Describing Few-Body Systems at the Limit of Nucleonic DoF

Deuteron at Extremely Small Distances

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Exploring QCD with Light Nuclei at EIC, 21-25 January, 2020, CFNS,
Stony Brook University, Stony Brook, NY

Motivation for Deuteron Studies

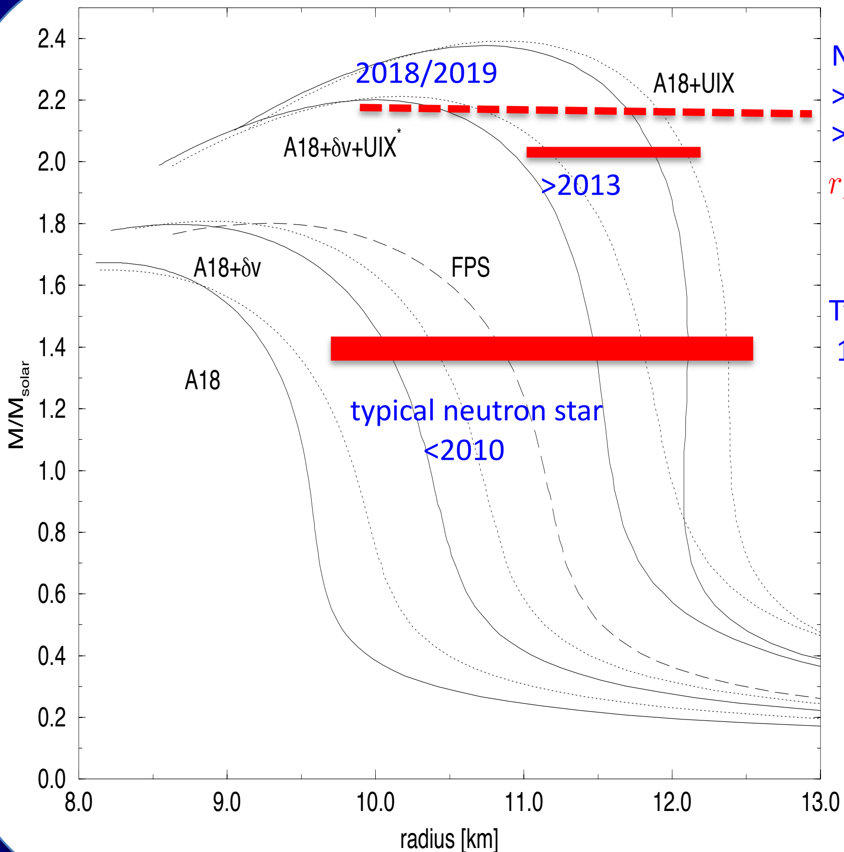
Probing NN Repulsive Core & Hadron-Quark Transition

- *strength of the core;*
- *non-nucleonic components, hidden color, gluons*
- *hadronization in the deuteron*

Processes to Study

- *quasi-elastic processes in d(e,eN)N reactions*
- *probing superfast quarks (x>1) in inclusive d(ee')X processes*
- *tagged DIS processes*

"Unreasonable" Persistence of Nucleons



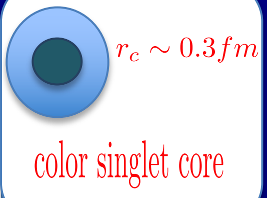
Neutron star masses
>2 solar masses
>2010

