Azimuthal Angular Asymmetries from the Soft Gluon Radiation Associated with Jet

Feng Yuan
Lawrence Berkeley National Laboratory

Reference: Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774; arXiv: 2105.xxxx, to be submitted

Soft gluon radiation leads to Sudakov Logarithms

Sudakov, 1956; Collins-Soper-Sterman 1985

- Differential cross section depends on $Q_1=q_T$, where $Q^2 >> Q_1^2 >> \Lambda^2_{QCD}$

$$\frac{d\sigma}{dQ_1^2} = \frac{1}{Q_1^2} f_1 \otimes f_2 \otimes \sum_i \alpha_s^i \ln^{2i-1} \frac{Q^2}{Q_1^2} + \cdots$$

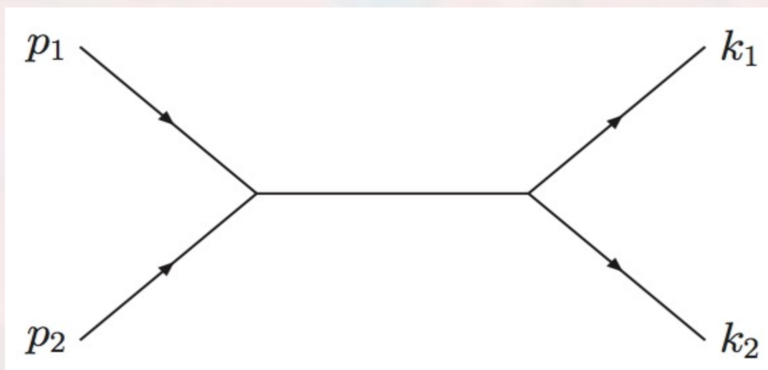
- Resummation of these large logs
  - In terms of transverse momentum dependent parton distributions and fragmentation functions and apply to
  - Semi-inclusive hadron production in DIS, Drell-Yan type of hard processes in pp collisions, e.g., Higgs, Z/W boson, …

Hard process with jet is different

- Final states carry color
  - Soft gluon radiation associated with the jet will contribute
- Jet algorithm will enter into the calculations as well
  - Only out of cone radiation contributes to the imbalance between the two jets

Leading double logs in dijet case

- Power counting: each incoming parton contributes to a half of the associated color factor



DL coefficient:
$A^{(1)}=(C_{p1}+C_{p2})/2$

Banfi-Dasgupta-Delenda, PLB 2008
Mueller-Xiao-Yuan, PRD 2013

Beyond the leading double logs

- Jet size-dependence is computed by averaging the azimuthal angle between the soft gluon and leading jet
- Matrix form due to colored final state Kidonakis-Sterman 1997

$$x_1 f_a(x_1, \mu = b_0/b_\perp) x_2 f_b(x_2, \mu = b_0/b_\perp) e^{-S_{\rm Sud}(Q^2, b_\perp)}$$
$$\mathrm{Tr}\left[\mathbf{H}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s\dagger}] \mathbf{S}_{ab\to cd} \exp[-\int_{b_0/b_\perp}^{Q} \frac{d\mu}{\mu}\gamma^{s}]\right]$$

(Sun, C.-P. Yuan, F. Yuan, PRL 2014)

$$S_{\rm Sud}(Q^2, b_\perp) = \int_{b_0^2/b_\perp^2}^{Q^2} \frac{d\mu^2}{\mu^2}\left[\ln\left(\frac{Q^2}{\mu^2}\right) A + B + D_1 \ln \frac{Q^2}{P_T^2 R_1^2} + D_2 \ln \frac{Q^2}{P_T^2 R_2^2}\right]$$

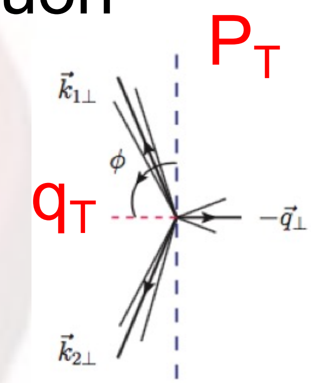
D: color-factor for the jet
R: jet size

see also, heavy quark pair resummation:
Zhu-Li-Li-Shao-Yang 2012
Catani-Grazzini-Torre 2014

