L)J_p|0\rangle = \alpha P_L u_p$$

Claudson, et al., NPB195 (1982) 297



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Easy calculation, BUT has systematic error due to higher order of ChPT

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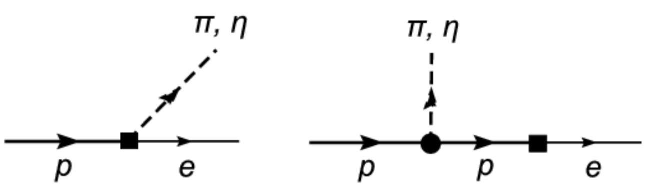

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Easy calculation, BUT has systematic error due to higher order of ChPT

- The "direct" method
  - Measurement of matrix element extracted from 3-pt function.
  - Rather expensive, while there is no uncertainty depending on models.
  - Provides each channels of decay mode.

S Aoki et al. (JLQCD), PRD62, 014506 (2000), Y. Aoki et al.(RBC), PRD75, 014507 (2007), Y. Aoki, A. Soni, ES, 1304.7424

Lattice QCD

- Monte Carlo simulation

  Theoretically rigorous calculation including quark-gluon dynamics

- Lattice fermion
  - Require "realistic" fermion for the precise calculation
    - Wilson-clover and staggered fermions may have large lattice artifacts (cut-off effect, operator mixing …)
    - Domain-wall (and also overlap fermion) is even better.

- Domain-Wall fermion (DWF) [Blum Soni, (97), CP-PACS(99), RBC(00), RBC/UKQCD. (05 --)]

• Setting 5th dimension, its size L_s
• Chiral fermion are localized on boundaries ⇒ Chiral symmetry (if $L_s \to \infty$).
• Good chiral sym. and its breaking effect is suppressed as $am_{res} \sim \exp(-L_s)$.

RBC/UKQCD efforts

- RBC(2007) Y. Aoki et al.(RBC), PRD75, 014507 (2007)
  - "Direct"/"indirect" method in Quench DW
  - Comparison between "direct" and "indirect" method
  - Non-perturbative renormalization (NPR)

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  - NPR

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- RBC/UKQCD(2013) Y. Aoki, A. Soni, ES, 1304.7424, appears in PRD
  - "Direct" method in Nf=2+1 QCD with dynamical DW
  - NPR
  - Estimate of all systematic errors
- This work
  - High precision using AMA

Error reduction techniques

Blum, Izubuchi, ES, PRD88 (2013), ES (lattice 2012)

- Covariant approximation averaging (CAA)
  - For original correlator O, (unbiased) improved estimator is defined as

$$\mathcal{O}^{(\mathrm{imp})} = \mathcal{O}^{(\mathrm{rest})} + \frac{1}{N_G}\sum_{g\in G}\mathcal{O}^{(\mathrm{appx}),g}, \quad \mathcal{O}^{(\mathrm{rest})} = \mathcal{O} - \mathcal{O}^{(\mathrm{appx})}$$

  - $\langle O\rangle = \langle O^{(\mathrm{imp})}\rangle$ if approximation has covariance under lattice symmetry g
  - Improved error $\mathrm{err}^{\mathrm{imp}} \simeq \mathrm{err}/\sqrt{N_G}$
  - Computational cost of $O^{(\mathrm{imp})}$ is cheap.

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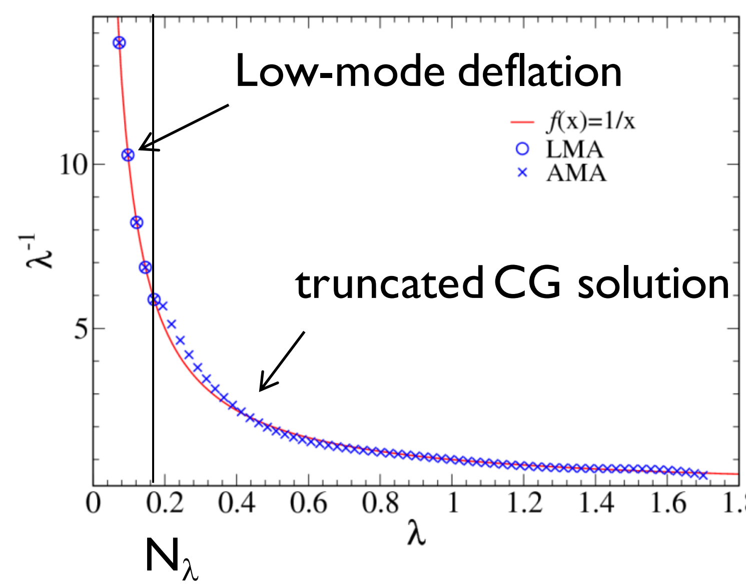
  - Computational cost of $O^{(\mathrm{imp})}$ is cheap.
- All-mode-averaging (AMA)
  - Relaxed CG solution for approximation

$$\mathcal{O}^{(\mathrm{appx})} = \mathcal{O}[S_l],\ S_l = \sum_{\lambda=1}^{N_\lambda} v_\lambda v_\lambda^\dagger \frac{1}{\lambda} + P_n(\lambda)|_{|\lambda|>N_\lambda}$$

  - $P_n(\lambda)$ is polynomial approximation of $1/\lambda$
    - Low mode part : # of eigen mode
    - Mid-high mode : degree of poly.