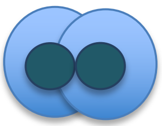
$r_{NN} \sim 0.6 - 0.8 fm$

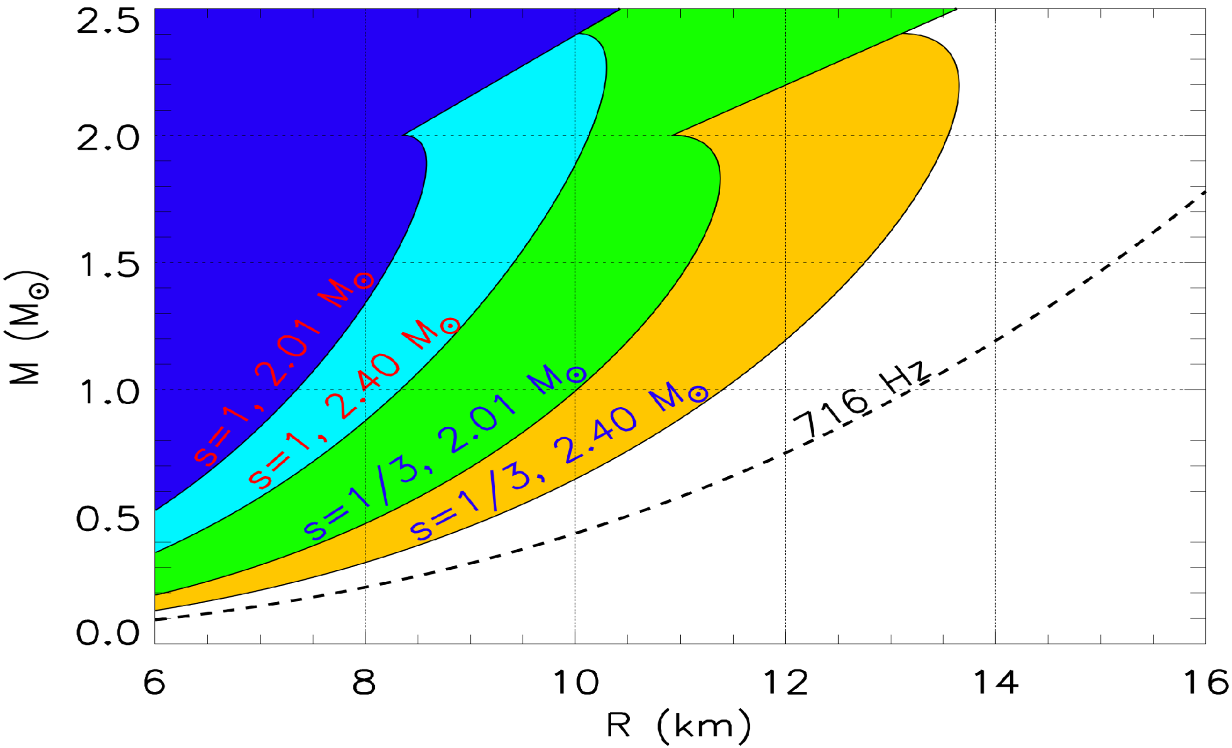
Typical neutron star
1.4 Solar mass

$r_{NN} \sim 0.7 - 1.0 fm$

H. Heiselberg,
V.Pandharipande
ARNPS 2000

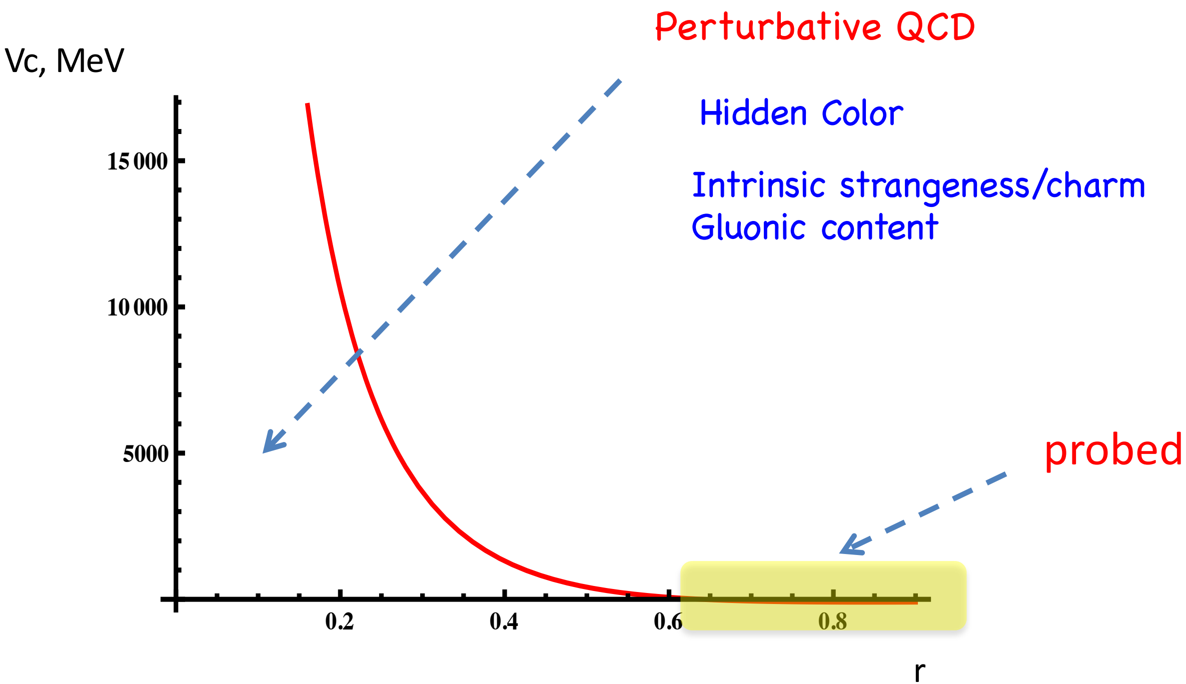






Lattimer, 2019

NN - Potential



~80% hidden color

Brodsky,Ji, Lepage, PRL 83

Probing the Deuteron at Short Distances

$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \cdots$$

$$\Psi_{hc} = \Psi_{N_c,N_c}$$

The NN core can be due to the orthogonality of

$$\langle \Psi_{N_c,N_c} \mid \Psi_{N,N} \rangle = 0$$

Conceptually: How to probe nuclei at short nucleon separations

- Probe bound nucleons at large internal momenta

- Need high energy probes to resolve such nucleons in nuclei

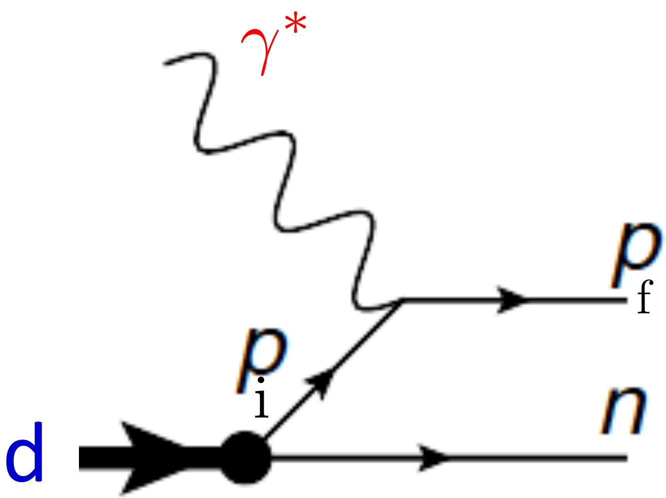
Problem: Non Relativistic Quantum Mechanics is increasingly problematics for description of high momentum component of nuclear wave function

Solutions: Problem is similar to the partonic model in which case nucleons are partons of the nucleus

Probing the Deuteron at Short Distances

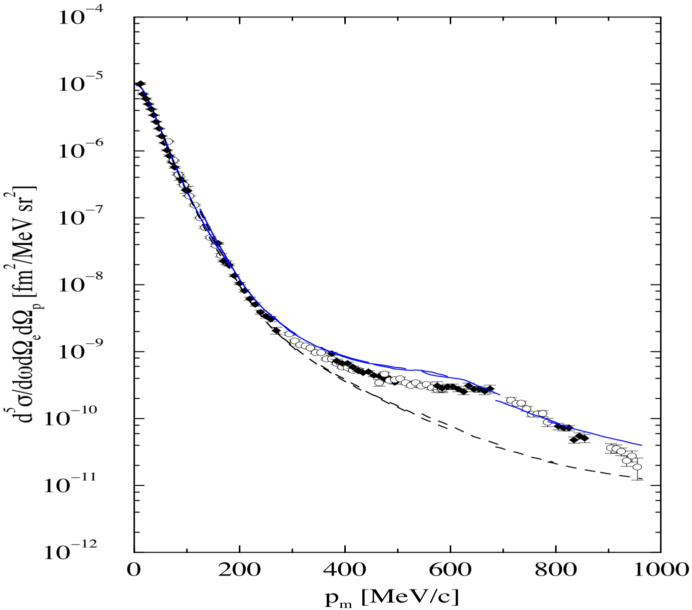
- quasi-elastic processes in d(e,eN)N reactions

Considering reaction: $e + d \to e' + p_f + n$

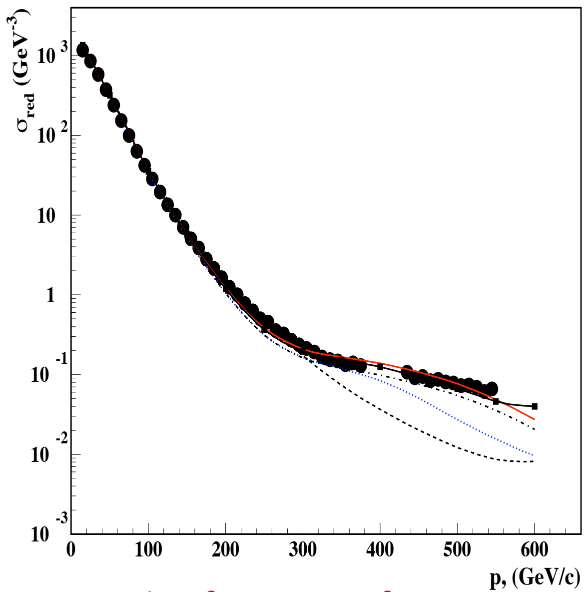


$$|p_i| = |p_f - q| > 300 \text{ MeV/c}$$

Impossibility to Probe Deuteron at Small Distances at low Q²

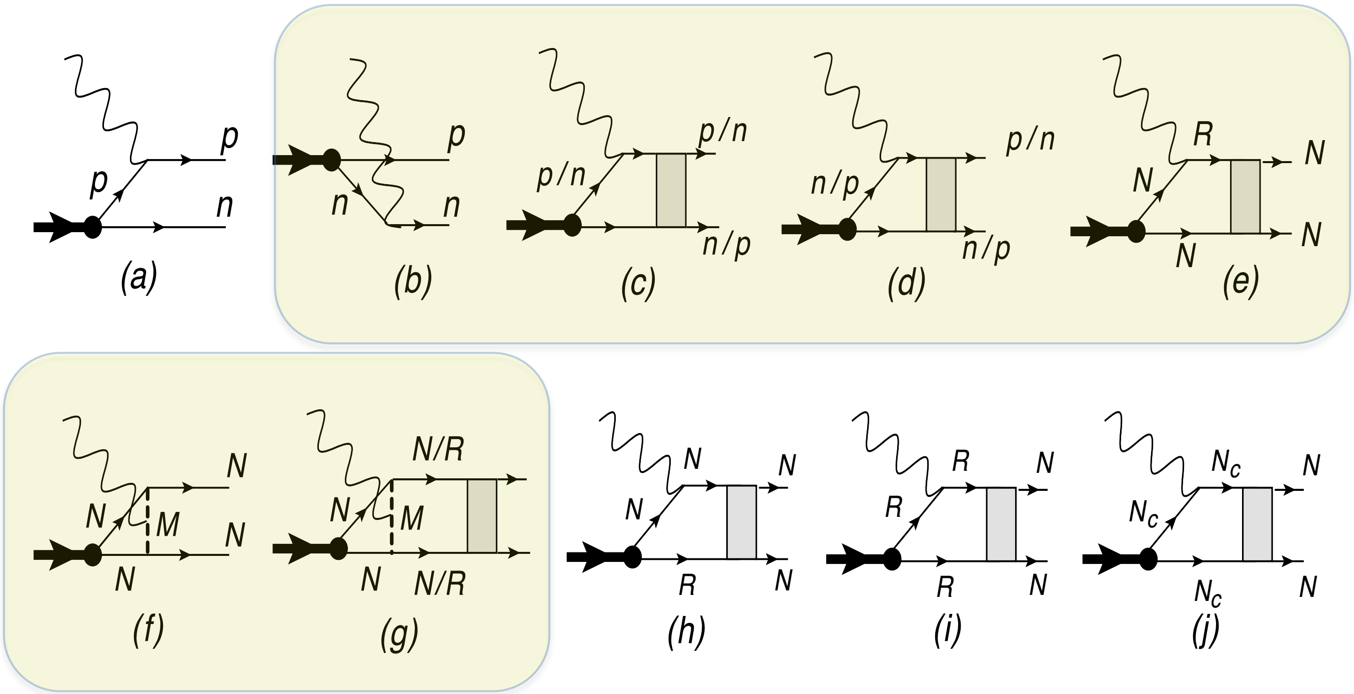


Mainz, $Q^2 = 0.33$ GeV²

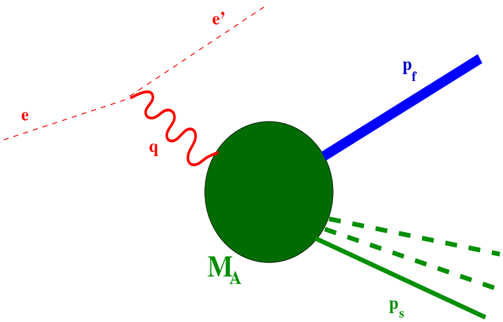


JLab, $Q^2 = 0.66$ GeV²

Impossibility to Probe Deuteron at Small Distances at low Q^2



#Theory of High Energy eA Scattering:



High Energy Approximations:

$$|\vec{q}| = q_3 \sim p_{f3} \gg p \sim M_N$$

$$Q^2 \geq few \quad \mathrm{GeV}^2$$

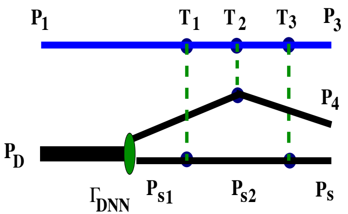
Both for QE/DIS

- Emergence of the small parameter

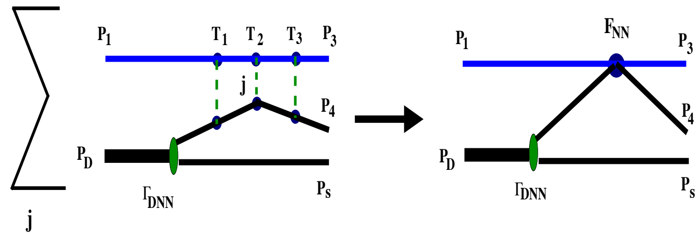
$$\frac{q_-}{q_+} = \frac{q_0 - q_3}{q_0 + q_3} \ll 1 \quad \mathcal{O}(\frac{q_-}{q_+})$$

