

Deeply Virtual Compton Scattering off He nuclei

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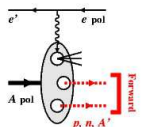
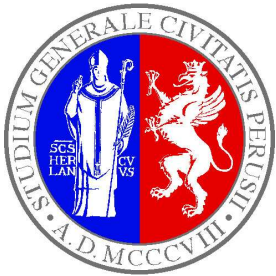
in collaboration with



Sara Fucini – Università di Perugia and INFN, Perugia, Italy

Matteo Rinaldi – Università di Perugia and INFN, Perugia, Italy

Michele Viviani – INFN, Pisa, Italy



Outline

The nucleus: *“a Lab for QCD fundamental studies”*

Realistic calculations: use of exact wave functions, solutions of the Schrödinger equation, with realistic NN potentials (Av18, Nijmegen, CD Bonn) and 3-body forces

GPDs of light nuclei (deuteron aside):

1 - GPDs for ^3He :

A complete impulse approximation realistic study is reviewed
(S.S. PRC 2004, PRC 2009; M. Rinaldi and S.S., PRC 2012, PRC 2013)
No data; proposals? Prospects at JLAB-12 and EIC;

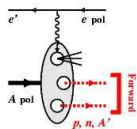
2 - DVCS off ^4He :

data available from JLab at 6 GeV; new data expected at 12 GeV;
our calculations (not yet realistic)

(**Coherent:** S. Fucini, S.S., M. Viviani, Phys.Rev. C98 (2018) no.1, 015203)

(**Incoherent:** S. Fucini, S.S., M. Viviani, arXiv:1909.12261 [nucl-th]) .

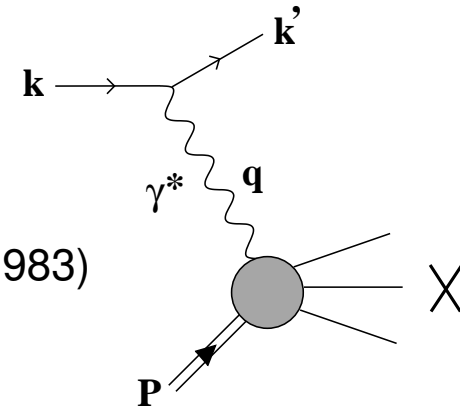
My point: *I do not know if realistic calculations will describe the data. I think they are necessary to distinguish effects due to “conventional” or to “exotic” nuclear structure*








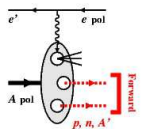
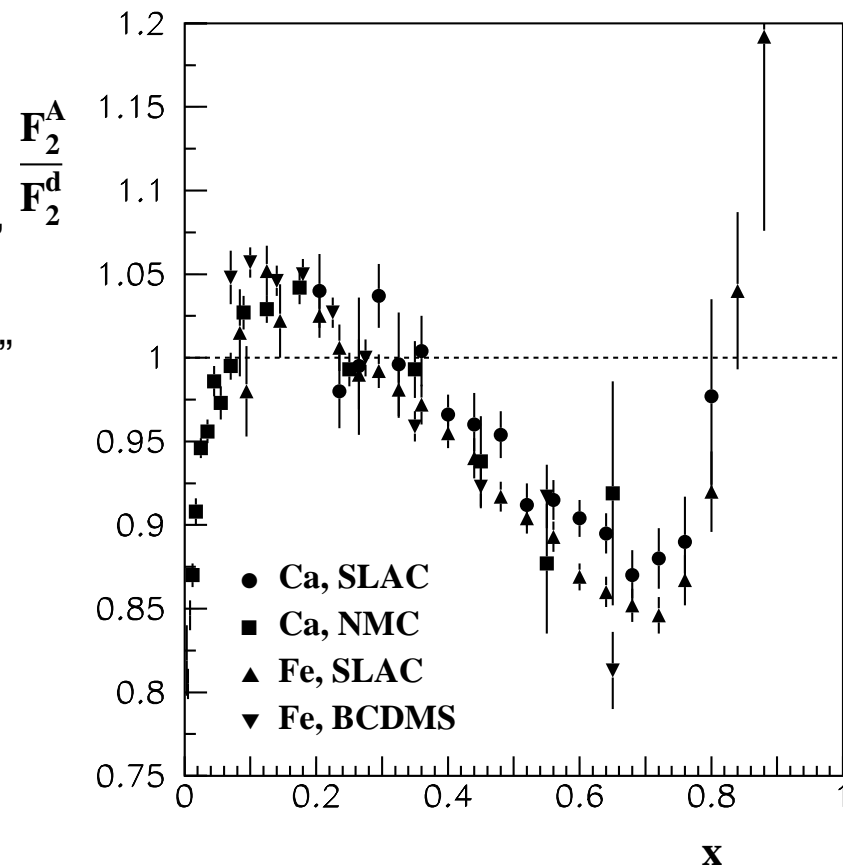
EMC effect in A-DIS

Measured in $A(e, e')X$, ratio of A to d SFs F_2 (EMC Coll., 1983)

One has $0 \leq x = \frac{Q^2}{2M\nu} \leq \frac{M_A}{M} \simeq A$



-  $x \leq 0.1$ "Shadowing region"
-  $0.1 \leq x \leq 0.2$ "Enhancement region"
-  $0.2 \leq x \leq 0.8$ "EMC (binding) region"
-  $0.8 \leq x \leq 1$ "Fermi motion region"
-  $x \geq 1$ "TERRA INCOGNITA"



EMC effect: explanations?

Situation: basically not understood. Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

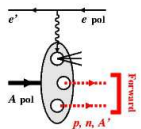
- the knowledge of nuclear parton distributions is crucial for the data analysis of heavy ions collisions;
- the partonic structure of the neutron is measured with nuclear targets and several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

Inclusive measurements cannot distinguish between models

One has to go beyond

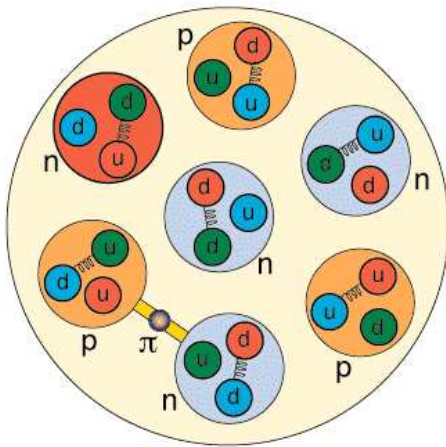
(R. Dupré and S.S., EPJA 52 (2016) 159)

- **SIDIS (TMDs)** - not treated here
- **Hard Exclusive Processes (GPDs)**



EMC effect: way out?

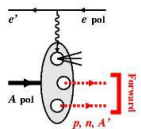
Question: Which of these transverse sections is more similar to that of a nucleus?



To answer, we should perform a *tomography...*

We can! M. Burkardt, PRD 62 (2000) 07153

Answer: Deeply Virtual Compton Scattering
& Generalized Parton Distributions (GPDs)



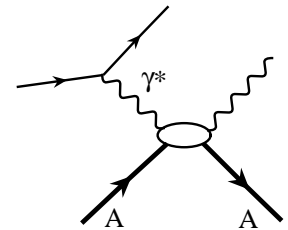
GPDs: a unique tool...

- not only **3D** structure, at **parton level**; many other aspects, e.g., contribution to the solution to the “**Spin Crisis**” (J.Ashman et al., **EMC collaboration**, **PLB 206, 364 (1988)**), yielding parton total angular momentum...

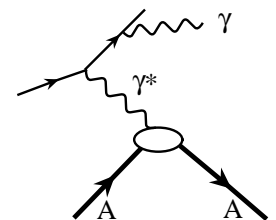
... but also an experimental challenge:

- Hard exclusive process \rightarrow small σ ;

- Difficult extraction:



DVCS



BH

$$T_{\text{DVCS}} \propto CFF \propto \int_{-1}^1 dx \frac{H_q(x, \xi, \Delta^2)}{x - \xi + i\epsilon} + \dots$$

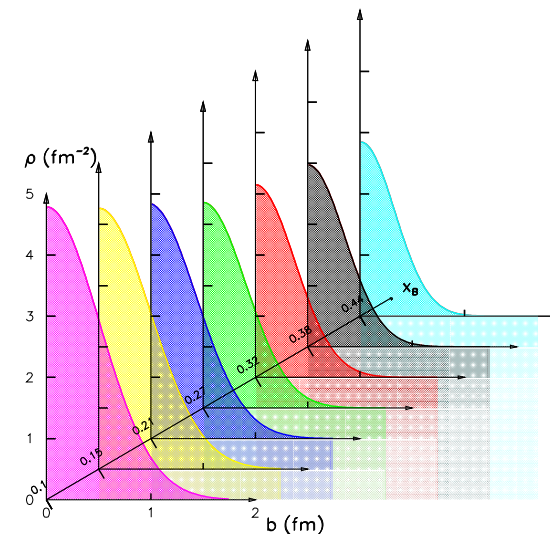
- Competition with the **BH** process! (σ asymmetries measured).

$$d\sigma \propto |T_{\text{DVCS}}|^2 + |T_{\text{BH}}|^2 + 2 \Re\{T_{\text{DVCS}} T_{\text{BH}}^*\}$$

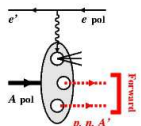
Nevertheless, for the proton, we have results:

(Guidal et al., **Rep. Prog. Phys.** 2013...

Dupré, Guidal, Niccolai, Vanderhaeghen Eur.Phys.J. A53 (2017) 171)



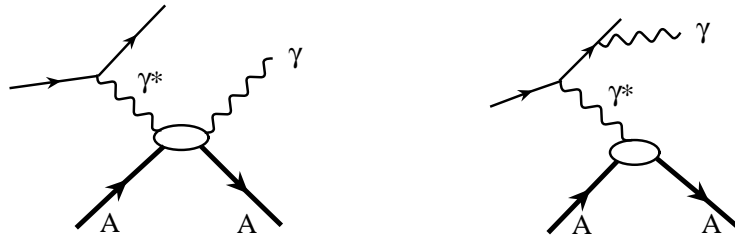
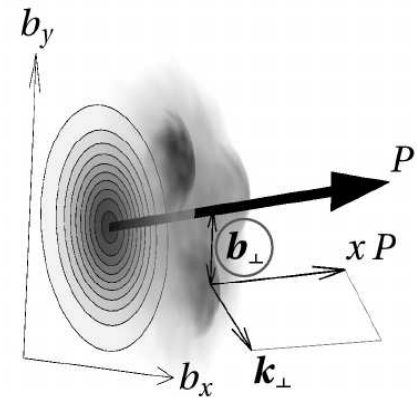
Deeply Virtual Compton Scattering off He nuclei – p.6



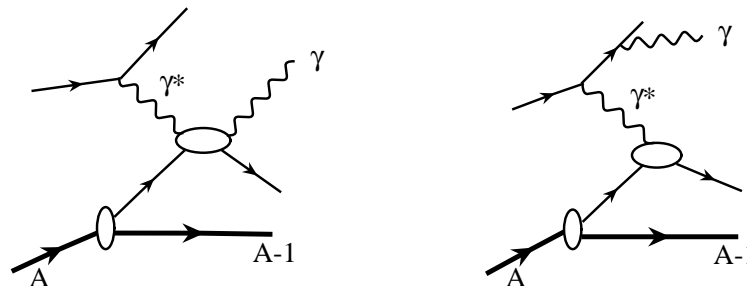
Nuclei and DVCS tomography

In impact parameter space, GPDs are *densities*:

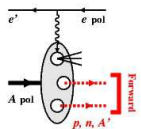
$$\rho_q(x, \vec{b}_\perp) = \int \frac{d\vec{\Delta}_\perp}{(2\pi)^2} e^{i\vec{b}_\perp \cdot \vec{\Delta}_\perp} H^q(x, 0, \Delta^2)$$