3. Lattice calculation and new technique

Lattice parameters in this work

- DWFs Nf=2+1
  - $24^3 \times 64$ size at a^{-1} = 1.73 GeV ⇒ 2.5 fm^3 box size
  - Light quark mass m=0.005, 0.01, 0.02 (m_π = 0.3 -- 0.6 GeV)
  - Strange quark mass m_s = 0.04 (m_K = 0.5 GeV)
  - 5th dimension, L_s = 16 in which am_{res} = 0.003, which means that there is good chiral symmetry on the lattice.
  - NPR of BV operators at μ=2GeV Y. Aoki et al. (RBC-UKQCD), PRD78, 054505 (2008)
  - APE + Gaussian smeared source and sink.
  - Three sorts of momentum

    n_p=(1,0,0), (1,1,0), (1,1,1)
  - AMA

    3×10^{-3} precision of truncated solver in N_G = 32 source locations

    Low-mode deflation (300 lowmodes) in light quark, but strange part is only using truncated solver.

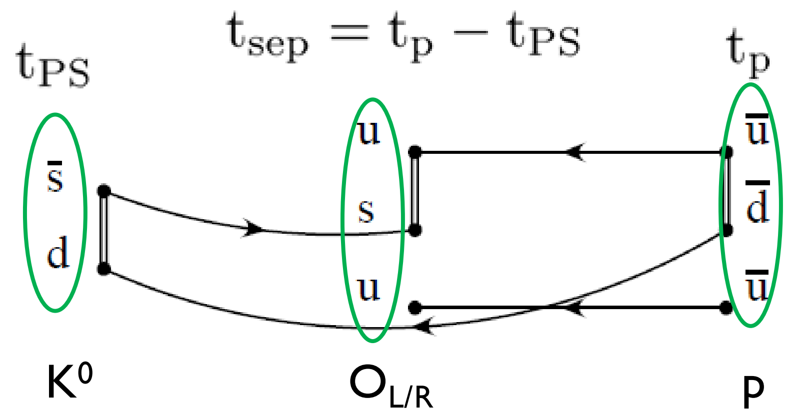
W_0 from 3-pt function

- (PS meson)-(BV operator)-(Nucleon)
- Location of operators

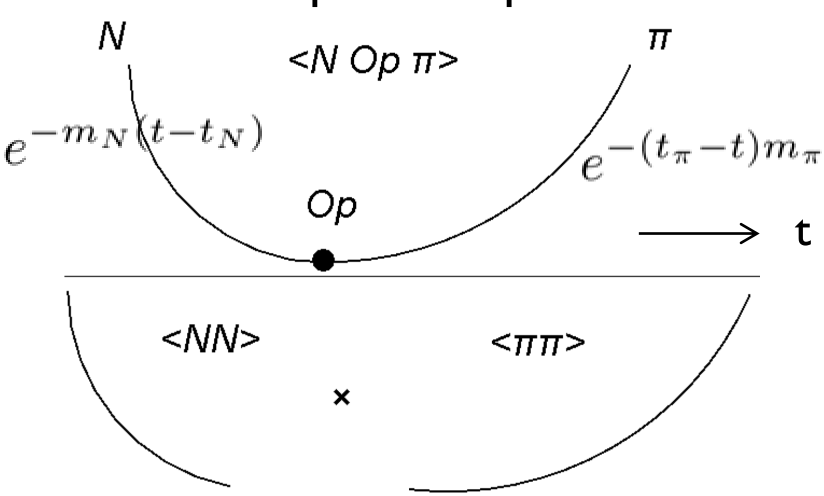
  which relies on signal region

  t_{sep} = 22: t=5(PS) and 27(p),

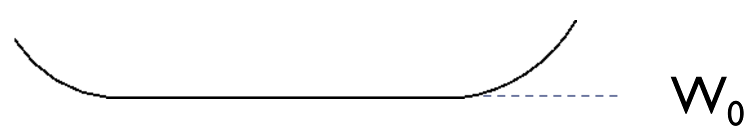
  t_{sep} = 18: t=5(PS) and 23(p).



- Comparison is good check of excited state contamination
- Ratio of 3-pt and 2-pt



