Factorization & Resummation of Fiducial Power Corrections

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Factorization & Resummation of Fiducial Power Corrections

based on
[2006.11382, 2102.08039]

in collaboration with
G. Billis, B. Dehnadi, M. Ebert, I. Stewart, F. Tackmann

Drell-Yan $q_T \equiv p_T^Z = p_T^{\ell\ell}$ spectrum, inclusive over decay products, with $L = \ln \frac{q_T}{Q}$:

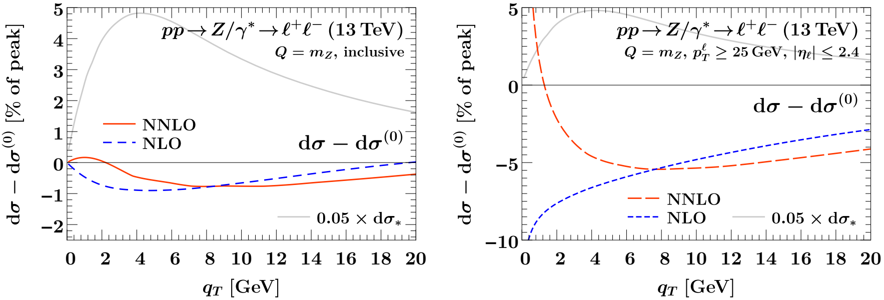
$$\frac{d\sigma}{dq_T} \sim \frac{\alpha_s}{q_T}\left[(L + \cdots) + \frac{q_T^2}{Q^2}(L + \cdots) + \mathcal{O}(q_T^2)\right] + \frac{\alpha_s^2}{q_T}\left[(L^3 + \cdots) + \frac{q_T^2}{Q^2}(L^3 + \cdots) + \mathcal{O}(q_T^2)\right] + \mathcal{O}(\alpha_s^3)$$

- Singular terms $\sim 1/q_T^2$ fully predicted by TMD factorization
- Subleading terms suppressed as $\mathcal{O}(q_T^2/Q^2)$
  - Finite as $q_T \to 0$, safe to include by a fixed-order matching (Y term)
  - Important progress on fixed-order structure, large-N_c properties, ...
    [Balitsky, Tarasov '17; Ebert, Moult, Stewart, Tackmann, Vita, Zhu '18]

With fiducial acceptance cuts on leptons, as applied in experiments:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}q_T} \sim \frac{\alpha_s}{q_T}\left[(L+\cdots) + \frac{q_T}{Q}(L+\cdots) + \frac{q_T^2}{Q^2}(L+\cdots) + \mathcal{O}(q_T^2)\right] + \frac{\alpha_s^2}{q_T}\left[(L^3+\cdots) + \frac{q_T}{Q}(L^3+\cdots) + \frac{q_T^2}{Q^2}(L^3+\cdots) + \mathcal{O}(q_T^2)\right] + \mathcal{O}(\alpha_s^3)$$

- First subleading terms only suppressed as $\mathcal{O}(q_T/Q)$ [Ebert, Tackmann '19]
- If included only by fixed-order matching, diverge logarithmically as $q_T \to Q$



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Question

Can these linear power corrections be factorized & resummed?

Factorization of fiducial power corrections

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(p_1^\mu)\ell^+(p_2^\mu)$:

- Factorize matrix element: $\mathcal{M}_{pp\to V+X} = \mathcal{M}^\mu_{V\to L} \langle X|J_{V\mu}|pp\rangle$
- Factorize cross section accordingly: $\dfrac{\mathrm{d}\sigma}{\mathrm{d}^4q} = L_{\mu\nu}(q)W^{\mu\nu}(q,P_a,P_b)$
- $L_{\mu\nu}$ only depends on q $\Rightarrow L_{\mu\nu}(q) = \left(g_{\mu\nu} - \dfrac{q_\mu q_\nu}{q^2}\right) L(q^2)$

▶ Factorized cross section simplifies to

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4q} = L(q^2)W(q,P_a,P_b)\,, \quad W = \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) W^{\mu\nu}$$

- Leading-power expansion of W for $q_T \ll Q$ given by inclusive TMD factorization
- Inclusive W only depends on

$$q^2 \equiv Q^2\,, \quad P_{a,b}\cdot q = E_{\rm cm}\sqrt{Q^2 + q_T^2}\, e^{\pm Y}$$

  - Intuitively: azimuthal symmetry implies quadratic corrections in q_T^2 only

Recap: Tensor decomposition for fiducial Drell-Yan

Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(p_1^\mu)\ell^+(p_2^\mu)$:

- Allow for generic fiducial cuts $\Theta(p_1, p_2)$ on lepton momenta
- Fiducial cross section takes the form

$$\frac{\mathrm{d}\sigma(\Theta)}{\mathrm{d}^4 q} = L_{\mu\nu}(q, \Theta)\, W^{\mu\nu}(q, P_a, P_b)$$

- Covariantly decompose $W^{\mu\nu}$ into structure functions:

$$\frac{\mathrm{d}\sigma(\Theta)}{\mathrm{d}^4 q} = \sum_{i=-1}^{7} (L \cdot P_i)(W \cdot P_i) \equiv \sum_{i=-1}^{7} L_i(q, \Theta)\, W_i(q, P_a, P_b)$$

  - Total of nine W_i allowed by $q_\mu W^{\mu\nu} = 0$ and $W^{*\mu\nu} = W^{\nu\mu}$
  - Decomposition independent of leptonic final state

Strategy

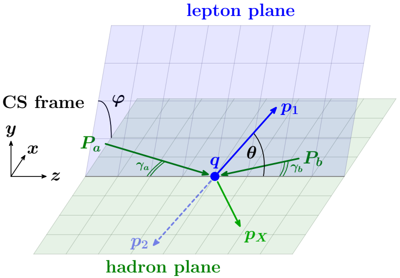
Choose projectors $P_i^{\mu\nu}$ such that power counting of L_i and W_i is transparent

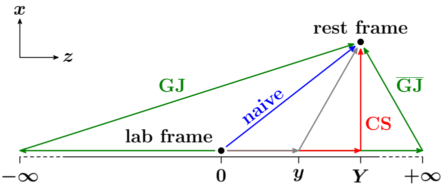
Convenient basis to build $P_i^{\mu\nu}$: Axes x^μ, y^μ, z^μ of the Collins-Soper rest frame

- Can be written as covariant linear combination of q^μ, P_a^μ, P_b^μ

  Fun fact: different definitions of CS frame (bisector criterion *vs.* sequence of boosts) are incompatible for nonzero hadron masses

  [...despite assertions to the contrary e.g. in Arnold, Metz, Schlegel '09]





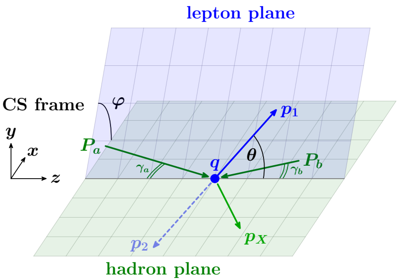
Convenient basis to build $P_i^{\mu\nu}$: Axes x^μ, y^μ, z^μ of the Collins-Soper rest frame

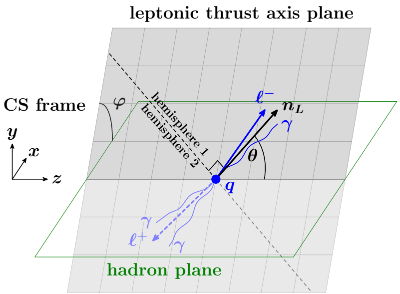
- Leptonic tensor projections become spherical harmonics $g_i(\theta, \varphi)$:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\, \mathrm{d}\cos\theta\, \mathrm{d}\varphi} = \frac{3}{16\pi} \sum_{i=-1}^{7} L_\pm(q^2)\, g_i(\theta, \varphi)\, W_i$$

