Sivers and Qiu-Sterman functions

from SIDIS and Drell-Yan data

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QCD Evolution Workshop 2021

HRT

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- We [M.Bury,A.Prokudin,AV] made extraction of Sivers function from SIDIS and DY
- We fit TSSA using TMD factorization
- Many smaller studies see [2103.03270]
- In this talk, I focus on **the relation between Qiu-Sterman and Sivers functions**

$$A_{UT}^{\sin(\phi_h-\phi_S)} = -M\frac{\sum_q e_q^2\int_0^\infty \frac{bdb}{2\pi}\,b\,J_1\left(\frac{b|P_{hT}|}{z}\right)R(b,Q)f_{1T,q\leftarrow h_1}^{\perp}(x,b)D_{1,q\to h_2}(z,b)}{\sum_q e_q^2\int_0^\infty \frac{bdb}{2\pi}\,J_0\left(\frac{b|P_{hT}|}{z}\right)R(b,Q)f_{1,q\leftarrow h_1}(x,b)D_{1,q\to h_2}(z,b)}$$

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- We [M.Bury,A.Prokudin,AV] made extraction of Sivers function from SIDIS and DY
- We fit TSSA using TMD factorization
- Many smaller studies see [2103.03270]
- In this talk, I focus on **the relation between Qiu-Sterman and Sivers functions**

Sivers function

$$A_{UT}^{\sin(\phi_h-\phi_S)} = -M\frac{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi}\, b\, J_1\left(\frac{b|P_{hT}|}{z}\right) \boxed{R(b,Q)}\,\boxed{f_{1T,q\leftarrow h_1}^{\perp}(x,b)}\,\boxed{D_{1,q\to h_2}(z,b)}}{\sum_q e_q^2 \int_0^\infty \frac{bdb}{2\pi} J_0\left(\frac{b|P_{hT}|}{z}\right) \boxed{R(b,Q)}\,\boxed{f_{1,q\leftarrow h_1}(x,b) D_{1,q\to h_2}(z,b)}}$$

All numerics is done by
`Artemide`

TMD evolution
independent nonperturbative

SV19
DY+SIDIS fit
[Scimemi,AV,1912.06532]

Sivers and Qiu-Sterman functions are related to each other
alike unpolarized TMDPDF to unpolarized PDF

collinear distributions
twist-2

N \ q	U	L	T
U	q		
L		Δq	
T			δq

$b \to 0$

$k_T \to \infty$

TMD distributions

N \ q	U	L	T
U	f_1		$h_1^{\perp}$
L		g_1	$h_{1L}^{\perp}$
T	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$

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Sivers and Qiu-Sterman functions are related to each other alike unpolarized TMDPDF to unpolarized PDF

collinear distributions
twist-2 & twist-3

N \ q	U	L	T
U	q		δT_ϵ
L		Δq	δT_g
T	T	ΔT	δq

$$\xleftarrow{\quad b \to 0 \quad}$$

$$\xleftarrow{\quad k_T \to \infty \quad}$$

TMD distributions

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Warning! There is no common labeling for twist-3 functions

Sivers and Qiu-Sterman functions are related to each other
alike unpolarized TMDPDF to unpolarized PDF

collinear distributions
twist-2

N \ q	U	L	T
U	q		δT_ϵ
L		Δq	δT_g
T	T	ΔT	δq

$b \to 0$

$k_T \to \infty$

TMD distributions

N \ q	U	L	T
U	f_1		$h_1^\perp$
L		g_1	$h_{1L}^\perp$
T	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

Topic of my talk

Warning! There is no common labeling for twist-3 functions

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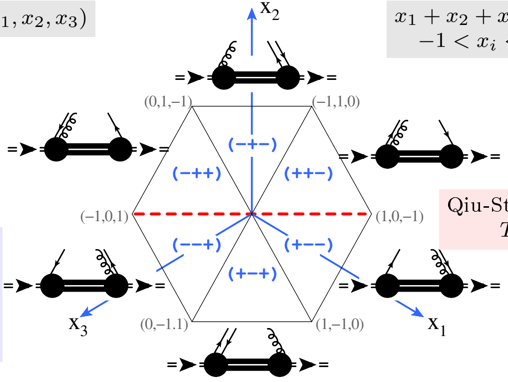
Twist-3 distributions are complicated

