TMD Parton Distributions up to N^{3}LL from Drell-Yan data

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3DSPIN

MAPPING THE PROTON IN 3D

Quick facts

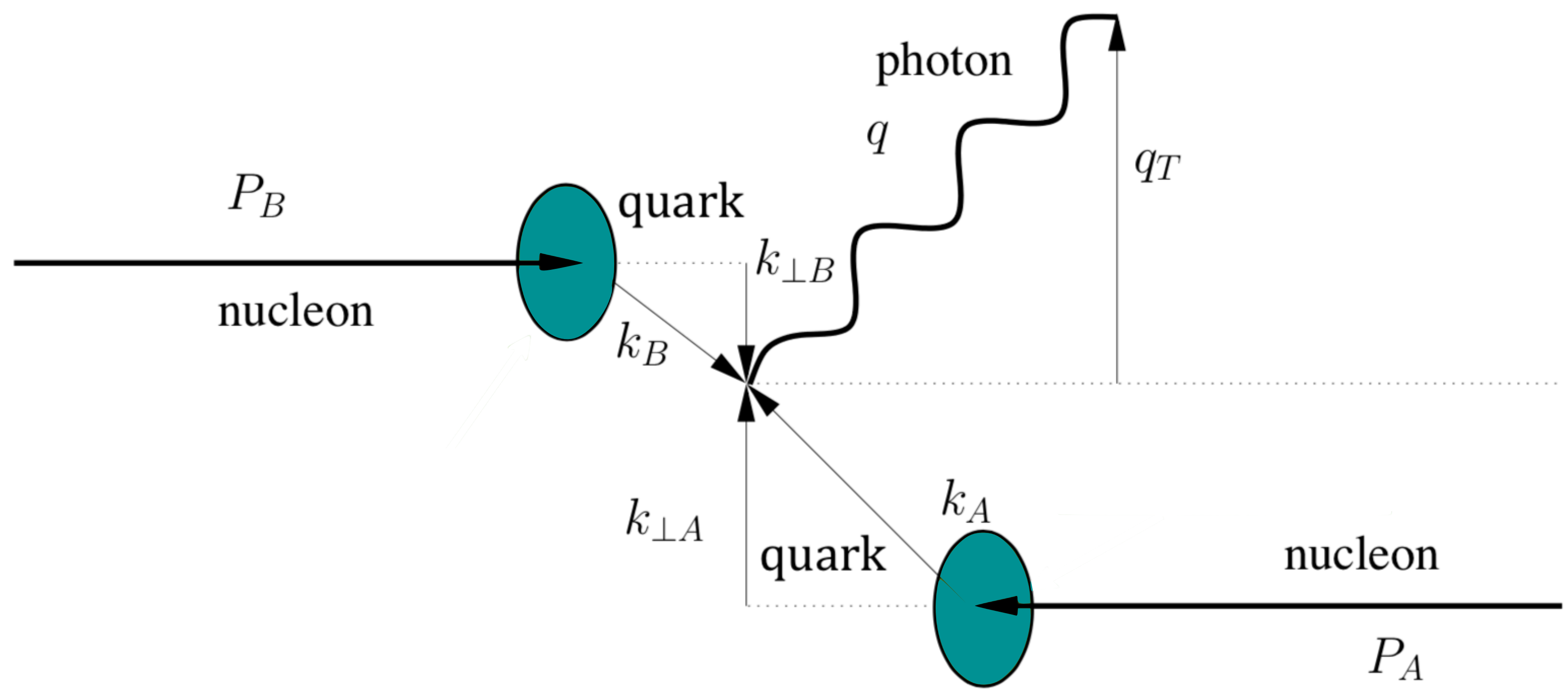
Perturbative accuracy: N^3LL
(first DY fit at this accuracy)

353 high-energy and low-energy Drell-Yan data:
the TMD "Sugarloaf" fit

$\chi^2 = 1.02$

no normalisation factors

TMD factorisation for DY



$$\left(\frac{d\sigma}{dq_T}\right)_{\text{res.}} \stackrel{\text{TMD}}{=} \sigma_0 H(Q) \int d^2\mathbf{b}_T e^{i\mathbf{b}_T\cdot\mathbf{q}_T} \underline{F_1(x_1,\mathbf{b}_T,Q,Q^2)}\,\underline{F_2(x_2,\mathbf{b}_T,Q,Q^2)}$$

quark TMD PDFs

TMD structure

$$F_{f/P}(x, \mathbf{b}_T; \mu, \zeta) = \sum_j C_{f/j}(x, b_{\boxed{*}}; \mu_b, \zeta_F) \otimes f_{j/P}(x, \mu_b) \qquad : A$$

$$\times \exp\left\{ K(b_{\boxed{*}}; \mu_b) \ln \frac{\sqrt{\zeta_F}}{\mu_b} + \int_{\mu_b}^{\mu} \frac{d\mu'}{\mu'} \left[\gamma_F - \gamma_K \ln \frac{\sqrt{\zeta_F}}{\mu'} \right] \right\} \qquad : B$$

$$\times \exp\left\{ g_{j/P}(x, b_T) + g_K(b_T) \ln \frac{\sqrt{\zeta_F}}{\sqrt{\zeta_{F,0}}} \right\} \qquad : C$$

$$\left(\mu_b = 2e^{-\gamma_E}/b_* \right)$$

- matching to collinear PDF at $b_T \ll 1/\Lambda_{QCD}$
- **perturbative**

- CS and RGE evolution to large b_T
- **perturbative**

- b_* prescription to avoid Landau pole
- f_{NP} “parametrises” the **non-perturbative** transverse modes
- **fit** f_{NP} to data

Perturbative accuracy

Accuracy	H and C	K and γ_F	γ_K	PDF and α_s evolution
LL	0	-	1	-
NLL	0	1	2	LO
NLL'	1	1	2	NLO
NNLL	1	2	3	NLO
NNLL'	2	2	3	NNLO
N^{3}LL	2	3	4	NNLO

NLL $\quad C^0 \quad \alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$

NLL' $\quad \left(C^0 + \alpha_S C^1\right) \quad \alpha_S^n \ln^{2n}\left(\frac{Q^2}{\mu_b^2}\right), \quad \alpha_S^n \ln^{2n-1}\left(\frac{Q^2}{\mu_b^2}\right)$

same logarithmic accuracy (difference = NNLL)

