INFRARED SAFETY IN COORDINATE SPACE (New year, new title)

G. Sterman

Based on work with O. Erdogan 1705.04539 (PRD) and ongoing

QCD Evolution 2021

- Motivation
- IR Safe weights and cross sections
- Weighted cross sections as coordinate integrals
- Path formulation of cross sections
- Delayed cancellations
- Comments

- Motivation

  - Evolutions interpolate between perturbative scales
  - We may want a space-time picture for the transition from perturbative to long-time dynamics
  - For example, a space-time analog of power corrections from resummed logs

In this talk, a look at weighted cross sections as coord. space integrals

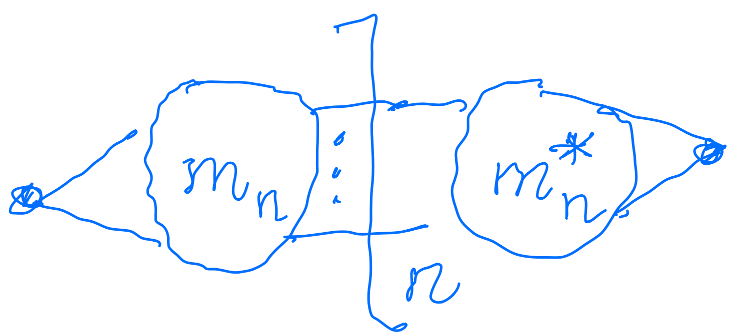
- Weights, cross sections and IR safety
(for simplicity in $\gamma^* \to$ hadrons)
(massless)

$$\sigma[q, f]$$

$$\propto \sum_n \langle 0 | J_\mu(0) | n \rangle \langle n | J^\mu(0) | 0 \rangle$$

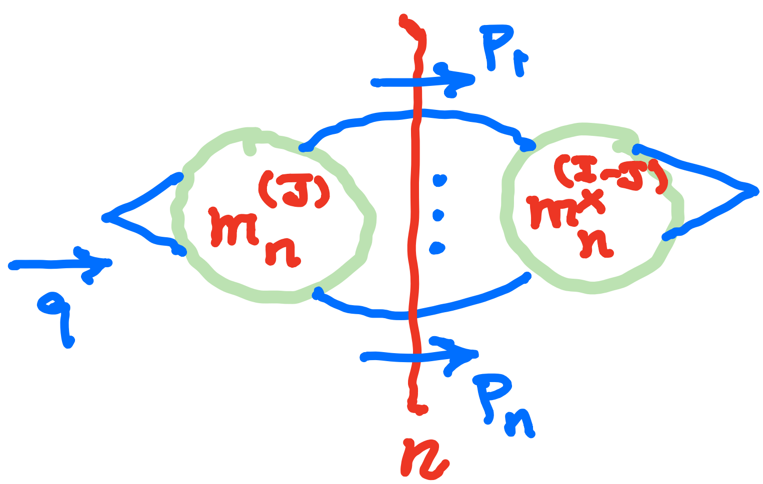
$$\cdot (2\pi) \, \delta^4(q - p_n)$$

$$\cdot f_n(p_1 \ldots p_n)$$ assume smooth



In perturbation theory (J^th order)

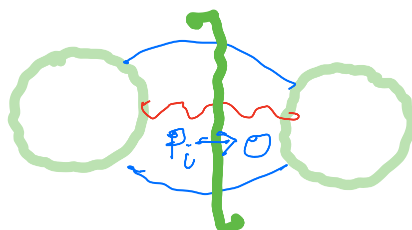
$$\sigma^{(J)}[q, f] = \sum_{\{n\}} \int \frac{d^4 p_i}{(2\pi)^4} \, m_n^{*(I-J)}(p_i) \cdot \prod_{a=1}^{n} (2\pi)\delta_+(p_i^2) \cdot f_n(\ldots p_i \ldots) \, m_n^{(J)}(p_i) \cdot (2\pi)^4 \delta^4\Big(q - \sum_{a \in n} p_a\Big)$$



$\sigma[q,t]$ is IR finite (safe) if two conditions are satisfied, relating f_n, f_{n-1}:

* $$\lim_{p_i \to 0} \frac{f_n(\ldots p_i \ldots) - f_{n-1}(\ldots \hat{p}_i \ldots)}{|\vec{p}_i|^\alpha} = 0$$

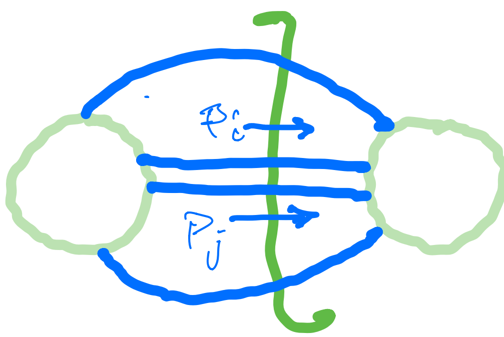
for some $\alpha > 0$



$p_i \to 0$

* $$\lim_{\theta_{ij} \to 0} \frac{f_n(\ldots p_i \ldots p_j \ldots) - f_{n-1}(\ldots p_i + p_j \ldots \hat{p}_j \ldots)}{|\theta_{ij}|^\beta} = 0$$

for some $\beta > 0$



$p_i \parallel p_j$

This is a large set.

