Reweighting the quark Sivers function with STAR jet data

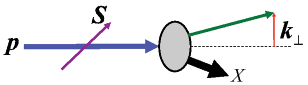
Carlo Flore

Laboratoire de Physique des 2 Infinis
Irène Joliot-Curie (IJCLab), CNRS, Orsay

QCD Evolution 2021
(virtual meeting)
10 May 2021

M. Boglione, U. D'Alesio, CF,
J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin,
Phys. Lett. B 815 (2021) 136135

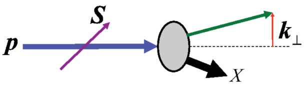
Introduction - the Sivers function



D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
& f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x, \text{-}\boldsymbol{k}_\perp) \\
&= \Delta^N f_{a/p^\uparrow}(x, k_\perp^2)\, \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp)\cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \qquad (TO - CA) \\
&= -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp a}(x, k_\perp^2)\, \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp)\cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \qquad (Amsterdam)
\end{aligned}
$$

Introduction - the Sivers function

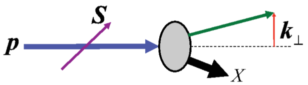


D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
& f_{a/p^\uparrow}(x,\boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x,\text{-}\boldsymbol{k}_\perp) \\
&= \Delta^N f_{a/p^\uparrow}(x,k_\perp^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\boldsymbol{S}}{|\boldsymbol{k}_\perp|} \qquad (TO-CA) \\
&= -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp a}(x,k_\perp^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\boldsymbol{S}}{|\boldsymbol{k}_\perp|} \qquad (Amsterdam)
\end{aligned}
$$

- genuine TMD distribution

Introduction - the Sivers function

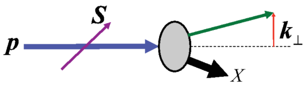


D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
& f_{a/p^\uparrow}(x,\boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x,\text{-}\boldsymbol{k}_\perp) \\
& = \Delta^N f_{a/p^\uparrow}(x,k_\perp^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (TO-CA) \\
& = -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp a}(x,k_\perp^2)\,\frac{(\hat{\boldsymbol{P}}\times\boldsymbol{k}_\perp)\cdot\boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (Amsterdam)
\end{aligned}
$$

- genuine TMD distribution
- express correlation between proton transverse polarization and parton intrinsic $\boldsymbol{k}_\perp$

Introduction - the Sivers function

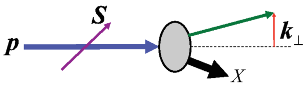


D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
& f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x, \text{-}\boldsymbol{k}_\perp) \\
&= \Delta^N f_{a/p^\uparrow}(x, k_\perp^2) \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (TO - CA) \\
&= -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp a}(x, k_\perp^2) \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (Amsterdam)
\end{aligned}
$$

- genuine TMD distribution
- express correlation between proton transverse polarization and parton intrinsic $\boldsymbol{k}_\perp$
- extracted from SIDIS azimuthal asymmetries: $A_{UT}^{\sin(\phi_h - \phi_S)} \equiv \frac{F_{UT}^{\sin(\phi_h - \phi_S)}}{F_{UU,T}} = \frac{\mathcal{C}\left[f_{1T}^{\perp q} D_1^q\right]}{\mathcal{C}\left[f_1^q D_1^q\right]}$

Introduction - the Sivers function

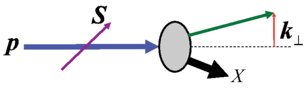


D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
& f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x, \text{-}\boldsymbol{k}_\perp) \\
&= \Delta^N f_{a/p^\uparrow}(x, k_\perp^2) \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (TO - CA) \\
&= -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp a}(x, k_\perp^2) \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (Amsterdam)
\end{aligned}
$$

- genuine TMD distribution
- express correlation between proton transverse polarization and parton intrinsic $\boldsymbol{k}_\perp$
- extracted from SIDIS azimuthal asymmetries: $A_{UT}^{\sin(\phi_h - \phi_S)} \equiv \frac{F_{UT}^{\sin(\phi_h - \phi_S)}}{F_{UU,T}} = \frac{\mathcal{C}\left[f_{1T}^{\perp q} D_1^q\right]}{\mathcal{C}\left[f_1^q D_1^q\right]}$
- modified universality/process dependence: $f_{1T}^{\perp q[\mathrm{DY}]}(x, k_\perp^2) = -f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_\perp^2)$

Introduction - the Sivers function



D.W. Sivers, PRD 41 (1990) 83 & PRD 43 (1991) 261

$$
\begin{aligned}
& f_{a/p^\uparrow}(x, \boldsymbol{k}_\perp) - f_{a/p^\uparrow}(x, \text{-}\boldsymbol{k}_\perp) \\
&= \Delta^N f_{a/p^\uparrow}(x, k_\perp^2)\, \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (TO-CA) \\
&= -\frac{2|\boldsymbol{k}_\perp|}{M} f_{1T}^{\perp a}(x, k_\perp^2)\, \frac{(\hat{\boldsymbol{P}} \times \boldsymbol{k}_\perp) \cdot \boldsymbol{S}}{|\boldsymbol{k}_\perp|} \quad (Amsterdam)
\end{aligned}
$$