Coherent DVCS: nuclear tomography

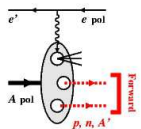
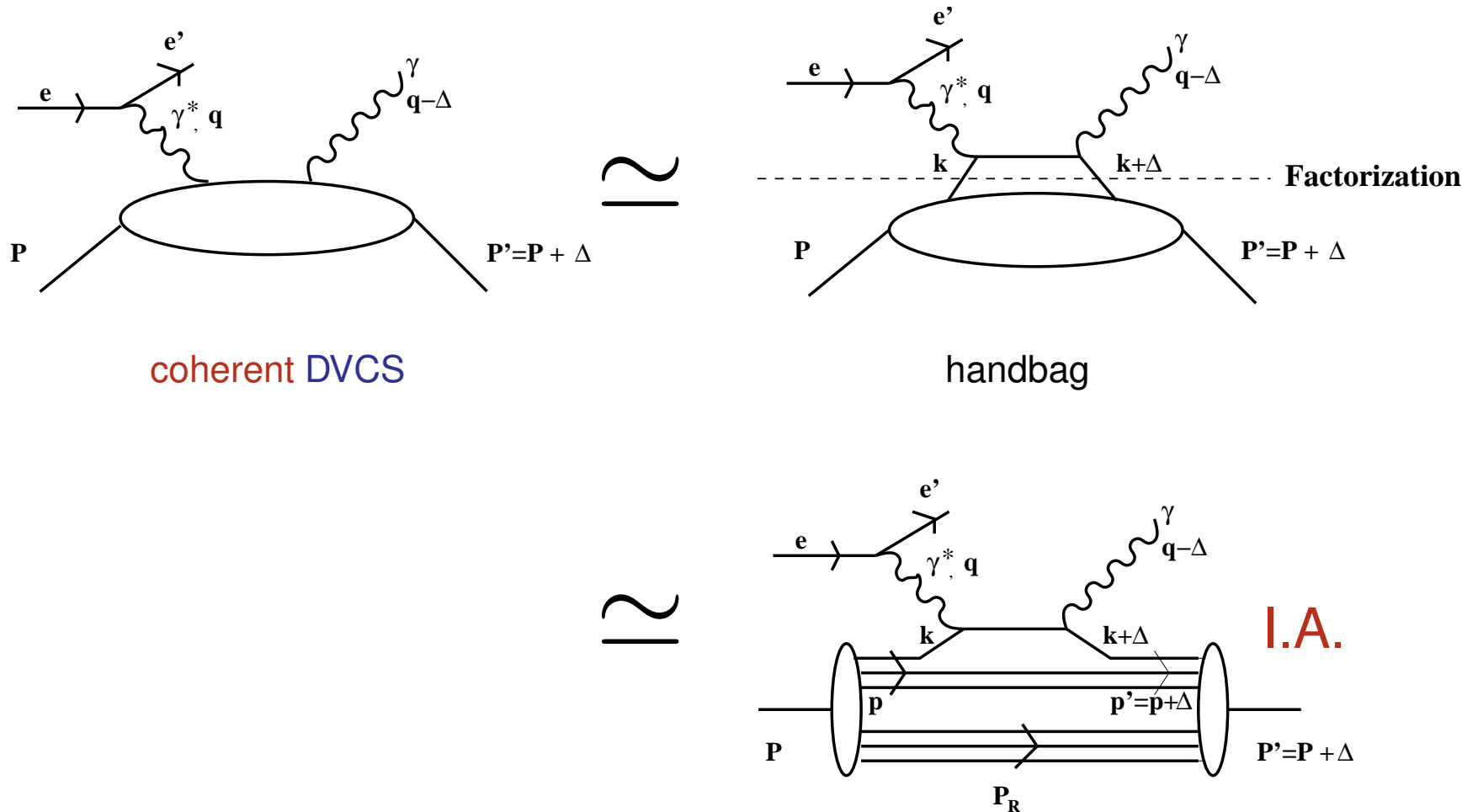


Incoherent DVCS: tomography of bound nucleons: realization of the EMC effect



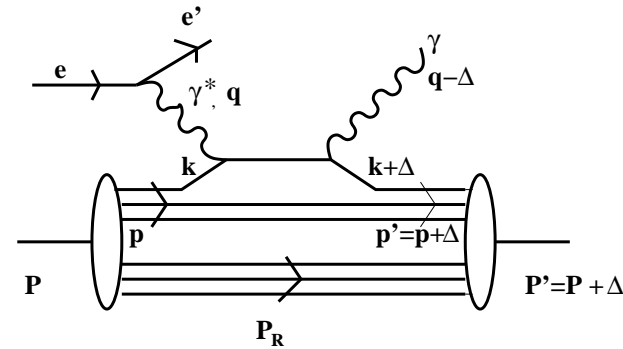
Nuclei: why? - not only tomography

ONE of the reasons is understood by studying coherent DVCS in the I.A. to the handbag contribution:



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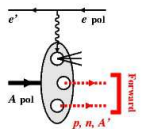


In a symmetric frame ($\bar{p} = (p + p')/2$) :

$$\begin{aligned} k^+ &= (x + \xi)\bar{P}^+ = (x' + \xi')\bar{p}^+ , \\ (k + \Delta)^+ &= (x - \xi)\bar{P}^+ = (x' - \xi')\bar{p}^+ , \end{aligned}$$

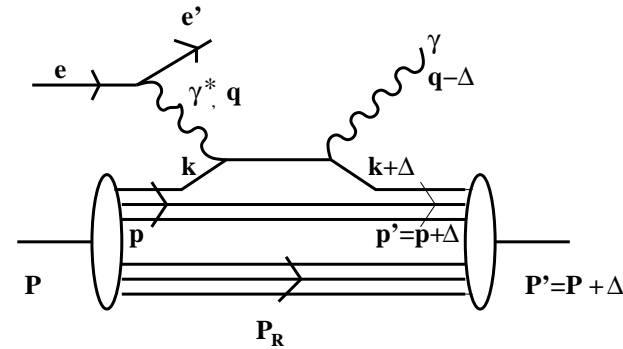
one has, for a given GPD

$$GPD_q(x, \xi, \Delta^2) \simeq \int \frac{dz^-}{4\pi} e^{ix\bar{P}^+ z^-} {}_A \langle P' S' | \hat{O}_q^+ | PS \rangle_A |_{z^+=0, z_\perp=0} .$$



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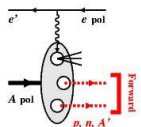
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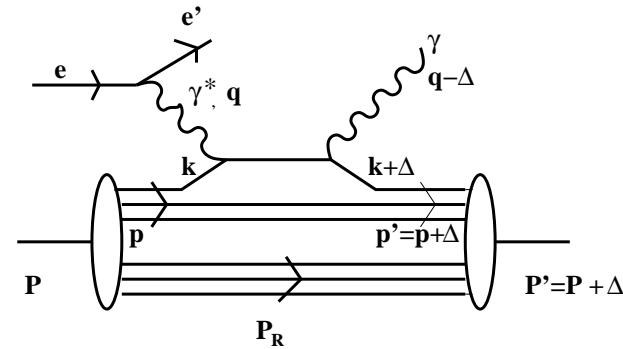
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By properly inserting complete sets of states for the interacting nucleon and the recoiling system :



Nuclei: why? - not only tomography

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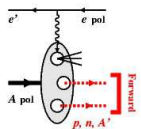
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one has, for a given GPD

$$\begin{aligned} GPD_q(x, \xi, \Delta^2) &= \int \frac{dz^-}{4\pi} e^{ix'\bar{p}^+ z^-} \langle P'S' | \sum_{\vec{P}'_R, S'_R, \vec{p}', s'} \{ |P'_R S'_R\rangle | p' s'\rangle \} \langle P'_R S'_R | \\ &\quad \langle p' s' | \hat{O}_q^+ \sum_{\vec{P}_R, S_R, \vec{p}, s} \{ |P_R S_R\rangle | ps\rangle \} \{ \langle P_R S_R | \langle ps | \} | PS \rangle , \end{aligned}$$

and, since $\{ \langle P_R S_R | \langle ps | \} | PS \rangle = \langle P_R S_R, ps | PS \rangle (2\pi)^3 \delta^3(\vec{P} - \vec{P}_R - \vec{p}) \delta_{S, S_R} \delta_{s, s_R}$,



Why nuclei?

a convolution formula can be obtained (S.S. PRC 70, 015205 (2004)):

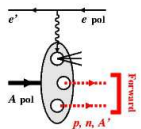
$$H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int \frac{d\bar{z}}{\bar{z}} h_N^A(\bar{z}, \xi, \Delta^2) H_q^N\left(\frac{x}{\bar{z}}, \frac{\xi}{\bar{z}}, \Delta^2\right)$$

in terms of $H_q^N(x', \xi', \Delta^2)$, the GPD of the free nucleon N , and of the light-cone off-diagonal momentum distribution:

$$h_N^A(z, \xi, \Delta^2) = \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(\bar{z} - \frac{\bar{p}^+}{\bar{P}^+}\right)$$

where $P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E)$, is the one-body off-diagonal spectral function for the nucleon N in the nucleus,

$$\begin{aligned} P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) &= \frac{1}{(2\pi)^3} \frac{1}{2} \sum_M \sum_{R,s} \langle \vec{P}' M | (\vec{P} - \vec{p}) S_R, (\vec{p} + \vec{\Delta}) s \rangle \\ &\times \langle (\vec{P} - \vec{p}) S_R, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_R^*) . \end{aligned}$$



Why nuclei?

The obtained expressions have the correct **limits**:

● the **x-integral** gives the f.f. $F_q^A(\Delta^2)$ in **I.A.**:

$$\int dx H_q^A(x, \xi, \Delta^2) = F_q^N(\Delta^2) \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) = F_q^A(\Delta^2)$$

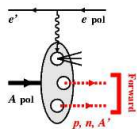
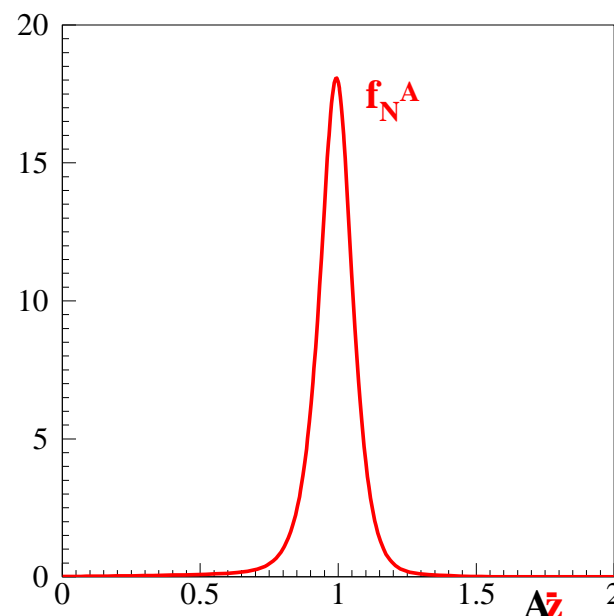
● **forward limit** (standard DIS):

$$q^A(x) \simeq \sum_N \int_x^1 \frac{d\tilde{z}}{\tilde{z}} f_N^A(\tilde{z}) q^N\left(\frac{x}{\tilde{z}}\right)$$

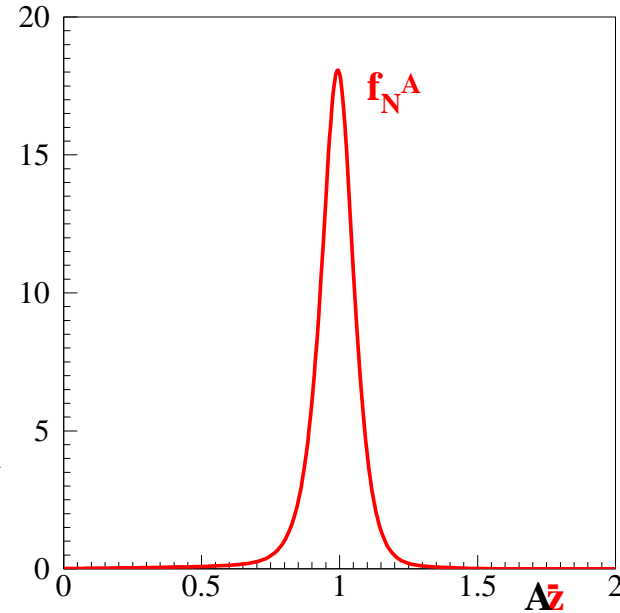
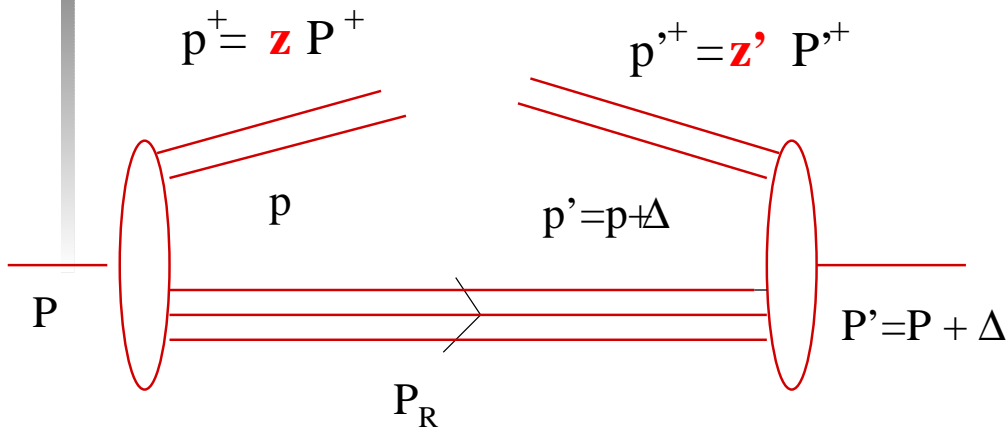
with the **light-cone momentum distribution**:

$$f_N^A(\tilde{z}) = \int dE d\vec{p} P_N^A(\vec{p}, E) \delta\left(\tilde{z} - \frac{p^+}{P^+}\right),$$

which is strongly peaked around $A\tilde{z} = 1$:



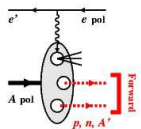
Why nuclei?



Since $z - z' = -x_B(1 - z)/(1 - x_B)$, $\xi \simeq x_B/(2 - x_B)$ can be tuned to have $z - z'$ larger than the width of the narrow nuclear light-cone momentum distribution $f_N^A(\bar{z} = (z + z')/2)$: in this case IA predicts a *vanishing* GPD, **at small x_B** .

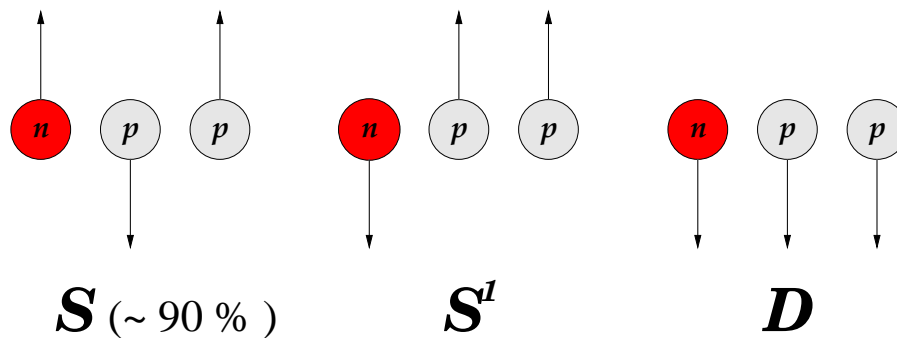
If DVCS were observed at this kinematics, **exotic** effects beyond IA, **non-nucleonic degrees of freedom**, would be pointed out (Berger, Cano, Diehl and Pire, PRL 87 (2001) 142302)

Similar effect predicted in DIS at $x_B > 1$, where DIS data are not accurate enough.