Non-perturbative: b*

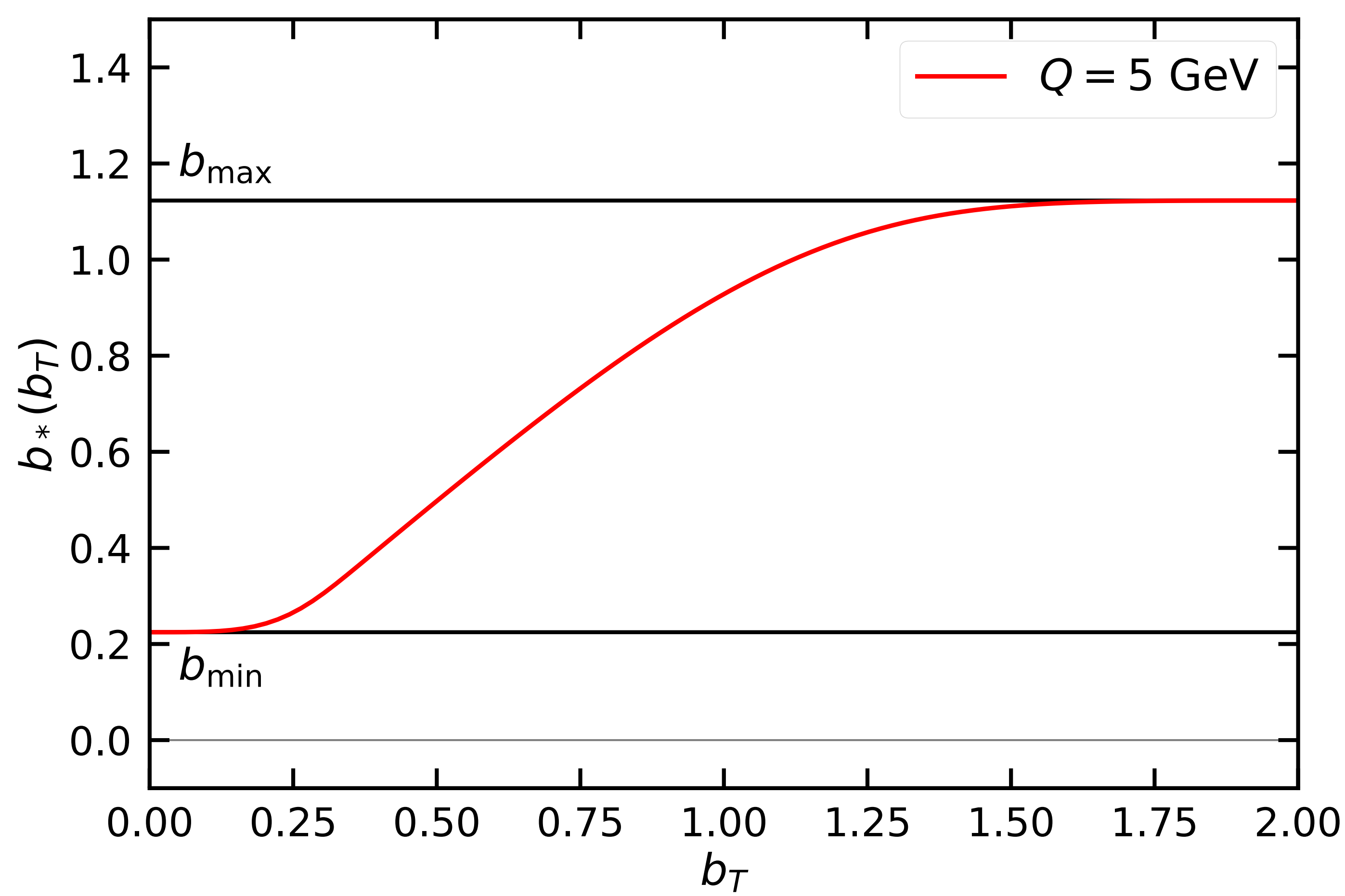
$$\alpha_s(\mu_b) = \alpha\left(\frac{2e^{-\gamma_E}}{b}\right) \gg 1 \qquad \text{for large } b \text{ values}$$

b*-prescription to avoid Landau pole at Λ_{QCD}

$$b_*(b) = b_{\max}\left(\frac{1-\exp\left(-\frac{b^4}{b_{\max}^4}\right)}{1-\exp\left(-\frac{b^4}{b_{\min}^4}\right)}\right)^{\frac{1}{4}}$$

$$b_{\max} = 2e^{-\gamma_E}$$

$$b_{\min} = 2e^{-\gamma_E}/Q$$



Non-perturbative: f_{NP}

$$F(x,b;\mu,\zeta) = \overbrace{\left[\frac{F(x,b;\mu,\zeta)}{F(x,b_*(b);\mu,\zeta)}\right]}^{f_{NP}} F(x,b_*(b);\mu,\zeta)$$

- depends on choice of b*
- determined through a fit to experimental data

$$f_{NP}(x,b,\zeta) = \Bigg[\underbrace{\frac{1-\boxed{\lambda}}{1+g_1(x)\frac{b^2}{4}}}_{\text{Q-Gaussian}} + \underbrace{\lambda \exp\left(-g_{1,B}(x)\frac{b^2}{4}\right)}_{\text{Gaussian}}\Bigg]$$

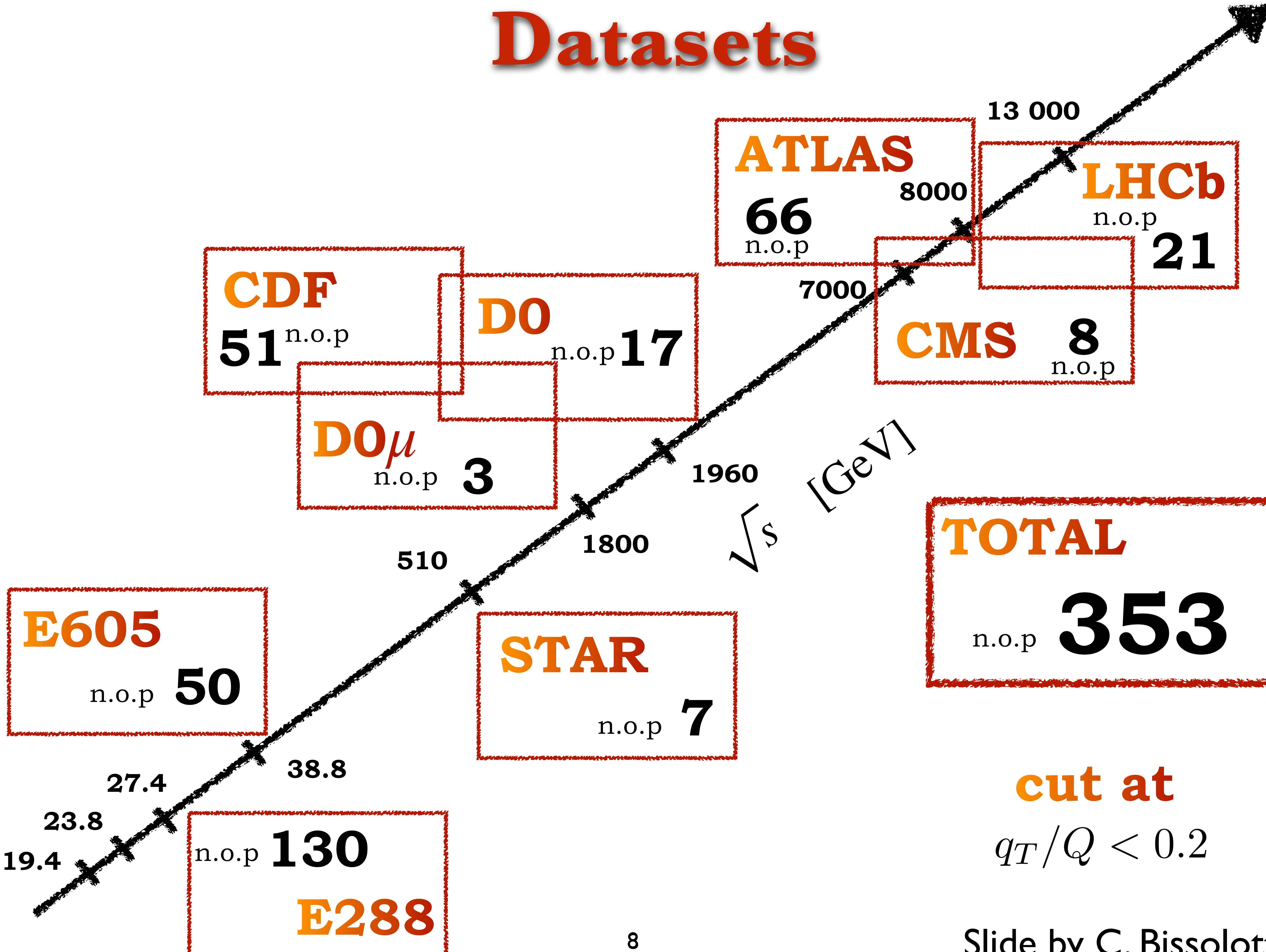
$$\times \quad \exp\left[-\left(\boxed{g_2}+\boxed{g_{2B}}b^2\right)\log\left(\frac{\zeta}{Q_0^2}\right)\frac{b^2}{4}\right]$$

$$g_1(x) = \frac{\boxed{N_1}}{x\boxed{\sigma}}\exp\left[-\frac{1}{2\sigma^2}\ln^2\left(\frac{x}{\boxed{\alpha}}\right)\right]$$

$$g_{1,B}(x) = \frac{\boxed{N_{1B}}}{x\boxed{\sigma_B}}\exp\left[-\frac{1}{2\sigma_B^2}\ln^2\left(\frac{x}{\boxed{\alpha_B}}\right)\right]$$