Coordinate Space versions

Fourier transforms:

$$\mathcal{M}(p_1 \ldots p_n) = \prod_i^n \int d^4x_i\, e^{ip_i \cdot x_i}\, \tilde{\mathcal{M}}(x_1 \ldots x_n)$$

and so on...

In m: $\tilde{\Delta}(z) = \dfrac{1}{-z^2+i\epsilon}$

In m^* $\tilde{\Delta}^*(z) = \dfrac{1}{-z^2-i\epsilon}$

For cut lines

$$\tilde{\Delta}_c(z) = \frac{1}{-z^2+i\epsilon z^0}$$

$$= \frac{1}{-2(z^+-i\epsilon)(z^--i\epsilon)-z_\perp^2}$$

$$\tilde{F}_n(r_1 \ldots r_n) = \prod_i \int d^4p_i\, e^{-ip_i \cdot r_i} \cdot \mathcal{F}_n(p_1 \ldots p_n) \cdot \delta^4(q-\sum p_i)$$

Example: thrust moments $(q_0 \to \infty)$

$$e^{-(1-T)N/Q}$$

$$F(\{r_i\}, \tfrac{N}{Q}) = \prod_i \frac{1}{(2\pi)^2} \frac{1}{r_i^+ + r_i^- - 2N/Q} \cdot \left(\frac{1}{r_i^+ - i\epsilon} + \frac{1}{r_i^- - i\epsilon}\right) \delta^2(r_{i\perp})$$

In these terms:

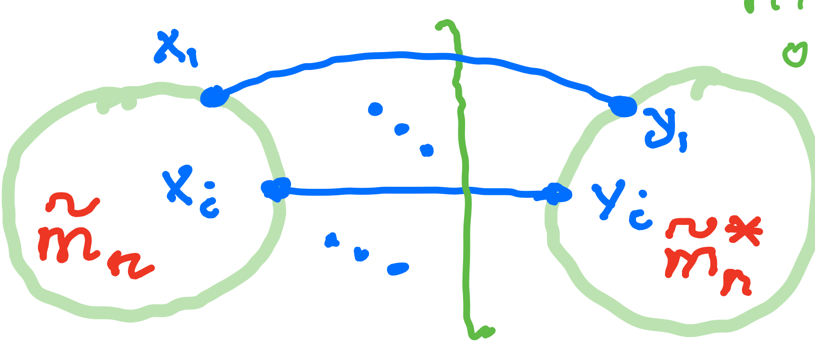
$$\sigma_n^{(J)}[q,f]$$

$$= \sum_I \prod_{i=1}^{n} \int dx_i \, dy_i \, \tilde{m}_n^{*(J-I)}(y_i)$$

$$\cdot \int d^4 r_i \, \tilde{\Delta}_e(y_i - r_i - x_i)$$

$$\cdot \tilde{F}_n(r_1 \dots r_n) \, \tilde{m}_n^{(I)}(x_i)$$

All infor. on weight here



$\otimes F_n(r_1 \dots r_n)$

- IR singularities in each n from $x_i, y_i \to \infty$
- But we know they cancel

Aim: exhibit manifest cancellation of IR

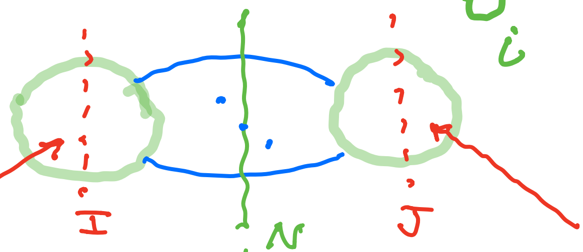
Approach: "Path" formulation for cross section (GS, Ozan Erdogan)

Coordinate analog of Light Cone Ordered Perturbation Theory

- order vertices in x^+ & do all minus LC integrals
- all lines onshell
- x^+ integrals give products of energy deficit denominators

LCOPT:

$$S_i = \sum_{a \in i} \frac{p_{a\perp}^2}{2p_a^+}$$



$$\prod_{\text{state } i} \frac{i}{Q^- - S_i + i\epsilon} \qquad f_N\,\delta(Q^- - S_N) \qquad \prod_{\text{state } j} \frac{1}{Q^- - S_j - i\epsilon}$$

IR Cancellation by summing over states in each order using

$$2\pi\,\delta(D) = \frac{i}{D+i\epsilon} - \frac{i}{D-i\epsilon}$$

IR divergences cancel after $\sum_N$ for IR safe f_N's.