GPDs for ^3He : why?

- ^3He is **theoretically well known**. Even a **relativistic treatment** may be implemented.
- ^3He has been used extensively as an **effective neutron target**, especially to unveil the **spin content** of the **free neutron**, due to its peculiar spin structure:

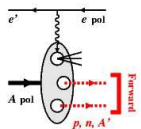


In S -wave
 $^3\vec{H}e = \vec{n}!$

^3He always **promising** when the **neutron angular momentum properties** have to be studied. To what extent for total J ?

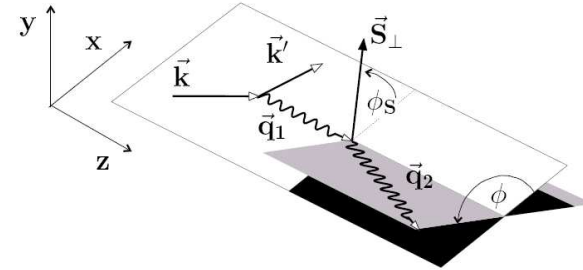
- ^3He is a **unique** target for GPDs studies. Examples:

- * access to the neutron information in coherent processes
- * HERE: heavier targets do not allow refined theoretical treatments. Test of the theory
- * HERE: Between ^2H (“not a nucleus”) and ^4He (a true one). Not isoscalar!



Extracting GPDs: ${}^3\text{He} \simeq p$

One measures asymmetries: $A = \frac{\sigma^+ - \sigma^-}{\sigma^+ + \sigma^-}$



● Polarized beam, unpolarized target:

$$\Delta\sigma_{LU} \simeq \sin\phi \left[F_1 \mathcal{H} + \xi(F_1 + F_2) \tilde{\mathcal{H}} + (\Delta^2 F_2 / M^2) \mathcal{E} / 4 \right] d\phi \quad \Rightarrow \quad H$$

● Unpolarized beam, longitudinally polarized target:

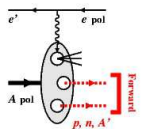
$$\Delta\sigma_{UL} \simeq \sin\phi \left\{ F_1 \tilde{\mathcal{H}} + \xi(F_1 + F_2) [\mathcal{H} + \xi / (1 + \xi) \mathcal{E}] \right\} d\phi \quad \Rightarrow \quad \tilde{H}$$

● Unpolarized beam, transversely polarized target:

$$\Delta\sigma_{UT} \simeq \cos\phi \sin(\phi_S - \phi) \left[\Delta^2 (F_2 \mathcal{H} - F_1 \mathcal{E}) / M^2 \right] d\phi \quad \Rightarrow \quad E$$

To evaluate cross sections, e.g. for experiments planning, one needs H, \tilde{H}, E

This is what we have calculated for ${}^3\text{He}$. H alone, already very interesting.



GPDs of ^3He in IA

H_q^A can be obtained in terms of H_q^N (S.S. PRC 70, 015205 (2004), PRC 79, 025207 (2009)):

$$H_q^A(x, \xi, \Delta^2) = \sum_N \int dE \int d\vec{p} \sum_S \sum_s P_{SS,ss}^N(\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} H_q^N(x', \Delta^2, \xi'),$$

and $\tilde{G}_M^{3,q}$ in terms of $\tilde{G}_M^{N,q}$ (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

$$\tilde{G}_M^{3,q}(x, \Delta^2, \xi) = \sum_N \int dE \int d\vec{p} \left[P_{+-,+-}^N - P_{+-,-+}^N \right] (\vec{p}, \vec{p}', E) \frac{\xi'}{\xi} \tilde{G}_M^{N,q}(x', \Delta^2, \xi'),$$

($\tilde{G}_M^q = H^q + E^q$) where $P_{SS,ss}^N(\vec{p}, \vec{p}', E)$ is the one-body, spin-dependent, off-diagonal spectral function for the nucleon N in the nucleus,

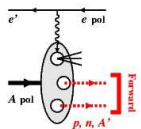
$$P_{SS',ss'}^N(\vec{p}, \vec{p}', E) = \frac{1}{(2\pi)^6} \frac{M\sqrt{ME}}{2} \int d\Omega_t \sum_{s_t} \langle \vec{P}' S' | \vec{p}' s', \vec{t}_{st} \rangle_N \langle \vec{p} s, \vec{t}_{st} | \vec{P} S \rangle_N,$$

evaluated by means of a **realistic** treatment based on **Av18 wave functions**

(“CHH” method in A. Kievsky *et al* NPA 577, 511 (1994); Av18 + UIX overlaps in E. Pace *et. al*, PRC 64, 055203 (2001)).

Nucleon GPDs in ^3He calculations given by an old version of the VGG model

(VGG 1999, x – and Δ^2 – dependencies factorized)



Nucleon off-shellness in I.A. :

In the forward limit $f_N^A(\tilde{z}) = \int dE d\vec{p} P_N^A(\vec{p}, E) \delta\left(\tilde{z} - \frac{p^+}{P^+}\right)$,

$$P_N^A(\vec{p}, E) = \sum_f \left| \begin{array}{c} \text{Diagram: } ^3\text{He} \text{ nucleus with incoming } \vec{p} \text{ and } E, \text{ and outgoing } \vec{p}_f, E_f^* \end{array} \right|^2$$

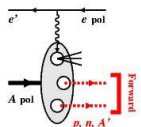
intrinsic overlaps

$$\sum_f \delta(E - E_{min} - E_f^*) \underbrace{S_A \langle \Psi_A; J_A \mathcal{M} \pi_A | \vec{p}, \sigma; \phi_f(E_f^*) \rangle}_{\text{intrinsic overlaps}} \underbrace{\langle \phi_f(E_f^*); \sigma \vec{p} | \pi_A J_A \mathcal{M}'; \Psi_A \rangle_{S_A}}_{\text{intrinsic overlaps}}$$

$$\tilde{z} = \frac{p_0 - p_3}{M_A} \quad p_0 = M_A - \sqrt{M_{A-1}^{*2} + p^2} \simeq M - E - T_f \longrightarrow p^2 \neq M^2$$

“Instant-Form” I.A.:

- off-shellness driven by nuclear dynamics
(all NN correlations included in the realistic wf)
- number SR fulfilled; momentum SR violated by 2 %



The calculation has the correct limits:

1 - Forward limit: the ratio:

$$R_q(x, 0, 0) = \frac{H_q^3(x, 0, 0)}{2H_q^p(x, 0, 0) + H_q^n(x, 0, 0)}$$

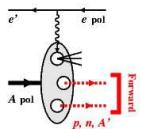
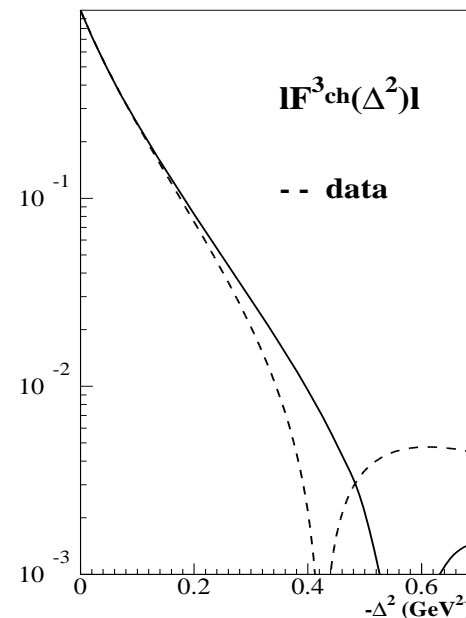
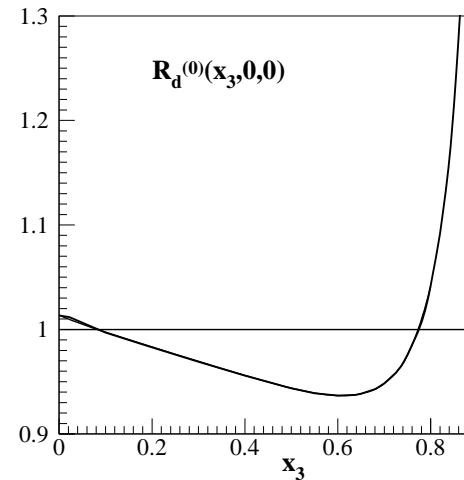
$$= \frac{q^3(x)}{2q^p(x) + q^n(x)}$$

shows an EMC-like behavior;

2 - Charge F.F.:

$$\sum_q e_q \int dx H_q^3(x, \xi, \Delta^2) = F^3(\Delta^2)$$

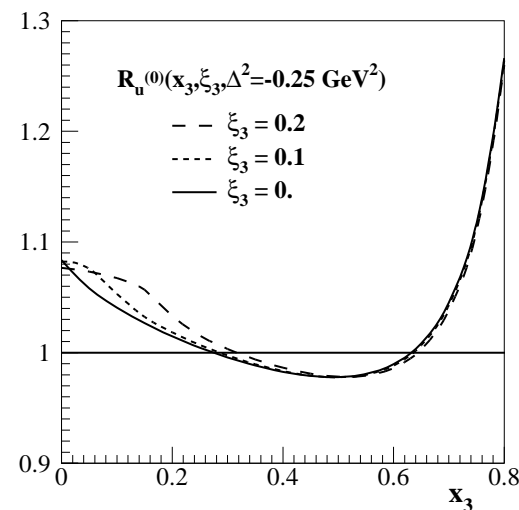
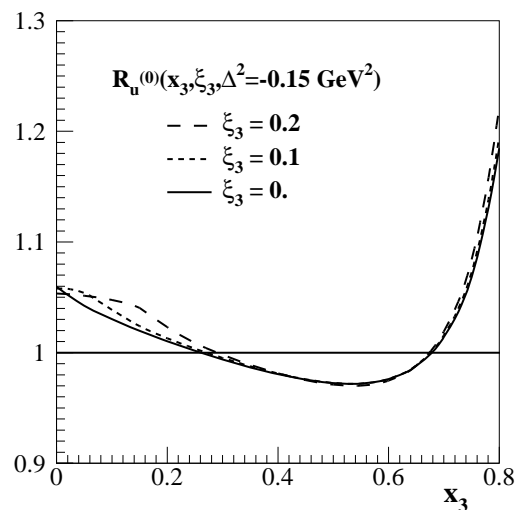
in good agreement with data in the region relevant to the coherent process, $-\Delta^2 \leq 0.2 \text{ GeV}^2$.



Nuclear effects - general features



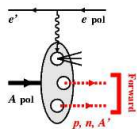
Nuclear effects grow with ξ at fixed Δ^2 , and with Δ^2 at fixed ξ :



$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2)}$$

$$H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2)$$

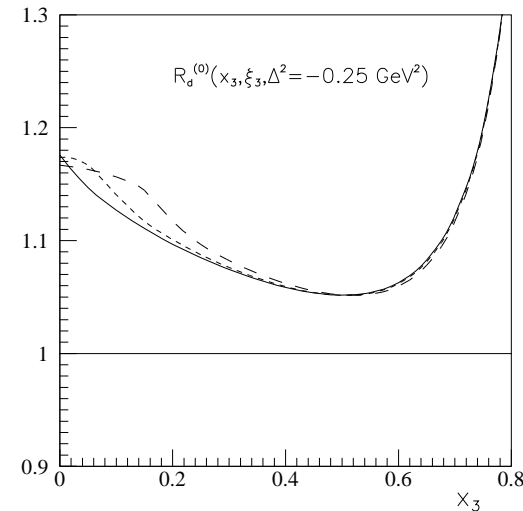
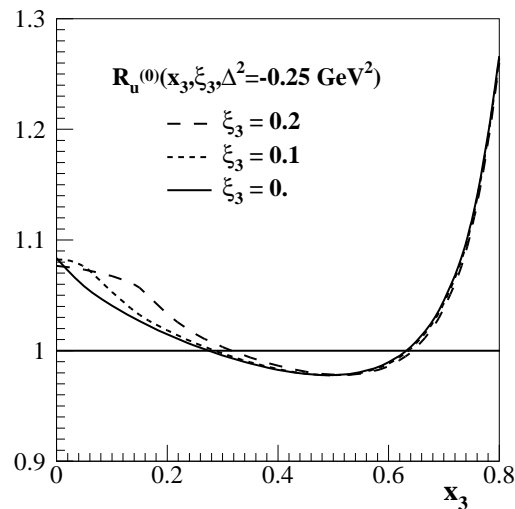
$R_q^{(0)}(x, \xi, \Delta^2)$ would be one if there were no nuclear effects;
as it is found also for the deuteron, there is **no factorization** into terms
dependent separately on Δ^2 and x, ξ (the factorization hypotheses has been
used to estimate nuclear **GPDs**), even if the nucleonic model is factorized



Nuclear effects - flavor dependence



Nuclear effects are bigger for the d flavor rather than for the u flavor:



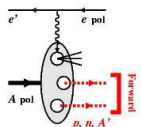
$$R_q^{(0)}(x, \xi, \Delta^2) = \frac{H_q^3(x, \xi, \Delta^2)}{2H_q^{3,p}(x, \xi, \Delta^2) + H_q^{3,n}(x, \xi, \Delta^2)}$$

$$H_q^{3,N}(x, \xi, \Delta^2) = \tilde{H}_q^N(x, \xi) F_q^3(\Delta^2)$$

$R_q^{(0)}(x, \xi, \Delta^2)$ would be one if there were no nuclear effects;