$\Rightarrow$

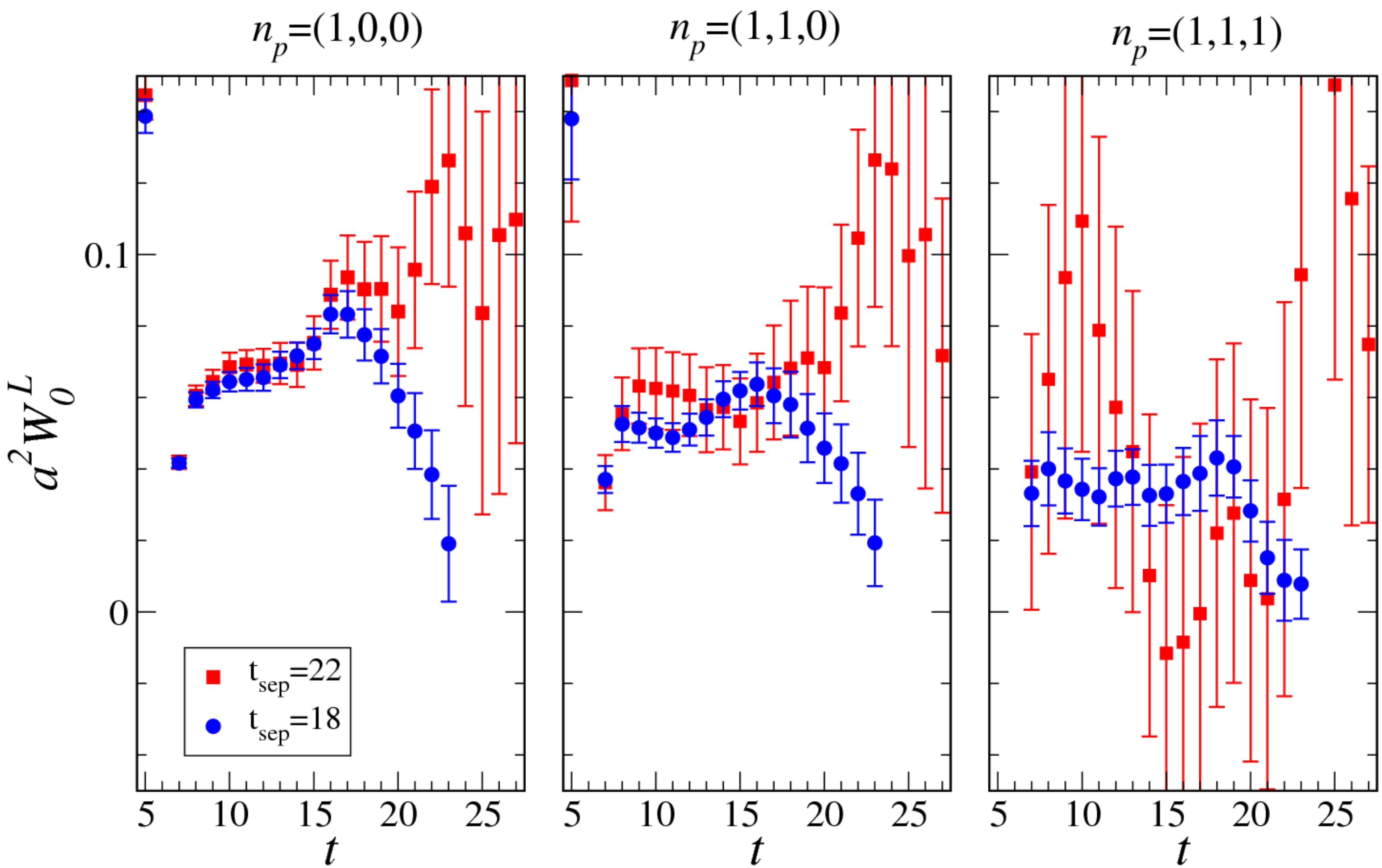


The signal appears as plateau.