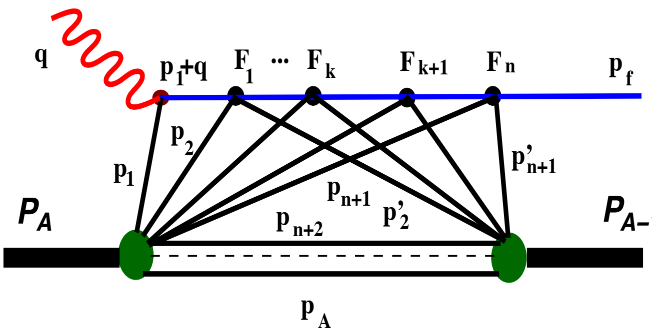
$$\frac{p_{f-}}{p_{f+}} = \frac{E_f - p_{f3}}{E_f + p_{f3}} \ll 1 \quad \mathcal{O}(\frac{p_{f-}}{p_{f+}})$$

Emergence of "effective" theory



$$\mathcal{O}\left(\frac{p_{1-}}{p_{1+}}\right)$$





Effective Feynman
Diagrammatic Rules

M.S. IJMS 2001

Wave function?

From Schroedinger Equation -> Feynman Diagrams-> Light-Front Wave Function

Schroedinger eq. → Lipmann-Schwinger Eq.

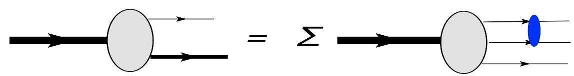
$$\left[-\sum_i \frac{\nabla_i^2}{2m} + \frac{1}{2}\sum_{i,j} V(x_i - x_j)\right]\psi(x_1,\cdots,x_A) = E\psi(x_1,\cdots,x_A)$$

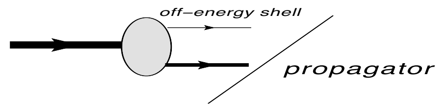
$$\left(\sum_i \frac{k_i^2}{2m} - E_b\right)\Phi(k_1,\cdots,k_A) = -\frac{1}{2}\sum_{i,j}\int U(q)\,\Phi(k_1,\cdots,k_i-q,\cdots k_j+q,\cdots,k_A)d^3q$$

Lipmann-Schwinger Eq → t - ordered diagrammatic method

$$\left(\sum_i \frac{k_i^2}{2m} - E_b\right)\Phi(k_1,\cdots,k_A) = -\frac{1}{2}\sum_{i,j}\int U(q)\,\Phi(k_1,\cdots,k_i-q,\cdots k_j+q,\cdots,k_A)d^3q$$

$$\Phi(k_1,\cdots k_A) = \frac{1}{\sum \frac{k_i^2}{2m} - E_b}\Gamma_{A\to N,A-1}$$

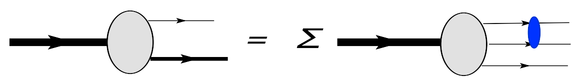


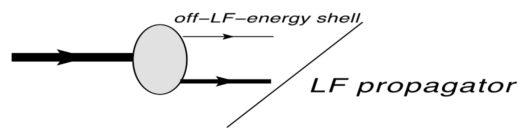


Weinberg Eq → τ - ordered diagrammatic method

$$\left(\sum \frac{k_{i\perp}^2+m^2}{\alpha_i} - M_A^2\right)\Phi_{LF}(k_1,\cdots k_A) = \frac{1}{2}\sum_{i,j}\int U_{LF}(q)\Phi_{LF}(k_1,\cdots k_A)\prod\frac{d\alpha_i}{\alpha_i}d^2k_{i\perp}$$

$$\Phi_{LF}(k_1,\cdots k_A) = \frac{1}{\sum \frac{k_{i\perp}^2+m^2}{\alpha_i} - M_A^2}\Gamma_{A\to N,A-1}$$





Deuteron is a pseudo-vector "particle"

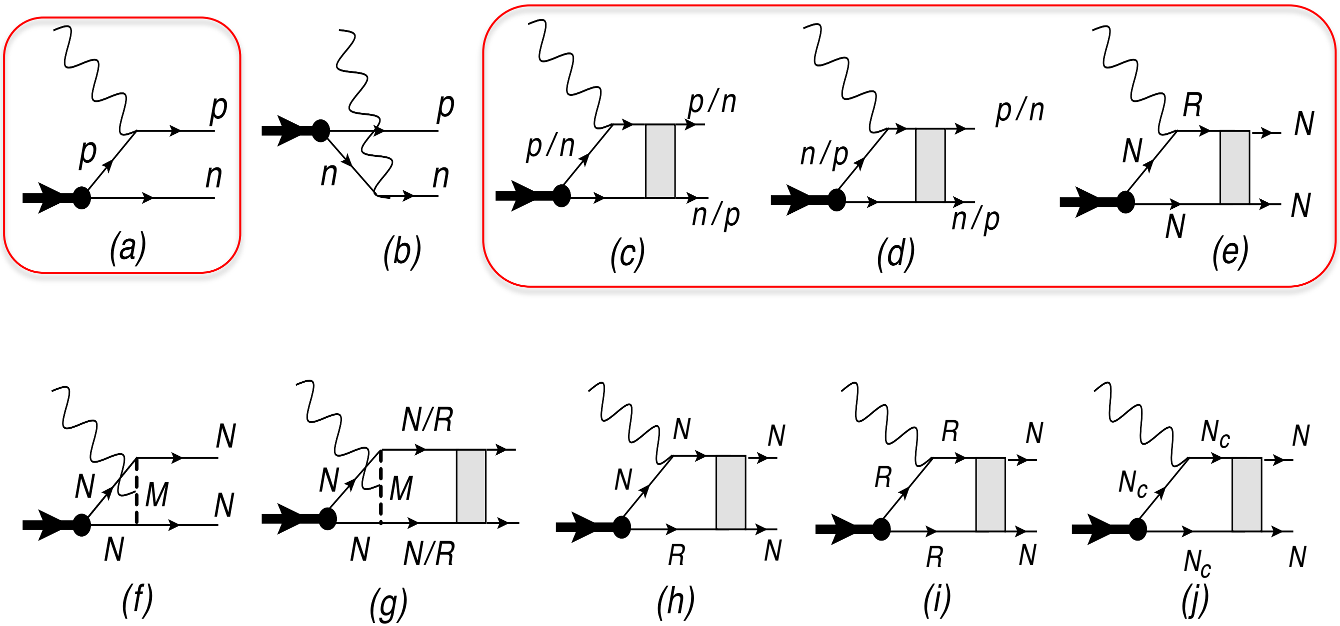
$$\Psi^{\lambda}_{s_2 s_1} = \chi^{\lambda}_{\mu(P_d)} \bar{u}_{(k_2,s_2)} \Gamma^{\mu} \left(u_{(k_1,s_1)}\right)^{C}$$

$$\Gamma_\mu = \boxed{F_1}\gamma_\mu + \boxed{F_2}\frac{(k_1-k_2)_\mu}{m} + F_3\frac{\Delta p_\mu}{m} + F_4\frac{(k_1-k_2)_\mu \Delta \not{p}}{m^2} + \boxed{F_5}\frac{1}{2m^2}\gamma_5 \epsilon^{\mu\nu\rho\gamma}(P_d)_\nu (k_1-k_2)_\rho \Delta p_\gamma + F_6\frac{\Delta p_\mu \Delta \not{p}}{m^2}$$

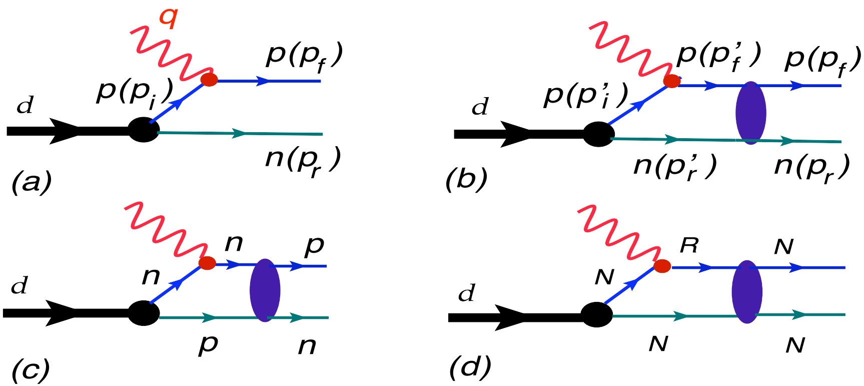
$$\Delta p^{\mu} = (0, \Delta p^{-}, 0)$$

$$\Delta p^{-} = \frac{1}{p_d^{+}}\left(M_d^2 - 4\frac{m_N^2 + p_\perp^2}{\alpha(2-\alpha)}\right)$$