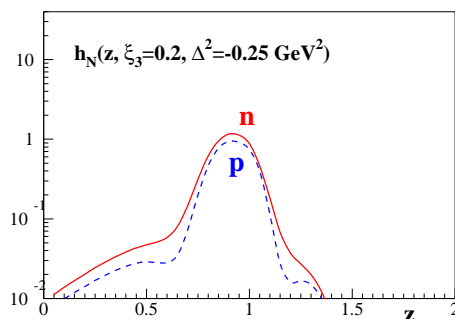
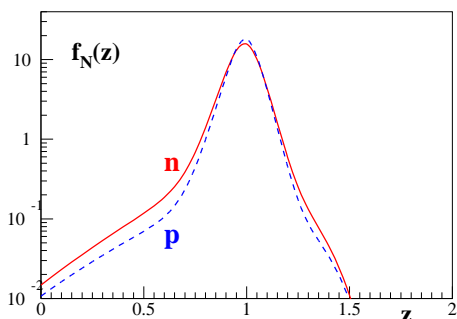
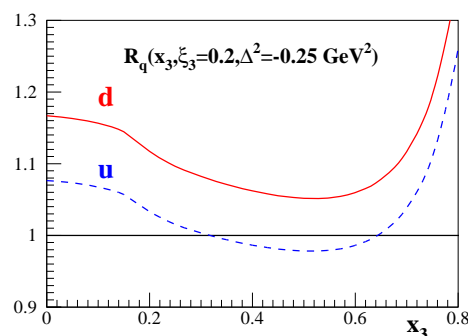
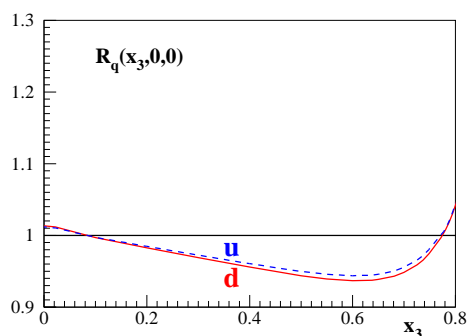


This is a typical **conventional, IA** effect (spectral functions are different for p and n in ${}^3\text{He}$, not isoscalar!); if (not) found, clear indication on the reaction mechanism of **DIS off nuclei**. Not seen in ${}^2\text{H}$, ${}^4\text{He}$



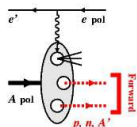
Nuclear effects - flavor dependence

- The **d** and **u** distributions follow the pattern of the **neutron** and **proton** light-cone momentum distributions, respectively:



- How to perform a flavor separation? Take **the triton** ${}^3\text{H}$!
Possible (see MARATHON@JLab). Possible for DVCS (ALERT).
Studied in **S.S. Phys. Rev. C 79 (2009) 025207**

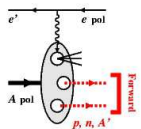
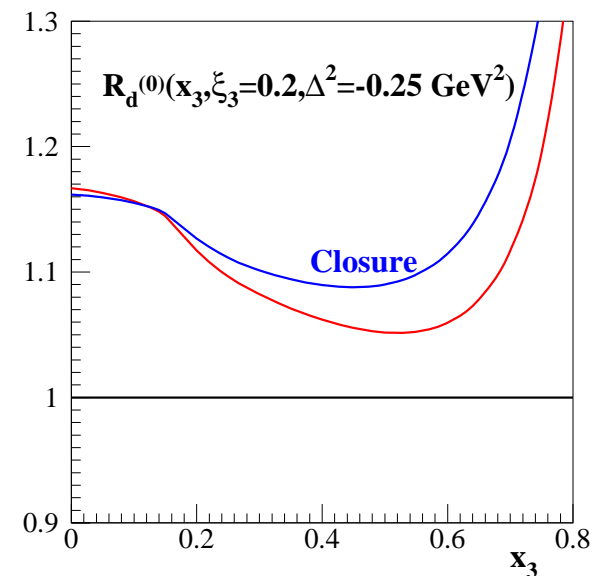
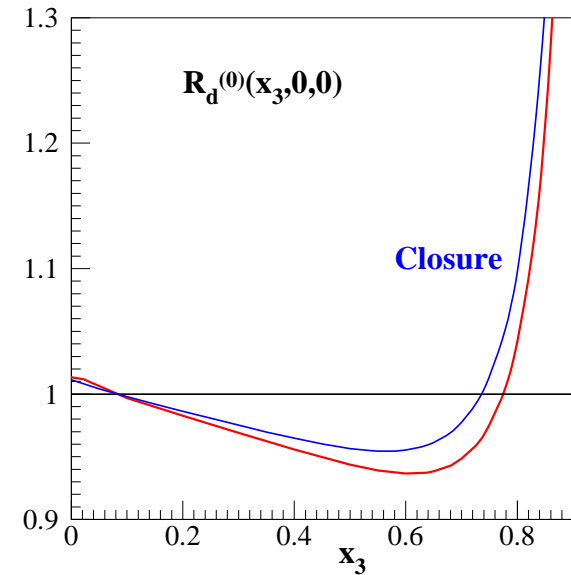
$$H_t, H_H \rightarrow H_u^H \simeq H_d^t, H_d^H \simeq H_u^t \text{ in the valence region...}$$



Nuclear effects - the binding

Nuclear effects are bigger than in the forward case:
dependence on the binding

- In calculations using $n(\vec{p}, \vec{p} + \vec{\Delta})$ instead of $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$, in addition to the **IA**, also the **Closure** approximation has been assumed;
- 5 % to 10 % **binding** effect between $x = 0.4$ and 0.7 - much **bigger** than in the forward case;
- for $A > 3$, the evaluation of $P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E)$ is **difficult** - such an effect is **not under control**: **Conventional** nuclear effects can be **mistaken for exotic** ones;
- for ^3He it is possible : this makes it a **unique** target, even among the **Few-Body systems**.



^3He calculations: summary

Our results, for ^3He : (S.S. PRC 2004, 2009; M. Rinaldi and S.S., PRC 2012, 2013)

- * I.A. calculation of H_3 , E_3 , \tilde{H}_3 , within AV18;
- * Interesting predictions: strong sensitivity to details of nuclear dynamics;
- * extraction procedure of the neutron information, able to take into account all the nuclear effects encoded in an IA analysis;

Coherent DVCS off ^3He would be:

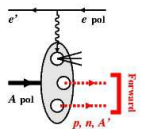
- * a test of IA; relevance of non-nucleonic degrees of freedom;
- * a test of the A - and isospin dependence of nuclear effects;
- * complementary to incoherent DVCS off the deuteron in extracting the neutron information (with polarized targets).

No data; no proposals at JLAB... difficult to detect slow recoils using a polarized target... But even unpolarized, ^3He would be interesting!

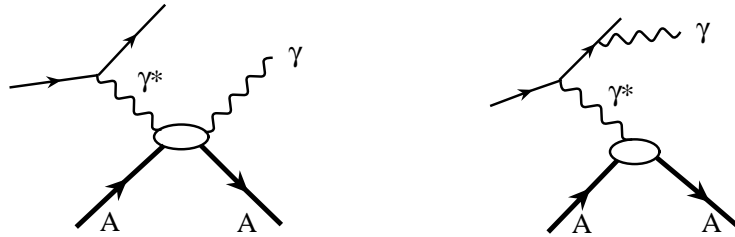
Together with ^3H , nice possibilities (flavor separation of nuclear effects, test of IA)

at the EIC, beams of polarized light nuclei will operate. $^3\vec{H}e$ can be used.

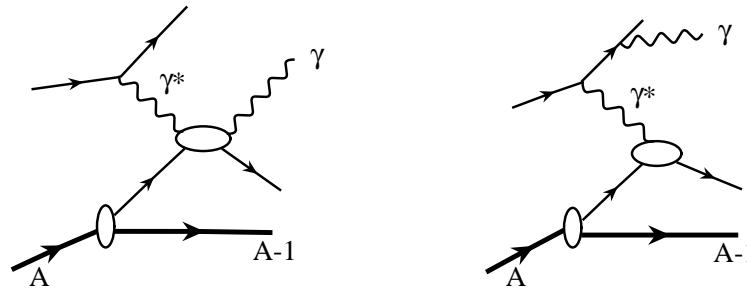
Our codes available to interested colleagues.



Nuclei and DVCS tomography

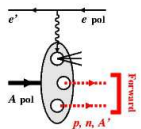


Coherent DVCS: nuclear tomography



Incoherent DVCS: tomography of bound nucleons: realization of the EMC effect

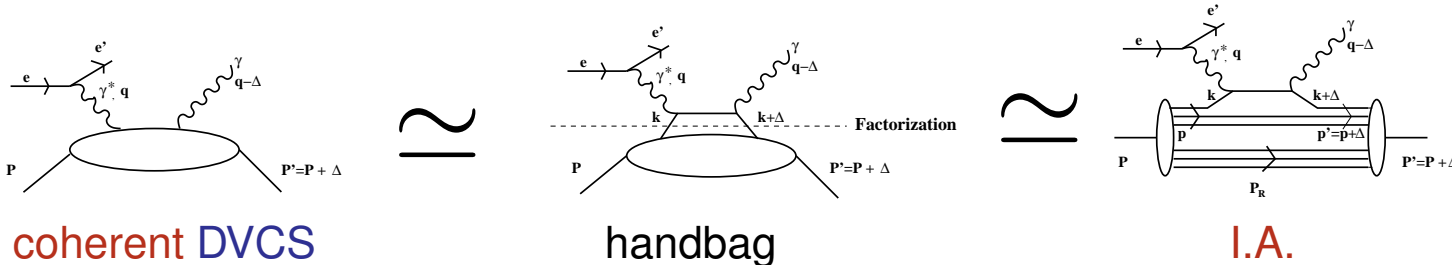
- Very difficult to distinguish coherent and incoherent channels (for example, in Hermes data, **Airapetian et al., PRC 2011**).
- Large energy gap between the photons and the slow-recoiling systems: very different detection systems required at the same time... **Very difficult...**
- But possible! CLAS, ^4He :** separation of coherent (**Hattawy et al., PRL 119, 202004 (2017)**) and incoherent (**Hattawy et al., PRL 123 (2019) no.3, 032502**) channels



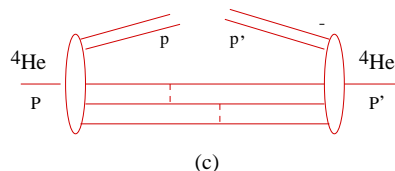
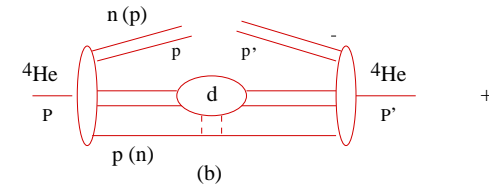
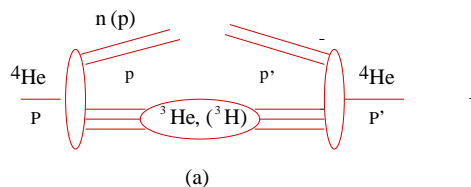
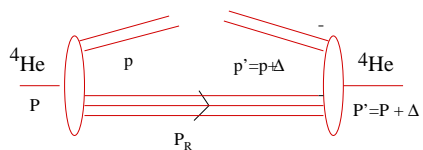
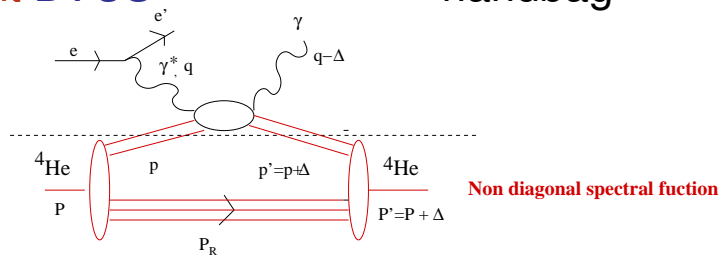
Our IA approach to coherent DVCS off ^4He

Realistic microscopic calculations are necessary. A collaboration is going on with Sara Fucini (Perugia, Ph.D. student), Michele Viviani (INFN Pisa).

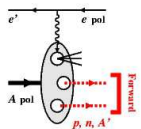
coherent DVCS in the Impulse Approximation (I.A.) to the handbag contribution:



I.A. :



a) worked out; b) is feasible; c) is really challenging



Coherent DVCS off ^4He : IA formalism

Convolution formula (E_q^N neglected) (S.Fucini, SS, M.Viviani PRC. 98 (2018) 015203):

$$H_q^{4He}(x, \Delta^2, \xi) = \sum_N \int_{|x|}^1 \frac{dz}{z} h_N^{4He}(z, \Delta^2, \xi) H_q^N\left(\frac{x}{z}, \Delta^2, \frac{\xi}{z}\right)$$

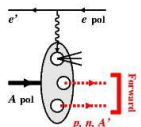
Non-diagonal light-cone momentum distribution:

$$\begin{aligned} h_N^{4He}(z, \Delta^2, \xi) &= \int dE \int d\vec{p} P_N^{4He}(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta(z - \bar{p}^+ / \bar{P}^+) \\ &= \frac{M_A}{M} \int dE \int_{p_{min}}^{\infty} dp \tilde{M} p P_N^{4He}(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(\tilde{z} \frac{\tilde{M}}{p} - \frac{p^0}{p} - \cos \theta\right) \end{aligned}$$

with $\xi_A = \frac{M_A}{M} \xi$, $\tilde{z} = z + \xi_A$, $\tilde{M} = \frac{M}{M_A} (M_A + \frac{\Delta^+}{\sqrt{2}})$ and M_{A-1}^{2*} is the squared mass of the final excited $A - 1$ -body state.