9 parameters

Datasets



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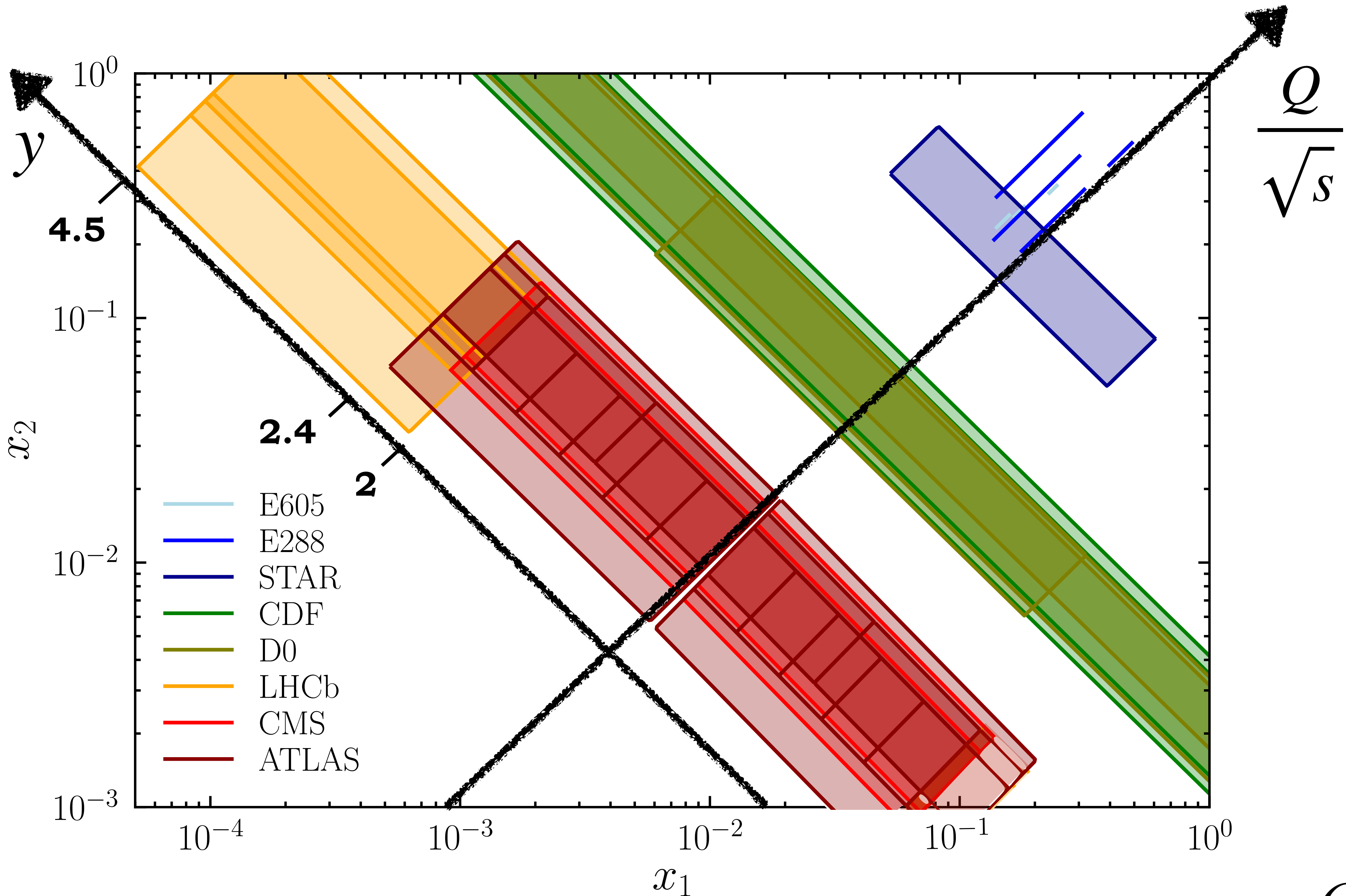
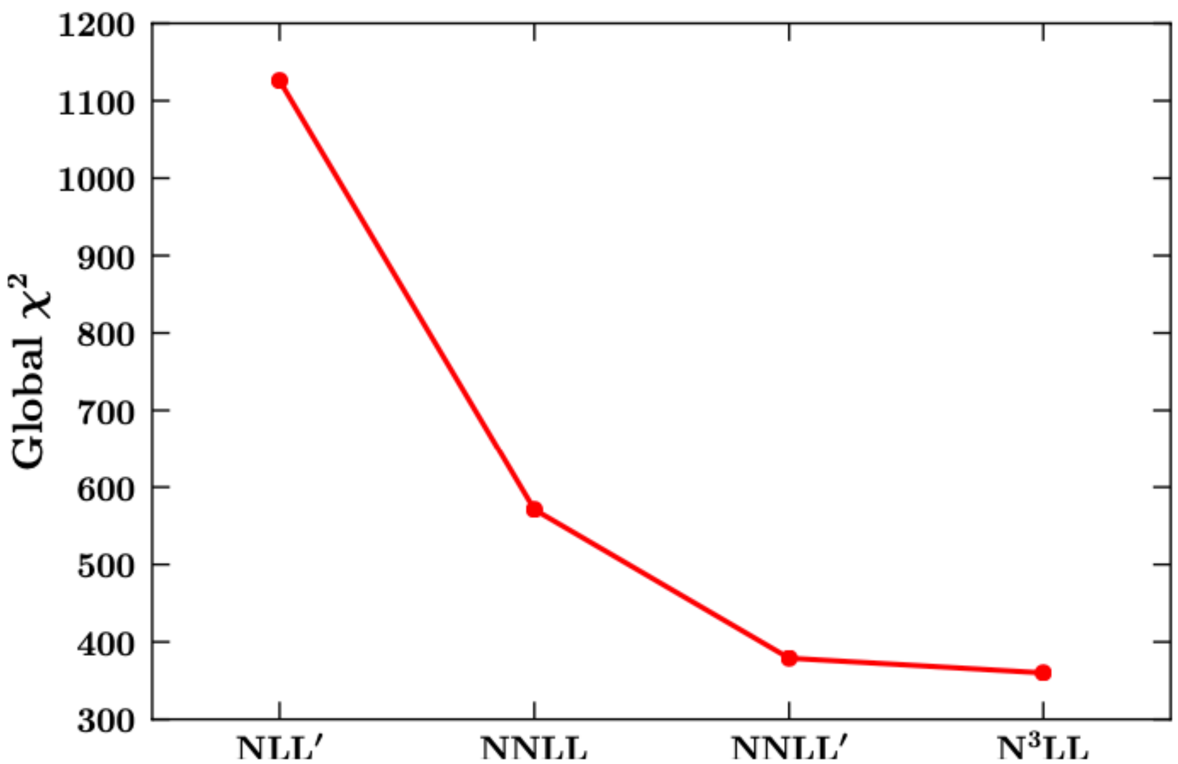
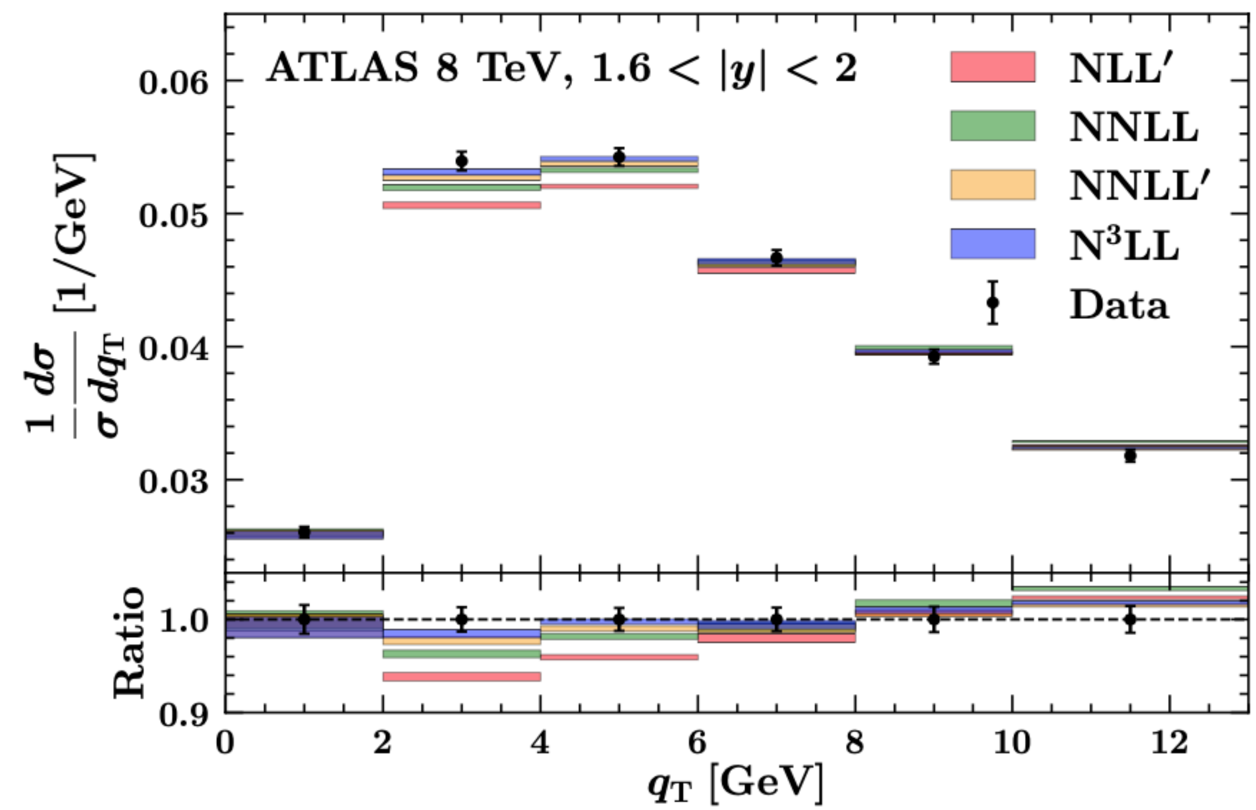
x-y-Q coverage