Impossibility to Probe Deuteron at Small Distances at low Q²



In high momentum transfer limit: $Q^2 > 1\text{-}2\ GeV^2$



Generalized Eikonal Approximation at large Q2, 1997-2010

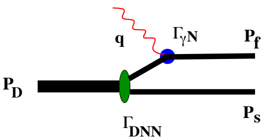
$$\langle s_f, s_r \mid A_0^\mu \mid s_d\rangle = -\bar{u}(p_r, s_r)\Gamma^\mu_{\gamma^* p}\frac{\not{p}_i + m}{p_i^2 - m^2} \cdot \bar{u}(p_f, s_f)\Gamma_{DNN} \cdot \chi^{s_d}$$

$$\begin{aligned}\langle s_f, s_r \mid A_1^\mu \mid s_d\rangle \quad &= \quad -\int \frac{d^4 p'_r}{i(2\pi)^4}\frac{\bar{u}(p_f, s_f)\bar{u}(p_r, s_r)F_{NN}[\not{p}'_r + m][\not{p}_d - \not{p}'_r + \not{q} + m]}{(p_d - p'_r + q)^2 - m^2 + i\epsilon} \\ &\times \frac{\Gamma_{\gamma^* N}[\not{p}_d - \not{p}'_r + m]\Gamma_{DNN}\chi^{s_d}}{((p_d - p'_r)^2 - m^2 + i\epsilon)(p'^2_r - m^2 + i\epsilon)}\end{aligned}$$

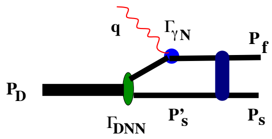
$$\Psi_d^{s_d}(s_1, p_1, s_2, p_2) = -\frac{\bar{u}(p_1, s_1)\bar{u}(p_2, s_2)\Gamma^{s_d}_{DNN}\chi_{s_d}}{(p_1^2 - m^2)\sqrt{2}\sqrt{(2\pi)^3(p_2^2 + m^2)^{\frac{1}{2}}}}$$

Some Results: $e + d \to e' + p + n$

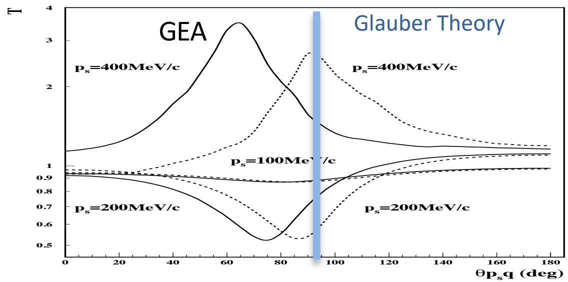
Frankfurt, M.S., Strikman, PRC 1997



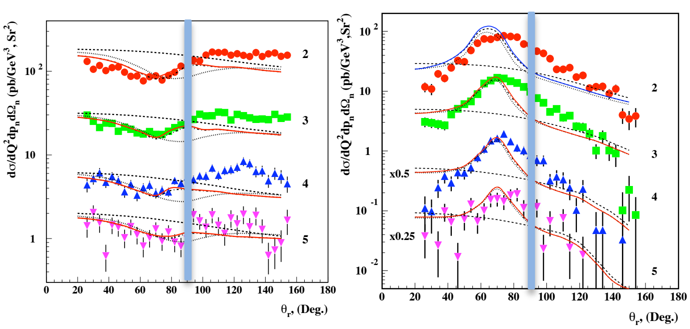
(a)



(b)