- Normalized W_i are equal to standard angular coefficients A_i

  Neat fact: Relation is robust against QED final-state radiation (⟶ backup)
  if (a) Born leptons are used and (b) production and decay still factorize
  [Point (b) investigated in Billis, JM, Tackmann, forthcoming; see talks by G. Billis at SCET '20, REF '20]





Relation to Collins-Soper angular coefficients and power expansion

Convenient basis to build $P_i^{\mu\nu}$: Axes x^μ, y^μ, z^μ of the Collins-Soper rest frame

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- Derive scaling of W_i by combining (sub)leading current matching in SCET with power expansion of x^μ, y^μ, z^μ in $\lambda \sim q_T/Q$
  [Compatible with FO and factorization analyses in literature, see e.g. Boer, Vogelsang '06]

- Checked explicitly that using a different tensor decomposition (rest frame) leaves scaling of W_i invariant $\Rightarrow$ difference is only $\mathcal{O}(\lambda^2)$

- Use azimuthal symmetry, $\Theta^{(0)}(q, \cos\theta, \cancel{\varphi})$, to derive power counting of L_i
  - Many g_i average out at leading power

Power corrections to fiducial q_T spectrum (or any azimuthally symmetric observable)

- Leading power:

$$\frac{d\sigma^{(0)}}{d^4q} = \sum_{i=-1,4} L_i^{(0)} W_i^{(0)}$$

- Next-to-leading power:

$$\frac{d\sigma^{(1)}}{d^4q} = \sum_{i=-1,2,4,5} L_i^{(1)} W_i^{(0)}$$

- Linear terms arise *uniquely* from L_i

i	W_i	L_i	$g_i(\theta,\varphi)$
-1	$\sim \lambda^0$	$\sim \lambda^0$	$1+\cos^2\theta$
4	$\sim \lambda^0$	$\sim \lambda^0$	$\cos\theta$
2	$\sim \lambda^0$	$\sim \lambda^1$	$\sin^2\theta\cos(2\varphi)$
5	$\sim \lambda^0$	$\sim \lambda^1$	$\sin^2\theta\sin(2\varphi)$
0	$\sim \lambda^2$	$\sim \lambda^0$	$1-\cos^2\theta$
1	$\sim \lambda^1$	$\sim \lambda^1$	$\sin(2\theta)\cos\varphi$
3	$\sim \lambda^{\geq 1}$	$\sim \lambda^1$	$\sin\theta\cos\varphi$
6	$\sim \lambda^{\geq 1}$	$\sim \lambda^1$	$\sin(2\theta)\sin\varphi$
7	$\sim \lambda^{\geq 1}$	$\sim \lambda^1$	$\sin\theta\sin\varphi$

- Extend LP factorization / resummation to capture all linear power corrections:

$$\frac{d\sigma^{(0+1)}}{d^4q} = \sum_{i=-1,2,4,5} (L_i^{(0)} + L_i^{(1)}) W_i^{(0)}$$

- $W_{-1,4}$ obey standard unpolarized TMD factorization:

$$W_{-1,4}^{(0)} = \sum_{a,b} H_{-1,4\,ab} \int d^2\vec{b}_T\, e^{i\vec{b}_T\cdot\vec{q}_T} f_a(x_a,\vec{b}_T) f_b(x_b,\vec{b}_T)$$

- $W_{2,5}$ (double Boer-Mulders effect) are suppressed by $\Lambda_{\rm QCD}^2/q_T^2$

- Leading power:

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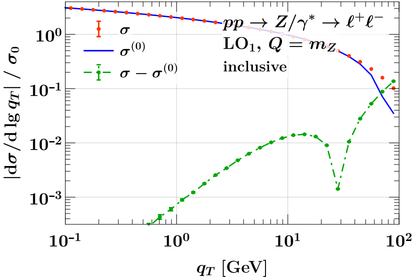
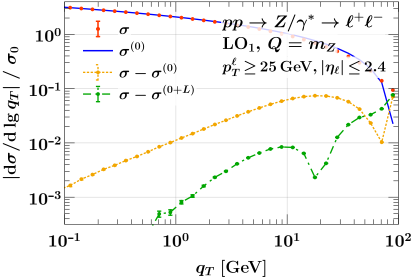
▸ Extend LP factorization / resummation to capture all fiducial power corrections:

$$\frac{d\sigma^{(0+L)}}{d^4q} \equiv \sum_{i=-1,2,4,5} L_i W_i^{(0)} = \frac{d\sigma^{(0+1)}}{d^4q}\left[1+\mathcal{O}(\lambda^2)\right]$$

- ▸ Using the exact L_i instead only induces $\mathcal{O}(\lambda^2)$ corrections
- ▸ More convenient than $d\sigma^{(0+1)}$ …and often the correct thing to do, see next section

Verify numerically:





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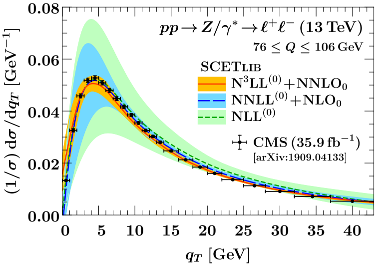
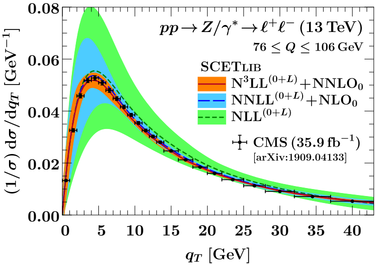
Resummed at leading power:



Resummed with fiducial p.c.s:

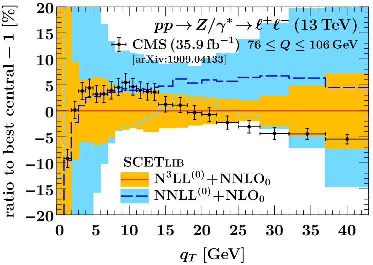


- Good agreement with data at $\mathrm{N^3LL{+}NNLO_0}$, except in nonperturbative regime $q_T \lesssim 1\,\mathrm{GeV}$
- Perturbative uncertainties greatly reduced at higher orders
- Perturbative convergence further improves with inclusion of fiducial corrections

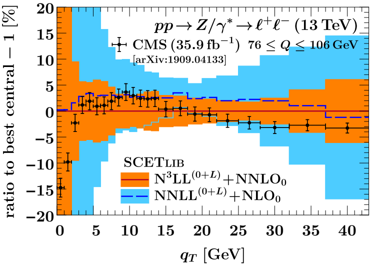
[Here: Comparison to CMS 13 TeV measurement (1909.04133)]

[See 2006.11382 and backup for comparison to ATLAS 8 TeV measurement (1512.02192) and results for ϕ^*]

Ratio to central $\mathrm{N^3LL}^{(0)}$+$\mathrm{NNLO_0}$:



Ratio to central $\mathrm{N^3LL}^{(0+L)}$+$\mathrm{NNLO_0}$:



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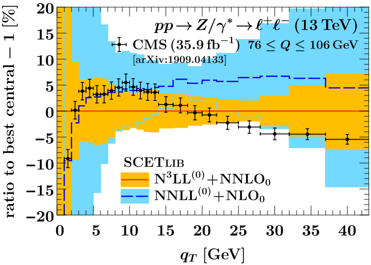

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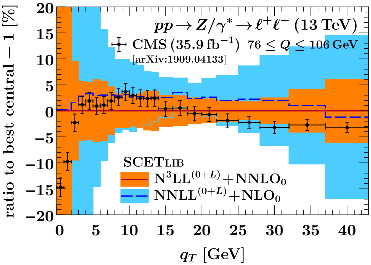
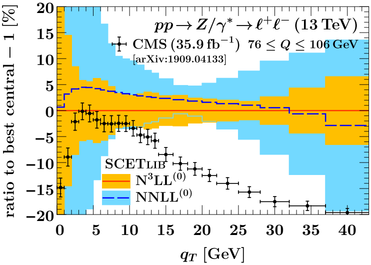