$T(x_1, x_2, x_3)$

$x_1 + x_2 + x_3 = 0$
$-1 < x_i < 1$



Qiu-Sterman function $T(-x, 0, x)$

Each region $x_i \lessgtr 0$ has its own partonic interpretation

Small-b matching at $\sim b$ order

[Scimemi,AV,1804.08148]

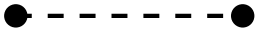
TMD operator: $\bar{q}(zn+b)[zn+b, \pm\infty n+b]\Gamma[\pm\infty n, 0]q(0)$

SIDIS$=+\infty$

DY$=-\infty$

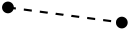
Small-b expansion

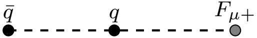
$=$



$\bar{q}(zn)[\lambda n, 0]\Gamma q(0)$ ~ 1

$+$





$\sim b$

$$b^\mu\Big\{\lim_{y\to zn}\frac{\partial}{\partial y^\mu}\bar{q}(y)[y,0]\Gamma q(0) - igz\Big[\int_0^1 v + \int_{\pm\infty}^1 + \int_{\pm\infty}^0\Big]\bar{q}(z)F_{\mu+}(vz)\Gamma q(0)\Big\}$$

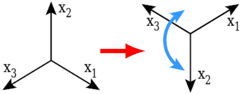
$+ \cdots$

- Parameterize matrix elements → distributions

$$\langle p,s|g\bar{q}(z_1 n)\gamma^+ F_{\mu+}(z_2 n)q(z_2 n)|p,s\rangle = 2p_+^2 \epsilon_T^{\mu\nu} s_\nu M \int [dx] e^{-ip^+ z_i x_i} T(x_1,x_2,x_3)$$

- Compute matrix element over OPE
- Use symmetries to simplify

$$T(x_1,x_2,x_3) = T(-x_3,-x_2,-x_1)$$



- Obtain

$$\begin{aligned} \text{(DY)} \qquad \Phi^{[\gamma^+]}_{q\leftarrow h}(x,\boldsymbol{b}) &= f_1(x) + ib_\mu \tilde{s}_T^\mu M\,\pi T(-x,0,x) + O(\boldsymbol{b}^2), \\ \text{(SIDIS)} \qquad \Phi^{[\gamma^+]}_{q\leftarrow h}(x,\boldsymbol{b}) &= f_1(x) - ib_\mu \tilde{s}_T^\mu M\,\pi T(-x,0,x) + O(\boldsymbol{b}^2). \end{aligned}$$

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From operators to distributions

- Parameterize matrix elements $\to$ distributions

$$\langle p,s|g\bar{q}(z_1 n)\gamma^+ F_{\mu+}(z_2 n)q(z_2 n)|p,s\rangle = 2p_+^2 \epsilon_T^{\mu\nu} s_\nu M \int [dx] e^{-ip^+ z_i x_i} T(x_1,x_2,x_3)$$

- Compute matrix element over OPE
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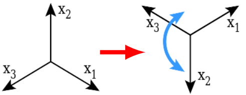
$$T(x_1,x_2,x_3) = T(-x_3,-x_2,-x_1)$$



- Obtain

$$\begin{aligned}
\text{(DY)} \qquad \Phi_{q\leftarrow h}^{[\gamma^+]}(x,\boldsymbol{b}) &= \underbrace{f_1(x)}_{f_1(x,\mathbf{b})} \underbrace{+ ib_\mu \tilde{s}_T^\mu M\, \pi T(-x,0,x)}_{f_{1T}^{\perp}(x,\mathbf{b})} + O(\boldsymbol{b}^2),\\
\text{(SIDIS)} \qquad \Phi_{q\leftarrow h}^{[\gamma^+]}(x,\boldsymbol{b}) &= f_1(x) - ib_\mu \tilde{s}_T^\mu M\, \pi T(-x,0,x) + O(\boldsymbol{b}^2).
\end{aligned}$$