- genuine TMD distribution
- express correlation between proton transverse polarization and parton intrinsic $\boldsymbol{k}_\perp$
- extracted from SIDIS azimuthal asymmetries: $A_{UT}^{\sin(\phi_h-\phi_S)} \equiv \frac{F_{UT}^{\sin(\phi_h-\phi_S)}}{F_{UU,T}} = \frac{\mathcal{C}\left[f_{1T}^{\perp q} D_1^q\right]}{\mathcal{C}\left[f_1^q D_1^q\right]}$
- modified universality/process dependence: $f_{1T}^{\perp q[\mathrm{DY}]}(x, k_\perp^2) = -f_{1T}^{\perp q[\mathrm{SIDIS}]}(x, k_\perp^2)$
- in the GPM, Sivers effect generates single spin asymmetries (SSAs) in $p^\uparrow p \to \text{jet}\ X$

Introduction - formalism

U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino *et al.*, PRD 73 (2006) 014020;
L. Gamberg, Z.-B. Kang, PLB 696 (2011) 109; U. D'Alesio *et al.*, PRD 96 (2017) 036011 ...

- Inclusive jet production in $p^{\uparrow}p$ collisions can be described in the GPM, where a factorized formulation in terms of TMDs is assumed as a starting point

Introduction - formalism

U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino *et al.*, PRD 73 (2006) 014020;
L. Gamberg, Z.-B. Kang, PLB 696 (2011) 109; U. D'Alesio *et al.*, PRD 96 (2017) 036011 ...

- Inclusive jet production in $p^{\uparrow}p$ collisions can be described in the GPM, where a factorized formulation in terms of TMDs is assumed as a starting point
- a color gauge invariant formulation of GPM (CGI-GPM) was developed, with inclusion of initial and final state interaction; process dependence is recovered

Introduction - formalism

U. D'Alesio, F. Murgia PRD 70 (2004) 074009; M. Anselmino *et al.*, PRD 73 (2006) 014020;
L. Gamberg, Z.-B. Kang, PLB 696 (2011) 109; U. D'Alesio *et al.*, PRD 96 (2017) 036011 ...

- Inclusive jet production in $p^\uparrow p$ collisions can be described in the GPM, where a factorized formulation in terms of TMDs is assumed as a starting point
- a color gauge invariant formulation of GPM (CGI-GPM) was developed, with inclusion of initial and final state interaction; process dependence is recovered
- A_N in $p^\uparrow p \to \text{jet}\, X$:

$$A_N \equiv \frac{d\sigma^\uparrow - d\sigma^\downarrow}{d\sigma^\uparrow + d\sigma^\downarrow} \equiv \frac{d\Delta\sigma}{2d\sigma}\,,$$

with

$$d\Delta\sigma^{\text{CGI-GPM}} = \frac{2\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a\, dx_b}{x_a\, x_b}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b} \left(-\frac{k_{\perp a}}{M_p}\right) f_{1T}^{\perp a}(x_a, k_{\perp a}) \cos\varphi_a$$
$$\times f_{b/p}(x_b, k_{\perp b})\, H^{\text{Inc}}_{ab\to cd}\,\delta(\hat{s}+\hat{t}+\hat{u})$$

and

$$d\sigma = \frac{\alpha_s^2}{s} \sum_{a,b,c,d} \int \frac{dx_a\, dx_b}{x_a\, x_b}\, d^2\boldsymbol{k}_{\perp a}\, d^2\boldsymbol{k}_{\perp b}\, f_{a/p}(x_a, k_{\perp a}) f_{b/p}(x_b, k_{\perp b})\, H^U_{ab\to cd}\, \delta(\hat{s}+\hat{t}+\hat{u})\,.$$

[note: $d\Delta\sigma^{\text{GPM}} = d\Delta\sigma^{\text{CGI-GPM}}\left[H^{\text{Inc}}_{ab\to cd} \to H^U_{ab\to cd}\right]$]

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

- Reweighting is a well established technique in the context of collinear PDFs fits

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

- Reweighting is a well established technique in the context of collinear PDFs fits
- it allows to assess the impact of new data sets on existing PDFs extractions, avoiding a new global fit

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

- Reweighting is a well established technique in the context of collinear PDFs fits
- it allows to assess the impact of new data sets on existing PDFs extractions, avoiding a new global fit
- used both in the Bayesian framework and in the Hessian approach

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

- Reweighting is a well established technique in the context of collinear PDFs fits
- it allows to assess the impact of new data sets on existing PDFs extractions, avoiding a new global fit
- used both in the Bayesian framework and in the Hessian approach
- applied to TMDs only for projections for future experiments.
  Never applied to a fit with actual data...