Azimuthal angular asymmetries

Catani-Grazzini-Sargsyan 2017

- Azimuthal angular asymmetries arise from soft gluon radiations
  - ϕ is defined as angle between total and different transverse momenta of the two final state particles
- Infrared safe but divergent
  - $<\cos(\phi)>$, $<\cos(2\phi)>$, … divergent, $\sim 1/q_T^2$
  - Integral is finite for small q_T-cutoff, resummation can be carried out for the harmonics
  - Examples discussed include Vj, top quark pair production



Azimuthal angular correlations in jet production processes

Hatta, Xiao, Yuan, Zhou, arXiv: 2010.10774;
arXiv: 2105.xxxx, to be submitted

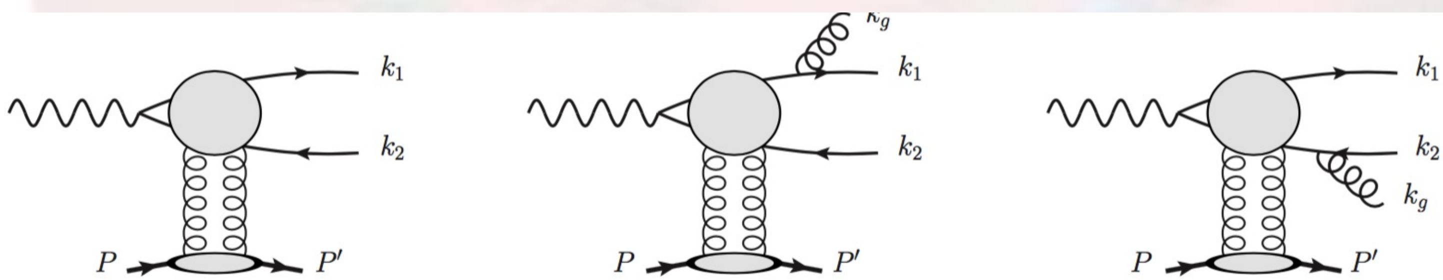
- Lepton plus jet production at the EIC
- Diffractive photoproduction of dijet
- Inclusive dijet in DIS

1. Diffractive dijet production

- Gluon radiation tends to be aligned with the jet direction

$$S_J(q_\perp) = \delta(q_\perp) + \frac{\alpha_s}{2\pi^2}\int dy_g \left(\frac{k_1\cdot k_2}{k_1\cdot k_g k_2\cdot k_g}\right)_{\vec{q}_\perp = -\vec{k}_{g\perp}}$$

$$S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|) + \cdots$$



Hatta-Xiao-Yuan-Zhou, 2010.10774
anisotropy was neglected in an earlier paper:
Hatta-Mueller-Ueda-Yuan, 1907.09491

Leading power contributions, explicit result at α_s

$$S_J(q_\perp) = S_{J0}(|q_\perp|) + 2\cos(2\phi) S_{J2}(|q_\perp|)$$

$$S_{J0}(q_\perp) = \delta(q_\perp) + \frac{\alpha_0}{\pi}\frac{1}{q_\perp^2}\,, \quad S_{J2}(q_\perp) = \frac{\alpha_2}{\pi}\frac{1}{q_\perp^2}\,,$$

where

$$\alpha_0 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_0}{R^2}\,, \quad \alpha_2 = \frac{\alpha_s C_F}{2\pi} 2\ln\frac{a_2}{R^2}\,.$$

a_0,a_2 are order 1 constants, so,

in the small-R limit, <cos(2ϕ)> goes to 1

Additional gluon radiation contributions,

- In the momentum space, it will be a convolution
  - $q_T=k_{g1}+k_{g2}+\ldots$
  - Dominant contributions will be ϕ-independent
- It is convenient to perform resummation in Fourier-b space

$$\widetilde{S}_J(b_\perp) = \int d^2q_\perp e^{iq_\perp\cdot b_\perp} S_J(q_\perp)$$
$$= \widetilde{S}_{J0}(|b_\perp|) - 2\cos(2\phi_b)\widetilde{S}_{J2}(|b_\perp|) + \cdots$$

$$\widetilde{S}_{J0}(b_\perp) = 1 + \alpha_0 \ln(\mu_b^2/P_\perp^2)\ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2$$