Sivers function **Qiu-Sterman distribution**

$$\begin{aligned}
\text{(DY)} \qquad f_{1T}^{\perp}(x,\boldsymbol{b}) &= \pi T(-x,0,x) + O(\boldsymbol{b}^2),\\
\text{(SIDIS)} \qquad f_{1T}^{\perp}(x,\boldsymbol{b}) &= -\pi T(-x,0,x) + O(\boldsymbol{b}^2).
\end{aligned}$$

- Result depends on Wilson line's direction

$$f_{1T}^{\perp}[\text{SIDIS}] = -f_{1T}^{\perp}[\text{DY}]$$

- **This is only the leading term!**

Warning! Different notations!

$$T(-x,0,x)\Big|_{\text{here}} = \frac{-1}{\pi} f_{1T}^{\perp(1)}(x)\Big|_{\text{PV20}} = \frac{T_F(x,x)}{2\pi M}\Big|_{\text{EKT20}} = -F_{FT}(x,x)\Big|_{\text{JAM20}}$$

PV20=[Bacchetta,et al,2004.14278]
EKT20=[Echevarria,Kang,Terry,2009.10710]
JAM20=[Cammarota,et al,2002.08384]

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Higher powers in b

[Moos,AV,2008.01744]

$$f_{1T}^{\perp}(x,b) = \pm\pi\left\{T(-x,0,x) + \sum_{n=1}^{\infty}\frac{\left(\frac{x^2b^2M^2}{4}\right)^n}{(n+1)!(n-1)!}\int_x^1 \frac{du}{u}\frac{1+(n-1)u+u^2}{1-u}\frac{\bar{u}^n}{u^n}T\left(\frac{-x}{u},0,\frac{x}{u}\right)\right\}$$

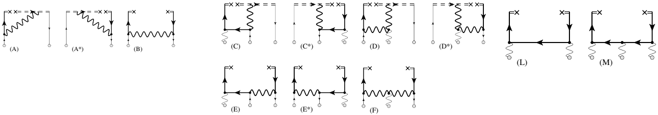
- No twist-2 contributions
- QS-function at all orders
- Target-mass corrections are suppressed
- $\text{TMD}_{\text{nuclei}} \sim \text{TMD}_{\text{hadron}}$

TMD distributions $f_{1T}^{\perp}$, g_{1T}, $h_1^{\perp}$, $h_{1L}^{\perp}$

Power	Collinear distributions					
1	$f(x)$	T_3				
b^2	$f(x)$	T_3	T_4	T_5		
b^4	$f(x)$	T_3	T_4	T_5	T_6	$\cdots$
b^6	$f(x)$	T_3	T_4	T_5	T_6	$\cdots$
	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$

Higher perturbative orders

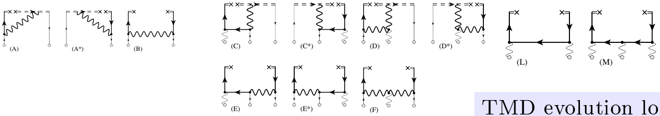
[Scimemi,Tarasov,AV,1901.04519]



$$f_{1T;q\leftarrow h;\mathrm{DY}}^{\perp}(x,\boldsymbol{b};\mu,\zeta)=\pi T(-x,0,x)+\pi a_s(\mu)\Big\{ -2\mathbf{L}_\mu P\otimes T + C_F\left(-\mathbf{L}_\mu^2+2\mathbf{l}_\zeta\mathbf{L}_\mu+3\mathbf{L}_\mu-\frac{\pi^2}{6}\right)T(-x,0,x) + \int d\xi\int_0^1 dy\delta(x-y\xi)\Big[\left(C_F-\frac{C_A}{2}\right)2\bar y T(-\xi,0,\xi)+\frac{3y\bar y}{2}\frac{G_+(-\xi,0,\xi)+G_-(-\xi,0,\xi)}{\xi}\Big]\Big\} +O(a_s^2)+O(\boldsymbol{b}^2),$$

Higher perturbative orders

[Scimemi,Tarasov,AV,1901.04519]



TMD evolution logs absent in ζ-presc.