The reweighting method

W.T. Giele, S. Keller PRD 58 (1998) 094023; R.D. Ball *et al.*, NPB 849 (2011) 112;
N. Sato, J. Owens, H. Prosper, PRD 89 (2014) 114020; H. Paukkunen, P. Zurita, JHEP 12 (2014) 100

- Reweighting is a well established technique in the context of collinear PDFs fits
- it allows to assess the impact of new data sets on existing PDFs extractions, avoiding a new global fit
- used both in the Bayesian framework and in the Hessian approach
- applied to TMDs only for projections for future experiments.
  Never applied to a fit with actual data...

...up to now!

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING:

- consider a model for a TMD depending on $\boldsymbol{a} = \{a_1, \cdots, a_n\}$ parameters with prior probability distribution $\pi(\boldsymbol{a})$

[if no prior knowledge exists, $\pi(\boldsymbol{a})$ is flat]

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING:

- consider a model for a TMD depending on $\boldsymbol{a} = \{a_1, \cdots, a_n\}$ parameters with prior probability distribution $\pi(\boldsymbol{a})$

  [if no prior knowledge exists, $\pi(\boldsymbol{a})$ is flat]

- for a set of data $\boldsymbol{y} = y_1, \cdots, y_{N_{\rm dat}}$, χ^2 is defined as

$$\chi^2[\boldsymbol{a}, \boldsymbol{y}] = \sum_{i,j=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)\, C_{ij}^{-1} (y_j[\boldsymbol{a}] - y_j)$$

[if only uncorrelated uncertainties: $\chi^2[\boldsymbol{a}, \boldsymbol{y}] = \sum_{i=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)^2 / \sigma_i^2$]

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING:

- consider a model for a TMD depending on $\boldsymbol{a} = \{a_1, \cdots, a_n\}$ parameters with prior probability distribution $\pi(\boldsymbol{a})$

  [if no prior knowledge exists, $\pi(\boldsymbol{a})$ is flat]

- for a set of data $\boldsymbol{y} = y_1, \cdots, y_{N_{\rm dat}}$, χ^2 is defined as

$$\chi^2[\boldsymbol{a},\boldsymbol{y}] = \sum_{i,j=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)\, C_{ij}^{-1} (y_j[\boldsymbol{a}] - y_j)$$

[if only uncorrelated uncertainties: $\chi^2[\boldsymbol{a},\boldsymbol{y}] = \sum_{i=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)^2/\sigma_i^2$]

- best fit $\boldsymbol{a}_0$ with associated χ^2_0; uncertainty represented by $\boldsymbol{a}_k$ parameter sets $(k = 1, \cdots, N_{\rm set})$ with $\chi^2_k \in [\chi^2_0, \chi^2_0 + \Delta\chi^2]$ given by

$$\chi^2_k[\boldsymbol{a}_k,\boldsymbol{y}] = \sum_{i,j=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}_k] - y_i)\, C_{ij}^{-1} (y_j[\boldsymbol{a}_k] - y_j)$$

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

Bayesian reweighting:

- consider a model for a TMD depending on $\boldsymbol{a} = \{a_1, \cdots, a_n\}$ parameters with prior probability distribution $\pi(\boldsymbol{a})$

  [if no prior knowledge exists, $\pi(\boldsymbol{a})$ is flat]

- for a set of data $\boldsymbol{y} = y_1, \cdots, y_{N_{\rm dat}}$, χ^2 is defined as

$$\chi^2[\boldsymbol{a}, \boldsymbol{y}] = \sum_{i,j=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)\, C_{ij}^{-1} (y_j[\boldsymbol{a}] - y_j)$$

[if only uncorrelated uncertainties: $\chi^2[\boldsymbol{a}, \boldsymbol{y}] = \sum_{i=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}] - y_i)^2 / \sigma_i^2$]

- best fit $\boldsymbol{a}_0$ with associated χ^2_0; uncertainty represented by $\boldsymbol{a}_k$ parameter sets $(k = 1, \cdots, N_{\rm set})$ with $\chi^2_k \in [\chi^2_0, \chi^2_0 + \Delta\chi^2]$ given by

$$\chi^2_k[\boldsymbol{a}_k, \boldsymbol{y}] = \sum_{i,j=1}^{N_{\rm dat}} (y_i[\boldsymbol{a}_k] - y_i)\, C_{ij}^{-1} (y_j[\boldsymbol{a}_k] - y_j)$$

- parameter sets $\boldsymbol{a}_k$ produced using Monte Carlo (MC) procedures

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

Bayesian reweighting (continued):

- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a}|\boldsymbol{y}) = \frac{\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})\,\pi(\boldsymbol{a})}{Z}$$

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

Bayesian reweighting (continued):

- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a}|\boldsymbol{y}) = \frac{\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})\,\pi(\boldsymbol{a})}{Z}$$

- we take $\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})d\boldsymbol{y}$ as probability to find new data confined in a differential volume $d\boldsymbol{y}$ around $\boldsymbol{y}$; weights are then defined as

$$w_k(\chi^2_k) = \exp\left\{-\frac{1}{2}\,\chi^2_k[\boldsymbol{a}_k, \boldsymbol{y}]\right\} / \sum_i w_i$$