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

Fit quality: convergence



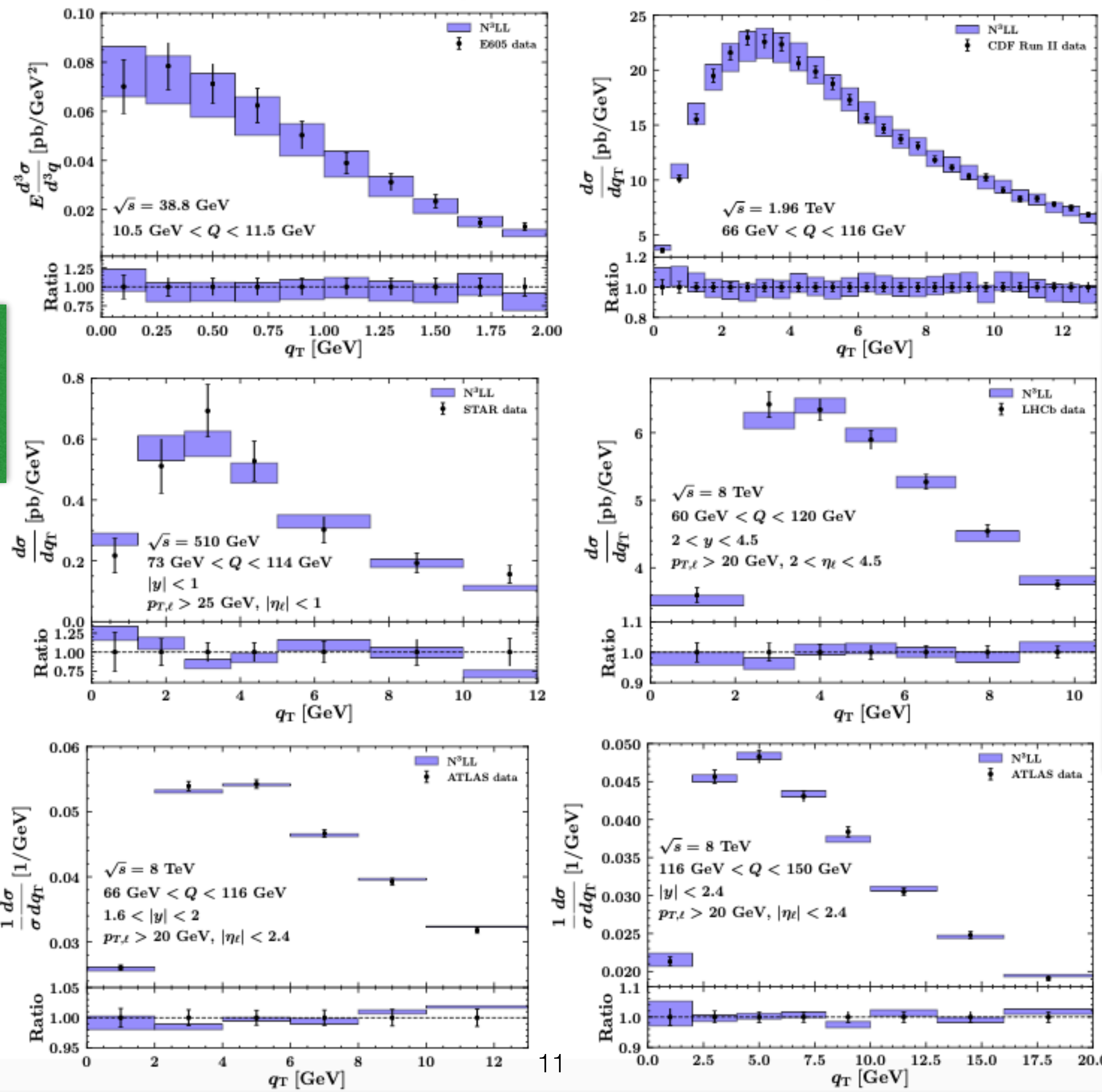


Order	NLL'	NNLL	NNLL'	N^{3}LL
χ^2 /d.o.f.	3.19	1.62	1.07	**1.02**

Fit quality: theory/data

No
ad-hoc
normalisation

when necessary,
total σ computed
with DYNNLO



Propagation of

experimental uncertainties
(corr. & uncorr.)