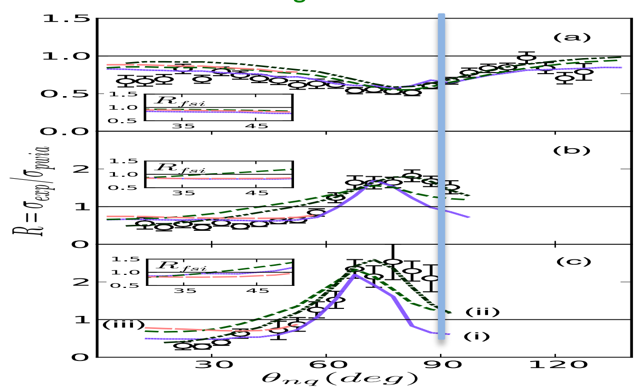


K. Egiyan et al PRL 2008

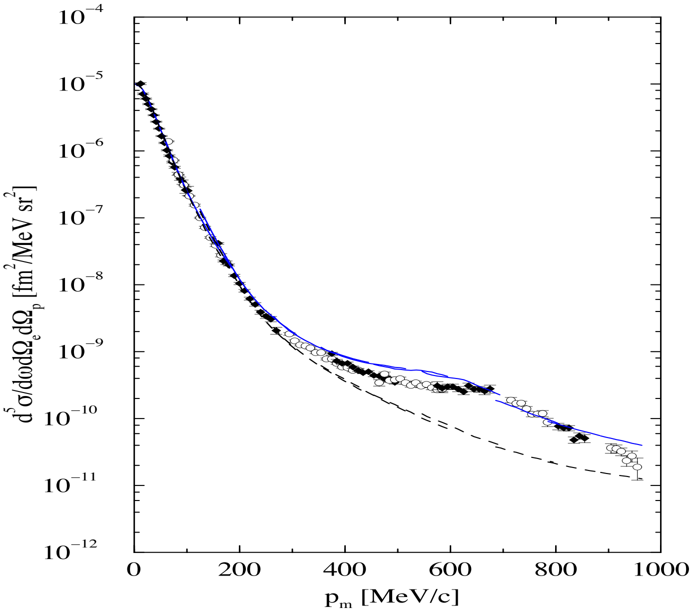


M.Sargsian, PRC 2010

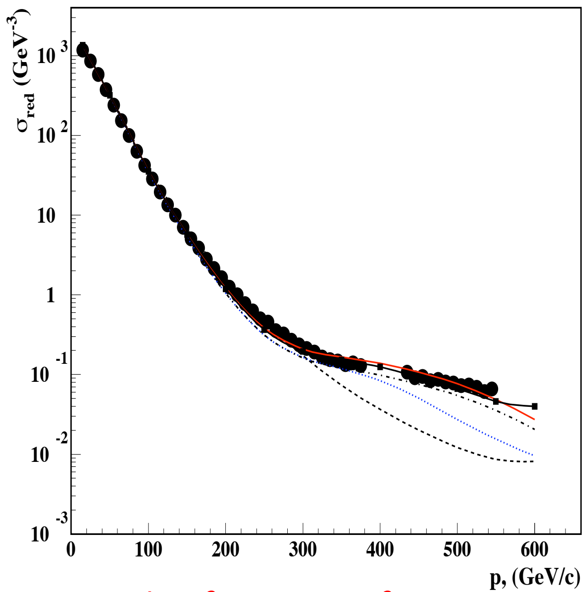
W. Boeglin et al PRL 2011



Impossibility to Probe Deuteron at Small Distances at low Q^2

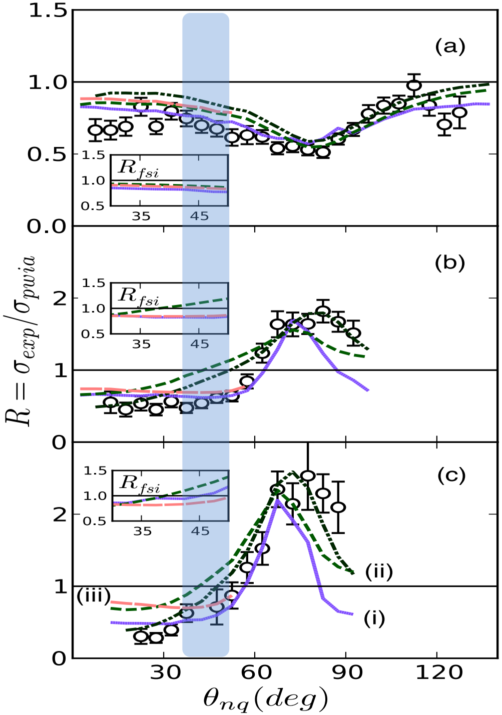


Mainz, $Q^2 = 0.33$ GeV^2



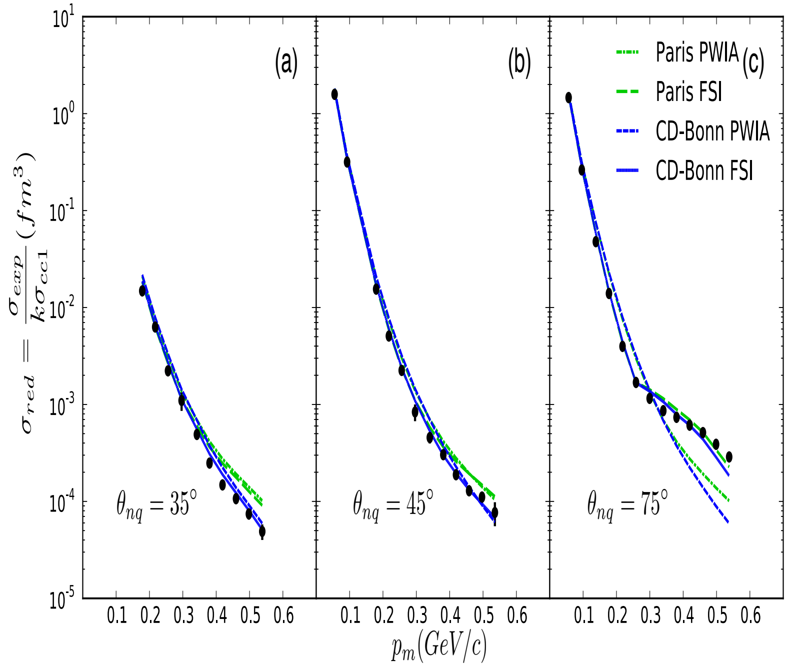
JLab, $Q^2 = 0.66$ GeV^2

Probing Deuteron at Small Distances at large Q^2



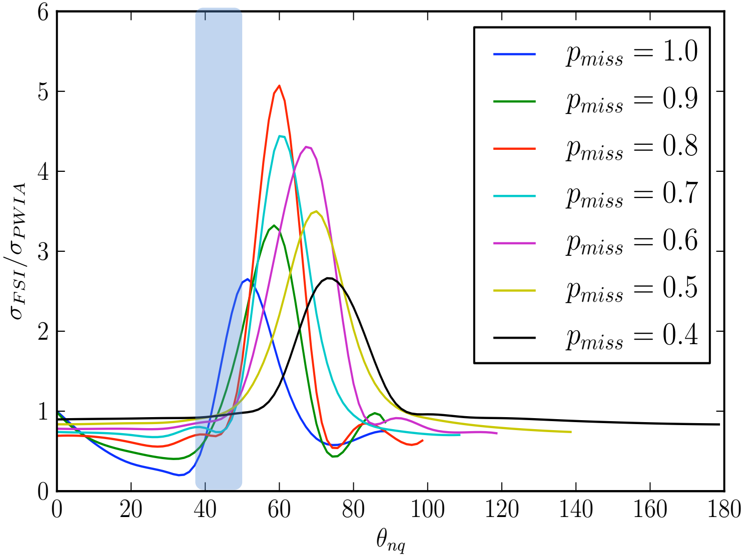
JLab, $Q^2 = 3.5$ GeV^2

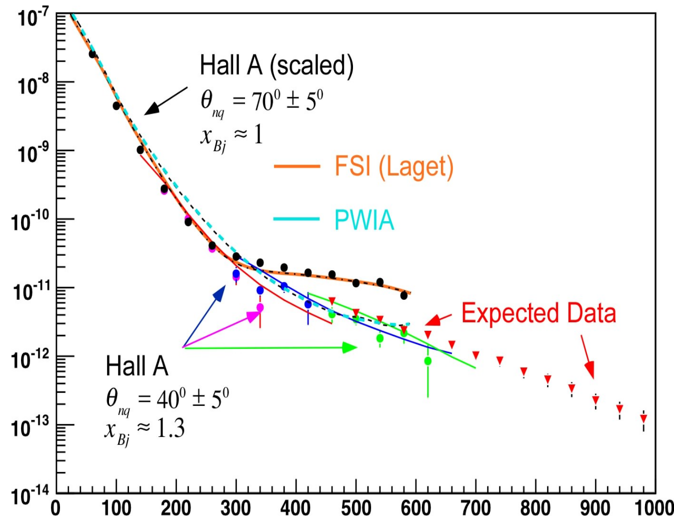
M.Sargsian, PRC 2010



Boeglin et al PRL 2011, deuteron probed at up to 550MeV/c

Probing Deuteron at Extreme Short Distances at large Q^2





JLab proposal $Q^2 = 4\ GeV^2$

Werner's talk

Extraction of Deuteron Light-Cone Momentum Distribution $\rho(\alpha, p_T)$

$$F_{2d}(x_{Bj}, Q^2) = \int_{x_{Bj}}^{2} F_{2p}^{bound}(\frac{x_{Bj}}{\alpha}, Q^2)\rho_d(\alpha)\frac{d\alpha}{\alpha} + \int_{x_{Bj}}^{2} F_{2n}^{bound}(\frac{x_{Bj}}{\alpha}, Q^2)\rho_d(\alpha)\frac{d\alpha}{\alpha}$$

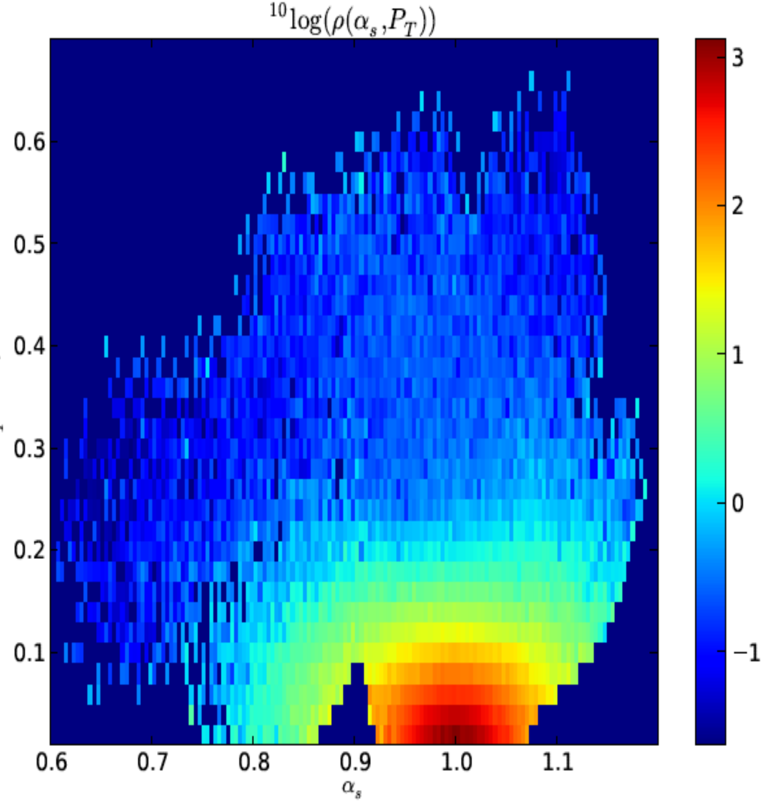
$$\rho_d(\alpha) = \int \rho(\alpha, p_T) d^2 p_T$$

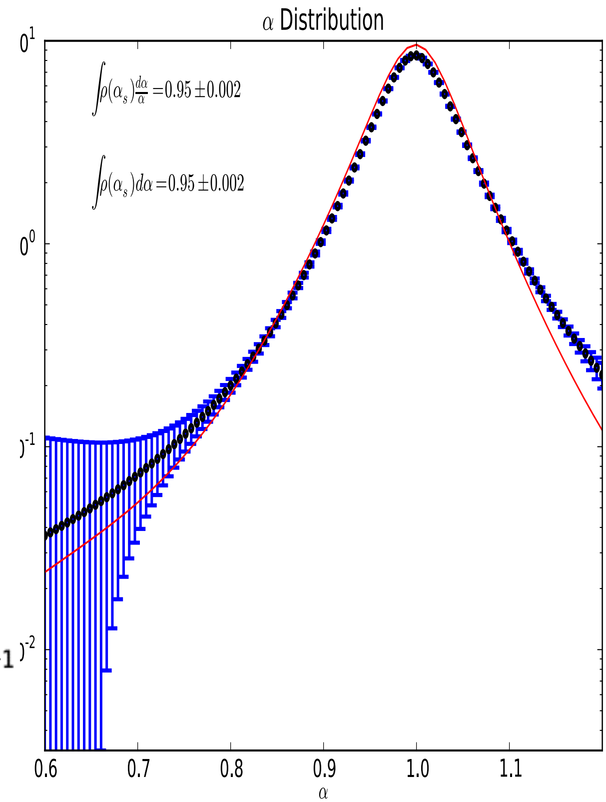
For d(e,e'p)n reaction:

$$\frac{d\sigma}{dEe' d\Omega_e d\Omega_s} = K\sigma_{eN}^{LC}(\alpha, p_t)\rho(\alpha, p_t),$$