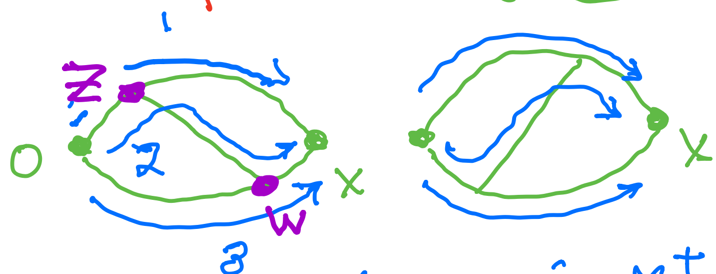
Coordinate approach offers a complementary perspective

Coordinate LCOPT

energy deficits

$\longrightarrow$ length deficits

In amplitude 

= 

- Order vertices in x^+
- Sum over complete sets of paths that cover diagram.
- Each term

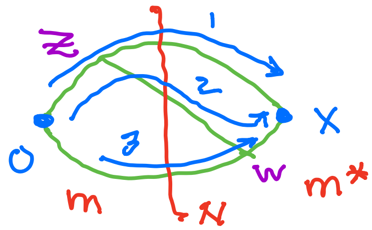
$$\prod_{\substack{\text{lines} \\ i}} \frac{\theta(x_i^+)}{2x_i^+} \prod_{\text{paths}} \frac{1}{\Delta x_p^- - i\epsilon}$$

$$\Delta x_p = x^- - \sum_{i\in p} \frac{x_{i\perp}^2}{2x_i^+}$$

E.G.

$$\Delta x_2 = x^- - \frac{(x_\perp - w_\perp)^2}{2(x^+ - w^+)} - \frac{(w_\perp - z_\perp)^2}{2(w^+ - z^+)} - \frac{z_\perp^2}{2z^+}$$

For Cross Section:



· Each term

$$\prod_{\substack{\text{lines } i\\ \text{in } m}} \frac{\theta(x_i^+)}{2x_i^+} \prod_{\substack{\text{lines}\\ \text{in } N}} \frac{1}{2(x_j^+ - r_j^+)}$$

$$\times \prod_{\substack{\text{lines}\\ \text{in } m^*}} \frac{\theta(-x_k^+)}{2x_k^+} \times \prod_{\text{paths}} \frac{1}{\Delta x_p(r) - i\epsilon}$$

$$\times F_N(\{r_j\}, j \in N)$$

$$\Delta x_p(r) = x^- - \sum_{\substack{i \in p\\ \text{in } m}} \frac{x_{i\perp}^2}{2x_i^+} - \sum_{\substack{j \in p\\ \text{in } N}} \frac{(x_{j\perp} - r_{j\perp})^2}{2(x_j^+ - r_j^+)} - r_j^-$$

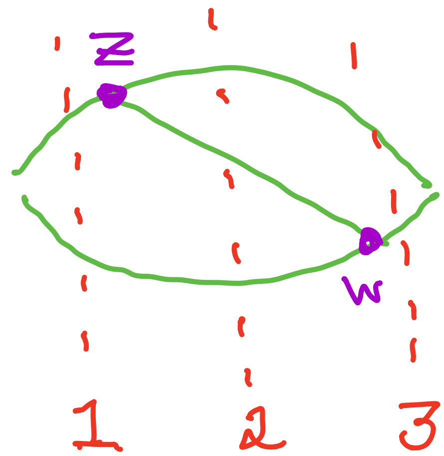
$$- \sum_{\substack{k \in p\\ \text{in } m^*}} \frac{x_{k\perp}^2}{2x_k^+}$$

Each path goes forward to the cut, the backward

For unweighted cross section
$(F_n = 1,\ \text{all } r_j = 0)$

Cancellation is pairwise
"largest time eqn"

E.G.



$w^+ > z^+$ state 2 cancels state 3

$z^+ > w^+$ state 2 cancels state 1

weight functions modify (delay) cancellations

But "locality" of the cancellation makes IR regulation unnecessary

Comments:

- In contemporary and planned facilities "complete" portraits of final states are accessible
- Final states are amenable to multiple metrics [weights]
- Here, I've described a suggestion for one way we might use these capabilities to probe evolution of jets in space-time.