All order resummation, in Fourier-b space

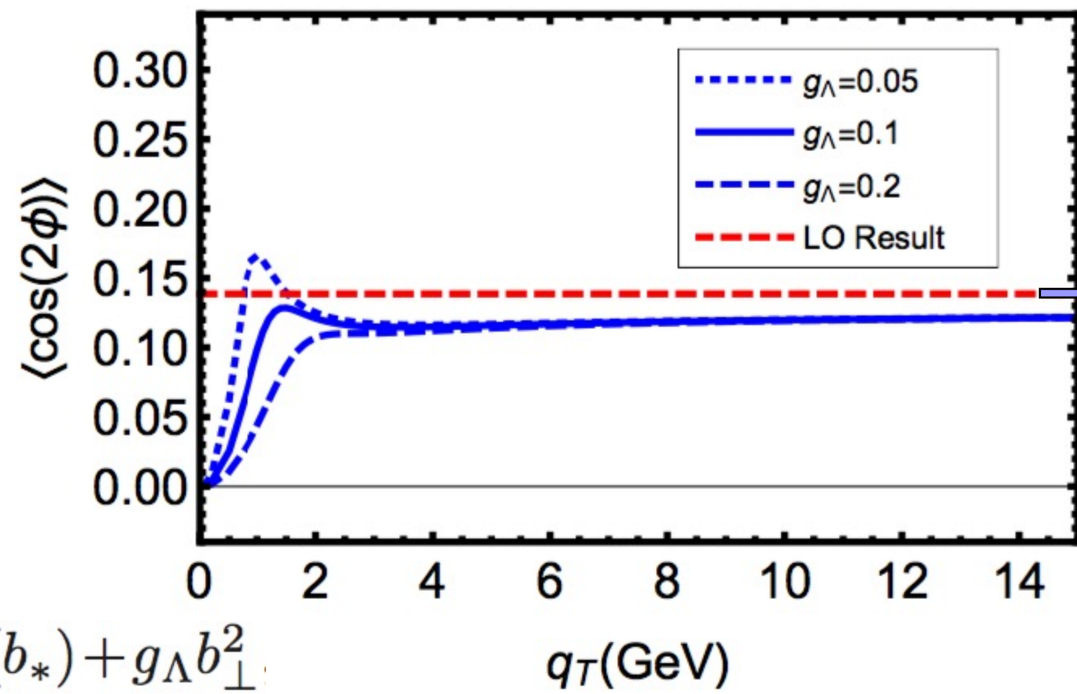
$$\widetilde{S}_{J0}(b_\perp) = e^{-\Gamma_0(b_\perp)} \ , \quad \widetilde{S}_{J2}(b_\perp) = \alpha_2 e^{-\Gamma_0(b_\perp)} \quad \Gamma_0(b_\perp) = \int_{\mu_b^2}^{P_\perp^2} \frac{d\mu^2}{\mu^2}\alpha_0$$

CMS Kinematics:

P_T~35GeV

R=0.4

y_1=y_2

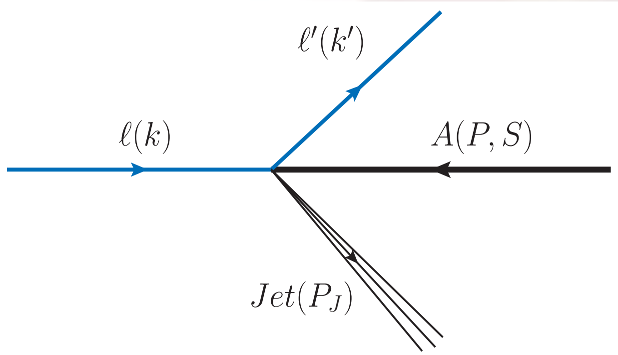


$\alpha_2/\alpha_0 \approx 0.14$

Non-pert. input:

$\Gamma_0(b_\perp) \Longrightarrow \Gamma_0(b_*) + g_\Lambda b_\perp^2$

2. Lepton-jet correlation in DIS



Quark distribution⊗soft factor

$$\frac{d^5\sigma(\ell p \to \ell' J)}{dy_\ell d^2k_{\ell\perp} d^2q_\perp} = \sigma_0 \int d^2k_\perp d^2\lambda_\perp x f_q(x, k_\perp, \zeta_c, \mu_F) \times H_{\rm TMD}(Q,\mu_F) S_J(\lambda_\perp,\mu_F)\,\delta^{(2)}(q_\perp - k_\perp - \lambda_\perp)\ .$$

Liu-Ringer-Vogelsang-Yuan 1812.08077, 2007.12866

(Lab frame)

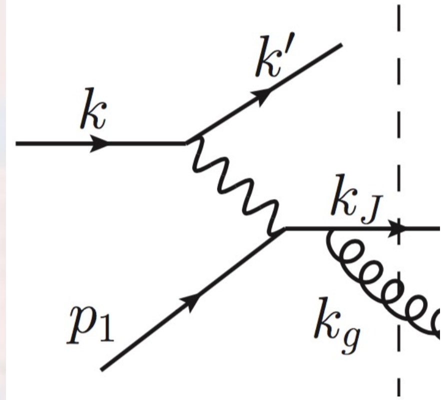
Total transverse momentum of the lepton+jet probes the TMD quark distribution

See also, Gutierrez-Reyes, Scimemi, Waalewijn, Zoppi, 1807.07573, 1904.04259

Nice results from a re-analysis of HERA data: Arratia for H1, Quintero for ZEUS

Soft gluon radiation

$$g^2 \int \frac{d^3 k_g}{(2\pi)^3 2E_{k_g}} \delta^{(2)}(q_\perp + k_{g\perp}) C_F S_g(k_J, p_1)$$
$$= \frac{\alpha_s}{2\pi^2} \frac{1}{q_\perp^2} \left[\ln \frac{Q^2}{q_\perp^2} + \ln \frac{Q^2}{k_{\ell\perp}^2} \right.$$
$$\left. + c_0 + 2c_1 \cos(\phi) + 2c_2 \cos(2\phi) + \cdots \right],$$



$$S_g(k_J, p_1) = \frac{2k_J \cdot p_1}{k_J \cdot k_g p_1 \cdot k_g}$$

- Small-R limit,

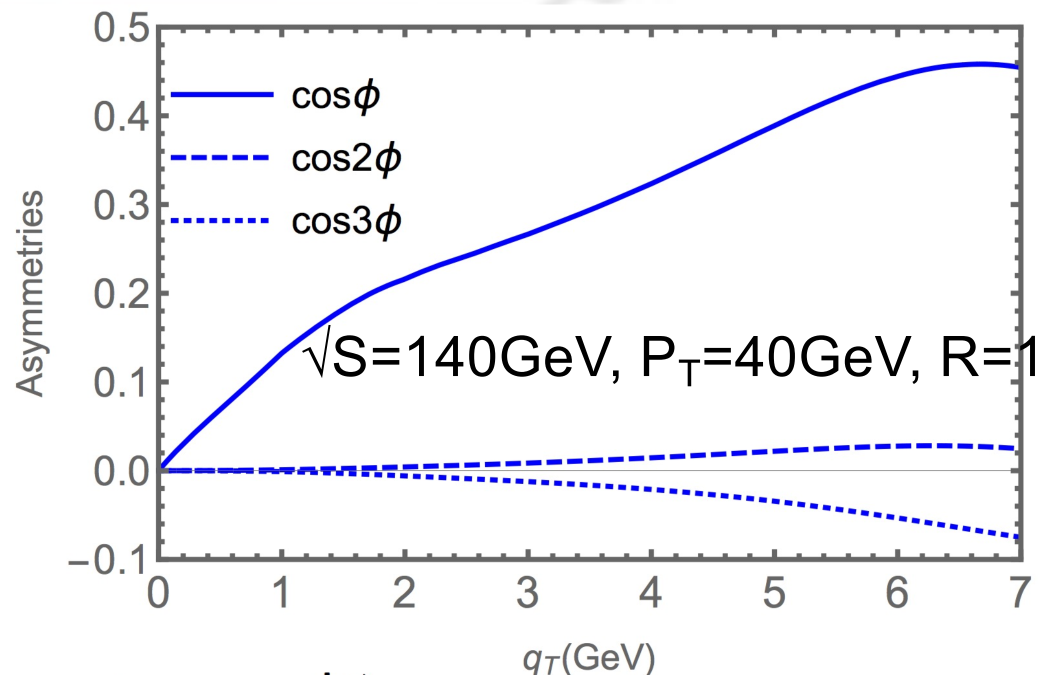
$$\ln \frac{1}{R^2} + 2\cos(\phi) \left(\ln \frac{1}{R^2} + 2\ln(4) - 2 \right) + 2\cos(2\phi) \left(\ln \frac{1}{R^2} - 1 \right)$$