For Inclusive D(e,e')X reaction:

$$\frac{d\sigma}{dE'_e d\Omega_e} = K\sum_N \sigma_{eN}\rho(\alpha)$$

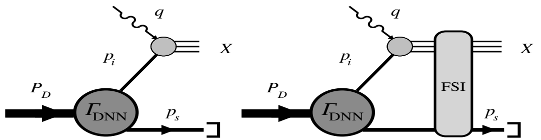


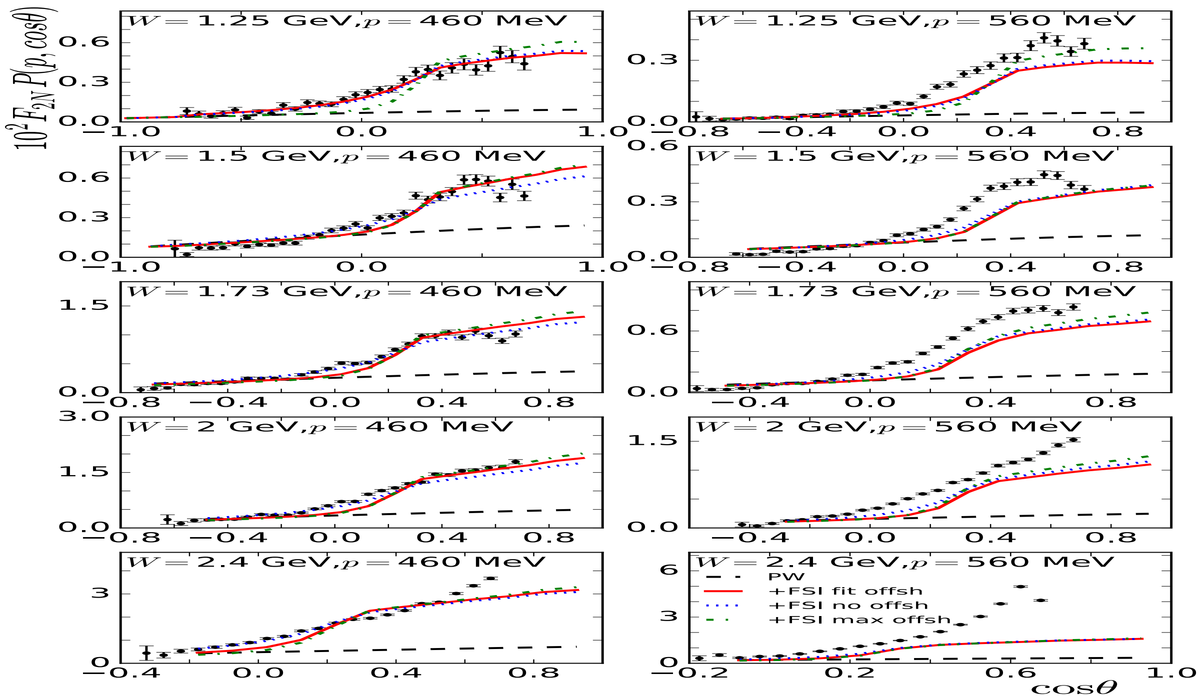


W.Boeglin & M.Sargsian, IJMP-E 2015

Extension to TDIS: $e + d \to e' + p_s + X$

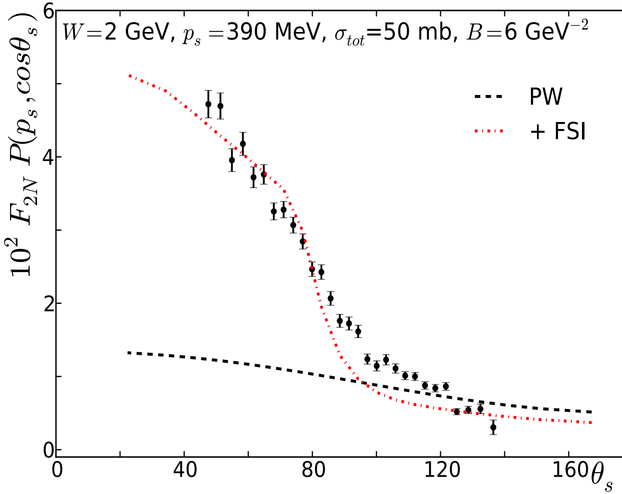
W.Cosyn & M.Sargsian, PRC 2011





W.Cosyn & M.Sargsian, PRC 2011

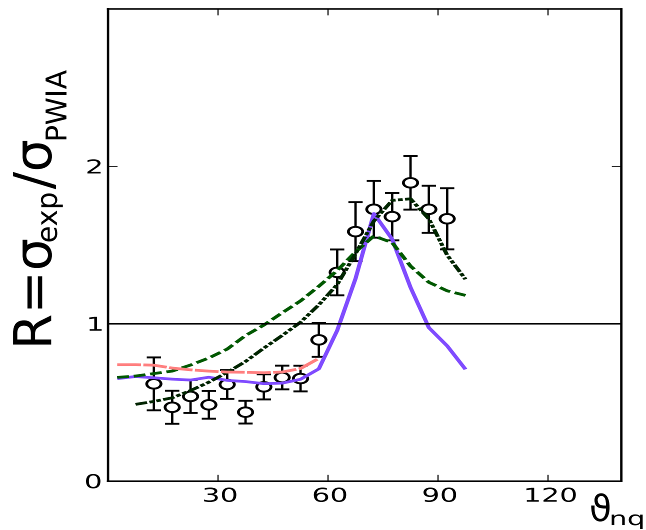
$$e + d \to e' + p_s + X$$



A.V. Klimenko et al PRC 2006

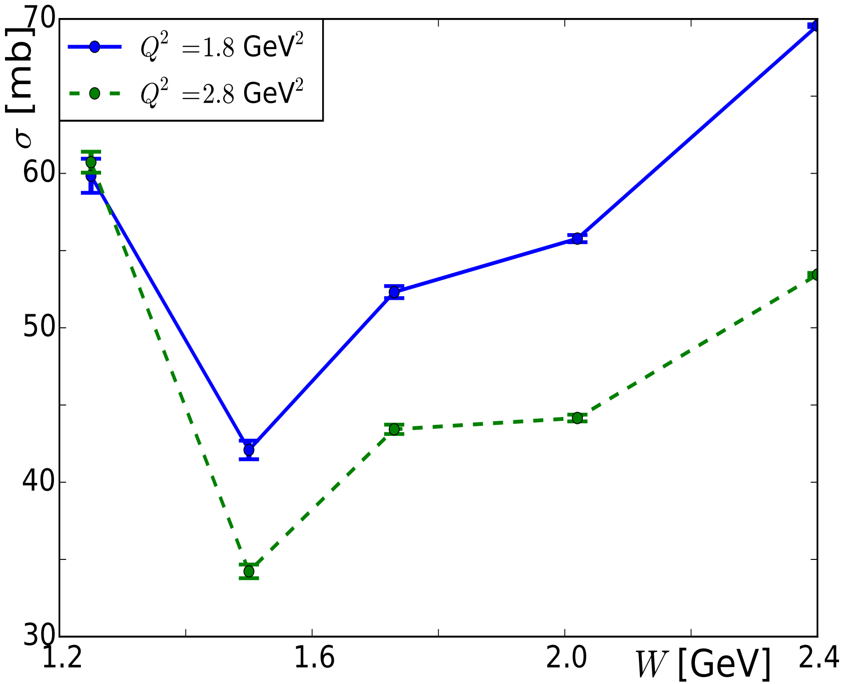
M.Sargsian, PRC 2010

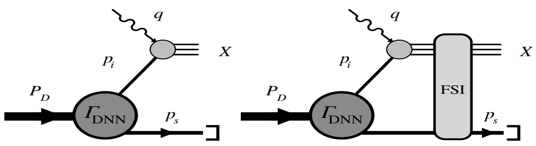
$$e + d \to e' + p + n$$



W. Boeglin et al PRL 2011

Hadronization Studies in $e + d \rightarrow e' + p_s + X$ TDIS:





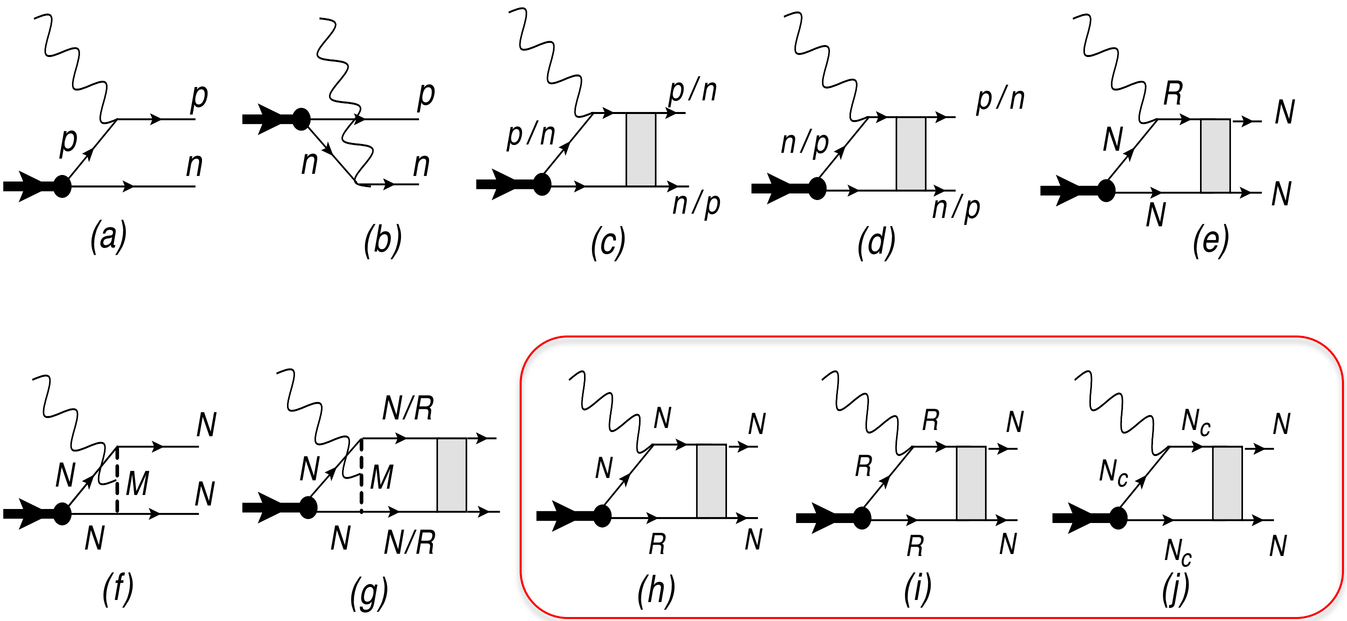
W.Cosyn & M.Sargsian, PRC 2011

W.Cosyn & M.Sargsian, IJMP 2017

Probing Deuteron at Core Distances

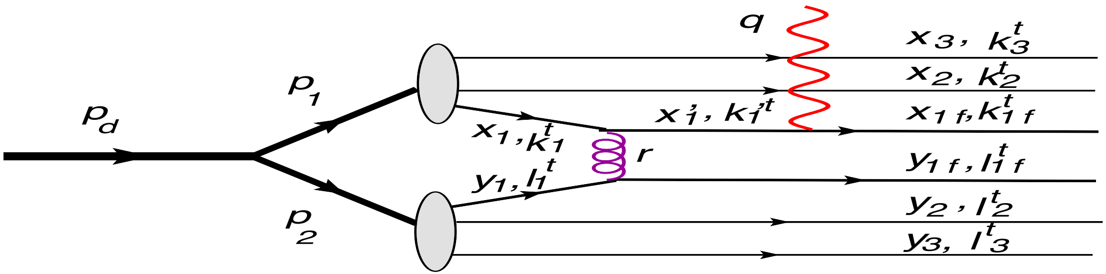
$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \cdots$$