+

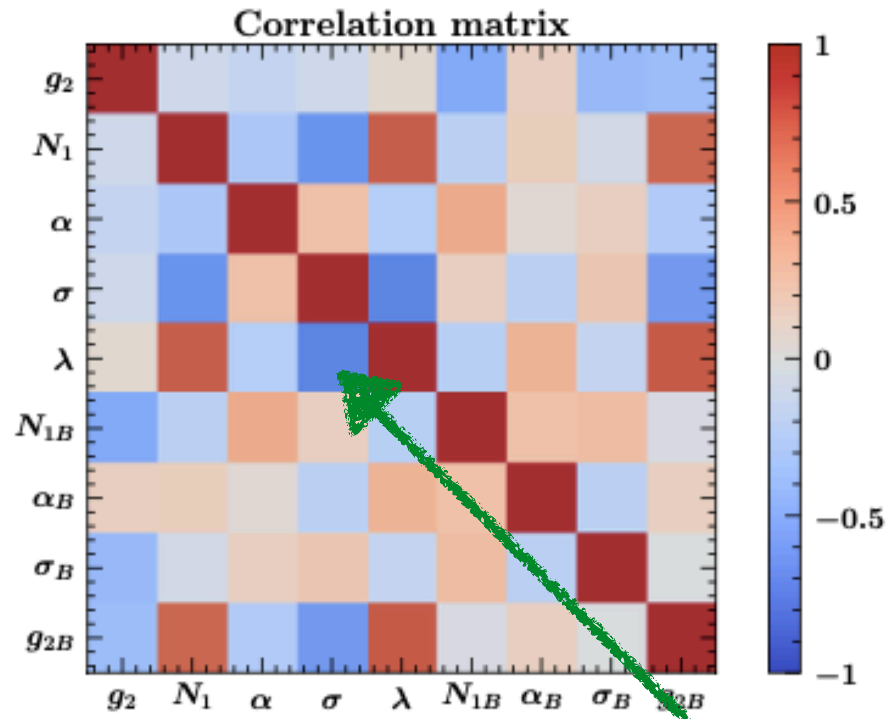
PDF uncertainty

via

Monte Carlo Sampling
(~200 replicas)

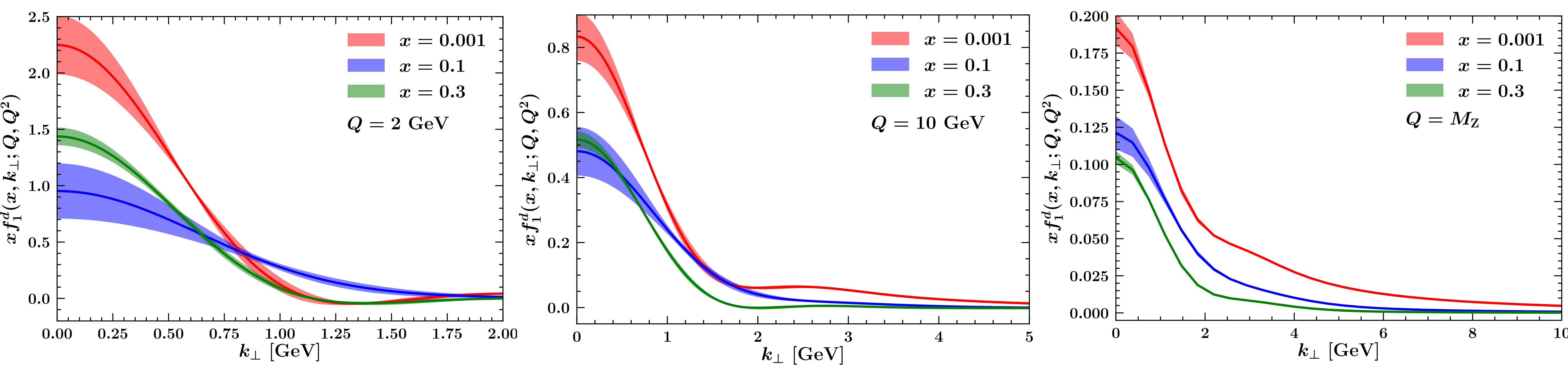
TMD distributions

Parameter	Value
g_2	0.036 ± 0.009
N_1	0.625 ± 0.282
α	0.205 ± 0.010
σ	0.370 ± 0.063
λ	0.580 ± 0.092
N_{1B}	0.044 ± 0.012
α_B	0.069 ± 0.009
σ_B	0.356 ± 0.075
g_{2B}	0.012 ± 0.003



$$f_{\mathrm{NP}}(x,b,\zeta) = \left[\frac{1-\lambda}{1+g_1(x)\frac{b^2}{4}} + \lambda \exp\left(-g_{1,B}(x)\frac{b^2}{4}\right)\right] \times \exp\left[-\left(g_2 + g_{2B}b^2\right)\log\left(\frac{\zeta}{Q_0^2}\right)\frac{b^2}{4}\right]$$

- $\lambda \sim 0.5$**:** gaussian and q-gaussian **equally important**
- g_{2B} (quartic term) small but significantly **different from zero**
- **correlations not very large** except for λ and σ



Test of x-dependence

Test: *x*-independent fit at N³LL with Davies, Webber, Stirling (1985) NP parameterisation:

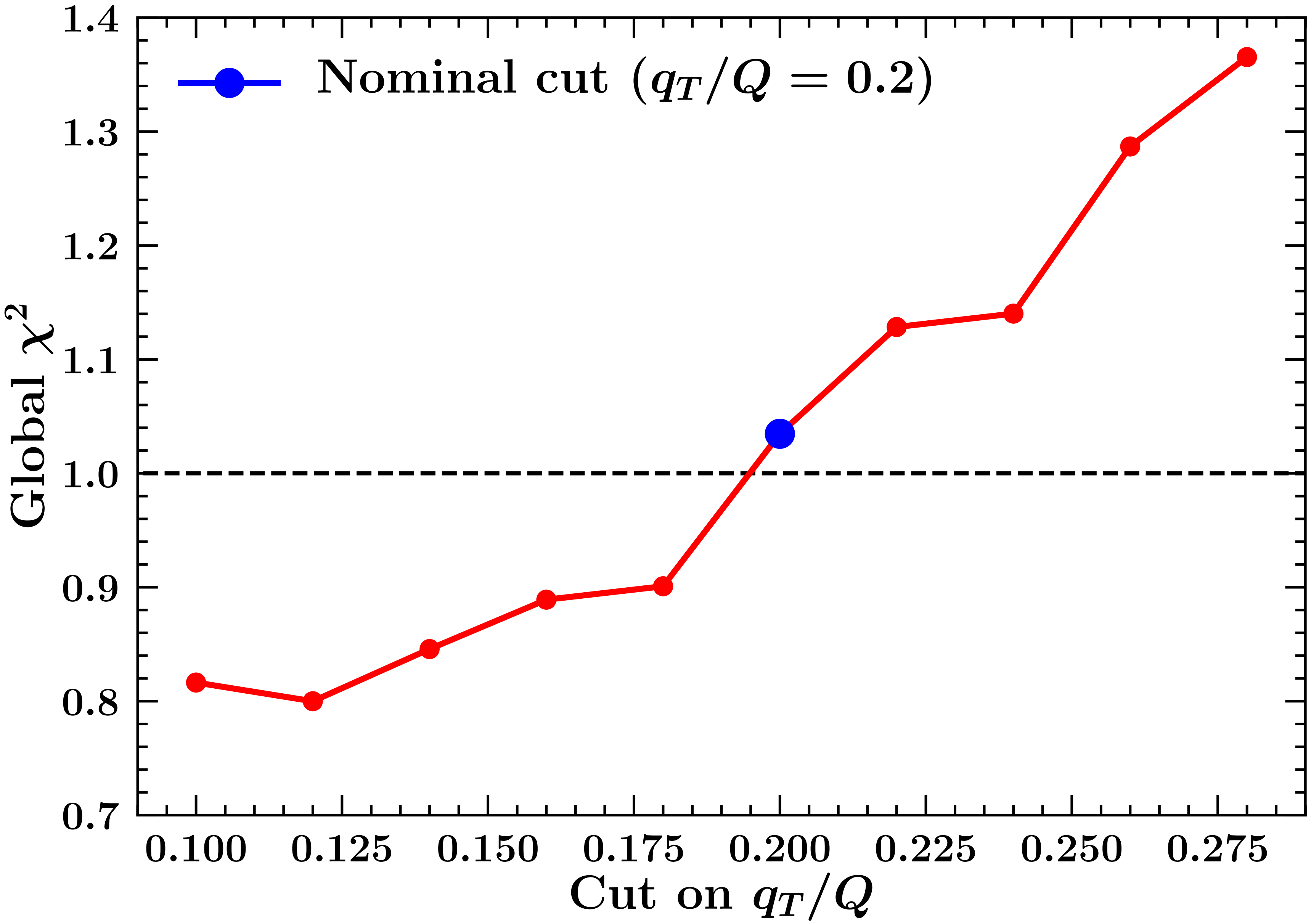
$$f_{\rm NP}^{\rm DWS}(b_T,\zeta) = \exp\left[-\frac{1}{2}\left(g_1 + g_2 \ln\left(\frac{\zeta}{2Q_0^2}\right)\right) b_T^2\right]$$