4. Preliminary results in high precision

Comparison with different t_{sep}

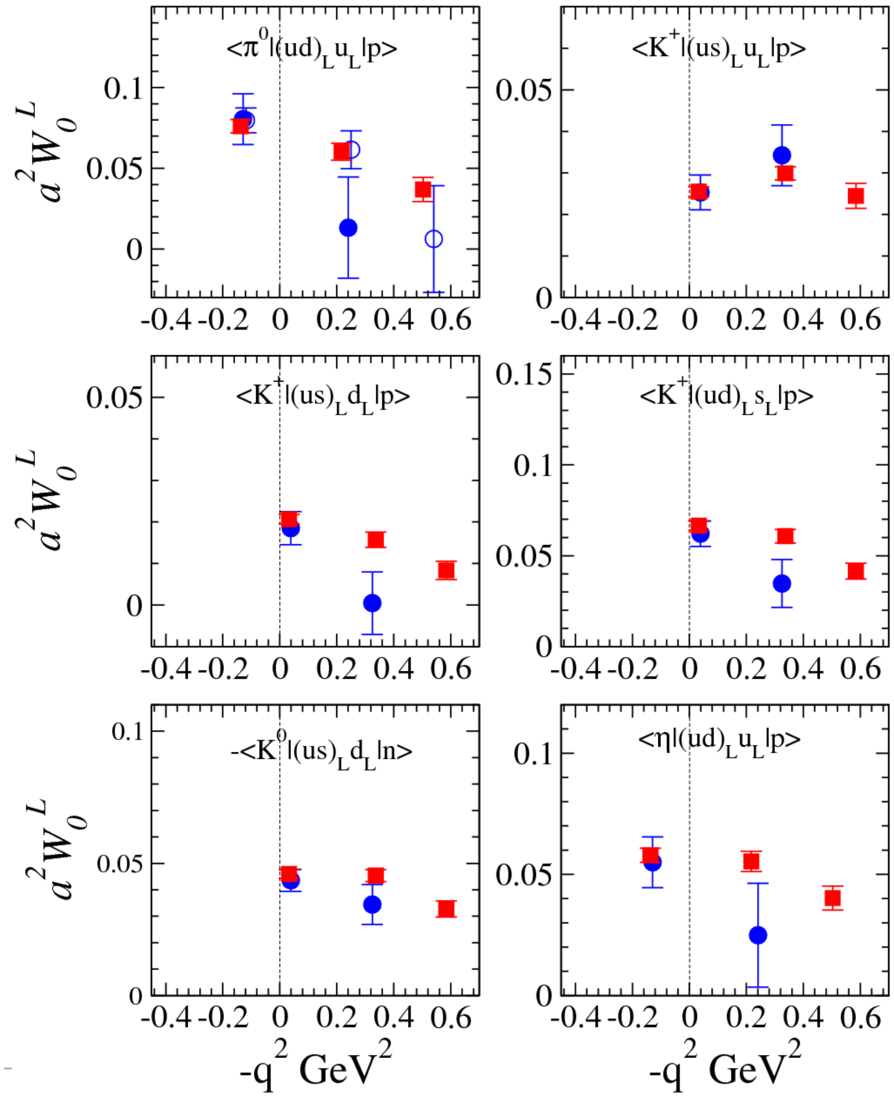
- Check of excited state contamination



Working on AMA

- Comparison with AMA, t_{sep}
  - Blue filled
    200 configs. × 2 src = 400 meas.
    t_{sep} = 22
  - Blue circle
    100 configs. with AMA, N_G = 32
    t_{sep} = 22
  - Red squared
    100 configs. with AMA, N_G = 32
    t_{sep} = 18

Statistical error can be reduced to 1/5 and more for t_{sep} = 18 in AMA.



Extrapolation into physical kinematics

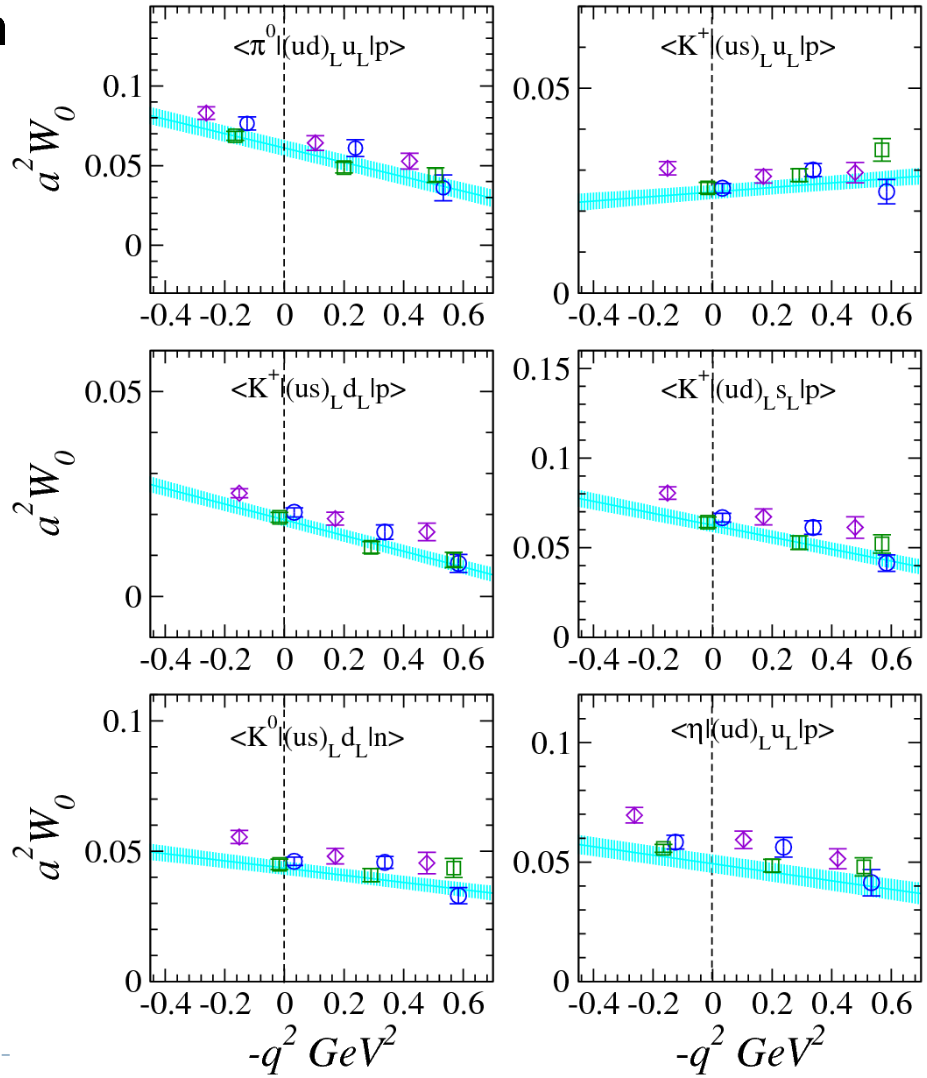
- Linear function in extrapolation

$$F_{W_0} = A_0 + A_1 \tilde{m}_{ud} + A_2 q^2$$

$A_{0,1,2}$ are free parameters for simultaneous fitting.