Limitatations of QE Processes



Deep Inelastic Processes off the Deuteron

MS in progress



$$A^\sigma = -\int \frac{d^4 p_2}{i(2\pi)^4} \times \{$$

$$N1: \quad \bar{u}(k_3)\bar{u}(k_2)\bar{u}(k_{1f})e_i\gamma^\sigma \frac{\not{k}'_1 + m_q}{k'^2_1 + m_q^2 - i\epsilon}[gT_c^F\gamma^\mu]\frac{\not{k}_1 + m_q}{k_1^2 + m_q^2 - i\epsilon}\Gamma_N(k_1, k_2, k_3)$$

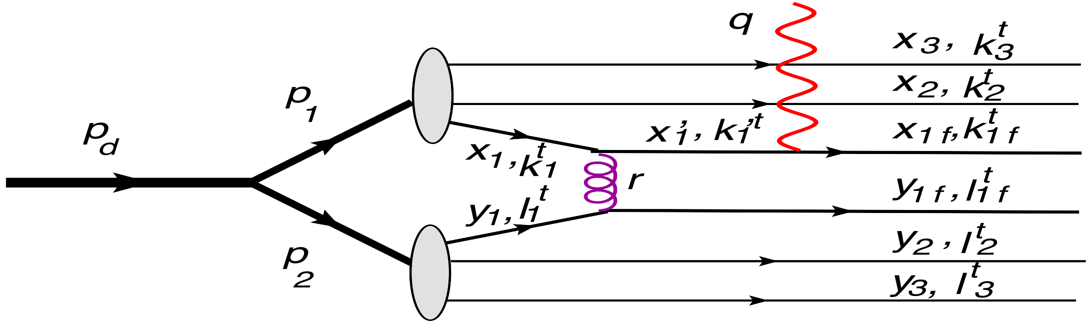
$$N2: \quad \bar{u}(l_3)\bar{u}(l_2)\bar{u}(l_{1f})[gT_c^F\gamma^\nu]\frac{\not{l}_1 + m_q}{l_1^2 + m_q^2 - i\epsilon}\Gamma_N(l_1, l_2, l_3)$$

$$g: \quad \frac{d_{\mu\nu}}{r^2 + i\epsilon}$$

$$d: \quad \frac{\not{p}_1 + m_N}{(p_d - p_2)^2 - m_N^2 + i\epsilon}\frac{\not{p}_2 + m_N}{(p_2)^2 - m_N^2 + i\epsilon}\Gamma_{dNN}\chi_d^m \}$$

Deep Inelastic Processes off the Deuteron

MS in progress

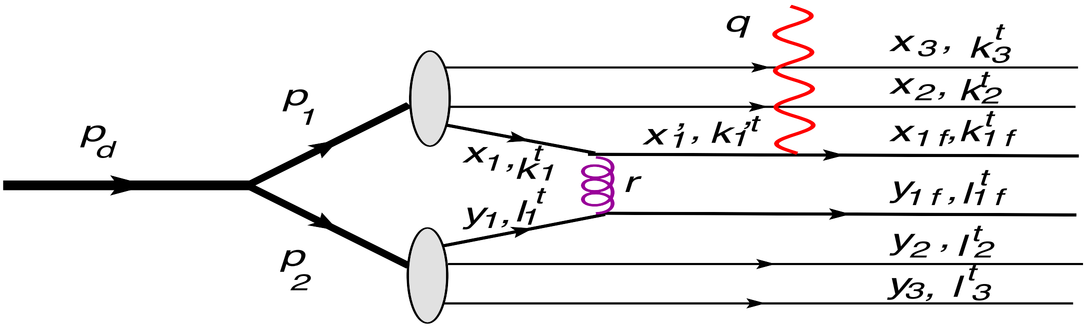


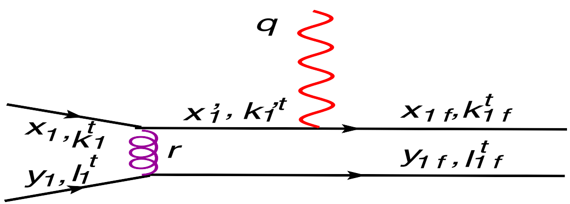
$$A^\sigma = \sum_{h_1,h_2} \int \frac{d\alpha}{\alpha} \frac{d^2 p_2}{2(2\pi)^3}$$

$$\left\{ \sum_{\eta_1,\lambda_1} H^\sigma_{(\eta_{1f},\eta_1),(\lambda_{1f},\lambda_1)} \frac{\psi_N^{h_1}(k_1,\eta_1;k_2,\eta_2;k_3,\eta_3)}{x_1\sqrt{2(2\pi)^3}} \frac{\psi_N^{h_2}(l_1,\lambda_1;l_2,\lambda_2;l_3,\lambda_3)}{y_1\sqrt{2(2\pi)^3}} \right\} \frac{\Psi_d^{h_1,h_2,m_d}(p_1,p_2)}{(1-\alpha)\sqrt{2(2\pi)^3}}$$

Amplitude of Hard Subprocess

MS in progress





$$H^{\sigma}_{(\eta_{1f},\eta_1),(\lambda_{1f},\lambda_1)} = g^2 \bar{u}_{\eta_{1f}}(k_{1f}) e_i \gamma^{\sigma} \frac{\not{k}_1' + m_q}{k_1'^2 - m_q^2 + i\epsilon} \gamma^{\mu} u_{\eta_1}(k_1) \bar{u}_{\lambda_{1f}}(l_{1f}) \gamma^{\nu} u_{\lambda_1}(l_1) \frac{d_{\mu\nu}}{r^2 + i\epsilon}$$

Deuteron Structure Function

- approximation: $\alpha \approx \frac{1}{2}$

$$H^{+}_{++;--} = -\frac{\sqrt{x_{1f}x_1}}{\sqrt{y_{1f}y_1}}\frac{1}{2}[x_1+y_1]\left[1-\frac{x_{1f}}{x_1+y_1}\right]^2\frac{8g^2e_i}{l^{t,2}_{1f}}\cdot p^+_d$$

$$A^+ = \qquad H^{+}_{(+,+);(-,-)}\sum_{h_1,h_2}\left\{\frac{\psi_N^{h_1}(k_1,+;k_2,\eta_2;k_3,\eta_3)}{x_1\sqrt{2(2\pi)^3}}\frac{\psi_N^{h_2}(l_1,-;l_2,\lambda_2;l_3,\lambda_3)}{y_1\sqrt{2(2\pi)^3}}\right\}$$

$$\times\int\frac{d\alpha}{\alpha}\frac{d^2p_2}{2(2\pi)^3}\frac{\Psi_d^{h_1,h_2,m_d}(p_1,p_2)}{(1-\alpha)\sqrt{2(2\pi)^3}}$$

$$W^{++}\equiv\frac{p^2_{d+}}{M_d^2}\frac{F_{2d}}{\nu} =$$

$$\frac{1}{4\pi M_d}\int[A^{\dagger,+}A^+](2\pi)^4\delta^4(q+p_d-k_{1f}-k_2-k_3-l_{1f}-l_2-l_3)\times$$

$$\frac{dx_{1f}}{x_{1f}}\frac{d^2k^t_{1f}}{2(2\pi)^3}\frac{dy_{1f}}{y_{1f}}\frac{d^2l^t_{1f}}{2(2\pi)^3}\prod_{i=2,3}\frac{dx_i}{x_i}\frac{d^2k^t_i}{2(2\pi)^3}\frac{dy_i}{y_i}\frac{d^2l^t_i}{2(2\pi)^3}$$

Deuteron Structure Function

- definition of PDF:

$$f_i(x) = \sum_{j=1}^{3} \int \frac{dx_j}{x_j} \frac{d^2k_{jt}}{2(2\pi)^3} \frac{dx_2}{x_2} \frac{d^2k_{2t}}{2(2\pi)^3} \frac{dx_3}{x_3} \frac{d^2k_{3t}}{2(2\pi)^3} \delta(1-x_j-x_2-x_3)\delta^2(p_t-k_{jt}-k_{2t}-k_{3t})\delta(x-x_j)\psi^2(k_j, k_2$$

$$F_{2d}(x_{Bj}, Q^2) = \sum_{i,j} x_{Bj} e_i^2 \int dx_1 dy_1 \frac{d^2 l_{1f,t}}{2(2\pi)^3} \frac{8\alpha_{QCD}}{l_{1f,t}^4} f_i(x_1, Q^2) f_j(y_1, l_{1f,t}^2) \times$$
$$\frac{1}{y_1^2}\left[1 - \frac{x_{Bj}}{x_1 + y_1}\right]^2 \Theta(x_1 + y_1 - x_{Bj}) \left[\sum_{h_1,h2} \int \frac{\Psi_d(\alpha, p_t)}{\alpha(1-\alpha)} \frac{d\alpha}{\sqrt{2(2\pi)^3}} \frac{d^2 p_t}{(2\pi)^2}\right]^2$$

where $x_{Bj} = \frac{Q^2}{2m_N\nu}$.