$$f_{1T;q\leftarrow h;\text{DY}}^{\perp}(x,\boldsymbol{b};\mu,\zeta) = \pi T(-x,0,x) + \pi a_s(\mu)\Big\{ -2\mathbf{L}_\mu P\otimes T + C_F\Big(-\mathbf{L}_\mu^2 + 2\mathbf{l}_\zeta\mathbf{L}_\mu + 3\mathbf{L}_\mu - \frac{\pi^2}{6}\Big)T(-x,0,x)$$

$$+ \int d\xi \int_0^1 dy\,\delta(x-y\xi)\Big[\Big(C_F - \frac{C_A}{2}\Big)2\bar{y}T(-\xi,0,\xi) + \frac{3y\bar{y}}{2}\frac{G_+(-\xi,0,\xi)+G_-(-\xi,0,\xi)}{\xi}\Big]\Big\}$$

$$+O(a_s^2)+O(\boldsymbol{b}^2),$$

tw-3 evolution
see next slide

gluon QS-function

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Evolution mixes everything!

1/N_c-suppressed

The only QS-term

$$\mu^2 \frac{d}{d\mu^2} T(-x,0,x) = 2a_s(\mu) P \otimes T = 2a_s \int d\xi \int_0^1 dy \delta(x - y\xi) \Bigg\{$$

$$\boxed{\left(C_F - \frac{C_A}{2}\right)} \Bigg[\boxed{\left(\frac{1+y^2}{1-y}\right)_+ T(-\xi,0,\xi)} + (2y-1)_+ T(-x,\xi,x-\xi) - \Delta T(-x,\xi,x-\xi) \Bigg]$$

$$+ \frac{C_A}{2} \Bigg[\left(\frac{1+y}{1-y}\right)_+ T(-x,x-\xi,\xi) + \Delta T(-x,x-\xi,\xi) \Bigg]$$

$$+ \frac{1-2y\bar{y}}{4} \frac{G_+(-\xi,0,\xi) + Y_+(-\xi,0,\xi) + G_-(-\xi,0,\xi) + Y_-(-\xi,0,\xi)}{\xi} \Bigg\},$$

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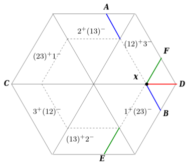
Evolution mixes everything!



$1/N_c$-suppressed

The only QS-term

$$\mu^2 \frac{d}{d\mu^2} T(-x,0,x) = 2a_s(\mu) P \otimes T = 2a_s \int d\xi \int_0^1 dy\, \delta(x-y\xi) \Big\{$$
$$\boxed{\left(C_F - \frac{C_A}{2}\right)} \Big[\left(\frac{1+y^2}{1-y}\right)_+ T(-\xi,0,\xi) + (2y-1)_+ T(-x,\xi,x-\xi) - \Delta T(-x,\xi,x-\xi) \Big]$$
$$\boxed{+\frac{C_A}{2}} \Big[\left(\frac{1+y}{1-y}\right)_+ T(-x,x-\xi,\xi) + \Delta T(-x,x-\xi,\xi) \Big]$$
$$+ \frac{1-2y\bar{y}}{4} \frac{G_+(-\xi,0,\xi) + Y_+(-\xi,0,\xi) + G_-(-\xi,0,\xi) + Y_-(-\xi,0,\xi)}{\xi} \Big\},$$

Non-QS terms

N_c-enchanted

gluon QS-functions

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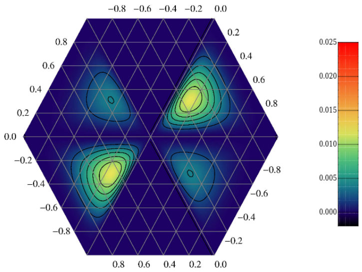
Sivers function has strong non-QS contribution

- Only at $b \to 0$
- Already logarithm term is strongly "non-QS"
- **Most probably it explains why Sivers function is "large"**
  - Model calculations show that QS is "small"



[Braun, et al, 1103.1269]

How to extract QS function from Sivers function?