Final result depends on the quark TMD

$$\frac{d^5\sigma}{dy_\ell d^2k_{\ell\perp}d^2q_\perp} = \sum_{n=1} 2\cos(n\phi)\int \frac{b_\perp db_\perp}{(2\pi)} J_n(|q_\perp||b_\perp|) \\ \times e^{-\text{ Sud}}\sum_q \sigma_0 x_q f_q(x_q,\mu_b) \qquad (15) \\ \times \int d|q'_\perp| J_n(|b_\perp||q'_\perp|)\frac{C_F\alpha_s c_n(q'^2_\perp)}{|q'_\perp|\pi}.$$

$$\text{Sud}(\mu_b^2, P_\perp^2, R) = \int_{\mu_b}^{Q}\frac{d\mu}{\mu}\left\{\frac{\alpha_s(\mu)C_F}{\pi}\left[\ln\frac{Q^2}{\mu^2}+\ln\frac{Q^2}{P_\perp^2} \right.\right. \\ \left.\left. -\frac{3}{2}+c_0(R)\right]\right\}, \qquad (14)$$

Estimate for EIC kinematics



- TMD quark follows SIYY parameterization
  - 1406.3073

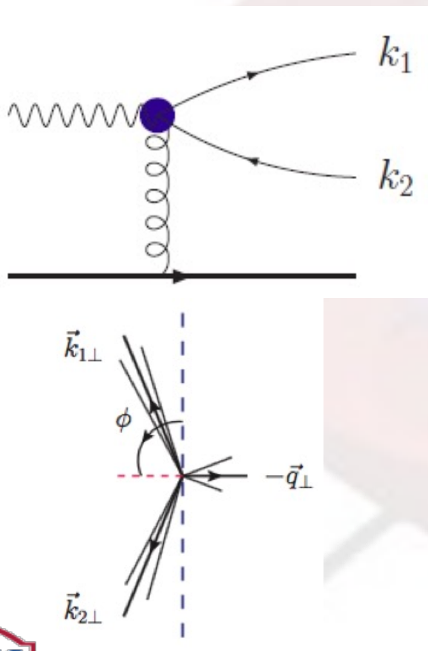
$$\mathrm{Sud}(b_\perp) \to \mathrm{Sud}(b_*) + \mathrm{Sud}^q_{\mathrm{NP}}(b_\perp) + \mathrm{Sud}^{\mathrm{jet}}_{\mathrm{NP}}(b_\perp)$$