Without fixed-order matching:



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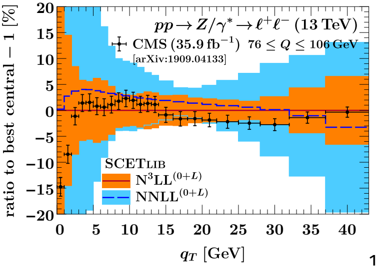
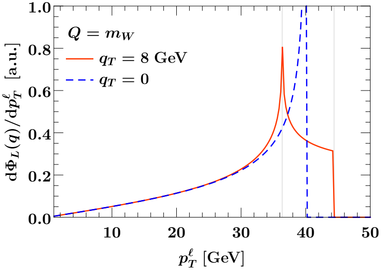


Application to leptonic observables



- Naive LP factorization:

$$\frac{d\sigma^{(0)}}{d^4q\, dp_T^\ell} = \sum_{i=-1} L_i^{(0)}(q, p_T^\ell)\, W_i^{(0)}(q, P_a, P_b)$$

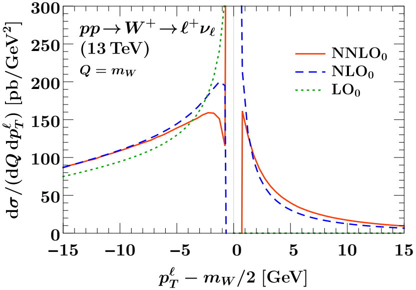
- LP kinematics bound $p_T^\ell \leq Q/2$
  - $d\sigma^{(0)}$ ill-defined for $\left| p_T^\ell - \frac{Q}{2} \right| \lesssim \frac{q_T}{2}$
- To obtain correct singular results as $p_T^\ell \sim Q/2$:
need to simultaneously expand L_i in $q_T \sim \left| p_T^\ell - Q/2 \right| \ll Q$
- In practice: more convenient to keep leptonic tensor exact

$$\boxed{\frac{d\sigma^{(0+L)}}{d^4q\, dp_T^\ell} = \sum_{i=-1,2,4,5} L_i W_i^{(0)}}$$

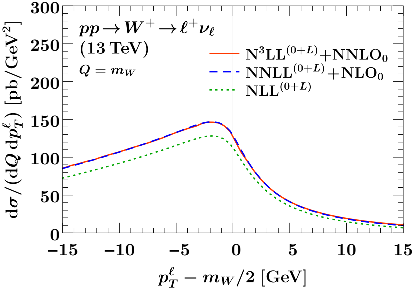
  - Avoids different expansions for $p_T^\ell \sim Q/2$ and $p_T^\ell \ll Q/2$
- Required in general when measurement induces additional small scale

p_T^ℓ spectrum with leptonic power corrections

Fixed order:



Resummed with leptonic corrections:



- Fixed order clearly breaks down as $p_T^\ell \to m_W/2$
  - Perturbative convergence lost in peak region
- Cured by including leptonic corrections in $\dfrac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}^4q\,\mathrm{d}p_T^\ell}$
  - Good perturbative convergence in peak region

[Balazs, Qiu, Yuan '95; Ellis, Ross, Veseli '97; Guzzi, Nadolsky, Wang '13] ($\to$ `RESBOS`)
[Scimemi, Vladimirov '18, Gutierrez-Reyes, Leal-Gomez, Scimemi '20] ($\to$ `Artemide`)

- Implement exact lepton kinematics in Collins-Soper frame (or use fixed $g_\perp^{\mu\nu}$)
- No formal justification / discussion of ambiguities in this choice

[Catani, de Florian, Ferrera, Grazzini '15; Camarda et al '19] ($\to$ `DYRes,DYTurbo`)
[Becher, Neumann' 20] (see Tobias' talk) ($\to$ `CuTe-MCFM`)

- Boost event to split total $\vec{q}_T$ among incoming partons
- Ambiguity from this choice is considered a $\mathcal{O}(q_T/Q)$ effect [Catani et al '15]
- Overlap removal in fixed-order matching unclear ...to us, at least – happy to discuss!

[Monni, Re, Torrielli '17; Bizon et al '17 '18] ($\to$ `RadISH`)

- Linear corrections only from fixed-order matching (not applicable for p_T^ℓ)
- Include a recoil prescription as of [Re, Rottoli, Torrielli, 2104.07509]

[This work] ($\to$ `SCETlib`)

- Formal justification: uniqueness of linear $\mathcal{O}(q_T/Q)$ corrections
  - TMD extractions that carry recoil can safely neglect FO matching of $\mathcal{O}(q_T^2/Q^2)$, should formally include Boer-Mulders function in the fit at order $\mathcal{O}(q_T/Q)$
- First results for p_T^ℓ at N^{3}LL$^{(0+L)}$ including fiducial power corrections

Resummation effects in the fiducial $gg \to H \to \gamma\gamma$ cross section

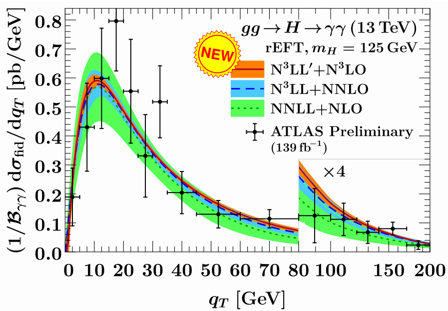
Consider $gg \to H \to \gamma\gamma$ with ATLAS fiducial cuts:

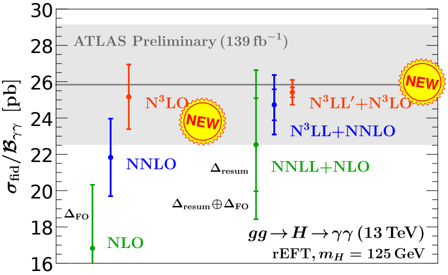
$$p_T^{\gamma 1} \geq 0.35\, m_H\,, \quad p_T^{\gamma 2} \geq 0.25\, m_H\,, \quad |\eta^\gamma| \leq 2.37\,, \quad |\eta^\gamma| \notin [1.37, 1.52]$$

Main results of 2102.08039:

- Computed fiducial spectrum in $q_T \equiv p_T^H = p_T^{\gamma\gamma}$ at N^{3}LL′+N^{3}LO
- Computed total fiducial cross section at N^{3}LO, and improved by resummation

[For many more details, see talk tomorrow at KITP Workshop “New Physics from Precision at High Energies”]





Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Compare fixed-order series, isolating the effect of $\int \mathrm{d}q_T \left[\frac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T}\right]$:

$$
\begin{aligned}
\sigma_{\rm incl}^{\rm FO} &= 13.80\,[1 + 1.291 \qquad\qquad + 0.783 \qquad\qquad + 0.299]\,\mathrm{pb} \\
\sigma_{\rm fid}^{\rm FO} &= 6.928\,[1 + 1.429 \qquad\qquad + 0.723 \qquad\qquad + 0.481]\,\mathrm{pb} \\
&= 6.928\,[1 + (1.300 + 0.129_{\rm fpc}) + (0.784 - 0.061_{\rm fpc}) + (0.331 + 0.150_{\rm fpc})]\,\mathrm{pb}
\end{aligned}
$$

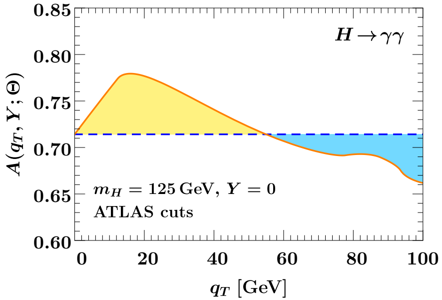
- Fiducial power corrections show no convergence, remainder is similar to inclusive