- Evolution effects cannot be dropped
- Mixture with gluon and ΔT is significant
- **There is a method...** $\to$ **slide 17.**

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Common ansatz for TMDs

- Unpolarized PDF

$$f_1(x,b) = [C(\ln(b)) \otimes q](x) f_{NP}(x,b)$$

  - C is known up to N^3LO
  - $q(x)$ and $q_g(x)$ are known from DIS, DY, ...
  - f_{NP} is parameterized by 2-5 parameters

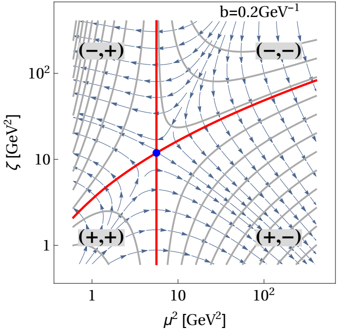
- Sivers function

$$f_{1T}^{\perp}(x,b) = [C(\ln(b)) \otimes T](x) f_{NP}(x,b)$$

  - Incorporates T, ΔT, $G^{\pm}$ each of which is a function of 2 variables
  - None of these functions is known
  - Practically, no twist-3 evolution code

Common ansatz for TMDs is not practical for Sivers function
(Boer-Mulders, worm-gears, pretzelocity)
Extract it as a whole NP function, without small-b matching!
(which is possible e.g. in ζ-prescription)

Reminder: ζ-prescription



In ζ-prescription a TMD is defined on the top of the equi-evolution line in the (μ, ζ)-evolution plane.

$$F(x, b) = F(x, b; \mu, \zeta(\mu, b))$$

- Evolution decouples from the (nonperturbative) TMD distribution (by definition)
- TMDs can be modeled without respect to small-b matching
- Perturbative order of evolution can be any (generally situation like in DIS)

Ansatz for Sivers function

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = \underbrace{N_q \frac{(1-x)x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)}}_{u,\ d,\ s,\ sea(=\bar{u},\bar{d},\bar{s})} \underbrace{\exp\left(-\frac{r_0+xr_1}{\sqrt{1+r_2x^2b^2}}b^2\right)}_{\text{Common for all flavors, Similar to unpol.}}$$

12 free parameters

- $\{r_0, r_1, r_2\}$ – TMD part
- $\{N_{u,d}, \beta_{u,d}, \epsilon_{u,d}\}$ – valence quarks
- $\{N_{s,sea}, \beta_s = \beta_{sea}\}$ – rest

No restrictions for parameters

- Data driven extraction
- Positivity (almost satisfied)

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Ansatz for Sivers function

$$f_{1T;q\leftarrow h}^{\perp}(x,b) = N_q \frac{(1-x)^{\alpha} x^{\beta_q}(1+\epsilon_q x)}{n(\beta_q,\epsilon_q)} \exp\left(-\frac{r_0 + x r_1}{\sqrt{1+r_2 x^2 b^2}} b^2\right)$$

$u,\ d,\ s,\ sea(=\bar{u},\bar{d},\bar{s})$

Sea is not constrained
$\beta_q = \beta_{sea},\quad \epsilon_s = \epsilon_{sea} = 0$

No resctrictions
typical fit $\alpha \in [0,5]$
fix $\alpha = 1$

Common for all flavors
Similar to unpol.

12 free parameters

- $\{r_0, r_1, r_2\}$ – TMD part
- $\{N_{u,d}, \beta_{u,d}, \epsilon_{u,d}\}$ – valence quarks
- $\{N_{s,sea}, \beta_s = \beta_{sea}\}$ – rest

No restrictions for parameters

- Data driven extraction
- Positivity (almost satisfied)

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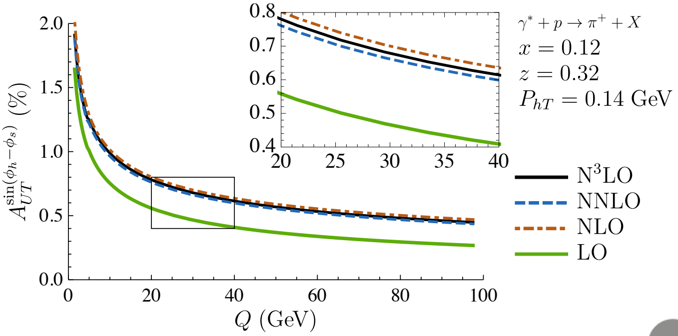
Absence of small-b matching $\neq$ tree order

We make extraction using N^3LO evolution!