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING (continued):

- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a}|\boldsymbol{y}) = \frac{\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})\,\pi(\boldsymbol{a})}{Z}$$

- we take $\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})d\boldsymbol{y}$ as probability to find new data confined in a differential volume $d\boldsymbol{y}$ around $\boldsymbol{y}$; weights are then defined as

$$w_k(\chi^2_k) = \exp\left\{-\frac{1}{2}\,\chi^2_k[\boldsymbol{a}_k,\boldsymbol{y}]\right\} / \sum_i w_i$$

- expectation value and variance for an observable (symmetric)

$$\mathrm{E}[\mathcal{O}] \simeq \sum_k w_k\,\mathcal{O}(\boldsymbol{a}_k) \qquad \mathrm{V}[\mathcal{O}] \simeq \sum_k w_k\,\left(\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}]\right)^2$$

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING (continued):

- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a}|\boldsymbol{y}) = \frac{\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})\,\pi(\boldsymbol{a})}{Z}$$

- we take $\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})d\boldsymbol{y}$ as probability to find new data confined in a differential volume $d\boldsymbol{y}$ around $\boldsymbol{y}$; weights are then defined as

$$w_k(\chi_k^2) = \exp\left\{-\frac{1}{2}\,\chi_k^2[\boldsymbol{a}_k, \boldsymbol{y}]\right\} / \sum_i w_i$$

- expectation value and variance for an observable (symmetric)

$$\mathrm{E}[\mathcal{O}] \simeq \sum_k w_k\,\mathcal{O}(\boldsymbol{a}_k) \qquad \mathrm{V}[\mathcal{O}] \simeq \sum_k w_k\left(\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}]\right)^2$$

- asymmetric errors calculated using `rv_discrete` function of Python SciPy; median as central value, uncertainties at 2σ CL

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING (continued):

- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a}|\boldsymbol{y}) = \frac{\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})\,\pi(\boldsymbol{a})}{Z}$$

- we take $\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})d\boldsymbol{y}$ as probability to find new data confined in a differential volume $d\boldsymbol{y}$ around $\boldsymbol{y}$; weights are then defined as

$$w_k(\chi^2_k) = \exp\left\{-\frac{1}{2}\,\chi^2_k[\boldsymbol{a}_k,\boldsymbol{y}]\right\} / \sum_i w_i$$

- expectation value and variance for an observable (symmetric)

$$\mathrm{E}[\mathcal{O}] \simeq \sum_k w_k\,\mathcal{O}(\boldsymbol{a}_k) \qquad \mathrm{V}[\mathcal{O}] \simeq \sum_k w_k\left(\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}]\right)^2$$

- asymmetric errors calculated using `rv_discrete` function of Python SciPy; median as central value, uncertainties at 2σ CL
- new data $\boldsymbol{y}_{\rm new}$ will change the weights $w_k \to w_k^{\rm new} \equiv w_k(\chi^2_k + \chi^2_{{\rm new},k})$ and so the $\mathcal{P}(\boldsymbol{a}|\boldsymbol{y})$

The reweighting method for TMDs

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

BAYESIAN REWEIGHTING (continued):

- Posterior density through Bayes theorem:

$$\mathcal{P}(\boldsymbol{a}|\boldsymbol{y}) = \frac{\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})\,\pi(\boldsymbol{a})}{Z}$$

- we take $\mathcal{L}(\boldsymbol{y}|\boldsymbol{a})d\boldsymbol{y}$ as probability to find new data confined in a differential volume $d\boldsymbol{y}$ around $\boldsymbol{y}$; weights are then defined as

$$w_k(\chi^2_k) = \exp\left\{-\frac{1}{2}\,\chi^2_k[\boldsymbol{a}_k,\boldsymbol{y}]\right\} / \sum_i w_i$$

- expectation value and variance for an observable (symmetric)

$$\mathrm{E}[\mathcal{O}] \simeq \sum_k w_k\,\mathcal{O}(\boldsymbol{a}_k) \qquad \mathrm{V}[\mathcal{O}] \simeq \sum_k w_k\,(\mathcal{O}(\boldsymbol{a}_k) - \mathrm{E}[\mathcal{O}])^2$$

- asymmetric errors calculated using `rv_discrete` function of Python SciPy; median as central value, uncertainties at 2σ CL
- new data $\boldsymbol{y}_{\text{new}}$ will change the weights $w_k \to w_k^{\text{new}} \equiv w_k(\chi^2_k + \chi^2_{\text{new},k})$ and so the $\mathcal{P}(\boldsymbol{a}|\boldsymbol{y})$

Let's reweight the Sivers function with STAR data!

Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148

- quark Sivers function parametrization:

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f_{q/p^\uparrow}^{(1)}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

where $q = u, d$, and where $\Delta^N f_{q/p^\uparrow}^{(1)}(x)$ is the Sivers first $k_\perp$-moment:

$$\Delta^N f_{q/p^\uparrow}^{(1)}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$
$$= N_q\,(1-x)^{\beta_q}$$

Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148

- quark Sivers function parametrization:

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f^{(1)}_{q/p^\uparrow}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

where $q = u, d$, and where $\Delta^N f^{(1)}_{q/p^\uparrow}(x)$ is the Sivers first $k_\perp$-moment:

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$
$$= N_q\,(1-x)^{\beta_q}$$

- 220 data points from SIDIS data, up to $x \lesssim 0.3$

Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148

- quark Sivers function parametrization:

$$\Delta^N f_{q/p^\uparrow}(x, k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2 \rangle_S} \Delta^N f^{(1)}_{q/p^\uparrow}(x) \frac{e^{-k_\perp^2/\langle k_\perp^2 \rangle_S}}{\pi \langle k_\perp^2 \rangle_S}$$

where $q = u, d$, and where $\Delta^N f^{(1)}_{q/p^\uparrow}(x)$ is the Sivers first $k_\perp$-moment:

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p} \Delta^N f_{q/p^\uparrow}(x, k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$
$$= N_q\,(1-x)^{\beta_q}$$

- 220 data points from SIDIS data, up to $x \lesssim 0.3$
- we adopt $\Delta\chi^2$ for 5 params at 2σ CL, *i.e.* $\Delta\chi^2 = 11.31$

Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, JHEP 07 (2018) 148

- quark Sivers function parametrization:

$$\Delta^N f_{q/p^\uparrow}(x,k_\perp) = \frac{4M_p k_\perp}{\langle k_\perp^2\rangle_S}\,\Delta^N f^{(1)}_{q/p^\uparrow}(x)\,\frac{e^{-k_\perp^2/\langle k_\perp^2\rangle_S}}{\pi\langle k_\perp^2\rangle_S}$$

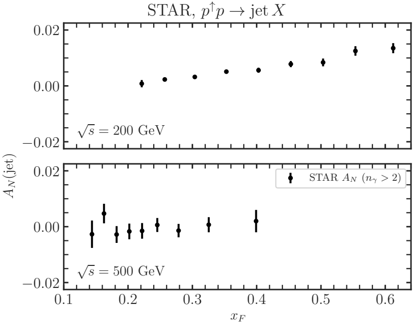
where $q = u, d$, and where $\Delta^N f^{(1)}_{q/p^\uparrow}(x)$ is the Sivers first $k_\perp$-moment:

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) = \int d^2\boldsymbol{k}_\perp \frac{k_\perp}{4M_p}\Delta^N f_{q/p^\uparrow}(x,k_\perp) \equiv -f_{1T}^{\perp(1)q}(x)$$
$$= N_q\,(1-x)^{\beta_q}$$

- 220 data points from SIDIS data, up to $x \lesssim 0.3$
- we adopt $\Delta\chi^2$ for 5 params at 2σ CL, *i.e.* $\Delta\chi^2 = 11.31$
- $2\cdot 10^5\,\boldsymbol{a}_k$ sets produced adopting Markov Chain MC with Metropolis-Hastings algorithm

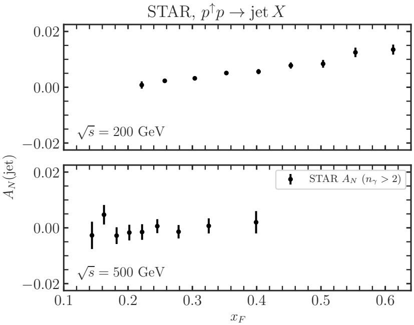
Reweighting the Sivers function (I)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135; STAR Coll., arXiv:2012.11428



Reweighting the Sivers function (I)

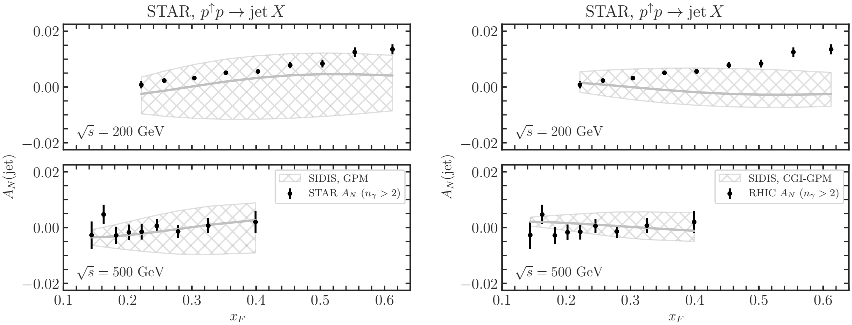
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135; STAR Coll., arXiv:2012.11428



- STAR data is for electromagnetic jets; $n_\gamma > 2$ not contaminated by single photon and isolated π^0 production; wide x_F coverage ($0.1 \lesssim x_F \lesssim 0.6$), complementary to SIDIS

Reweighting the Sivers function (I)