Lattice data is good fitted with linear func. χ^2/dof ~ 1--2

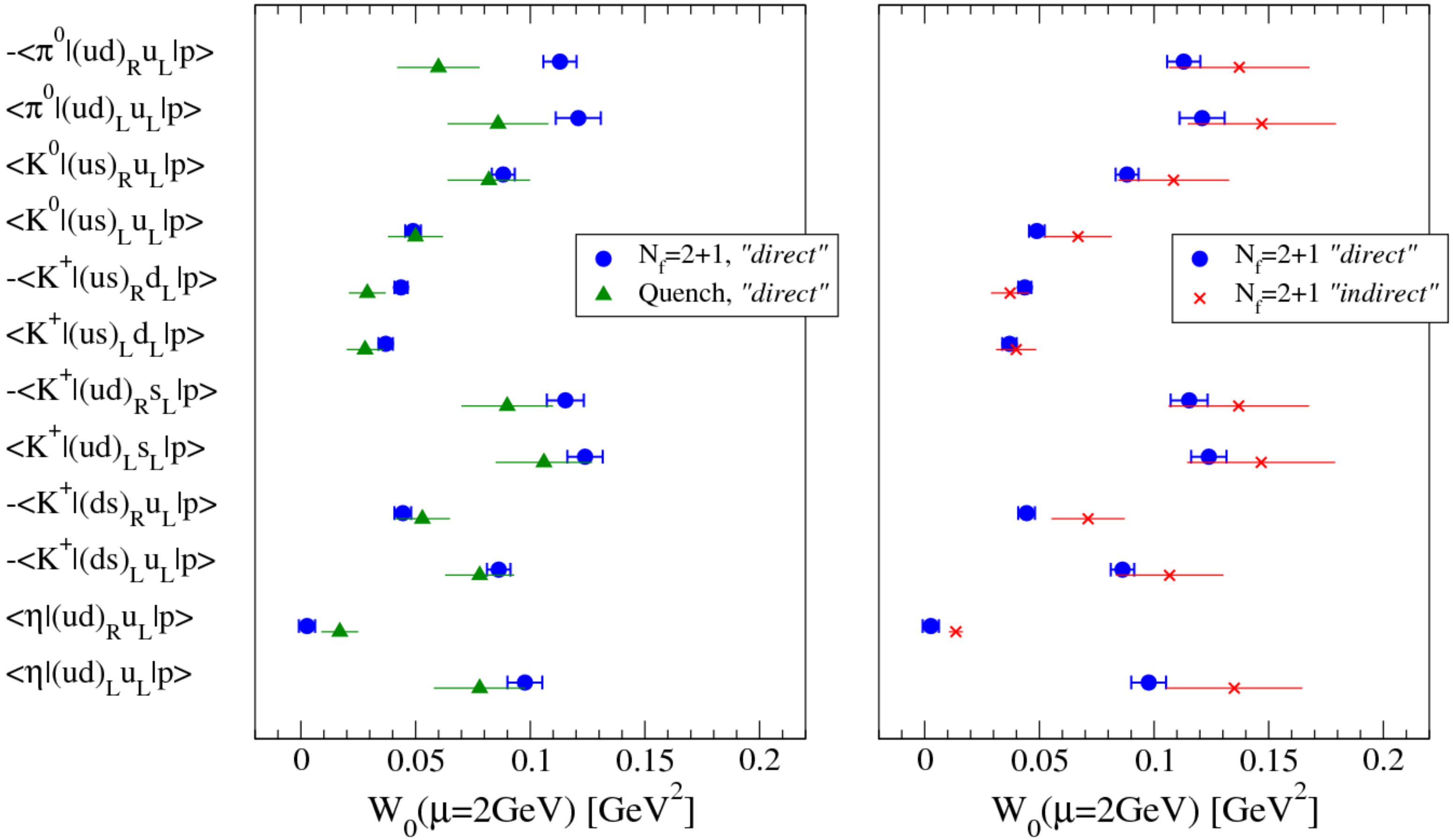
At physical point ($q^2 = 0$), which gives $W_0 = A_0$