with and without ATLAS data

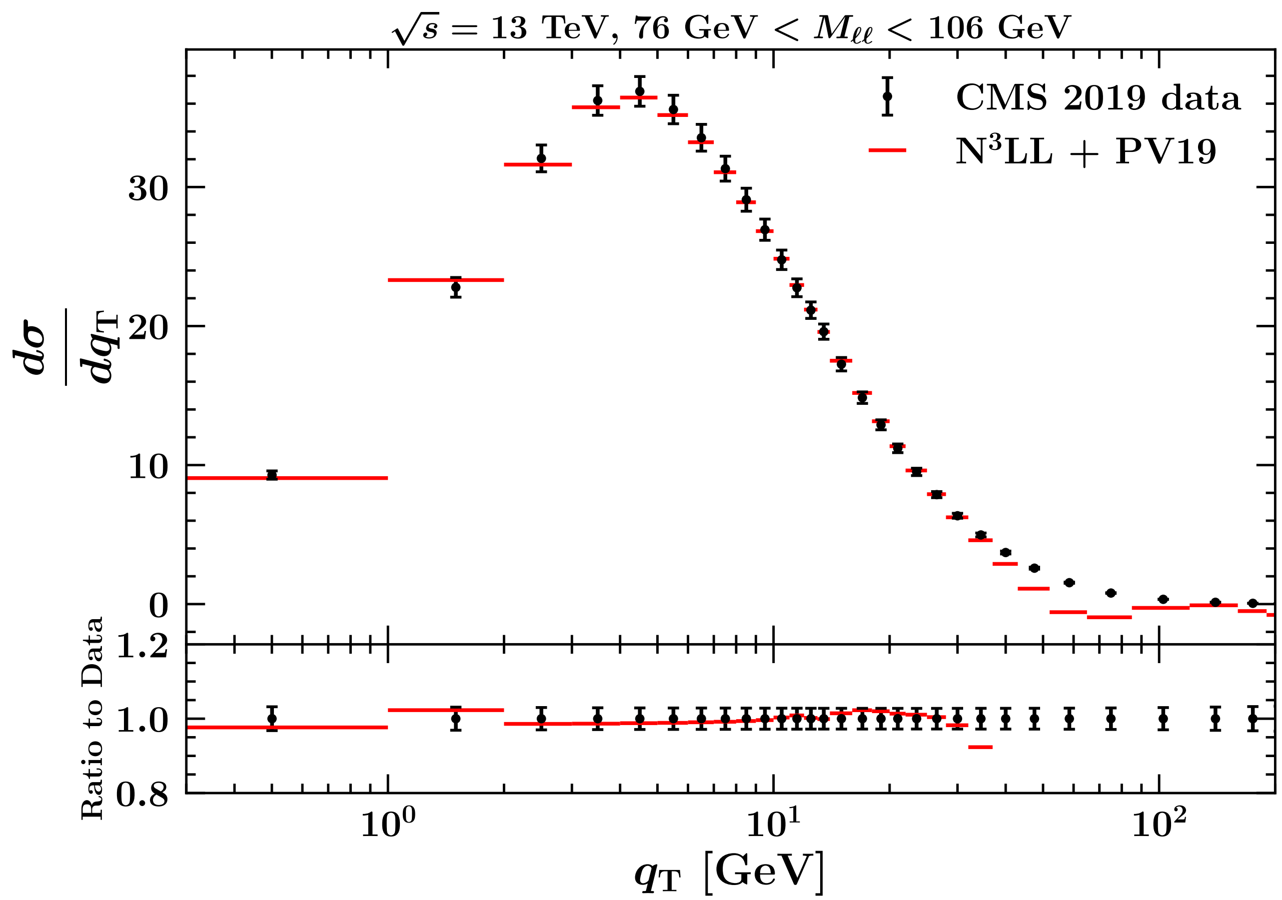
	Full dataset	No y-differential data
Global $\chi^2/N_{\rm dat}$	1.339	0.895
g_1	0.304	0.207
g_2	0.028	0.093

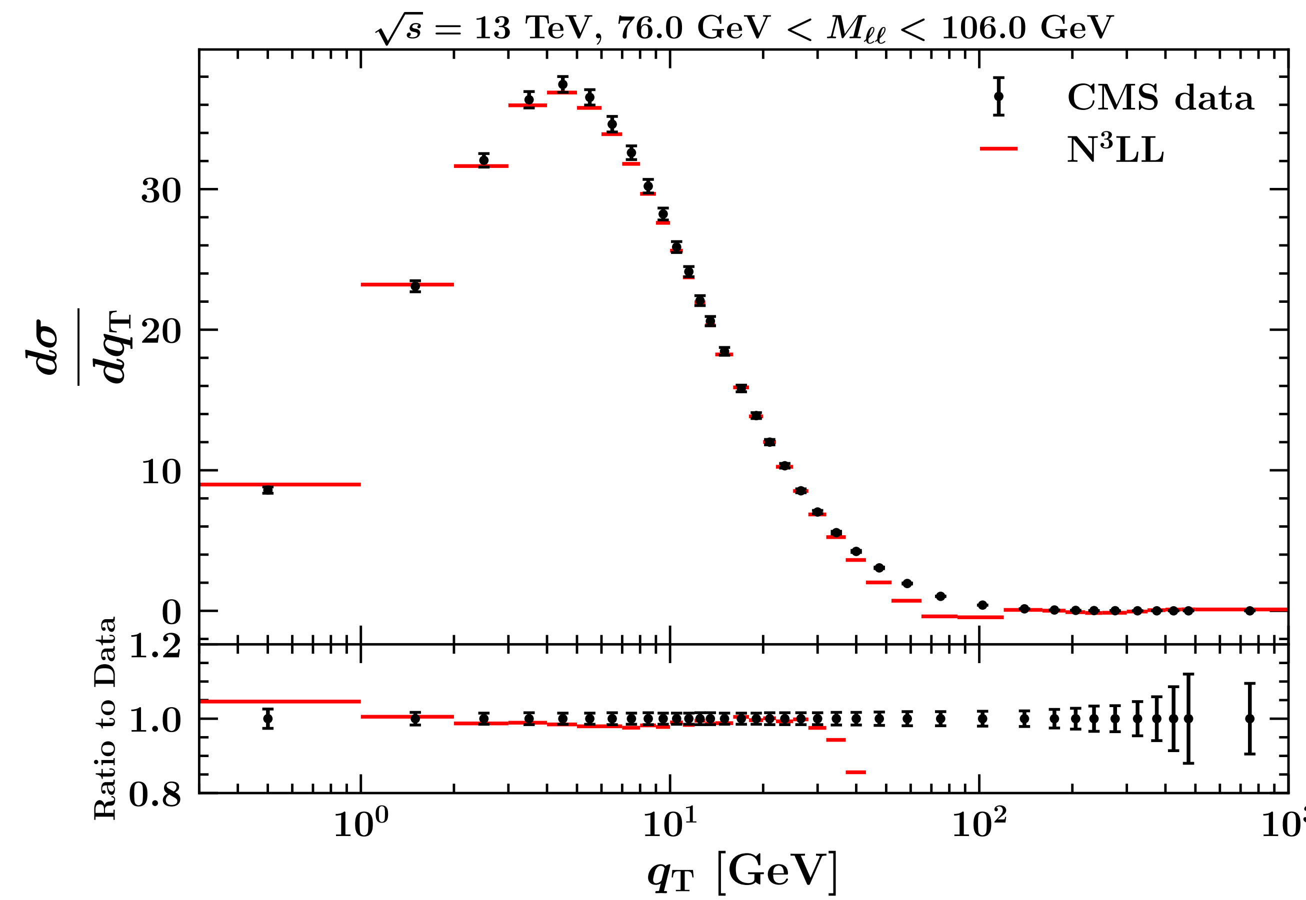
- χ^2 significantly higher for full dataset (1.339 vs. 1.020)
  - x-dependence required to describe data
- χ^2 significantly lower without ATLAS data
  - x-dependence at N³LL driven by ATLAS data