N^3LO =	C_V (cancel)	γ_V	Γ_{cusp}	CS-kernel at small-b	unpol.TMD at small-b
	α_s^3	α_s^3	α_s^4	α_s^3	α_s^2



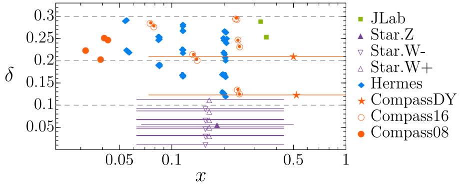
TMD factorization is valid only at $q_T/Q \ll 1$

we use $q_T < 0.3Q$ $(q_T = p_T/z)$

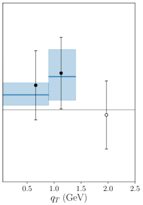
$\langle Q \rangle > 2\text{GeV}$

- ☹ Too many data do not fulfill requirement
  - COMPASS and JLab "1D" binning (integral over p_T or z)
- ☺ The latest HERMES 3D
- ☺ All points from DY (except 1) are valid

Only p_T-differential points are taken



Example: πDY



last point:

$4.3 < Q < 8.5$GeV,

$1.4 < q_T < 5.0$GeV

- SIDIS = 63 pt.
- DY = 13 pt.
- Total=76 pt.

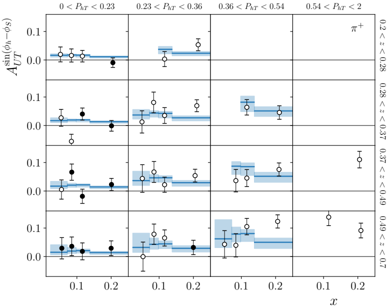
$$\chi^2/N_{pt} = 0.9$$

No principal problems with SIDIS+DY

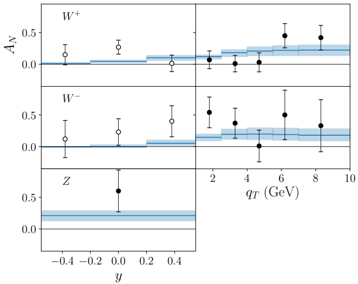
(reminder: large error-bars)

Example of data description

HERMES π^+



STAR $W^\pm/Z$



Filled points = in fit, Open point = prediction

Actually, we can explain more data (up to $q_T < 0.4Q$ in SIDIS)

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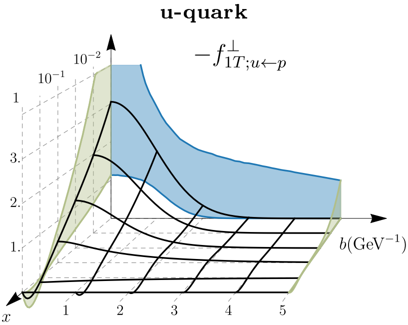
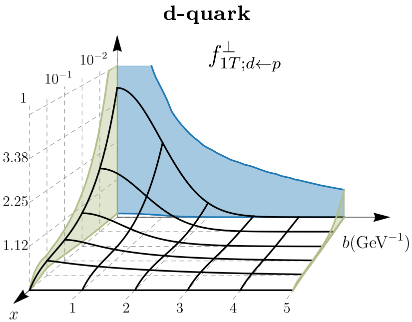
Sivers function





- Notably huge uncertainties
- Not sign definite
- Hope for data by JLab12GeV, and EIC

Extracting Qiu-Sterman function

$$f^{\perp}_{1T,q\leftarrow h}(x,b) = -\pi\Big\{T_q(-x,0,x;\mu) + a_s(\mu)\Big[\cancel{-2\mathbf{L}_\mu P\otimes T} - C_F\frac{\pi^2}{6}T(-x,0,x;\mu) + \tag{4.8}$$
$$\int_{-1}^{1}d\xi\int_0^1 dy\delta(x-y\xi)\left(-\frac{\bar y}{N_c}T_q(-\xi,0,\xi;\mu) + \frac{3y\bar y}{2\xi}G^{(+)}(-\xi,0,\xi;\mu)\right) + \mathcal{O}(a_s^2)\Big] + \mathcal{O}(b^2)\Big\},$$

- Fix $\mu = c_0/b$ ($\mathbf{L}_\mu = 0$)
  - Only QS-functions! (at this order)
- Fix b small (negligible power corections)
- Invert the formula
  - Gluon QS is still present...