4. Preliminary results in high precision

Comparison with previous results

- Quench and "indirect" method



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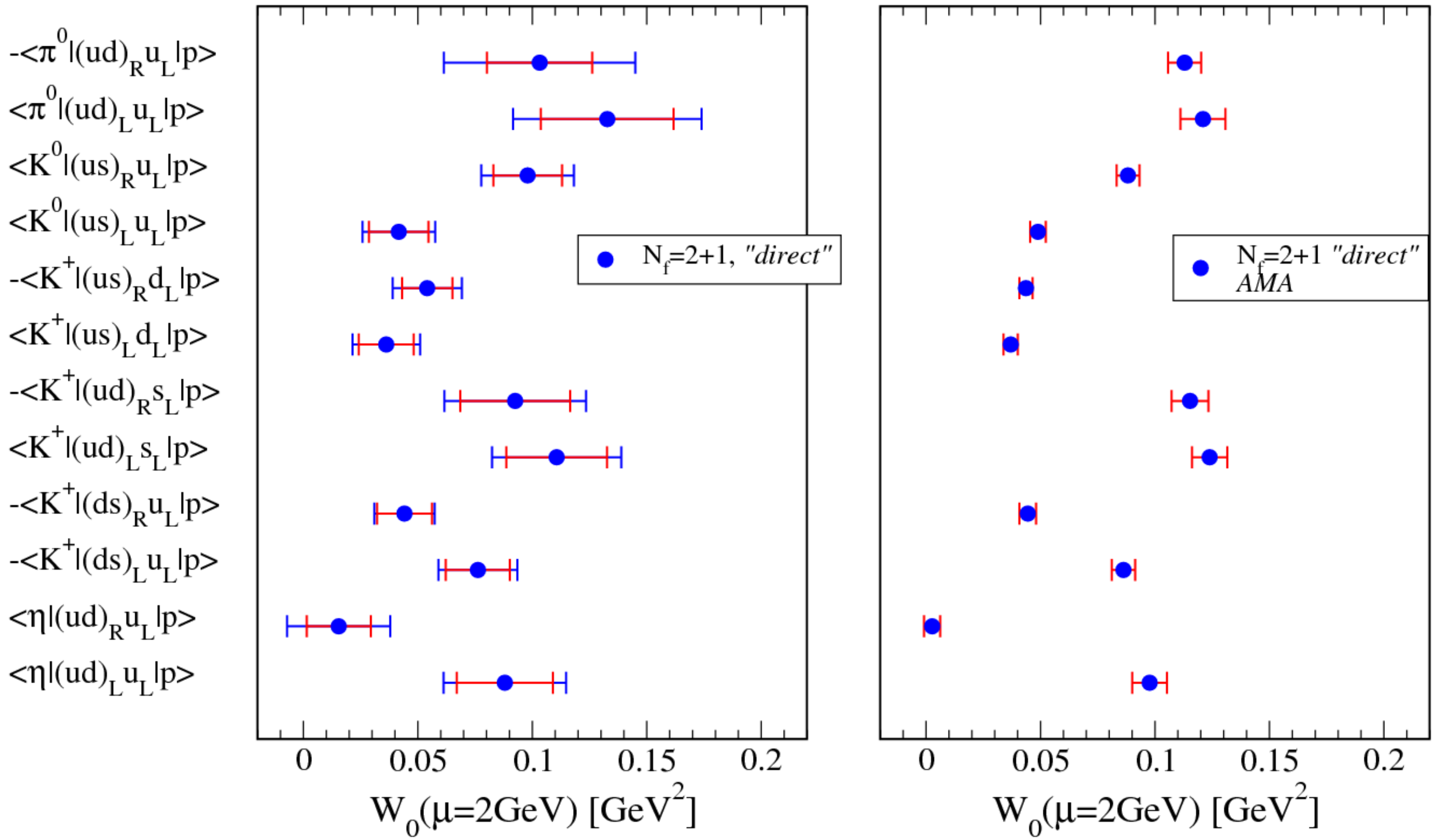


- red bar: only stat. error in this work, blue bar: sys. + stat. error in 1304.7424.
- Stat. error reduction to factor 5--6 and more by using t_{sep} = 18 with AMA.

4. Preliminary results in high precision
BChPT and lattice results

cyan line: lattice results
red line: BChPT at LO

BChPT in LO at $-q^2$

$$W_0(p \to \pi^0) \simeq \frac{\beta}{\sqrt{2} f_0}\left[1 - (D+F)\frac{-q^2 + m_N^2}{-q^2 - m_N^2}\right]$$

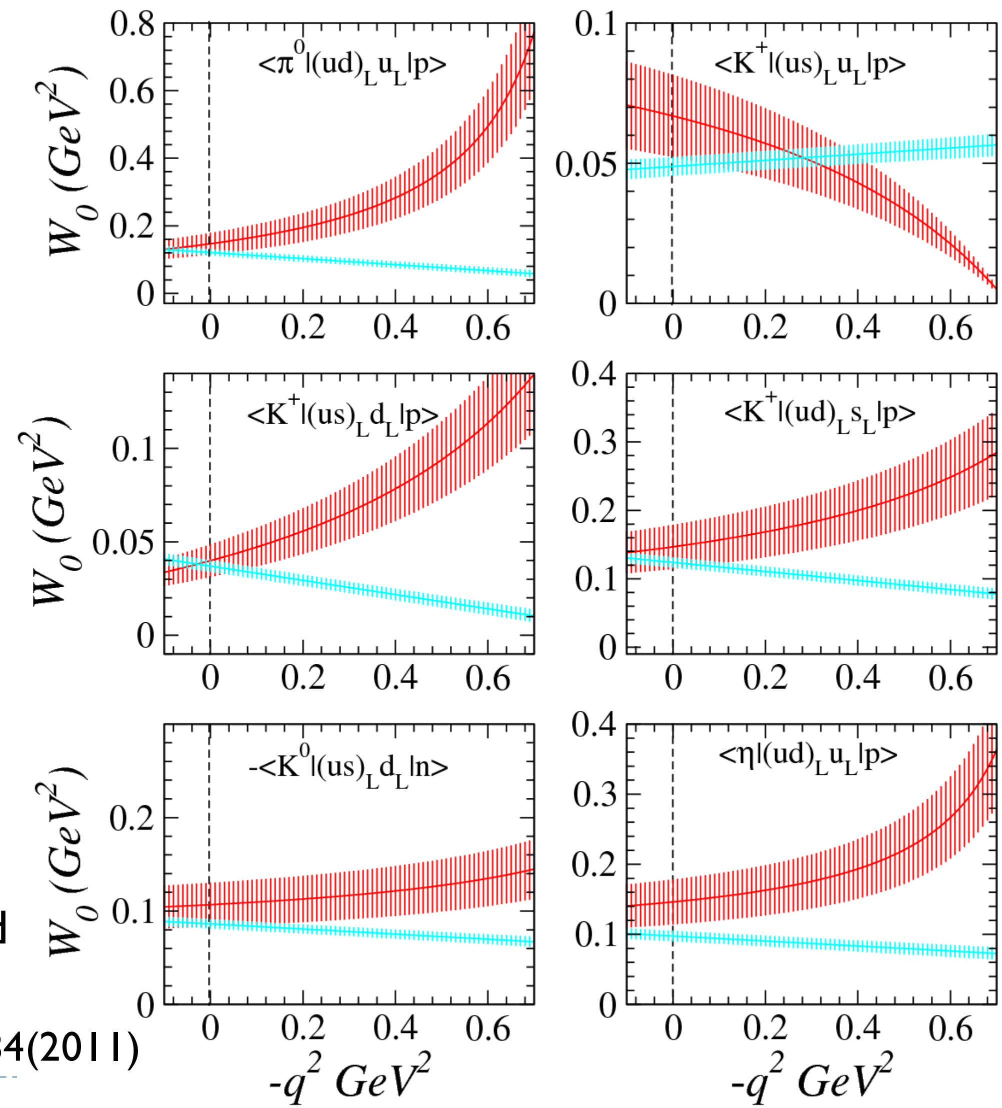
Error of BChPT is coming from LECs computed in lattice QCD:
β = 0.0120(13)(23) GeV^3