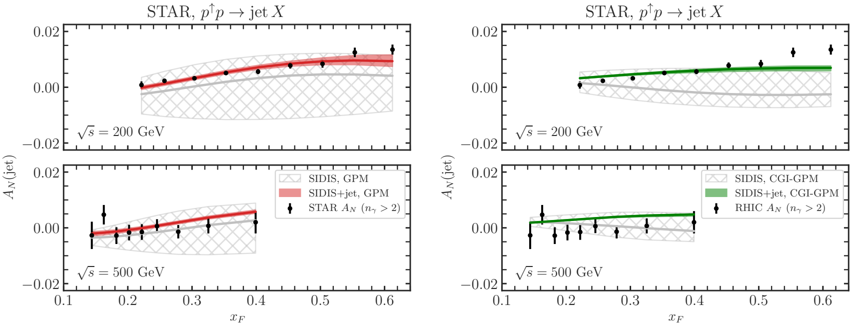
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135;
STAR Coll., arXiv:2012.11428



- STAR data is for electromagnetic jets; $n_\gamma > 2$ not contaminated by single photon and isolated π^0 production; wide x_F coverage ($0.1 \lesssim x_F \lesssim 0.6$), complementary to SIDIS
- SIDIS based predictions in the GPM (left) and in the CGI-GPM (right) describe the data within errors

Reweighting the Sivers function (I)

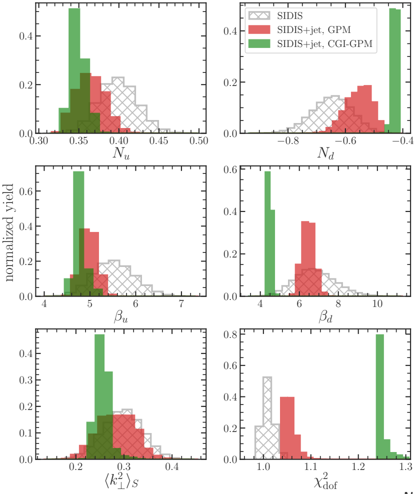
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135; STAR Coll., arXiv:2012.11428



- STAR data is for electromagnetic jets; $n_\gamma > 2$ not contaminated by single photon and isolated π^0 production; wide x_F coverage ($0.1 \lesssim x_F \lesssim 0.6$), complementary to SIDIS
- SIDIS based predictions in the GPM (left) and in the CGI-GPM (right) describe the data within errors
- reweighted curves show reduced uncertainties

Reweighting the Sivers function (II)

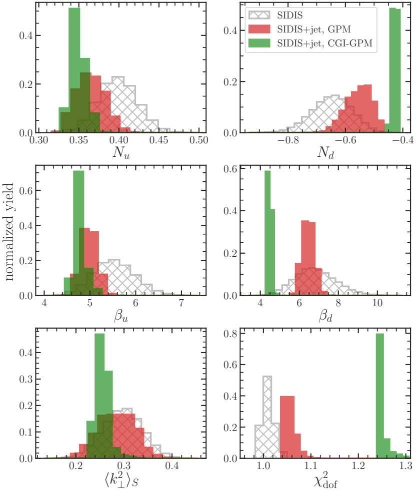
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q\,(1-x)^{\beta_q} \qquad (q = u, d)$$

Reweighting the Sivers function (II)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

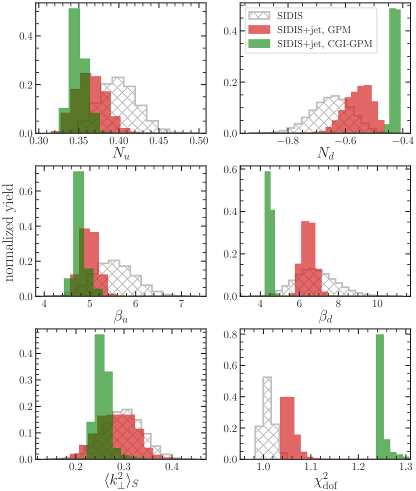


- $\langle k_\perp^2 \rangle_S$ Gaussian width does not vary much

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q\,(1-x)^{\beta_q} \qquad (q=u,d)$$

Reweighting the Sivers function (II)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

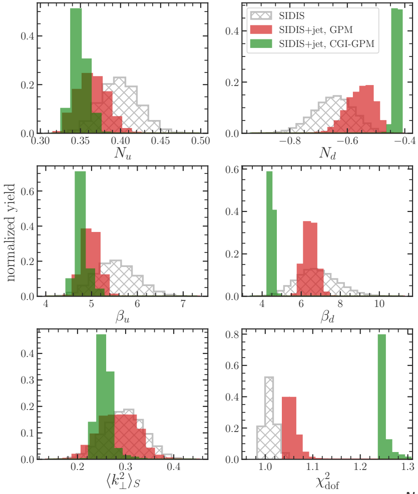


- $\langle k_\perp^2 \rangle_S$ Gaussian width does not vary much
- β_q parameters change in a different way when applying GPM or CGI-GPM formalisms

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q\,(1-x)^{\beta_q} \qquad (q = u, d)$$

Reweighting the Sivers function (II)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