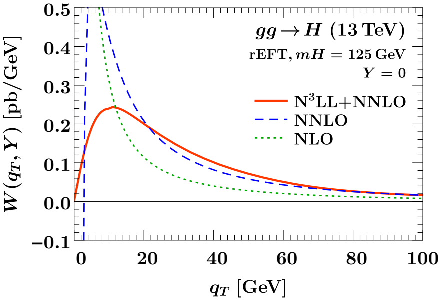
Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Two ways to understand this:

1. Acceptance acts as a weight in the q_T integral





$$\sigma_{\rm incl} = \int \mathrm{d}q_T\, W(q_T) \qquad \sigma_{\rm fid} = \int \mathrm{d}q_T\, A(q_T)\, W(q_T)$$

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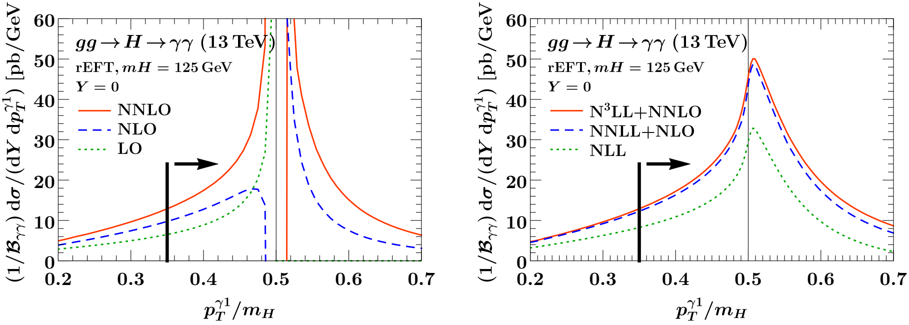
Two ways to understand this:

1. Acceptance acts as a weight in the q_T integral
2. We're cutting on the resummation-sensitive photon p_T



- Leaves behind logarithms of $\frac{p_L}{m_H} = \frac{p_T^{\text{cut}} - m_H/2}{m_H} = 0.15$

Key point

Fiducial power corrections induce resummation effects *in the total cross section*

Compare fixed-order series, isolating the effect of $\int \mathrm{d}q_T \left[\frac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{(0)}}{\mathrm{d}q_T}\right]$:

$$\begin{aligned}
\sigma_{\mathrm{incl}}^{\mathrm{FO}} &= 13.80\,[1 + 1.291 + 0.783 + 0.299]\,\mathrm{pb} \\
\sigma_{\mathrm{fid}}^{\mathrm{FO}} &= 6.928\,[1 + 1.429 + 0.723 + 0.481]\,\mathrm{pb} \\
&= 6.928\,[1 + (1.300 + 0.129_{\mathrm{fpc}}) + (0.784 - 0.061_{\mathrm{fpc}}) + (0.331 + 0.150_{\mathrm{fpc}})]\,\mathrm{pb}
\end{aligned}$$

- Fiducial power corrections show no convergence, remainder is similar to inclusive

After resummation of $\sigma^{(0)} + \sigma^{\mathrm{fpc}}$, at successive matched orders:

$$\begin{aligned}
\sigma_{\mathrm{incl}}^{\mathrm{res}} &= 24.16\,[1 + 0.756 + 0.207 + 0.024]\,\mathrm{pb} \\
\sigma_{\mathrm{fid}}^{\mathrm{res}} &= 12.89\,[1 + 0.749 + 0.171 + 0.053]\,\mathrm{pb}
\end{aligned}$$

NOTE Checked explicitly that in our profile scale setup, $\sigma_{\mathrm{incl}}^{\mathrm{res}}$ and $\sigma_{\mathrm{incl}}^{\mathrm{FO}}$ *agree* within Δ_{resum}

- *Differ* in the fiducial case $\Rightarrow$ resummation effect is resolved

Summary & Outlook

- Showed that linear power corrections are uniquely predicted by q_T factorization
- Straightforward extension of LP factorization / resummation:

$$\frac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}^4 q}(\Theta) = \sum_{i=-1,2,4,5} L_i(q, \Theta)\, W_i^{(0)}(q, P_a, P_b)$$

- Also holds for leptonic fiducial power corrections, such as p_T^ℓ near the peak
  - Yields the *actual* leading-power result in such singular regions
- Showed the numerical impact in data comparison for q_T (and ϕ^*)
  - Improves perturbative convergence and agreement with data
  - Significantly reduces impact of fixed-order matching
- Observed, explained, and resummed large fiducial power corrections in gluon-fusion Higgs production induced by the experimental acceptance
  - Even *total* fiducial cross sections are sensitive to q_T resummation effects

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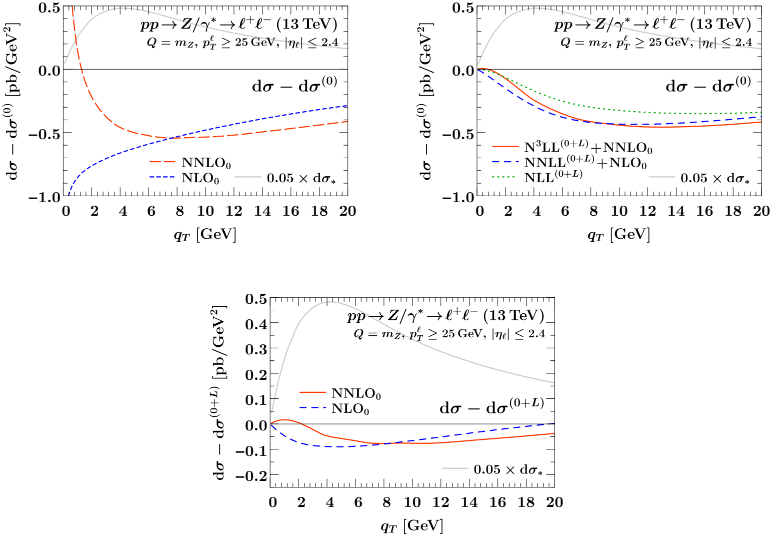
Thank you for your attention!

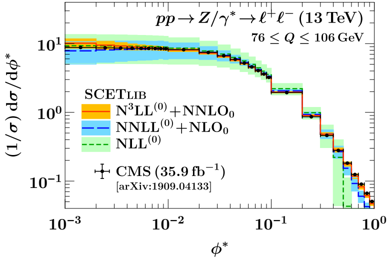
Backup

More on Drell-Yan

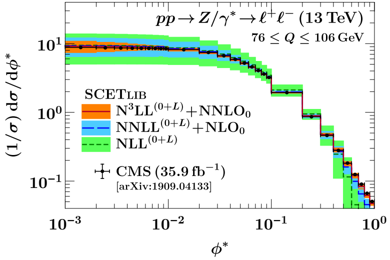
Effect of (un)resummed fiducial corrections



Resummed at leading power:



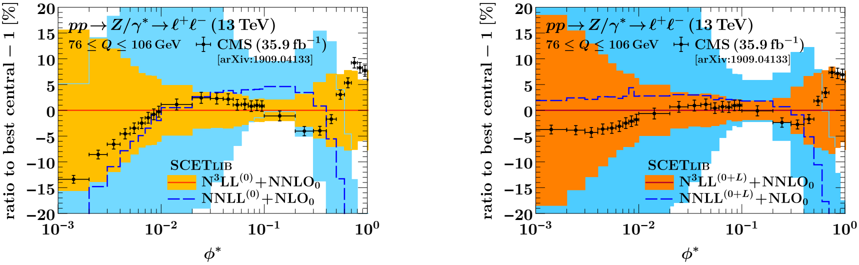
Resummed with fiducial corrections:



- Good agreement with data at $N^3LL^{(0+L)}+NNLO_0$, except in first bins due to nonperturbative effects
- After including fiducial p.c.s, FO matching only matters for $\phi^* \gtrsim 0.5$
- Including fiducial p.c.s renders shape perturbatively shable