One needs therefore the **non-diagonal spectral function** and a **model for nucleon GPDs**.

Well known GPDs model of Goloskokov-Kroll (EPJA 47 212 (2011)) used for the nucleonic part. In principle valid at Q^2 values larger than those of interest here.






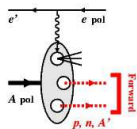
Coherent DVCS off ^4He : our nuclear model input

$$\begin{aligned}
 P(\vec{p}, \vec{p} + \vec{\Delta}, E) &= n_0(\vec{p}, \vec{p} + \vec{\Delta})\delta(E^*) + P_1(\vec{p}, \vec{p} + \vec{\Delta}, E) \\
 &= n_0(|\vec{p}|, |\vec{p} + \vec{\Delta}|, \cos \theta_{\vec{p}, \vec{p} + \vec{\Delta}})\delta(E^*) + P_1(|\vec{p}|, |\vec{p} + \vec{\Delta}|, \cos \theta_{\vec{p}, \vec{p} + \vec{\Delta}}, E) \\
 &\simeq a_0(|\vec{p}|)a_0(|\vec{p} + \vec{\Delta}|)\delta(E^*) + n_1(|\vec{p}|, |\vec{p} + \vec{\Delta}|)\delta(E^* - \bar{E})
 \end{aligned}$$

with $n_1(|\vec{p}|) = n(|\vec{p}|) - n_0(|\vec{p}|)$, $E = E_{min} + E^*$, $n_0(|\vec{p}|) = |a_0(|\vec{p}|)|^2$, and

$$a_0(|\vec{p}|) = \langle \Phi_3(1, 2, 3) \chi_4 \eta_4 | j_0(|\vec{p}| R_{123,4}) \Phi_4(1, 2, 3, 4) \rangle$$

-  $n_0(p)$, “ground”, and $n(p)$, “total” momentum distributions, evaluated realistically through 4-body and 3-body variational CHH wave functions, within the Av18 NN interaction, including UIX three-body forces.
-  \bar{E} , average excitation energy of the recoiling system, given by the model diagonal spectral function, also based on Av18+UIX, described in **M. Viviani et al., PRC 67 (2003) 034003**, update of **Ciofi & Simula, PRC 53 (1996) 1689**.
-  In summary: realistic Av18 + UIX momentum dependence; the dependence on E , angles and Δ is modelled and not yet realistic



Limits

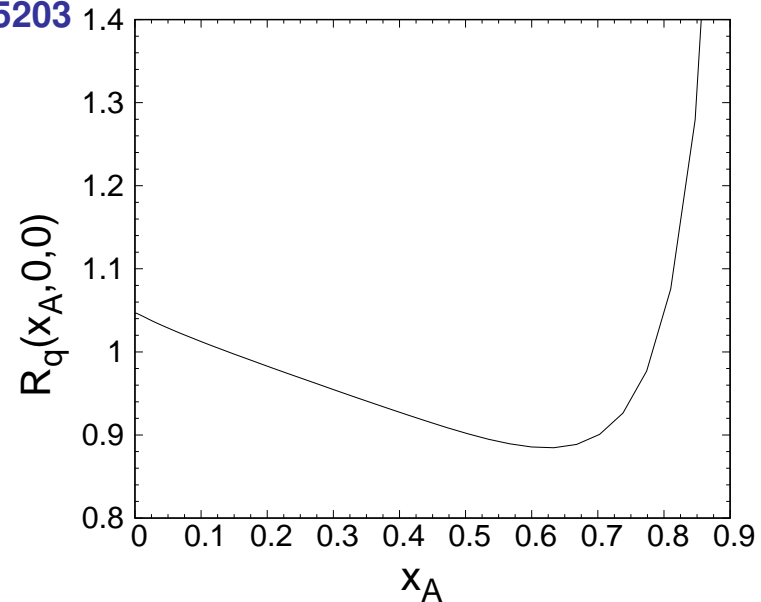
S.Fucini, SS., M. Viviani PRC 98 (2018) 015203

1 - Forward limit: the ratio:

$$R_q(x, 0, 0) = \frac{H_q^{4He}(x, 0, 0)}{2H_q^p(x, 0, 0) + 2H_q^n(x, 0, 0)}$$

$$= \frac{q^{4He}(x)}{2q^p(x) + 2q^n(x)}$$

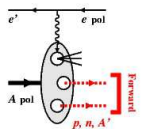
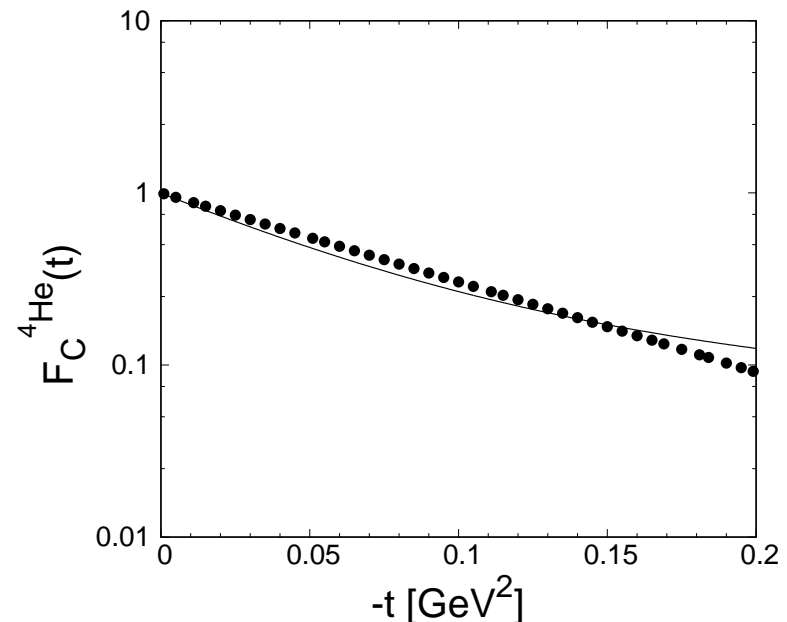
shows an EMC-like behavior;



2 - Charge F.F.:

$$\sum_q e_q \int dx H_q^{4He}(x, \xi, \Delta^2) = F_C^{4He}(\Delta^2)$$

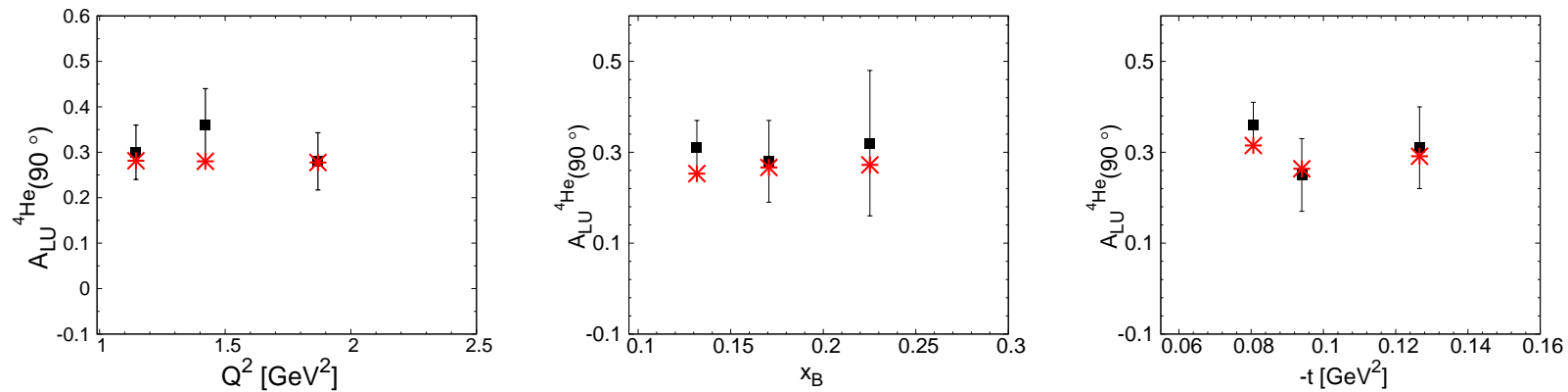
reasonable agreement with data in the region relevant to the coherent process, $-t = -\Delta^2 \leq 0.2 \text{ GeV}^2$.



Comparison with EG6 data: A_{LU}

S. Fucini, S.S., M. Viviani PRC 98 (2018) 015203

^4He azimuthal beam-spin asymmetry $A_{LU}(\phi) = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-}$, for $\phi = 90^\circ$:

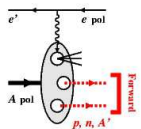


results of this approach (stars) vs EG6 data (squares)

From left to right, the quantity is shown in the experimental Q^2 , x_B and t bins, respectively: very good agreement

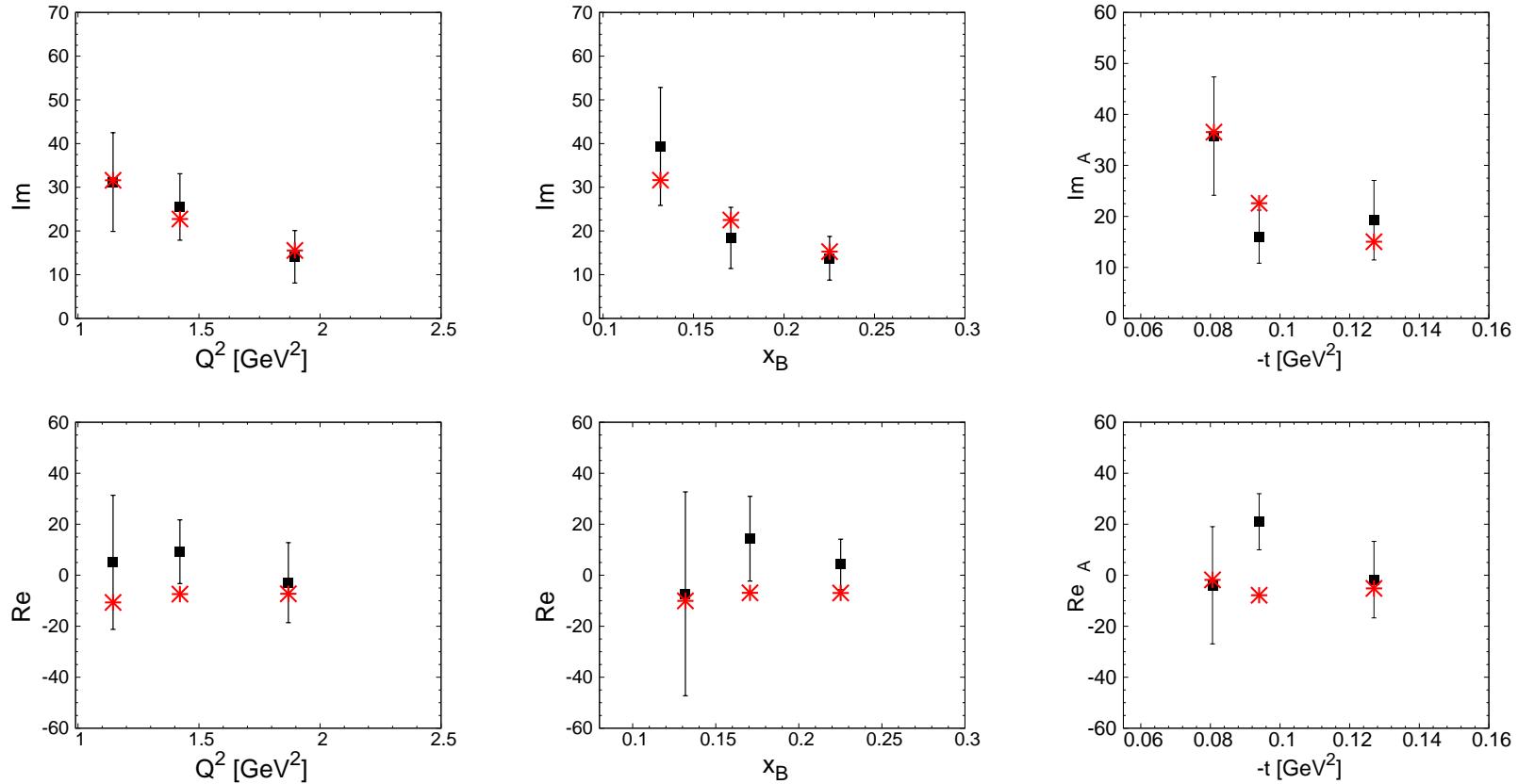
$$A_{LU}(\phi) = \frac{\alpha_0(\phi) \Im m(\mathcal{H}_A)}{\alpha_1(\phi) + \alpha_2(\phi) \Re e(\mathcal{H}_A) + \alpha_3(\phi) (\Re e(\mathcal{H}_A)^2 + \Im m(\mathcal{H}_A)^2)}$$

$\Re e(\mathcal{H}_A)$ and $\Im m(\mathcal{H}_A)$ experimentally extracted fitting these data using explicit forms for the kinematic factors α_i (Belitsky et al. PRD 2009)