Dependence on q_T cut



Preliminary: CMS data





Very good agreement in the TMD region

The Nanga Parbat framework

Nanga Parbat: a TMD fitting framework

Nanga Parbat is a fitting framework aimed at the determination of the non-perturbative component of TMD distributions.

Download

You can obtain NangaParbat directly from the github repository:

https://github.com/MapCollaboration/NangaParbat

For the last development branch you can clone the master code:

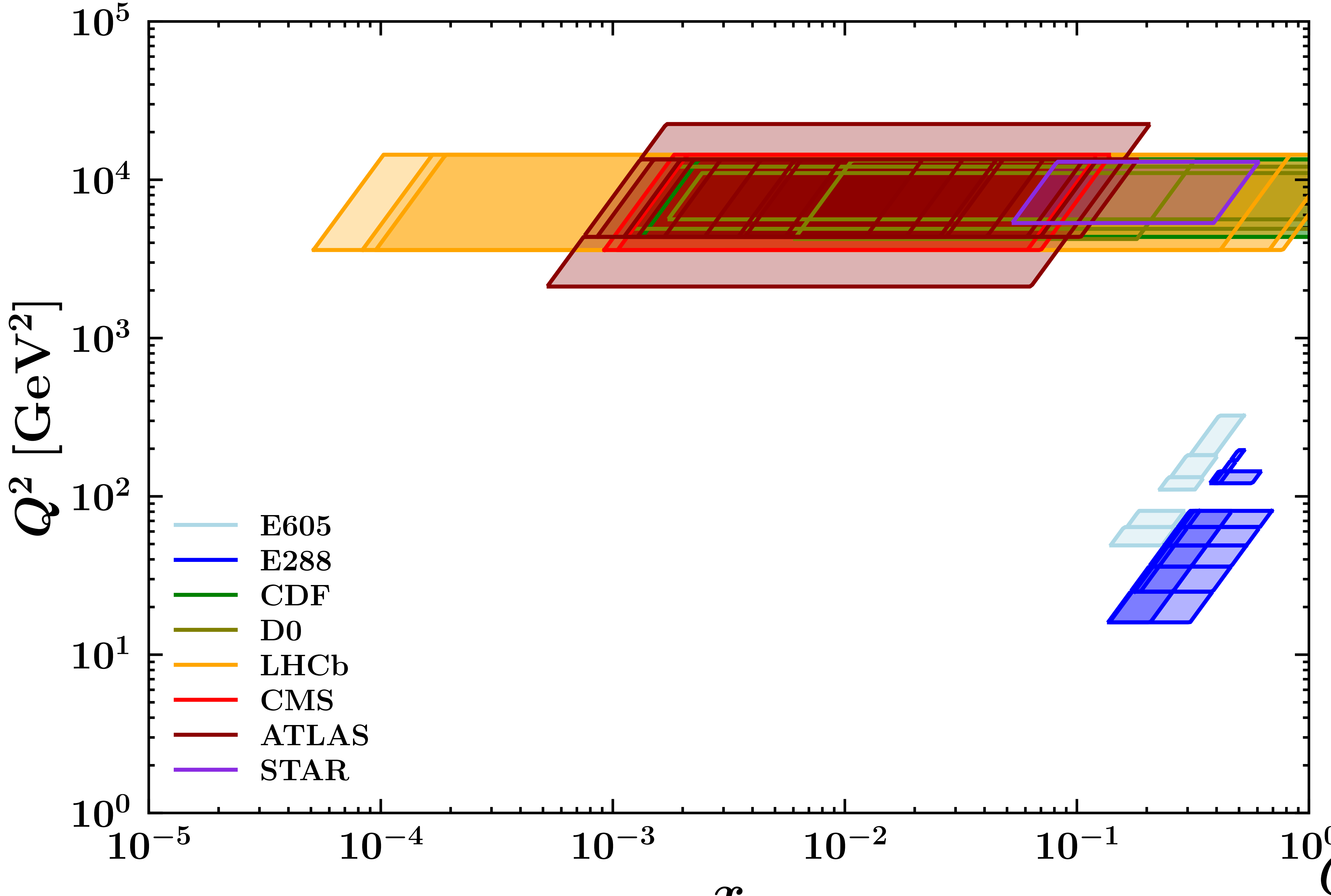
```
git clone git@github.com:MapCollaboration/NangaParbat.git
```

Conclusions

- Extraction of **TMD PDFs** from Drell-Yan data at **N^3LL** perturbative accuracy
- Low-energy (FNAL, RHIC) and high-energy (LHCb, CMS, ATLAS) data: **353 points**
- Very good description of entire dataset ($\chi^2 = 1.02$) **without *ad hoc* normalisation**
- Nice **convergence** of the perturbative series (from NLL to N^3LL)
- ***x*-dependence** mostly constrained by *rapidity-binned on-peak* ATLAS data
- check of **TMD validity range** through variation of q_T/Q cut
- **Plans for the future**: matching with Y-term for DY + fit including SIDIS data
    - TMD FFs
    - further constraints on x-dependence
    - flavour dependence

Backup

x-Q^2 coverage



$$x_{1,2} = \frac{Q}{\sqrt{s}} e^{\pm y}$$

Datasets

Experiment	N_{dat}	Observable	$\sqrt{s}$ [GeV]	Q [GeV]	y or x_F	Lepton cuts
E605	50	$Ed^3\sigma/d^3q$	38.8	7 - 18	$x_F = 0.1$	-
E288 200 GeV	30	$Ed^3\sigma/d^3q$	19.4	4 - 9	$y = 0.40$	-
E288 300 GeV	39	$Ed^3\sigma/d^3q$	23.8	4 - 12	$y = 0.21$	-
E288 400 GeV	61	$Ed^3\sigma/d^3q$	27.4	5 - 14	$y = 0.03$	-
STAR 510	7	$d\sigma/dq_T$	510	73 - 114	$\|y\| < 1$	$p_{T\ell} > 25$ GeV $\|\eta_\ell\| < 1$
CDF Run I	25	$d\sigma/dq_T$	1800	66 - 116	Inclusive	-
CDF Run II	26	$d\sigma/dq_T$	1960	66 - 116	Inclusive	-
D0 Run I	12	$d\sigma/dq_T$	1800	75 - 105	Inclusive	-
D0 Run II	5	$(1/\sigma)d\sigma/dq_T$	1960	70 - 110	Inclusive	-
D0 Run II (μ)	3	$(1/\sigma)d\sigma/dq_T$	1960	65 - 115	$\|y\| < 1.7$	$p_{T\ell} > 15$ GeV $\|\eta_\ell\| < 1.7$
LHCb 7 TeV	7	$d\sigma/dq_T$	7000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
LHCb 8 TeV	7	$d\sigma/dq_T$	8000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
LHCb 13 TeV	7	$d\sigma/dq_T$	13000	60 - 120	$2 < y < 4.5$	$p_{T\ell} > 20$ GeV $2 < \eta_\ell < 4.5$
CMS 7 TeV	4	$(1/\sigma)d\sigma/dq_T$	7000	60 - 120	$\|y\| < 2.1$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.1$
CMS 8 TeV	4	$(1/\sigma)d\sigma/dq_T$	8000	60 - 120	$\|y\| < 2.1$	$p_{T\ell} > 15$ GeV $\|\eta_\ell\| < 2.1$
ATLAS 7 TeV	6 6 6	$(1/\sigma)d\sigma/dq_T$	7000	66 - 116	$\|y\| < 1$ $1 < \|y\| < 2$ $2 < \|y\| < 2.4$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.4$
ATLAS 8 TeV on-peak	6 6 6 6 6 6	$(1/\sigma)d\sigma/dq_T$	8000	66 - 116	$\|y\| < 0.4$ $0.4 < \|y\| < 0.8$ $0.8 < \|y\| < 1.2$ $1.2 < \|y\| < 1.6$ $1.6 < \|y\| < 2$ $2 < \|y\| < 2.4$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.4$
ATLAS 8 TeV off-peak	4 8	$(1/\sigma)d\sigma/dq_T$	8000	46 - 66 116 - 150	$\|y\| < 2.4$	$p_{T\ell} > 20$ GeV $\|\eta_\ell\| < 2.4$
Total	353	-	-	-	-	-