Ratio to central $\mathrm{N^3LL^{(0)}}$+$\mathrm{NNLO_0}$:

Ratio to central $\mathrm{N^3LL^{(0+L)}}$+$\mathrm{NNLO_0}$:



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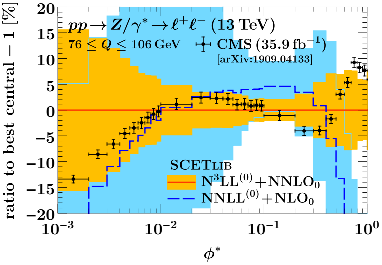
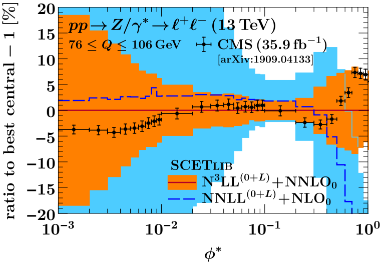
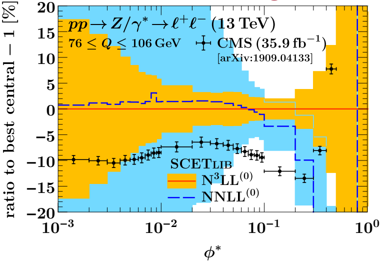


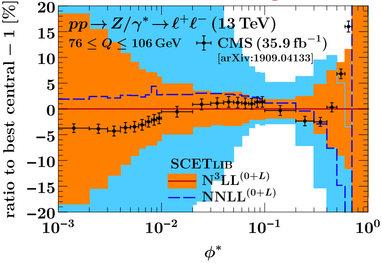
Ratio to central $\mathrm{N^3LL^{(0+\mathit{L})}}$+$\mathrm{NNLO_0}$:



Without fixed-order matching:



Without fixed-order matching:



Expansion of $W_{\mu\nu}$:

- Match QCD current onto SCET: $J_V^\mu(x) \sim \gamma_\perp^\mu C_V^{(0)}(q^2)\mathcal{O}_{q\bar{q}}^{(0)}(x)\,[1+\mathcal{O}(\lambda)]$
- Hadronic tensor at leading-power: $W^{\mu\nu} \propto g_\perp^{\mu\nu} W^{(0)}\,[1+\mathcal{O}(\lambda)]$
- No hard operators contribute at $\mathcal{O}(\lambda)$ to inclusive spectrum
  [Feige, Kolodrubetz, Moult, Stewart '17; Moult, Stewart, Vita '19]

Expansion of polarization vectors:

- Standard lightcone directions: $n_a^\mu = (1,0,0,1)\,, \quad n_b^\mu = (1,0,0,-1)$
- Rest frame unit vectors, covariant form:

$$x^\mu = n_\perp^\mu + \frac{q_T}{Q}\frac{n_a^\mu + n_b^\mu}{2} + \mathcal{O}(\lambda^2)\,, \quad y^\mu = \epsilon_\perp^{\mu\nu} n_{\perp\nu}\,, \quad z^\mu = \frac{n_a^\mu - n_b^\mu}{2}$$

Expansion of structure functions:

$$W_{-1} = (x^\mu x^\nu + y^\mu y^\nu)W_{\mu\nu} = g_\perp^{\mu\nu} g_{\perp\,\mu\nu} W^{(0)} + \cdots \quad = \mathcal{O}(\lambda^{-2})$$
$$W_0 = z^\mu z^\nu W_{\mu\nu} \quad \sim n_{a,b}^\mu n_{a,b}^\nu\, g_{\perp\mu\nu} W^{(0)} + \cdots = \mathcal{O}(\lambda^0)$$

- Easy to obtain (bound on) scaling of all W_i in this fashion

Full list of hadronic structure functions

$$
\begin{aligned}
W_{-1} &= (x_\mu x_\nu + y_\mu y_\nu)\, W^{\mu\nu} && = W_{++} + W_{--} \\
W_0 &= 2\, z_\mu z_\nu\, W^{\mu\nu} && = 2W_{00} \\
W_1 &= -(x_\mu z_\nu + x_\nu z_\mu)\, W^{\mu\nu} && = -\frac{1}{\sqrt{2}}\big(W_{+0} + W_{0+} + W_{-0} + W_{0-}\big) \\
W_2 &= 2\,(y_\mu y_\nu - x_\mu x_\nu)\, W^{\mu\nu} && = -2\big(W_{+-} + W_{-+}\big) \\
W_3 &= 2\mathrm{i}\,(y_\mu z_\nu - y_\nu z_\mu)\, W^{\mu\nu} && = -\sqrt{2}\big(W_{+0} + W_{0+} - W_{-0} - W_{0-}\big) \\
W_4 &= 2\mathrm{i}\,(x_\mu y_\nu - x_\nu y_\mu)\, W^{\mu\nu} && = 2\big(W_{++} - W_{--}\big) \\
W_5 &= -(x_\mu y_\nu + x_\nu y_\mu)\, W^{\mu\nu} && = -\mathrm{i}\big(W_{+-} - W_{-+}\big) \\
W_6 &= -(y_\mu z_\nu + y_\nu z_\mu)\, W^{\mu\nu} && = -\frac{\mathrm{i}}{\sqrt{2}}\big(W_{+0} - W_{0+} - W_{-0} + W_{0-}\big) \\
W_7 &= -2\mathrm{i}\,(x_\mu z_\nu - x_\nu z_\mu)\, W^{\mu\nu} && = -\mathrm{i}\sqrt{2}\big(W_{+0} - W_{0+} + W_{-0} - W_{0-}\big)
\end{aligned}
$$

- Sometimes more intuitive to use polarization basis: $W_{\lambda\lambda'} = \epsilon^{\mu}_{\lambda}\epsilon^{*\nu}_{\lambda'} W_{\mu\nu}$

$$W_{-1} = (x_\mu x_\nu + y_\mu y_\nu)\, W^{\mu\nu}$$
$$W_0 = 2\, z_\mu z_\nu\, W^{\mu\nu}$$
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$$W_7 = -2\mathrm{i}\,(x_\mu z_\nu - x_\nu z_\mu)\, W^{\mu\nu}$$

i	W_i	L_i	$g_i(\theta,\varphi)$
-1	$\sim\lambda^0$	$\sim\lambda^0$	$1+\cos^2\theta$
4	$\sim\lambda^0$	$\sim\lambda^0$	$\cos\theta$
2	$\sim\lambda^0$	$\sim\lambda^1$	$\sin^2\theta\cos(2\varphi)$
5	$\sim\lambda^0$	$\sim\lambda^1$	$\sin^2\theta\sin(2\varphi)$
0	$\sim\lambda^2$	$\sim\lambda^0$	$1-\cos^2\theta$
1	$\sim\lambda^1$	$\sim\lambda^1$	$\sin(2\theta)\cos\varphi$
3	$\sim\lambda^{\geq 1}$	$\sim\lambda^1$	$\sin\theta\cos\varphi$
6	$\sim\lambda^{\geq 1}$	$\sim\lambda^1$	$\sin(2\theta)\sin\varphi$
7	$\sim\lambda^{\geq 1}$	$\sim\lambda^1$	$\sin\theta\sin\varphi$

- Sometimes more intuitive to use polarization basis: $W_{\lambda\lambda'} = \epsilon^\mu_\lambda \epsilon^{*\nu}_{\lambda'} W_{\mu\nu}$

- Consider $p(P_a^\mu)p(P_b^\mu) \to Z/\gamma^*(q^\mu) \to \ell^-(p_1^\mu)\ell^+(p_1^\mu)$
- Kinematic structure of leptonic tensor:

$$L_{\mu\nu}(p_1, p_2) \propto L_+(q^2)\big(p_1^\mu p_2^\nu + p_1^\nu p_2^\mu - g^{\mu\nu} p_1 \cdot p_2\big) + \mathrm{i}L_-(q^2)\,\epsilon^{\mu\nu\rho\sigma} p_{1\rho} p_{2\sigma}$$