Y. Aoki et al. (RBC-UKQCD), PRD78, 054505 (2008)

From low to high q^2 region, discrepancy becomes bigger.

give important suggestion for induced N decay scenario etc.

Davoudias, et al. PRL105(2010), PRD84(2011)



Summary

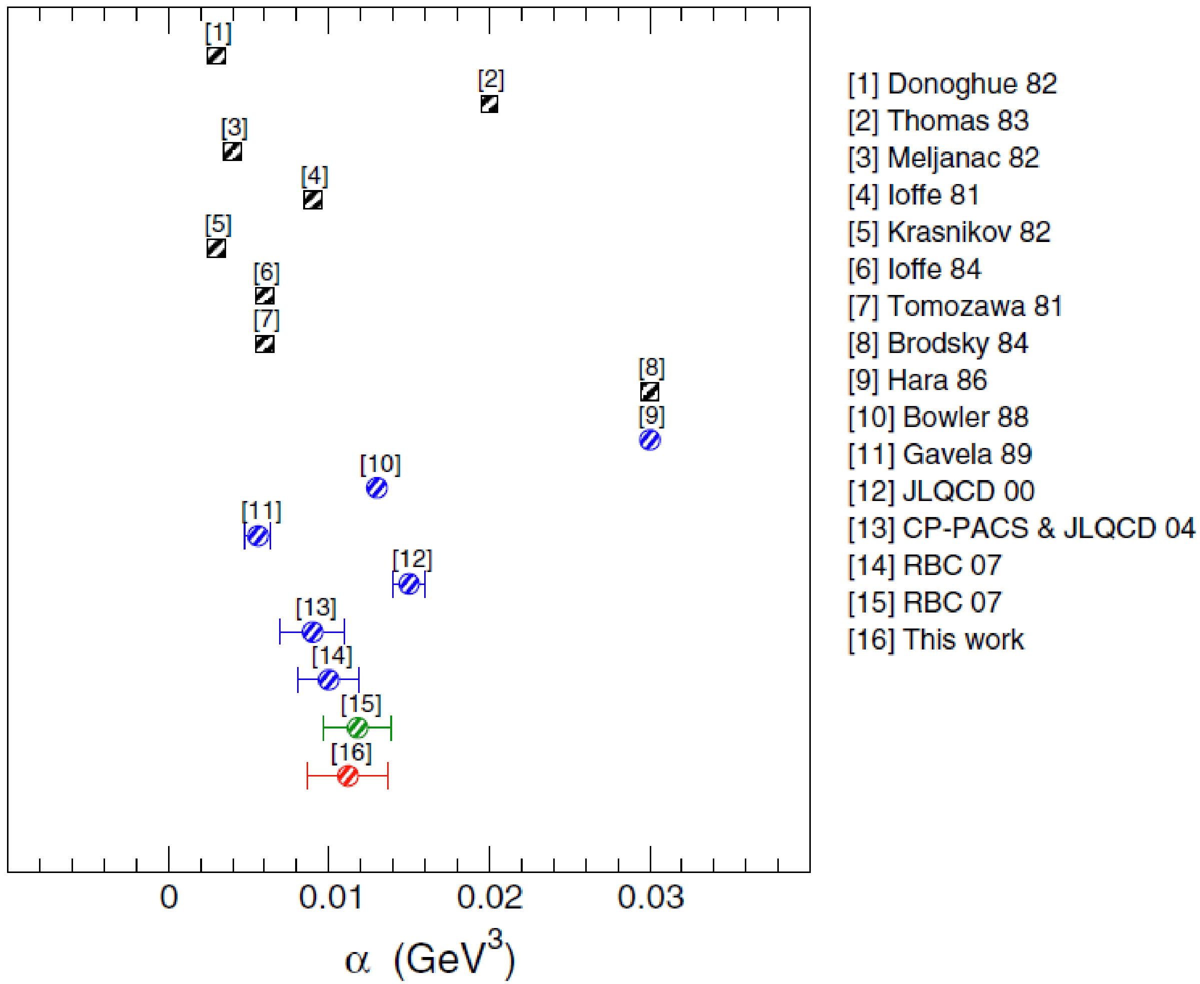
- Perform the lattice calculation precisely using AMA.
- Statistical error is less than 10% (factor three improvement from previous work).
- Linear function for q^2 and m is good fitting with lattice data.
- Lattice result is compatible to BChPT at $q^2=0$, but in high q^2 there may be significant discrepancy.
- Estimate of systematic error (finite size, …) is under way.
- Simulation in physical point gives final results (in near future).