Comparison with EG6 data: $\Im m(\mathcal{H}_A)$ & $\Re e(\mathcal{H}_A)$

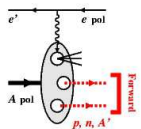
S. Fucini, S.S., M. Viviani PRC 98 (2018) 015203



$$\Im m(\mathcal{H}_A) = H_A(\xi, \xi, t) - H_A(-\xi, \xi, t),$$

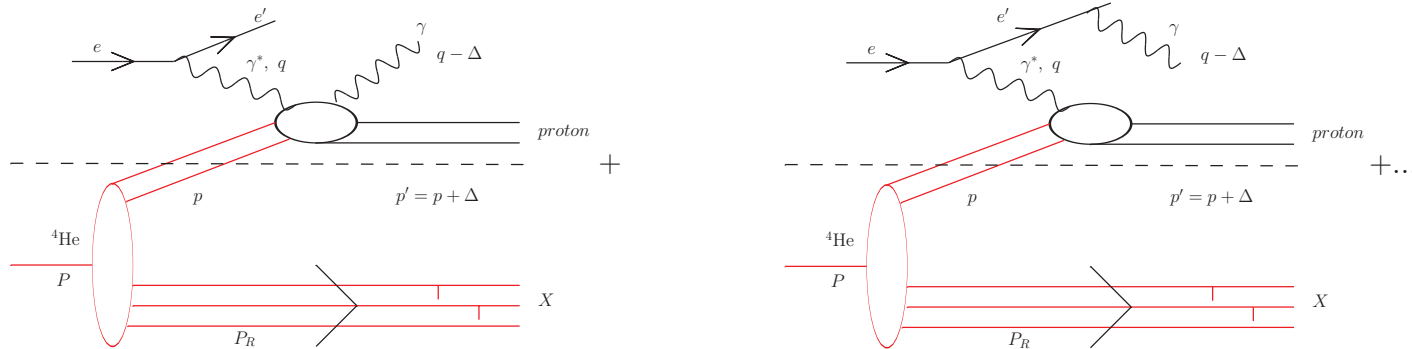
$$\Re e(\mathcal{H}_A) = \mathcal{P} \int_0^1 dx [H_A(x, \xi, t) - H_A(-x, \xi, t)] \left(\frac{1}{x - \xi} + \frac{1}{x + \xi} \right)$$

Very good agreement for $\Im m(\mathcal{H}_A)$, good agreement for $\Re e(\mathcal{H}_A)$
(data weakly sensitive to $\Re e(\mathcal{H}_A)$)



Our IA approach to incoherent DVCS off ^4He

S. Fucini, S.S., M. Viviani - arXiv:1909.12261 [nucl-th]



$$A_{LU}^{4,p} = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} \quad d\sigma^{\lambda,4} = \int dE \int d\vec{p} \frac{p \cdot k}{p_0 E_k} P^{4,p}(\vec{p}, E) d\sigma^{\lambda,p}$$

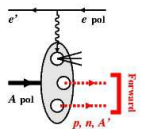
In IA, Instant Form approach, the **diagonal spectral function** $P^{4,p}(\vec{p}, E)$ arises:

- off-shellness driven by nuclear dynamics:

$$p_0 = M_A - \sqrt{M_{A-1}^{*2} + p^2} \simeq M - E - T_f \longrightarrow p^2 \neq M^2$$

$$\xi = Q^2 / [(p + p') \cdot (q + q')] \neq x_B / (2 - x_B)$$

- number SR fulfilled; momentum SR violated by 2 %
(polynomiality violated)



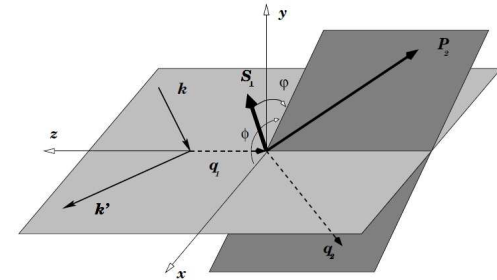
Incoherent DVCS off ^4He : formalism, ingredients

General structure of the differential cross section ($i = DVCS, BH, Int$):

$$\frac{d\sigma_i^{\lambda,4}}{d\text{kin}} \propto \int dE \int d\vec{p} P^{4,p}(\vec{p}, E) g(\text{kin}, \vec{p}, E) A_i(\text{kin}, \vec{p}, E)$$

$$d\text{kin} = dx_B dQ^2 dt d\Phi$$

$g(\text{kin}, \vec{p}, E)$: a complicated function



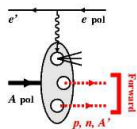
$$A_{BH} = T_{BH}^2, \quad A_{DVCS} = T_{DVCS}^2, \quad A_{Int} = Int_{BH-DVCS} \quad \text{for a bound proton}$$

$$A_{LU}^{4,p} \simeq \frac{\int dE \int d\vec{p} P^{4,p}(\vec{p}, E) g(\text{kin}, \vec{p}, E) Int_{BH-DVCS}(\text{kin}, \vec{p}, E)}{\int dE \int d\vec{p} P^{4,p}(\vec{p}, E) g(\text{kin}, \vec{p}, E) T_{BH}^2(\text{kin}, \vec{p}, E)}$$

$T_{BH}^2, T_{DVCS}^2, Int_{BH-DVCS}$ for a moving bound nucleon; our expressions, obtained generalizing the ones at leading twist for nucleons at rest (**Belitski et al. (2002)**); $T_{BH}^2 = c_0^{bound} + c_1^{bound}(\cos \Phi) + c_2^{bound} \cos(2\Phi)$

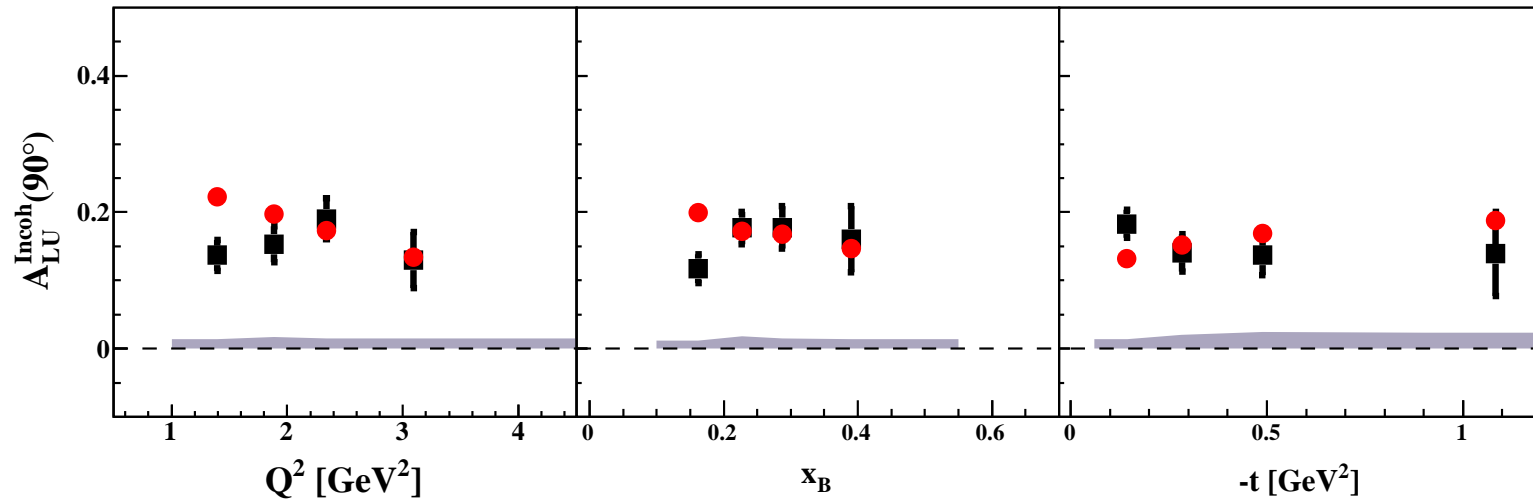
In $Int_{BH-DVCS}$, the H GPD in $\Im m(\mathcal{H}_N)$ evaluated in the GK model;

Av18-based model of the diagonal spectral function $P^{4,p}(\vec{p}, E)$
(**M. Viviani et al., PRC 67 (2003) 034003**)



Results for A_{LU}

S. Fucini, S.S., M. Viviani - arXiv:1909.12261 [nucl-th]

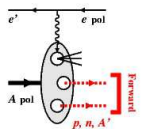


Calculations are performed in each experimental bin, at values of x_B , t and Q^2 corresponding to the experimental EG6 analysis. A strong dependence on the experimental kinematics is found.


EG6 data correctly reproduced using conventional ingredients up to data corresponding to low values of Q^2 ...

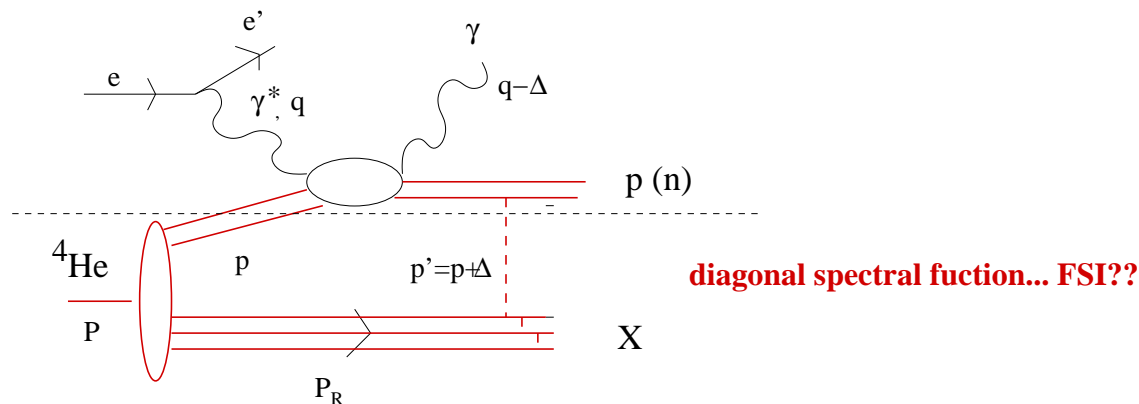
Possible important t/Q^2 effects to be added (other GPDs involved?)...


Complicated interplay between t and Q^2 to realize if this could be due to FSI effects or to not conventional effects... Work in progress.

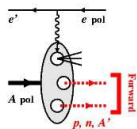
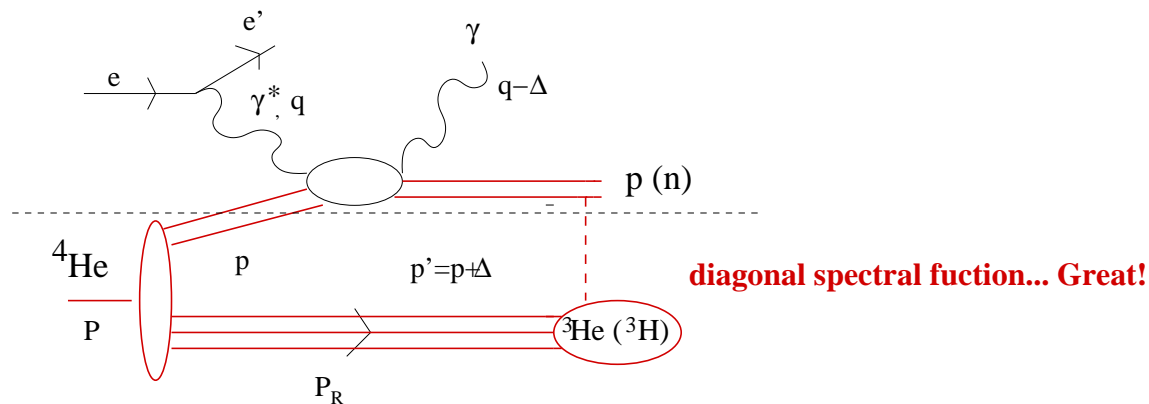


Incoherent DVCS off ^4He : beyond IA; FSI?

 $^4\text{He}(e, e' \gamma p(n))X$

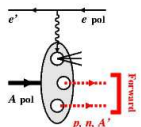


 **Tagged! e.g., $^4\text{He}(e, e' \gamma p)^3\text{H}$ (arXiv:1708.00835 [nucl-ex]) \rightarrow EIC!!!**



The quest for covariance

- Mandatory to achieve polynomiality for GPDs, and sum rules in DIS: number of particle and momentum sum rule not fulfilled at the same time in not covariant IA calculations
- Numerically not very relevant for forward Physics. It becomes relevant for non-diagonal observables at high momentum transfer. Example: form factors (well known since a long time, see, i.e., **Cardarelli et al., PLB 357 (1995) 267**)
- I do not expect big problems in the coherent case at low t ;
Crucial for incoherent at higher t , as well as finite t corrections (target mass corrections at least for scalar nuclei under control)
- Certainly it has to be studied.
For ^3He , formal developments available in a Light-Front framework (**A. Del Dotto, E. Pace, S.S., G. Salmè, PRC 95 (2017) 014001**).
Calculations in progress, starting from a diagonal, spin-independent spectral function.
 ^4He ... Later (very cumbersome).



Conclusions

GPDs of He nuclei:



1 - GPDs for ^3He :

A complete impulse approximation realistic study is available
(S.S. PRC 2004, PRC 2009; M. Rinaldi and S.S., PRC 2012, PRC 2013)

- * No data; proposals? Prospects at JLAB-12 and EIC;
- * planned LF calculation



2 - DVCS off ^4He :

* Coherent and Incoherent channel: a calculation (not yet realistic) with basic ingredients (GK model plus a model spectral function based on Av18 + UIX) describe well most of the data available from JLab at 6 GeV; (S. Fucini, S.S., M. Viviani, PRC 98 (2018) 015203; arXiv:1909.12261 [nucl-th]).

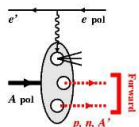
Straightforward and workable approach, suitable for planning new measurements.