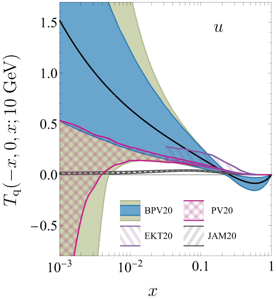
Pro: Exact relation (no modeling!)
Cons: Fixed value of μ

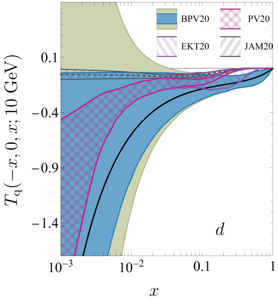
$$b = 0.11/\text{GeV} \Rightarrow \mu = 10\text{GeV}$$

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi}\left(1 + C_F a_s(\mu_b)\frac{\pi^2}{6}\right) f_{1T;q\leftarrow h}^{\perp}(x,b)$$

$$-\frac{a_s(\mu_b)}{\pi}\int_x^1 \frac{dy}{y}\left[\frac{\bar{y}}{N_c} f_{1T;q\leftarrow h}^{\perp}\left(\frac{x}{y},b\right) + \frac{3y^2\bar{y}}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2) + \mathcal{O}(b^2)$$

$$\boxed{G^{(+)} = \pm(|T_d| + |T_u|)}$$





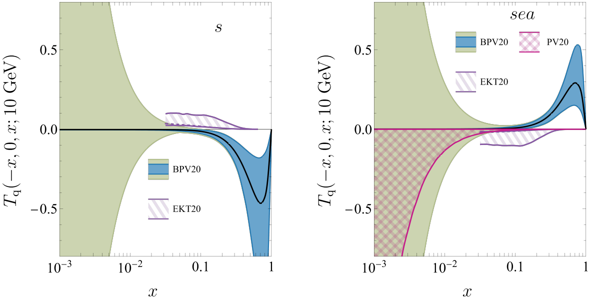
Extracting Qiu-Sterman function

$$b = 0.11/\text{GeV} \Rightarrow \mu = 10\text{GeV}$$

$$T_q(-x,0,x;\mu_b) = -\frac{1}{\pi}\left(1 + C_F a_s(\mu_b)\frac{\pi^2}{6}\right) f_{1T;q\leftarrow h}^{\perp}(x,b)$$

$$-\frac{a_s(\mu_b)}{\pi}\int_x^1 \frac{dy}{y}\left[\frac{\bar y}{N_c} f_{1T;q\leftarrow h}^{\perp}\left(\frac{x}{y},b\right) + \frac{3y^2\bar y}{2x} G^{(+)}\left(-\frac{x}{y},0,\frac{x}{y};\mu_b\right)\right] + \mathcal{O}(a_s^2) + \mathcal{O}(b^2)$$

$$\boxed{G^{(+)} = \pm(|T_d| + |T_u|)}$$



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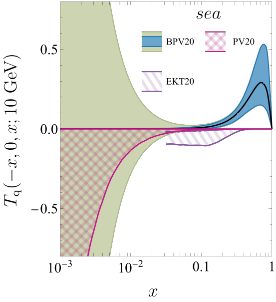
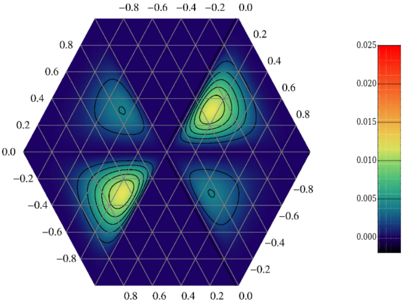




Large-x for anti-quarks (bug?/feature?)

- Main source of asymmetry in DY
- Similar to models
- Can be made smaller (within $\chi^2/N_{pt} = 1$)
- No “common” sense restrictions valence vs. sea (since it is interference term)

Waiting JLab measurements...