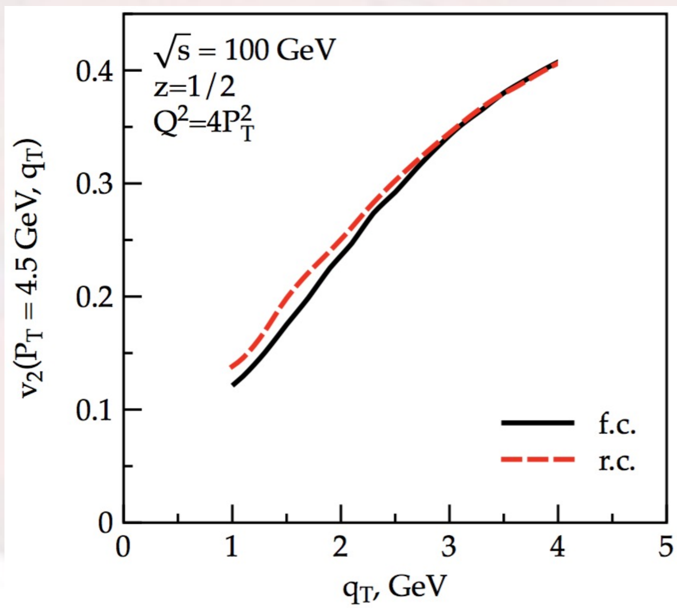
$$\mathrm{Sud}^q_{\mathrm{NP}}(b_\perp) = 0.106\, b_\perp^2 + 0.42 \ln(Q/Q_0) \ln(b_\perp/b_*)$$

$$\mathrm{Sud}^{\mathrm{jet}}_{\mathrm{NP}}(b_\perp) = g_\Lambda b_\perp^2$$

3. Inclusive Dijet in DIS

- Cos(2ϕ) anisotropy was proposed to study the linearly polarized gluon distribution





CGC calculation:
Dumitru-Lappi-Skokov,
1508.04438

see also,
Boer-Brodsky-Mulders-Pisano
1011.4225
Metz-Zhou, 1105.1991
Boer et al., 1702.08195,
1605.07934
Mantysaari et al.,
1902.05087, 1912.05586

Three contributions to cos(2ϕ) asymmetry

$$\frac{d^4\sigma}{d\Omega} = \sigma_0 \int \frac{d^2\vec{b}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}_\perp} \left[\widetilde{W}_0^{\gamma^* p}(|b_\perp|) - 2\cos(2\phi_b)\widetilde{W}_2^{\gamma^* p}(|b_\perp|)\right]$$

$$\widetilde{W}_0^{\gamma^* p}(b_\perp) = x_g\, f_g(x_g, \mu_b) e^{-\mathrm{Sud}_{\mathrm{pert}}^{\gamma^* p}(b_*) - \mathrm{Sud}_{\mathrm{NP}}^{\gamma^* p}(b_\perp)}$$

$$\widetilde{W}_2^{\gamma^* p}(b_\perp) = e^{-\mathrm{Sud}_{\mathrm{pert}}^{\gamma^* p}(b_*) - \mathrm{Sud}_{\mathrm{NP}}^{\gamma^* p}(b_\perp)}$$

$$\times \left[x_g f_g(x_g, \mu_b) \left(\alpha_2^{\gamma g} + \frac{\sigma_2}{\sigma_0} g_h(b_\perp) \right) + \frac{\sigma_2}{\sigma_0} \int \frac{dx'}{x'} x_g f_i(x', \mu) C_{h/i}^{(1)}\left(\frac{x_g}{x'}\right) \right]$$

Soft gluon from jet

Intrinsic linearly polarized gluon

Collinear splitting contribution

- Two loop calculation: Gutierrez-Reyes, et al., 1907.03780

- Numerically, contribution from soft gluon with jet is sizable
  - This can also be studied in real photon scattering process, where there is no linearly polarized gluon contribution
- The difference between the transverse and longitudinal photons purely comes from the linearly polarized gluon distribution

$$\frac{\sigma_2^L}{\sigma_0^L} = \frac{1}{2} \qquad \frac{\sigma_2^T}{\sigma_0^T} = -\frac{\epsilon_f^2 P_\perp^2}{\epsilon_f^4 + P_\perp^4}$$

Conclusion

- Soft gluon radiation can generate a sizable azimuthal asymmetry between the total and different transverse momenta of two final particles
- It provides an opportunity to explore QCD dynamics in the final state soft gluon radiation
- This physics has to be understood before we can apply the dijet azimuthal correlations to study the nucleon/nucleus tomography