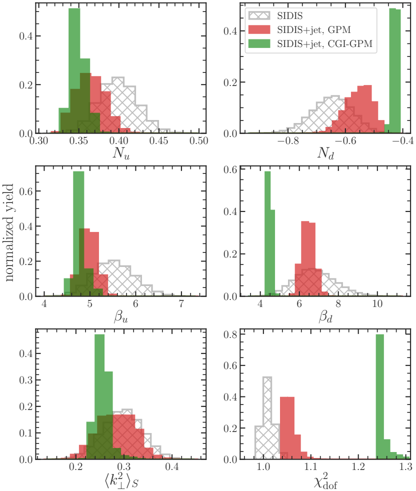


- $\langle k_\perp^2 \rangle_S$ Gaussian width does not vary much
- β_q parameters change in a different way when applying GPM or CGI-GPM formalisms
- while N_u changes slightly, N_d smaller in size in the CGI-GPM, but $f_{1T}^{\perp d}$ less suppressed in the large-x region

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q\,(1-x)^{\beta_q} \qquad (q = u, d)$$

Reweighting the Sivers function (II)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

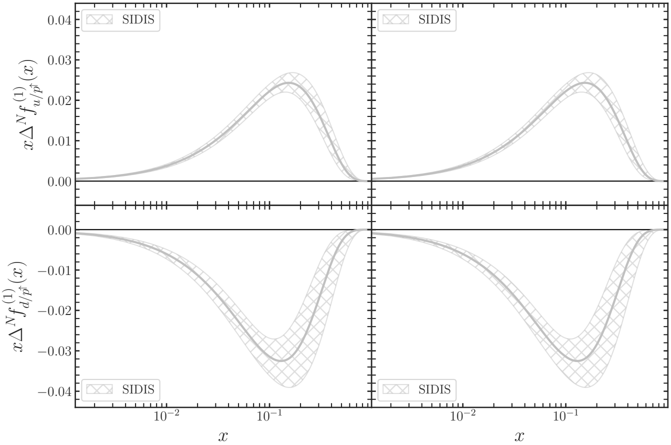


- $\langle k_\perp^2 \rangle_S$ Gaussian width does not vary much
- β_q parameters change in a different way when applying GPM or CGI-GPM formalisms
- while N_u changes slightly, N_d smaller in size in the CGI-GPM, but $f_{1T}^{\perp d}$ less suppressed in the large-x region
- $\chi^2_{\rm dof}$ after the reweighting for $N_{\rm dat}^{\rm SIDIS+jet} = 238$ slightly favors the GPM approach

$$\Delta^N f^{(1)}_{q/p^\uparrow}(x) \equiv -f_{1T}^{\perp(1)q}(x) = N_q\,(1-x)^{\beta_q} \qquad (q = u, d)$$

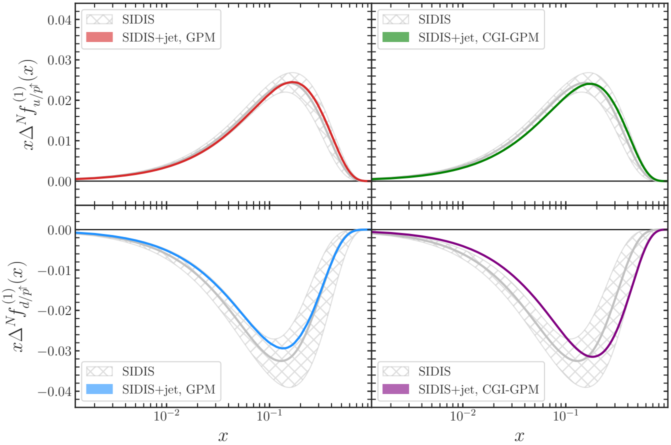
Reweighting the Sivers function (III)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



Reweighting the Sivers function (III)

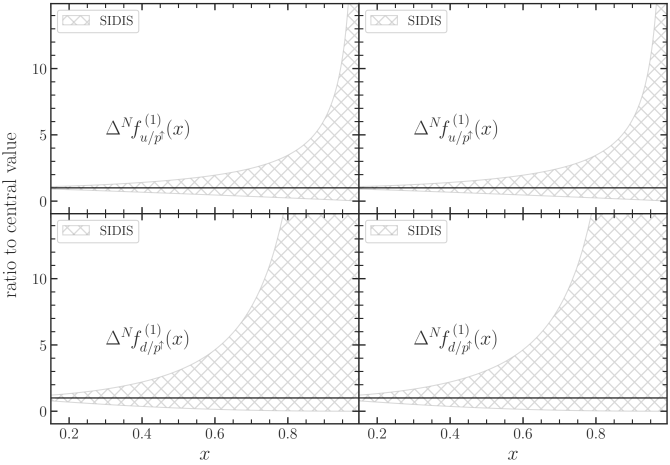
M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