- * New data expected at 12 GeV will require much more precise nuclear description (in progress)

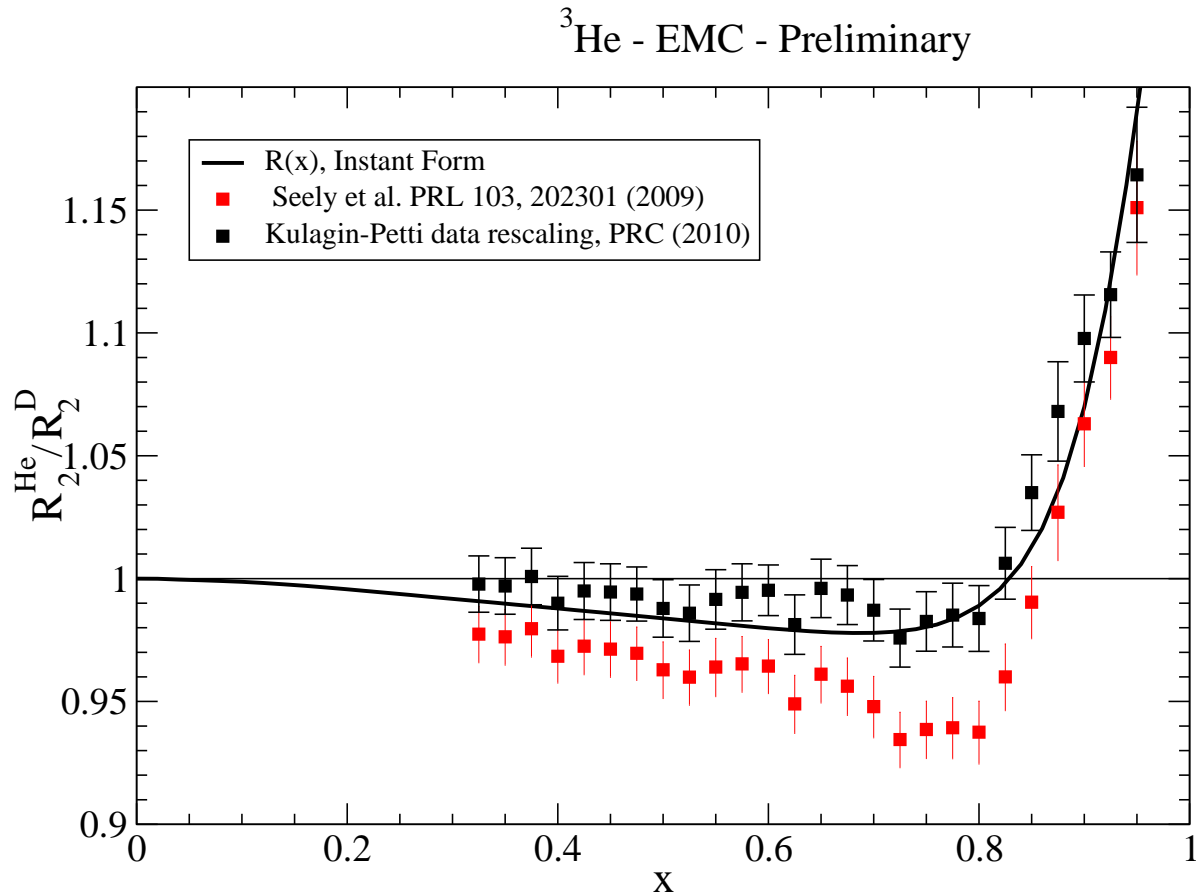
Our spirit: introduce new ingredients one at a time

Occam's razor: "*Frustra fit per plura quod potest fieri per pauciora*"

(*It is futile to do with more things what can be done with fewer*)



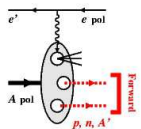
Backup: ^3He EMC effect IF - preliminary



Red squares: Seely et al. (E03103), Hall A JLab, PRL 103 (2009) 202301

Black squares: reanalysis (currently accepted) Kulagin and Petti, PRC 82 (2010) 054614

particle SR fulfilled; momentum SR violated by 2 %



Backup: ^4He EMC effect IF - preliminary

EMC effect: PRELIMINARY

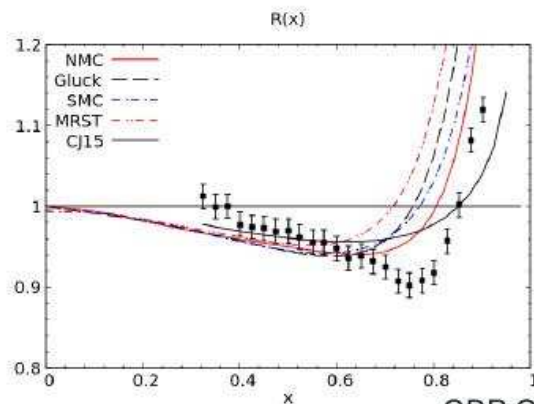
$$R(x) = \frac{F_2^{^4\text{He}}(x)}{F_2^d(x)} \quad x \in [0 : M_A/M]$$

where the **function structures** F_2 for $A = ^4\text{He}, d$ are defined as

$$F_2^A(x) = \sum_N \int_x^{M_A/M} dz f_N^A(z) F_2^N\left(\frac{x}{z}, Q^2\right)$$

in terms of the *light cone momentum distribution*

$$f_N^A(z) = \int d\vec{p} \int dE P_N^A(\vec{p}, E) \frac{p^+}{p_0} \delta\left(z - \sqrt{2} \frac{p^+}{M_A}\right)$$

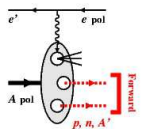


- Our model isn't predictive at **small x**
- Good agreement in the **valence region**
- Strong dependence on the model for F_2^N at **large x**
- Need to better unravel the Q^2 dependence of $R(x)$

Data from **Seely et al., PRL (2009)**

GDR QCD 2019

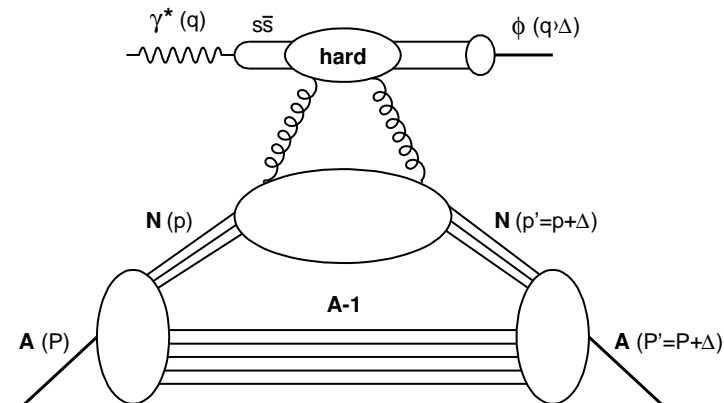
12 / 23



Backup: Many other issues...

x —moments of GPDs (ffs of energy momentum tensor): information on spatial distribution of energy, momentum and forces experienced by the partons. Predicted an A dependence stronger than in IA (not seen at HERMES); M. Polyakov, PLB 555, 57 (2003); H.C. Kim et al. PLB 718, 625 (2012)...

Gluon GPDs in nuclei



For GPDs, shadowing (low x_B) stronger than for PDFs

A. Freund and M. Strikman, PRC 69, 015203 (2004)...

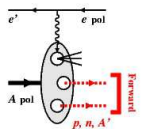
Exclusive ϕ — electroproduction, unique source of information, studied by ALERT, waiting for EIC...

Deuteron: an issue aside.

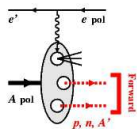
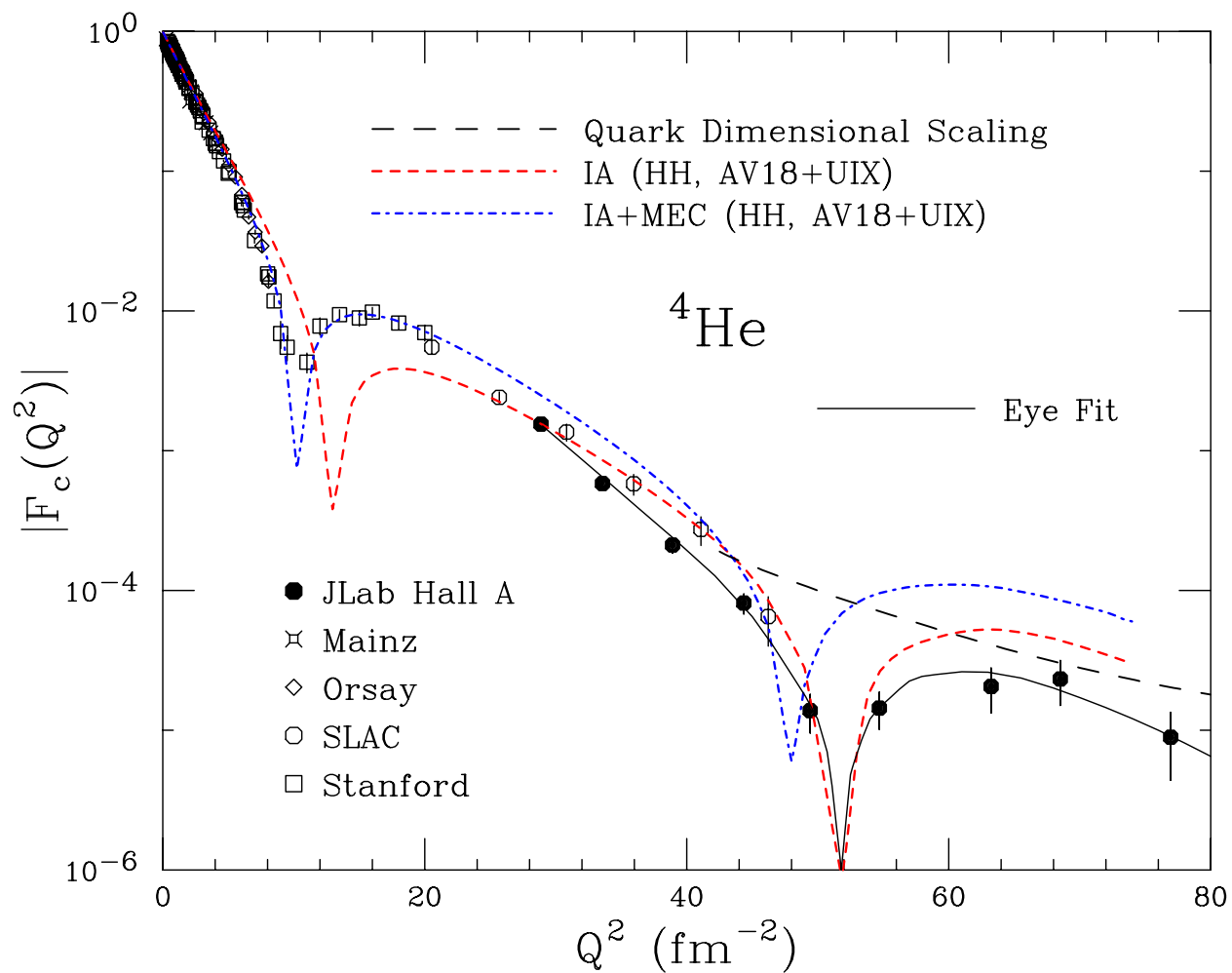
Extraction of the neutron information; access to a new class of distribution ($J = 1$)

Studied by different collaborations (by ALERT too, coherent and incoherent DVCS)

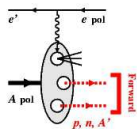
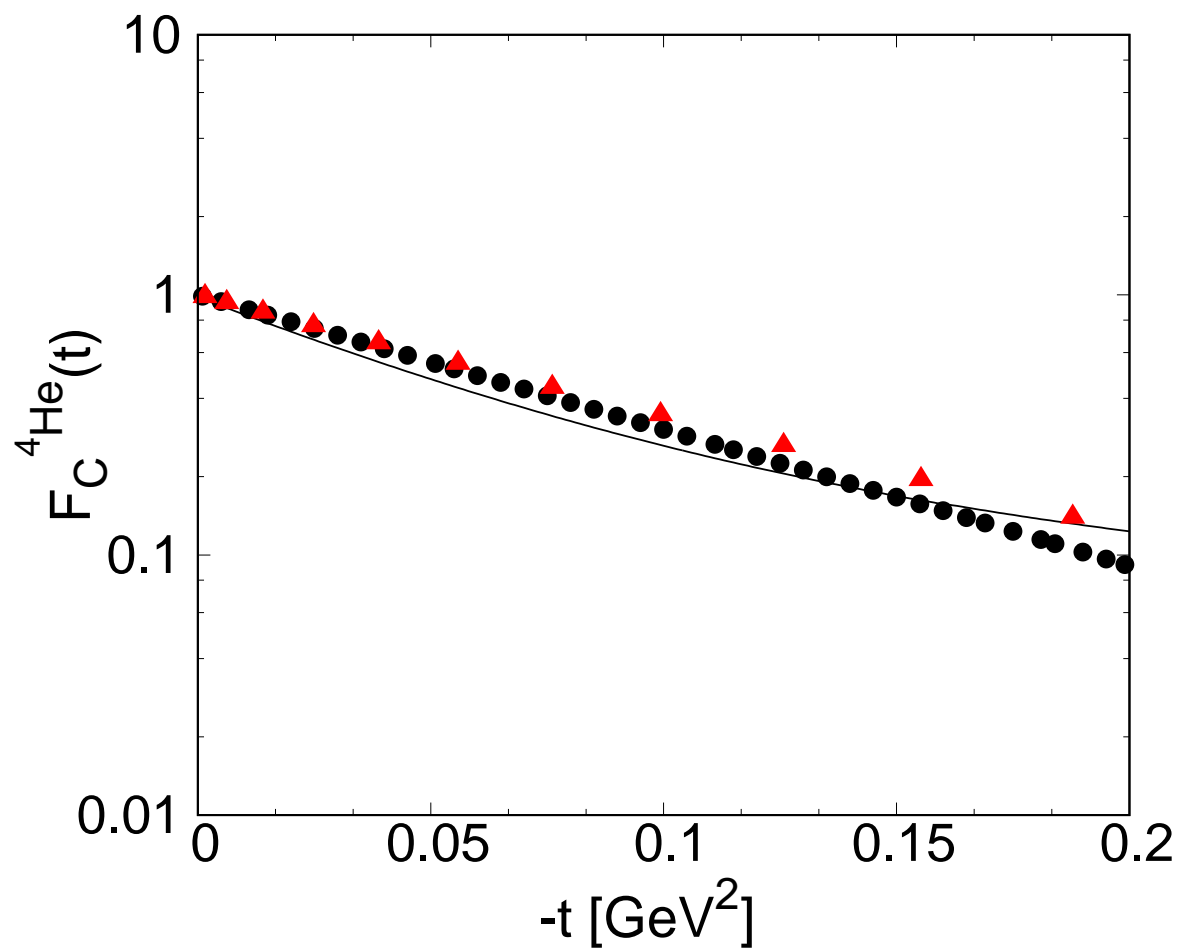
theory: Cano and Pire EPJA 19,423 (2004); Taneja et al. PRD 86,036008 (2012)...