Only data with
$q_T/Q \leq 0.2$

DYNNLO
for total σ

Experimental uncertainties

$$m_i \pm \sigma_{i,\text{stat}} \pm \sigma_{i,\text{unc}} \pm \sigma^{(1)}_{i,\text{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\text{corr}}$$

statistic

systematic

the central value of
the *i-th* measurement

Slide by C. Bissolotti

Experimental uncertainties

$$m_i \pm \sigma_{i,\text{stat}} \pm \sigma_{i,\text{unc}} \pm \sigma^{(1)}_{i,\text{corr}} \pm \cdots \pm \sigma^{(k)}_{i,\text{corr}}$$

uncorrelated **correlated**

additive **multiplicative**

$$\chi^2 = \sum_{i,j=1}^{n} (m_i - t_i)\, V^{-1}_{ij}\, (m_j - t_j)$$

$$\sigma^{(l)}_{i,\text{corr}} \equiv \delta^{(l)}_{i,\text{corr}} m_i$$

covariance matrix

$$V_{ij} = s_i^2 \delta_{ij} + \left(\sum_{l=1}^{k_a} \delta^{(l)}_{i,\text{add}} \delta^{(l)}_{j,\text{add}} + \sum_{l=1}^{k_m} \delta^{(l)}_{i,\text{mult}} \delta^{(l)}_{j,\text{mult}} \right) m_i m_j$$

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χ^2 chisquare

systematic shift

$$\boxed{\text{shift}} \quad d_i = \sum_{\alpha=1}^{k} \lambda_\alpha \sigma_{i,\mathrm{corr}}^{(\alpha)}$$

$$\frac{\partial \chi^2}{\partial \lambda_\alpha} = 0$$

nuisance parameters

$$\bar{t}_i = t_i + d_i$$

shifted prediction

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{m_i - \bar{t}_i}{s_i} \right)^2 + \sum_{\alpha=1}^{k} \lambda_\alpha^2$$

recover the form of
the uncorrelated definition

penalty term

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Experiment		$\chi^2_D/N_{\rm dat}$	$\chi^2_\lambda/N_{\rm dat}$	$\chi^2/N_{\rm dat}$
E605	$7\,\text{GeV} < Q < 8\,\text{GeV}$	0.419	0.068	0.487
	$8\,\text{GeV} < Q < 9\,\text{GeV}$	0.995	0.034	1.029
	$10.5\,\text{GeV} < Q < 11.5\,\text{GeV}$	0.191	0.137	0.328
	$11.5\,\text{GeV} < Q < 13.5\,\text{GeV}$	0.491	0.284	0.775
	$13.5\,\text{GeV} < Q < 18\,\text{GeV}$	0.491	0.385	0.877
E288 200 GeV	$4\,\text{GeV} < Q < 5\,\text{GeV}$	0.213	0.649	0.862
	$5\,\text{GeV} < Q < 6\,\text{GeV}$	0.673	0.292	0.965
	$6\,\text{GeV} < Q < 7\,\text{GeV}$	0.133	0.141	0.275
	$7\,\text{GeV} < Q < 8\,\text{GeV}$	0.254	0.014	0.268
	$8\,\text{GeV} < Q < 9\,\text{GeV}$	0.652	0.024	0.676
E288 300 GeV	$4\,\text{GeV} < Q < 5\,\text{GeV}$	0.231	0.555	0.785
	$5\,\text{GeV} < Q < 6\,\text{GeV}$	0.502	0.204	0.706
	$6\,\text{GeV} < Q < 7\,\text{GeV}$	0.315	0.063	0.378
	$7\,\text{GeV} < Q < 8\,\text{GeV}$	0.056	0.030	0.086
	$8\,\text{GeV} < Q < 9\,\text{GeV}$	0.530	0.017	0.547
	$11\,\text{GeV} < Q < 12\,\text{GeV}$	1.047	0.167	1.215
E288 400 GeV	$5\,\text{GeV} < Q < 6\,\text{GeV}$	0.312	0.065	0.377
	$6\,\text{GeV} < Q < 7\,\text{GeV}$	0.100	0.005	0.105
	$7\,\text{GeV} < Q < 8\,\text{GeV}$	0.018	0.011	0.029
	$8\,\text{GeV} < Q < 9\,\text{GeV}$	0.437	0.039	0.477
	$11\,\text{GeV} < Q < 12\,\text{GeV}$	0.637	0.036	0.673
	$12\,\text{GeV} < Q < 13\,\text{GeV}$	0.788	0.028	0.816
	$13\,\text{GeV} < Q < 14\,\text{GeV}$	1.064	0.044	1.107
STAR		0.782	0.054	0.836
CDF Run I		0.480	0.058	0.538
CDF Run II		0.959	0.001	0.959
D0 Run I		0.711	0.043	0.753
D0 Run II		1.325	0.612	1.937
D0 Run II (μ)		3.196	0.023	3.218
LHCb 7 TeV		1.069	0.194	1.263
LHCb 8 TeV		0.460	0.075	0.535
LHCb 13 TeV		0.735	0.020	0.755
CMS 7 TeV		2.131	0.000	2.131
CMS 8 TeV		1.405	0.007	1.412
ATLAS 7 TeV	$0 < \|y\| < 1$	2.581	0.028	2.609
	$1 < \|y\| < 2$	4.333	1.032	5.365
	$2 < \|y\| < 2.4$	3.561	0.378	3.939
ATLAS 8 TeV on-peak	$0 < \|y\| < 0.4$	1.924	0.337	2.262
	$0.4 < \|y\| < 0.8$	2.342	0.247	2.590
	$0.8 < \|y\| < 1.2$	0.917	0.061	0.978
	$1.2 < \|y\| < 1.6$	0.912	0.095	1.006
	$1.6 < \|y\| < 2$	0.721	0.092	0.814
	$2 < \|y\| < 2.4$	0.932	0.348	1.280
ATLAS 8 TeV off-peak	$46\,\text{GeV} < Q < 66\,\text{GeV}$	2.138	0.745	2.883
	$116\,\text{GeV} < Q < 150\,\text{GeV}$	0.501	0.003	0.504
Global		**0.88**	**0.14**	**1.02**

- Substantial contribution from χ^2_λ
- Low-energy data have lower χ^2 compared to high-energy data (larger normalisation uncertainties)
- LHCb well described
- ATLAS data well described except for low-rapidity bins
- Limited contribution from CMS (8 points)