  - Contains a parity-even (L_+) and parity-odd (L_-) component

- Parameterize $p_{1,2}^\mu$ in terms of CS angles θ, φ and project:

$$L_i(q, \Theta) = P_i^{\mu\nu} L_{\mu\nu} \propto L_{\pm(i)}(q^2) \int_{-1}^{1} \mathrm{d}\cos\theta \int_0^{2\pi} \mathrm{d}\varphi\, g_i(\theta, \varphi)\, \Theta(q, \theta, \varphi)$$

  - Projection onto $P_i^{\mu\nu}$ encoded in spherical harmonics $g_i(\theta, \varphi)$

- Expansion of $L_i(q, \Theta)$ in $q_T \ll Q$ depends on observable Θ
- *Often* (e.g. fiducial q_T spectrum), Θ is azimuthally symmetric at leading power:

$$\Theta(q, \theta, \varphi) = \Theta^{(0)}(q, \theta)\big[1 + \mathcal{O}(\lambda)\big]$$

  - All L_i except for $i = -1, 0, 4$ average out

Backup

Final-state QED radiation and lepton definitions

Two assumptions and one problem

- Assume there is still only a single V exchanged
- Also assume we know what particles $i \in \boldsymbol{L}$ came from the decay of the V, as opposed to initial-state radiation $i \in \boldsymbol{X}$

$\Rightarrow$ $\mathrm{d}\sigma \propto L_{\mu\nu}(q, \mathcal{O})\, W^{\mu\nu}(q, P_a, P_b)$ still holds, but $L^{\mu\nu}$ more complicated

- One option for a "lepton" in the presence of QED FSR: the directly observed bare lepton at the scale m_e ...something you can't do in QCD

- Another option: lepton is dressed with photons within some radius, adding their four-momenta ...similar to a QCD jet

$\Rightarrow$ Problem with both:

  - Momentum is lost $\Rightarrow$ no connection between q^μ and $\tilde{p}_1^\mu + \tilde{p}_2^\mu$
  - Angular momentum is lost $\Rightarrow$ basis of $g_i(\cos\theta, \varphi)$ no longer complete

- Basically: Defined by having no photon escape, i.e., enforcing $q^\mu = \tilde{p}_1^\mu + \tilde{p}_2^\mu$
- Formally: As an IR-safe *Born projection* of full L onto Born-like 2-body $\tilde{L}$
  - Many options, so precise definition depends on used Born projection

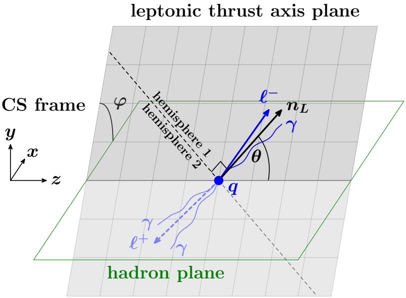
Example for illustration

- Pick $\vec{n}_L$ to be the (leptonic) thrust axis of L:
  [similar in spirit to Hall & Thaler, 1805.11622, where an event shape is minimized for photon isolation]

$$\vec{n}_L : \qquad \min_{\vec{n}} \sum_{i \in L} (E_i - |\vec{n} \cdot \vec{p}_i|) = Q - \max_{\vec{n}} \sum_i |\vec{n} \cdot \vec{p}_i|$$

- Take $\tilde{p}_{1,2}$ along $\vec{n}_L$:

$$\tilde{p}^\mu_{1,2} = \frac{Q}{2}(1, \pm \vec{n}_L)_{\rm CS} \equiv \frac{Q}{2}(t^\mu \pm n_L^\mu)$$



$\Rightarrow$ Born-like 2-body configuration

Angular Decomposition for Born Leptons

- All-order leptonic tensor $F^{\mu\nu}(\tilde{p}_1, \tilde{p}_2)$ may be complicated, but at the end of the day only depends on $\tilde{p}_{1,2}$ with $q^\mu = \tilde{p}_1^\mu + \tilde{p}_2^\mu$

$\Rightarrow$ Most general allowed structure (using current conservation $q_\mu F^{\mu\nu} = 0$)

$$F^{\mu\nu}(\tilde{p}_1, \tilde{p}_2) = 12\pi \Big[(t^\mu t^\nu - g^{\mu\nu} - n_L^\mu n_L^\nu) F_+(q^2) + (t^\mu t^\nu - g^{\mu\nu}) F_0(q^2) + \mathrm{i}\epsilon^{\mu\nu}{}_{\rho\sigma} n_L^\rho t^\sigma F_-(q^2) \Big]$$

$$F_\pm(q^2) = L_\pm(q^2) + \mathcal{O}(\alpha_{\mathrm{em}}) \,, \qquad F_0(q^2) = \mathcal{O}(\alpha_{\mathrm{em}})$$

- Contracting $L_i = P_i^{\mu\nu} F_{\mu\nu}$ yields

$$L_{-1}(q^2, \theta, \varphi) = \tfrac{3}{16\pi} \Big[F_+(q^2)\, g_{-1}(\theta, \varphi) + 2 F_0(q^2) \Big]$$

$$L_0(q^2, \theta, \varphi) = \tfrac{3}{16\pi} \Big[F_+(q^2)\, g_0(\theta, \varphi) + F_0(q^2) \Big]$$

$$L_i(q^2, \theta, \varphi) = \tfrac{3}{16\pi} F_\pm(q^2)\, g_i(\theta, \varphi) \qquad (i \geq 1)$$

$\Rightarrow$ Closes under spherical harmonics of degree ≤ 2!

Discussion

$\Rightarrow$ Can measure *exactly* the same W_i even with FSR:

$$\frac{16\pi}{3}\frac{d\sigma}{d^4q\, d\cos\theta\, d\varphi} = 2F_0\, W_{\text{incl}} + \sum_{i=-1}^{7} F_{\pm(i)}\, g_i(\theta,\varphi)\, W_i$$

$$= \left(F_+ + \frac{3}{2}F_0\right) W_{\text{incl}} \left[1 + \cos^2\theta + \frac{\tilde{A}_0}{2}(1 - 3\cos^2\theta) + \sum_{i=1}^{7} \tilde{A}_i\, g_i(\theta,\varphi)\right]$$

with $\tilde{A}_0 = \frac{F_+ W_0 + F_0\, W_{\text{incl}}}{\left(F_+ + \frac{3}{2}F_0\right) W_{\text{incl}}}$, $\tilde{A}_{i\geq 1} = \frac{F_{\pm(i)} W_i}{\left(F_+ + \frac{3}{2}F_0\right) W_{\text{incl}}}$

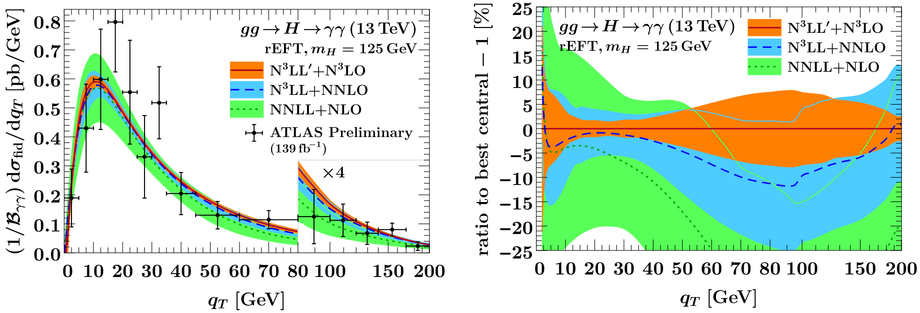
- Holds for *any Born projection* (with $n_L \propto \tilde{p}_1 - \tilde{p}_2$), only the $F_{\pm,0}$ need to be calculated in QED for each specific projection
- The $F_{\pm,0}(q^2)$ only depend on q^2 $\Rightarrow$ no large logs …here
- Experimental analyses effectively invert *some* QED parton shower (e.g. PHOTOS) to define their Born leptons, making it hard to compare different analyses