Thank you for your attention.

Backup

Comparison of α

In model calculation, there are model dependence on α, β which is about 0005 -- 0.03 GeV^3 .
For p decay, this is factor 10 difference.



Results of each channels

- Error budgets
  - 12 principal channels
  - Statistical error.
  - χ: Chiral extrapolation
    + Finite Volume.
  - $O(a^2)$: Lattice artifacts
  - ΔZ, Δa^{-1} : error of NPR, lattice spacing

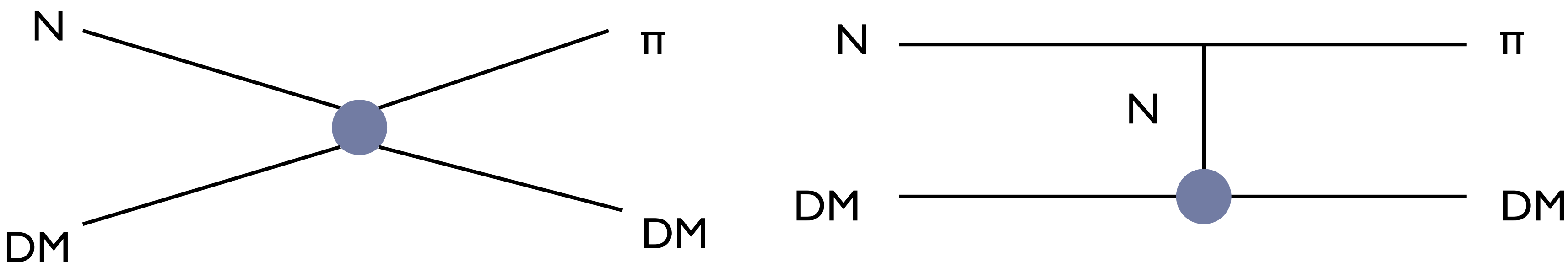
  - Pion channel : 30% stat. and sys. error

  - Keon channel : 10--20 % for stat. error, 5--10 % sys error

		systematic error budget			
matrix element	$W_0(\mu = 2\text{GeV})$ GeV^2	χ	$\mathcal{O}(a^2)$	ΔZ	Δa^{-1}
$\langle \pi^0 \| (ud)_R u_L \| p \rangle$	−0.103 (23) (34)	0.033	0.005	0.008	0.004
$\langle \pi^0 \| (ud)_L u_L \| p \rangle$	0.133 (29) (28)	0.026	0.007	0.011	0.005
$\langle K^0 \| (us)_R u_L \| p \rangle$	0.098 (15) (12)	0.007	0.005	0.008	0.003
$\langle K^0 \| (us)_L u_L \| p \rangle$	0.042 (13) (8)	0.007	0.002	0.003	0.001
$\langle K^+ \| (us)_R d_L \| p \rangle$	−0.054 (11) (9)	0.008	0.003	0.004	0.002
$\langle K^+ \| (us)_L d_L \| p \rangle$	0.036 (12) (7)	0.007	0.002	0.003	0.001
$\langle K^+ \| (ud)_R s_L \| p \rangle$	−0.093 (24) (18)	0.016	0.005	0.008	0.003
$\langle K^+ \| (ud)_L s_L \| p \rangle$	0.111 (22) (16)	0.012	0.006	0.009	0.004
$\langle K^+ \| (ds)_R u_L \| p \rangle$	−0.044 (12) (5)	0.003	0.002	0.004	0.002
$\langle K^+ \| (ds)_L u_L \| p \rangle$	−0.076 (14) (9)	0.006	0.004	0.006	0.003
$\langle \eta \| (ud)_R u_L \| p \rangle$	0.015 (14) (17)	0.017	0.001	0.001	0.001
$\langle \eta \| (ud)_L u_L \| p \rangle$	0.088 (21) (16)	0.014	0.004	0.007	0.003

Induced N decay scenario

- DM scattering induces the proton decay



$$\mathcal{L}_{\rm IND} = \frac{1}{\Lambda^3} u_R u_R d_R \Psi_R \Phi$$

Davoudias, et al. PRL105(2010), PRD84(2011)

- DM has baryon number, $n_\Phi + n_\Psi = -1$
- These masses are roughly estimated as $m_\Phi = m_\Psi = 2$--3 GeV
- PS meson is energetic, whose momentum is around 1 GeV
  $p_\pi \sim 0.8$ GeV, $p_K \sim p_\eta \sim 0.7$ GeV
- $q^2 \sim 0.6$ GeV^2 (pi), ~ 0.45 GeV^2 (K,η)
- In this region, BChPT at LO is not appropriate, because higher order correction should be significant.