Conclusion

Sivers function → Qiu-Sterman function

- The small-b asymptotic for Sivers is well studied
  - LP at NLO
  - N^{k}LP at LO

Sivers function from DY+SIDIS

- N^{3}LO theory (ζ-prescription)
- Unbiased ansatz for Sivers function
- Conservative cut for data

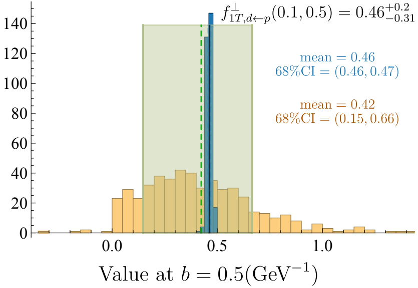
- Agreement between SIDIS and DY (accounting large error-bands)
- Many supplementary studies see [2103.03270]

Results available at

- `artemide`
- also with `TMDlib2` (set BPV20)

Backup slides

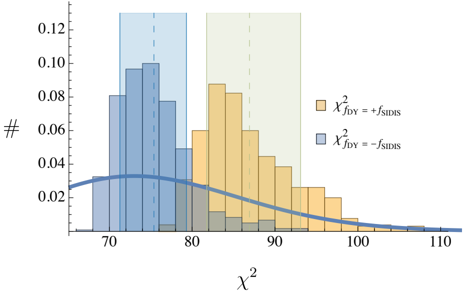
Poor data = large uncertainties



- Uncertainties estimated by replica method
  - Fitting 300 replicas of pseudo data
- Large and (often) asymmetric uncertainties
- Uncertainty due to unpol.TMD are non-negligible but much smaller then due to data

Check sign-change

$$f_{1T}^{\perp}(SIDIS) = -f_{1T}^{\perp}(DY)$$



$$f_{1T}^{\perp}(sea) \to -f_{1T}^{\perp}(sea)$$

Main source for large DY asymmetry are anti-quarks!

Current status of the small-b matching

refs. are defined in [V.Moos,AV,2008.01744]

Name	Function	Twist of leading matching	Twist-2 distributions in matching	Twist-3 distributions in matching	Order of leading power coef.function	Ref.
unpolarized	$f_1(x,b)$	tw-2	$f_1(x)$	–	N^{3}LO (α_s^3)	[21, 22]
Sivers	$f_{1T}^{\perp}(x,b)$	tw-3	–	$T(-x,0,x)$	NLO (α_s^1)	[23]
helicity	$g_{1L}(x,b)$	tw-2	$g_1(x)$	$\mathcal{T}_g(x)$	NLO (α_s^1)	[16, 17]
worm-gear T	$g_{1T}(x,b)$	tw-2/3	$g_1(x)$	$\mathcal{T}_g(x)$	LO (α_s^0)	[13, 14]
transversity	$h_1(x,b)$	tw-2	$h_1(x)$	$\mathcal{T}_h(x)$	NNLO (α_s^2)	[19]
Boer-Mulders	$h_1^{\perp}(x,b)$	tw-3	–	$\delta T_\epsilon(-x,0,x)$	LO (α_s^0)	[14]
worm-gear L	$h_{1L}^{\perp}(x,b)$	tw-2/3	$h_1(x)$	$\mathcal{T}_h(x)$	LO (α_s^0)	[13, 14]
pretzelosity	$h_{1T}^{\perp}$	tw-3/4	–	$\mathcal{T}_h(x)$	LO (α_s^0)	eq.(4.8)

- Twist-2 and twist-3 contributions at all powers of b^2 (tree)
- Typical expression (here for Sivers function):

$$f_{1T}^{\perp}(x,b)=\pm\pi\Big\{T_q(x)+\sum_{n=1}^{\infty}\left(\frac{x^2b^2M^2}{4}\right)^n\int_0^1 du\int dy\frac{\delta(x-uy)}{(n+1)!(n-1)!}\left(\frac{\bar u}{u}\right)^n\frac{1+(n-1)u+u^2}{1-u}T_q(y)\Big\}$$

  - $T(x)=T(-x,0,x)$ is Qiu-Sterman function
  - $\text{TMDs}_{proton}\sim\text{TMDs}_{nuclei}$
- Non-trivial matching for pretzelosity
  - Leading term: $h_{1T}^{\perp}(x,b)=-x^2\int_x^1\frac{du}{u}\frac{1-u^2}{u}\mathcal{T}_h\left(\frac{x}{u}\right)+\text{tw-4}$

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