$\Rightarrow$ Proposed definition using thrust would be an obvious tool-independent choice

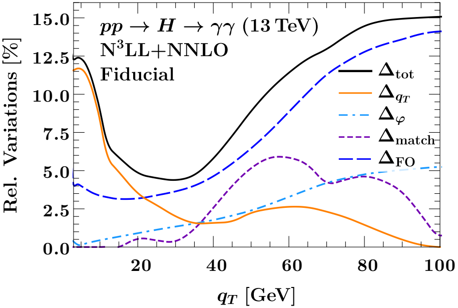
Backup

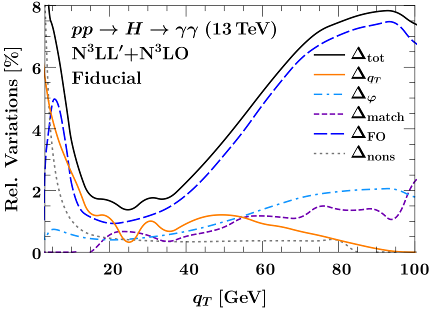
Detailed results for $gg \to H \to \gamma\gamma$

Fiducial q_T spectrum for $gg \to H \to \gamma\gamma$ at N^{3}LL′+N^{3}LO



- Total uncertainty is $\Delta_{tot} = \Delta_{q_T} \oplus \Delta_{\varphi} \oplus \Delta_{match} \oplus \Delta_{FO} \oplus \Delta_{nons}$
  [See also Ebert, JM, Stewart, Tackmann, 2006.11382 for details]
- Observe excellent perturbative convergence & uncertainty coverage
  - Crucial to consider *every* variation to probe all parts of the prediction
  - Three-loop beam function has noticeable efffect on central value and band
- Divide $H \to \gamma\gamma$ branching ratio $\mathcal{B}_{\gamma\gamma}$ out of data [LHC Higgs Cross Section WG, 1610.07922]
- Data are corrected for other production channels, photon isolation efficiency
  [ATLAS, 1802.04146]



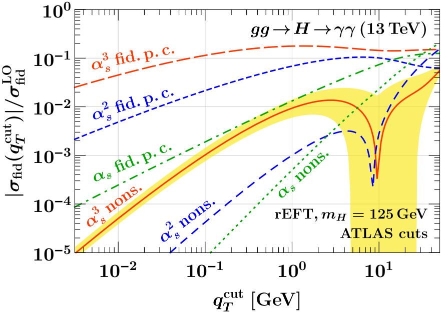


N^{3}LO: $\sigma_{\rm fid}/\mathcal{B}_{\gamma\gamma} = (25.16 \pm 1.78_{\rm FO} \pm 0.12_{\rm nons})\,\text{pb}$

N^{3}LL′+N^{3}LO: $\sigma_{\rm fid}/\mathcal{B}_{\gamma\gamma} = (25.41 \pm 0.59_{\rm FO} \pm 0.21_{q_T} \pm 0.17_{\varphi} \pm 0.06_{\rm match} \pm 0.20_{\rm nons})\,\text{pb}$

- Δ_{q_T} 36 independent scale variations in $W^{(0)}$ factorization
- Δ_φ Vary phase of hard scale over $\arg \mu_H \in \{\pi/4, 3\pi/4\}$
- $\Delta_{\rm match}$ Vary transition points governing resummation turn-off
- $\Delta_{\rm FO}$ Vary $\mu_R/m_H \in \{1/2, 2\}$ (dominates over μ_F due to overall α_s^2)
- $\Delta_{\rm nons}$ Uncertainty on nonsingular extraction

Size comparison of different sources of power corrections



$$\sigma_{\rm fid}(q_T^{\rm cut}) \equiv \int_0^{q_T^{\rm cut}} {\rm d}q_T \, \frac{{\rm d}\sigma_{\rm fid}}{{\rm d}q_T}$$

Backup

References & implementation details for $gg \to H \to \gamma\gamma$

Leading-power factorization & resummation to N^{3}LL$'$

At leading power in $q_T \ll m_H$, the hadronic dynamics for $gg \to H$ factorize as:

$$W^{(0)}(q_T, Y) = H(m_H^2, \mu) \int \mathrm{d}^2\vec{k}_a\, \mathrm{d}^2\vec{k}_b\, \mathrm{d}^2\vec{k}_s\, \delta\big(q_T - |\vec{k}_a + \vec{k}_b + \vec{k}_s|\big) \times B_g^{\mu\nu}(x_a, \vec{k}_a, \mu, \nu)\, B_{g\,\mu\nu}(x_b, \vec{k}_b, \mu, \nu)\, S(\vec{k}_s, \mu, \nu)$$

To reach N^{3}LL$'$ for $W^{(0)}$, implemented in SCETlib:

- Three-loop soft and hard function …includes in particular the three-loop virtual form factor
  [Li, Zhu, '16] [Baikov et al. '09; Lee et al. '10; Gehrmann et al. '10]
- Three-loop unpolarized and two-loop polarized beam functions
  [Ebert, Mistlberger, Vita '20; Luo, Yang, Zhu, Zhu '20]
  [Luo, Yang, Zhu, Zhu '19; Gutierrez-Reyes, Leal-Gomez, Scimemi, Vladimirov '19]
- Four-loop cusp, three-loop noncusp anomalous dimensions
  [Brüser, Grozin, Henn, Stahlhofen '19; Henn, Korchemsky, Mistlberger '20; v. Manteuffel, Panzer, Schabinger '20] [Li, Zhu, '16; Moch, Vermaseren, Vogt '05; Idilbi, Ma, Yuan '06; Vladimirov '16]
- N^{3}LL solutions to virtuality/rapidity RGEs in b_T space
- Hybrid profile scales for fixed-order matching
  [Lustermans, JM, Tackmann, Waalewijn '19]

Leading-power factorization & resummation to N^{3}LL$'$

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- Use $\mu_{\mathrm{FO}} = \mu_R = \mu_F = m_H$ for central predictions
- Gluon form factor contains large "timelike" logarithms $\ln \dfrac{-m_H^2 - \mathrm{i}0}{\mu^2}$
  [Ahrens, Becher, Neubert, Yang '08]
  - Resummed by hard evolution from $\mu_H = -\mathrm{i}m_H$:

$$W(q_T, Y) = H(m_H^2, \mu_H)\, U_H(Q, \mu_H, \mu_{\mathrm{FO}}) \left[\frac{W(q_T, Y)}{H(m_H^2, \mu_{\mathrm{FO}})}\right]_{\mathrm{FO}}$$

[Ebert, JM, Tackmann '17]

Efficient evaluation of beam function finite terms in SCETlib

- Beam function kernels are large expressions of HPLs and rational prefactors:

$$I_{ij}^{(n)}(z) = \sum_a \frac{P_a(z)}{Q_a(z)} H_{w_a}(z)\,, \qquad w_a = \left(\tfrac{\pm 1}{0}, \cdots, \tfrac{\pm 1}{0}\right) \text{ up to weight 5}$$

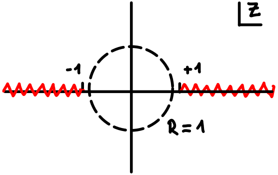
- Many tools for numerically evaluating *individual* HPLs on the market …
  [e.g. Gehrmann, Remiddi '01; Buehler, Duhr '11; Ablinger, Blümlein, Round, Schneider '18]

! But big sum is slow and has uncontrolled floating-point cancellations, in particular in limits $z \to 0, 1$ relevant for convolution $I_{ij}^{(n)} \otimes f_j$ against PDFs

Key idea

Implement the kernels *directly* as smart series expansions,
using algebraic methods inspired by those developed for individual HPLs