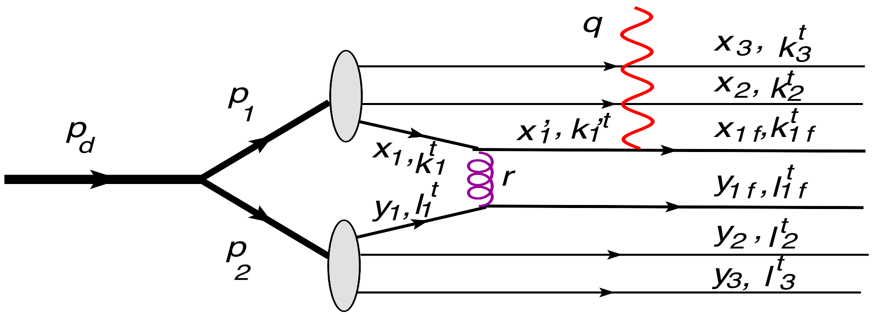
- Parametrically:

$$F_{2d}(x_{Bj}, Q^2) \sim \sum_{i,j} x_{Bj} e_i^2 \int dx_1 dy_1 f_i(x_1, Q^2) f_j(y_1, l_{1f,t}^2) \frac{1}{y_1^2}\left[1 - \frac{x_{Bj}}{x_1 + y_1}\right]^2 \Theta(x_1 + y_1 - x_{Bj})$$

Deuteron Structure Function

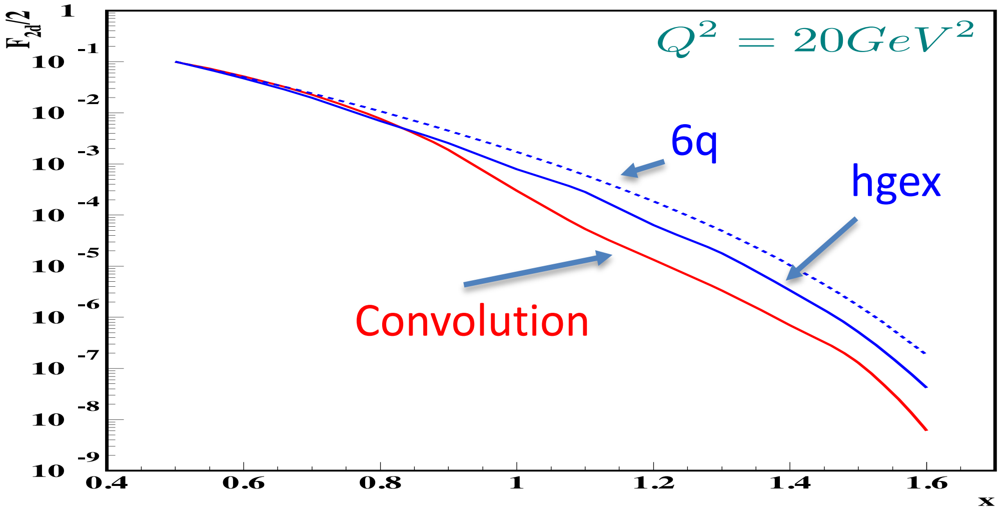
Why Double Partonic Distributions?

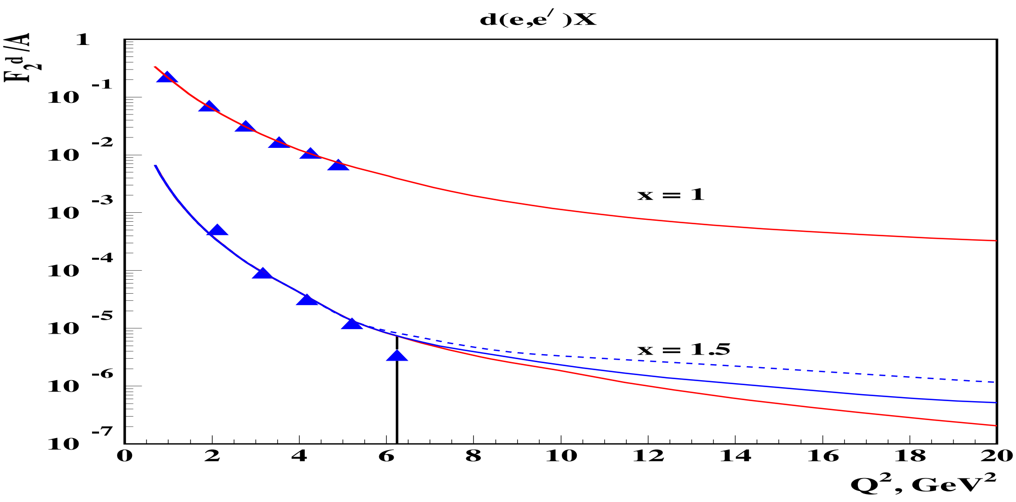
$$F_{2d}(x_{Bj},Q^2) \sim \sum_{i,j} x_{Bj} e_i^2 \int dx_1 dy_1 f_i(x_1,Q^2) f_j(y_1,l^2_{1f,t}) \frac{1}{y_1^2}\left[1-\frac{x_{Bj}}{x_1+y_1}\right]^2 \Theta(x_1+y_1-x_B$$



Repulsive core effect is due to:

$$f_i(x,Q^2)\ |_{x\to 1} \ \longrightarrow\ 0$$


$F_{2d}/2$
$Q^2 = 20GeV^2$
6q
hgex
Convolution
x
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{-8}
10^{-9}
0.4
0.6
0.8
1
1.2
1.4
1.6

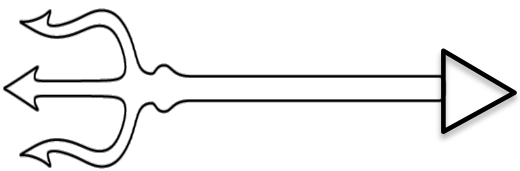

d(e,e′)X
F_2^d/A
x = 1
x = 1.5
Q^2, GeV^2
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
0
2
4
6
8
10
12
14
16
18
20

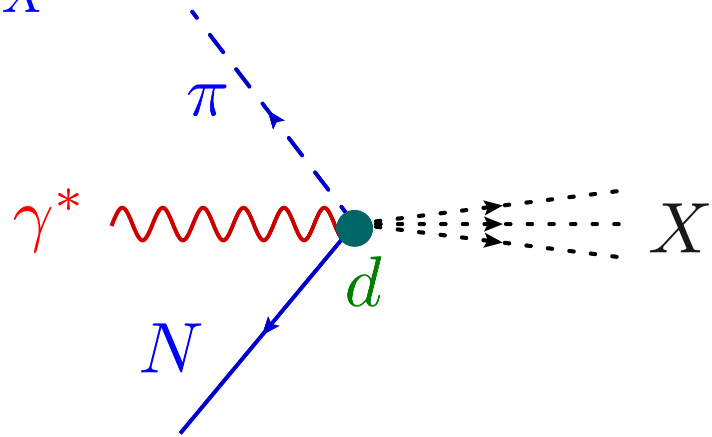
Probing Deuteron at Core Distances at large Q²

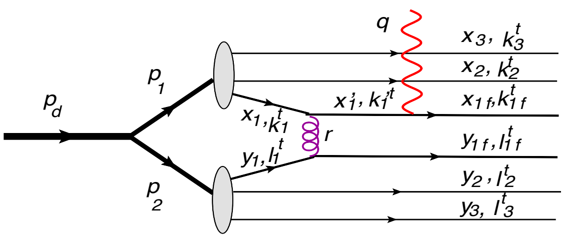
$$\Psi_d = \Psi_{pn} + \Psi_{\Delta\Delta} + \Psi_{NN^*} + \Psi_{hc} \cdots$$

$$e + d \to e' + \Delta_{backward} + X$$

$$e + d \to e' + N^*_{backward} + X$$







Some Outlook

- Deuteron deserves a special status for studies of the almost all issues relevant to contemporary high energy nuclear physics

- Quasielastic Deuteron Break-Up at extreme large missing momenta

- Tagged DIS off the deuteron

- Inclusive DIS Deuteron at $x>1$,

- Baryon + Meson Production in the target Fragmentation region