Backup: ^4He FF



Backup: ^4He FF - IA



Backup: $\tilde{G}_M^{3,q}$ calculation: correct limits

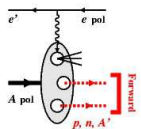
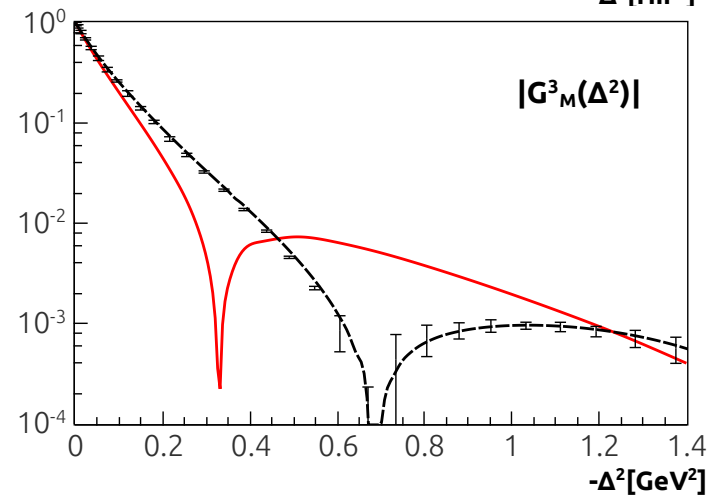
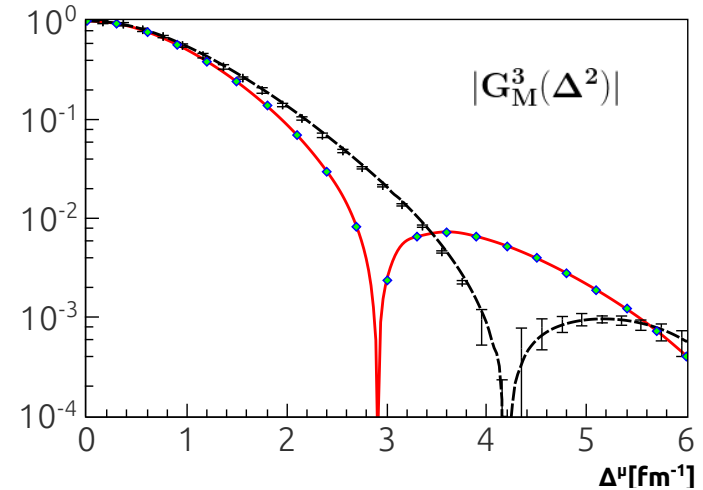
For \tilde{G}_M^3 (M. Rinaldi, S.S. PRC 85, 062201(R) (2012); PRC 87, 035208 (2013)):

1 - Forward limit: no control on $E_q^3(x, 0, 0)$
no possible check;

2 - Magnetic F.F.:

$$\sum_q \int dx \tilde{G}_M^{3,q}(x, \xi, \Delta^2) = G_M^3(\Delta^2)$$

- in perfect agreement with previous IA, Av18 calculations (L.E. Marcucci et al. PRC 58 (1998))
- in good agreement with data in the region relevant to the coherent process, $-\Delta^2 \ll 0.15 \text{ GeV}^2$
- To have agreement at higher Δ^2 , effects beyond IA are necessary: not important for the coherent channel!



Backup: Nuclear effects - the binding

General **IA** formula: $H_q^A(x, \xi, \Delta^2) \simeq \sum_N \int_x^1 \frac{dz}{z} h_N^A(z, \xi, \Delta^2) H_q^N\left(\frac{x}{z}, \frac{\xi}{z}, \Delta^2\right)$

where

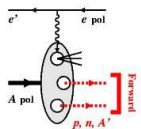
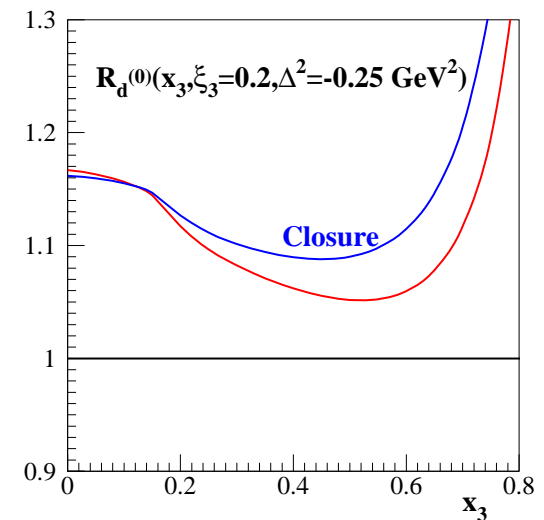
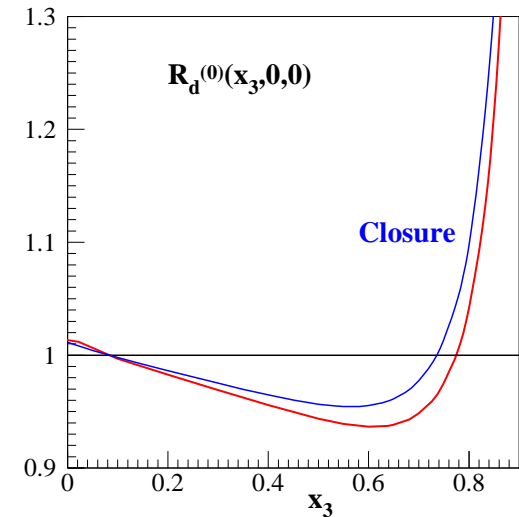
$$h_N^A(z, \xi, \Delta^2) = \int dE d\vec{p} P_N^A(\vec{p}, \vec{p} + \vec{\Delta}, E) \delta\left(z + \xi - \frac{p^+}{P^+}\right)$$

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) = \bar{\sum}_M \sum_{s,f} \langle \vec{P}' M | \vec{P}_f, (\vec{p} + \vec{\Delta}) s \rangle \\ \times \langle \vec{P}_f, \vec{p} s | \vec{P} M \rangle \delta(E - E_{min} - E_f^*)$$

using the **Closure Approximation**, $E_f^* = \bar{E}$:

$$P_N^3(\vec{p}, \vec{p} + \vec{\Delta}, E) \simeq \bar{\sum}_M \sum_s \langle \vec{P}' M | a_{\vec{p}+\vec{\Delta},s} a_{\vec{p},s}^\dagger | \vec{P} M \rangle \\ \delta(E - E_{min} - \bar{E}) = \\ = n(\vec{p}, \vec{p} + \vec{\Delta}) \delta(E - E_{min} - \bar{E}) ,$$

Spectral function substituted by a **Momentum distribution**
(forward case in C. Ciofi, S. Liuti PRC 41 (1990) 1100)

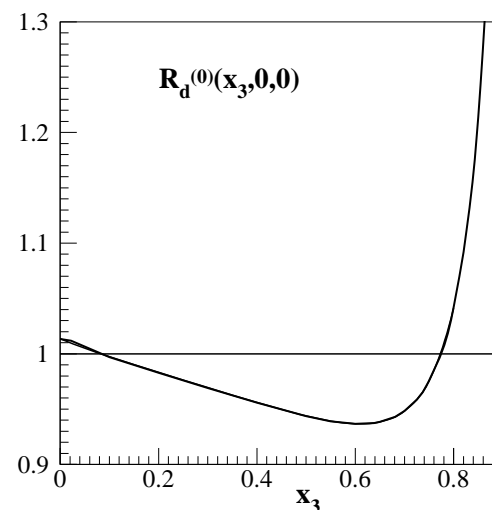


Backup: Dependence on the NN interaction

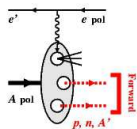
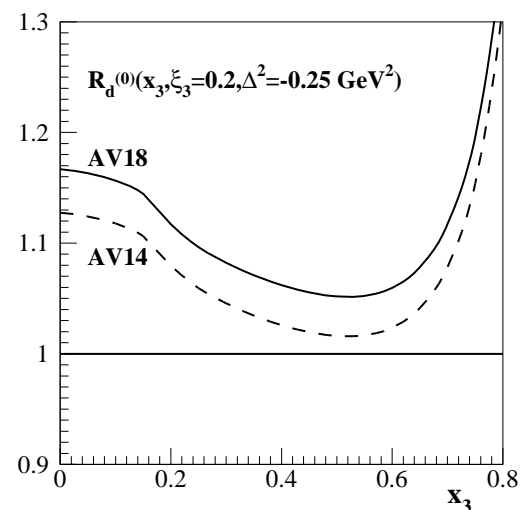
Nuclear effects are bigger than in the forward case: dependence on the potential



Forward case: Calculations using the **AV14** or **AV18** interactions are **indistinguishable**

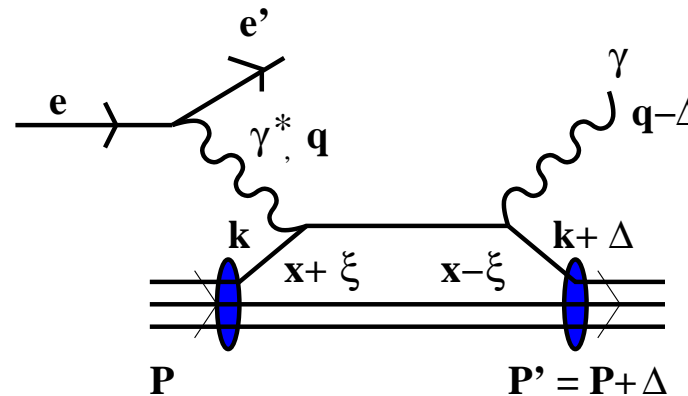


Non-forward case: Calculations using the **AV14** and **AV18** interactions **do differ**:



GPDs: Definition (X. Ji PRL 78 (97) 610)

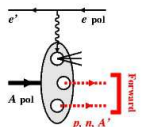
For a $J = \frac{1}{2}$ target,
in a hard-exclusive process,
(handbag approximation)
such as (coherent) DVCS:



the GPDs $H_q(x, \xi, \Delta^2)$ and $E_q(x, \xi, \Delta^2)$ are introduced:

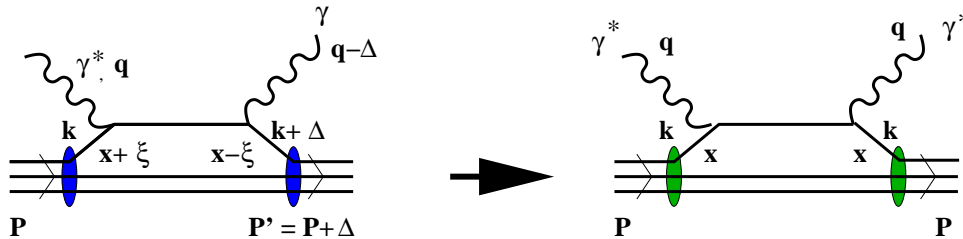
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle = H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) + E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{i\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots$$

- $\Delta = P' - P$, $q^\mu = (q_0, \vec{q})$, and $\bar{P} = (P + P')^\mu / 2$
- $x = k^+ / P^+$; $\xi = \text{"skewness"} = -\Delta^+ / (2\bar{P}^+)$
- $x \leq -\xi \longrightarrow$ GPDs describe *antiquarks*;
 $-\xi \leq x \leq \xi \longrightarrow$ GPDs describe $q\bar{q}$ *pairs*; $x \geq \xi \longrightarrow$ GPDs describe *quarks*



GPDs: constraints

when $P' = P$, i.e., $\Delta^2 = \xi = 0$, one recovers the usual PDFs:



$$H_q(x, \xi, \Delta^2) \Rightarrow H_q(x, 0, 0) = q(x); \quad E_q(x, 0, 0) \text{ unknown}$$

the x -integration yields the q-contribution to the Form Factors (ffs)

$$\int dx \int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle P' | \bar{\psi}_q(-\lambda n/2) \gamma^\mu \psi_q(\lambda n/2) | P \rangle =$$

$$\int dx H_q(x, \xi, \Delta^2) \bar{U}(P') \gamma^\mu U(P) + \int dx E_q(x, \xi, \Delta^2) \bar{U}(P') \frac{\sigma^{\mu\nu} \Delta_\nu}{2M} U(P) + \dots$$

$$\Rightarrow \int dx H_q(x, \xi, \Delta^2) = F_1^q(\Delta^2) \quad \int dx E_q(x, \xi, \Delta^2) = F_2^q(\Delta^2)$$

$$\Rightarrow \text{Defining } \boxed{\tilde{G}_M^q = H_q + E_q} \quad \text{one has } \int dx \tilde{G}_M^q(x, \xi, \Delta^2) = G_M^q(\Delta^2)$$