drastic reduction of uncertainties

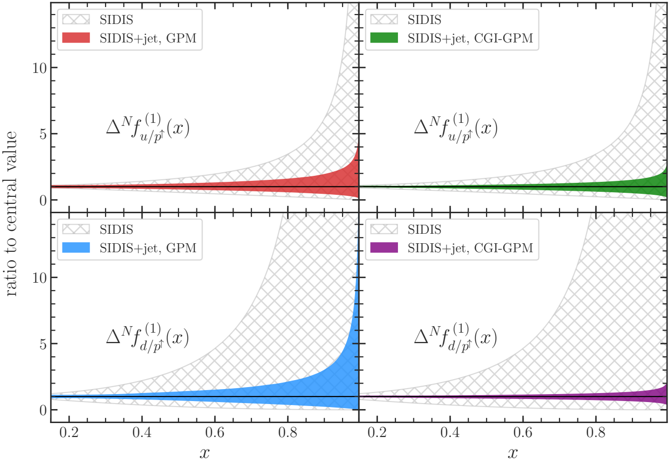
Reweighting the Sivers function (III)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



Reweighting the Sivers function (III)

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



STAR data allows to constrain the Sivers function at large *x*!

Conclusions and outlook

Conclusions and outlook

- We applied for the first time a reweighting procedure to a TMD density, the quark Sivers function

  Bayesian reweighting extended to the case of asymmetric uncertainties

Conclusions and outlook

- We applied for the first time a reweighting procedure to a TMD density, the quark Sivers function

  Bayesian reweighting extended to the case of asymmetric uncertainties

- STAR data allows to improve and extend the knowledge on the Sivers function at large x

  Complementarity of SIDIS and jet data

Conclusions and outlook

- We applied for the first time a reweighting procedure to a TMD density, the quark Sivers function

  Bayesian reweighting extended to the case of asymmetric uncertainties

- STAR data allows to improve and extend the knowledge on the Sivers function at large x

  Complementarity of SIDIS and jet data

- Natural extension of this exploratory study will be a global analysis including also A_N for inclusive pion production

  Simultaneous reweighting of Sivers, transversity and Collins functions

Conclusions and outlook

- We applied for the first time a reweighting procedure to a TMD density, the quark Sivers function

  Bayesian reweighting extended to the case of asymmetric uncertainties

- STAR data allows to improve and extend the knowledge on the Sivers function at large x

  Complementarity of SIDIS and jet data

- Natural extension of this exploratory study will be a global analysis including also A_N for inclusive pion production

  Simultaneous reweighting of Sivers, transversity and Collins functions

- Forthcoming measurements from COMPASS, JLab and the EIC will play a crucial role in unravelling the nucleon structure in its full complexity

Thank you

Backup

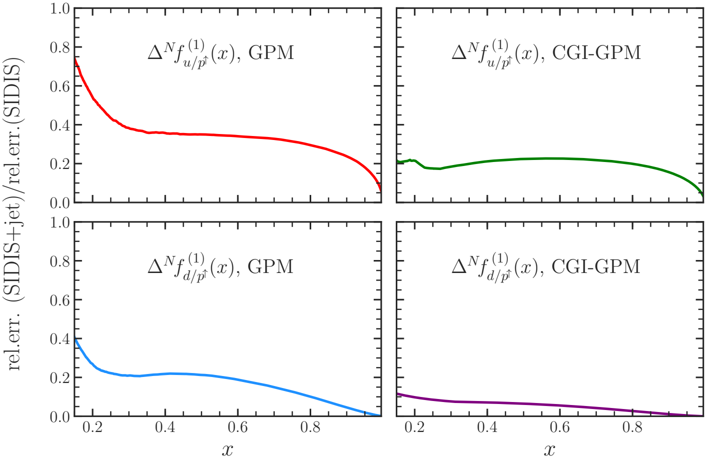
Reweighting the Sivers function - parameters

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135

	SIDIS	SIDIS+jet, GPM	SIDIS+jet, CGI-GPM
N_u	$0.40^{+0.05}_{-0.04}$	$0.36^{+0.04}_{-0.03}$	$0.35^{+0.02}_{-0.01}$
N_d	$-0.65^{+0.13}_{-0.15}$	$-0.55^{+0.07}_{-0.10}$	$-0.43^{+0.01}_{-0.02}$
β_u	$5.52^{+0.93}_{-0.83}$	$4.98^{+0.34}_{-0.30}$	$4.79^{+0.28}_{-0.19}$
β_d	$6.77^{+2.29}_{-1.85}$	$6.45^{+0.63}_{-0.52}$	$4.48^{+0.17}_{-0.13}$
$\langle k_\perp^2 \rangle_S$	$0.30^{+0.08}_{-0.08}$	$0.28^{+0.07}_{-0.07}$	$0.26^{+0.03}_{-0.02}$
χ^2_{dof}	$1.01^{+0.03}_{-0.02}$	$1.05^{+0.03}_{-0.01}$	$1.25^{+0.04}_{-0.01}$

Reweighting the Sivers function - relative errors

M. Boglione, U. D'Alesio, CF, J.O. Gonzalez-Hernandez, F. Murgia, A. Prokudin, PLB 815 (2021) 136135



Ratio of relative errors