1. Separate branch cuts by subtractions
   - Much more complex due to rational terms
   - ▶ Treat $Q_a(z)$ as additional primitives
2. Remap variables, push out remaining branch cut
   - Improves convergence radii of series



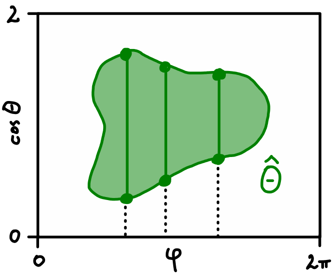
▶ Get $I_{ij}^{(3)}, P_{ij}^{(2)}, \ldots$ at machine precision in $\mathcal{O}(50\text{k})$ cycles for any z, ≥ 100 times faster than naive implementation (and much more precise)

Implementation of fiducial power corrections in SCETlib

... relies on fast & stable (for $q_T \to 0$) algorithm for evaluating the acceptance:

$$A(q_T, Y; \Theta) = \frac{1}{4\pi} \int \mathrm{d}\cos\theta \, \mathrm{d}\varphi \, \hat{\Theta}(q^\mu, \cos\theta, \varphi)$$

- $\hat{\Theta}(q_T = 0, Y, \cos\theta, \cancel{\varphi})$ is trivial
- For $q_T \neq 0$, analytically solve generic $\hat{\Theta}$ for bounds in θ at given q_T, Y, φ
- Do remaining 1D integral over φ adaptively
  - Takes $\mathcal{O}(1\,\mathrm{ms})$ on $2.50\,\mathrm{GHz}$ CPU for 10^{-7} target precision



Backup

Extracting the $gg \to H$ nonsingular cross section

Assume we dealt with this ...

$$\frac{d\sigma^{\text{sing}}}{dq_T} \equiv \frac{d\sigma^{(0+L)}}{dq_T} = \int dY\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y)$$

To match to FO and be able to integrate to the total cross section, we still need:

$$\frac{d\sigma_{\text{FO}}^{\text{nons}}}{dq_T} = \int dY\, A(q_T, Y; \Theta) \left[W_{\text{FO}}^{(2)}(q_T, Y) + \cdots\right] = \left[\frac{d\sigma_{\text{FO}_1}}{dq_T} - \frac{d\sigma_{\text{FO}}^{\text{sing}}}{dq_T}\right]_{q_T > 0}$$

$$\Rightarrow \quad \sigma = \int_0^{q_T^{\text{off}}} dq_T \left[\frac{d\sigma^{\text{sing}}}{dq_T} + \frac{d\sigma_{\text{FO}}^{\text{nons}}}{dq_T}\right] + \int_{q_T^{\text{off}}} dq_T\, \frac{d\sigma_{\text{FO}_1}}{dq_T}$$

Challenges:

- Obtaining stable $H + 1j$ results for $q_T \to 0$ is *hard* ...in particular at NNLO$_1$
- Dropping the nonsingular below $q_T \leq q_T^{\text{cut}}$ is not viable, either ...as we'll see shortly
  - In the context of q_T subtractions: crucial to use differential subtraction, not slicing

Assume we dealt with this ...

$$\frac{\mathrm{d}\sigma^{\rm sing}}{\mathrm{d}q_T} \equiv \frac{\mathrm{d}\sigma^{(0+L)}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y)$$

To match to FO and be able to integrate to the total cross section, we still need:

$$\frac{\mathrm{d}\sigma^{\rm nons}_{\rm FO}}{\mathrm{d}q_T} = \int \mathrm{d}Y\, A(q_T, Y; \Theta) \Big[W^{(2)}_{\rm FO}(q_T, Y) + \cdots\Big] = \left[\frac{\mathrm{d}\sigma_{\rm FO_1}}{\mathrm{d}q_T} - \frac{\mathrm{d}\sigma^{\rm sing}_{\rm FO}}{\mathrm{d}q_T}\right]_{q_T>0}$$

Key idea

Fit nonsingular data to known form at subleading power and integrate *analytically*:

$$q_T \mathrm{nons} \frac{\mathrm{d}\sigma^{\rm nons}_{\rm FO}}{\mathrm{d}q_T}\bigg|_{\alpha_s^n} = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \left(a_k + b_k \frac{q_T}{m_H} + c_k \frac{q_T^2}{m_H^2} + \cdots\right) \ln^k \frac{q_T^2}{m_H^2}$$

- Include higher-power b_k, c_k to get unbiased a_k
- Allows us to use more precise data at higher q_T as lever arm in the fit

Assume we dealt with this ...

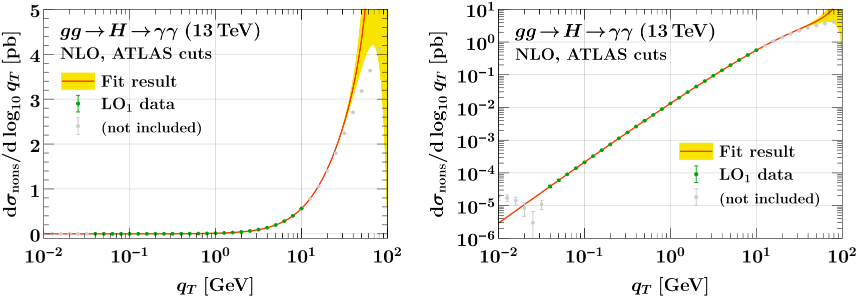
$$\frac{d\sigma^{\rm sing}}{dq_T} \equiv \frac{d\sigma^{(0+L)}}{dq_T} = \int dY\, A(q_T, Y; \Theta)\, W^{(0)}(q_T, Y)$$

To match to FO and be able to integrate to the total cross section, we still need:

$$\frac{d\sigma^{\rm nons}_{\rm FO}}{dq_T} = \int dY\, A(q_T, Y; \Theta) \left[W^{(2)}_{\rm FO}(q_T, Y) + \cdots\right] = \left[\frac{d\sigma_{{\rm FO}_1}}{dq_T} - \frac{d\sigma^{\rm sing}_{\rm FO}}{dq_T}\right]_{q_T>0}$$

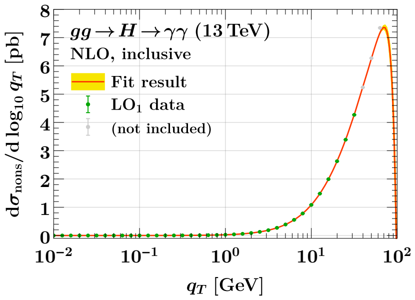
Fixed-order inputs:

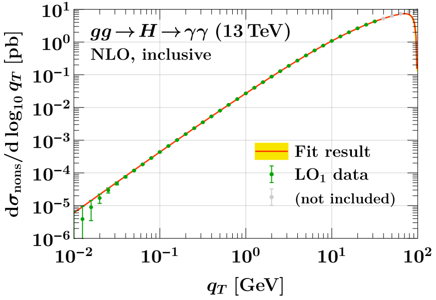
- NLO contribution to $W(q_T, Y)$ at $q_T > 0$ is easy
- At NNLO, renormalize & implement bare analytic results for $W(q_T, Y)$
  [Dulat, Lionetti, Mistlberger, Pelloni, Specchia '17]
- At N^{3}LO, use existing binned NNLO$_1$ results from `NNLOjet`
  [Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '15-16; as used in Chen et al. '18; Bizoń et al. '18]
- Use N^{3}LO total inclusive cross section as additional fit constraint on underflow
  [Mistlberger '18]



Fit procedure:

- Perform separate χ^2 fits of $\{a_k^{\mathrm{incl,fid}}\}$ to inclusive and fiducial nonsingular data
  [generated by our analytic implementation]
- Increase fit window to larger q_T until p value decreases
- Include subleading log coefficients at next higher power until p value decreases
- Also test intermediate combination to ensure fit is stable
  [procedure follows Moult, Rothen, Stewart, Tackmann, Zhu ’15-’16]





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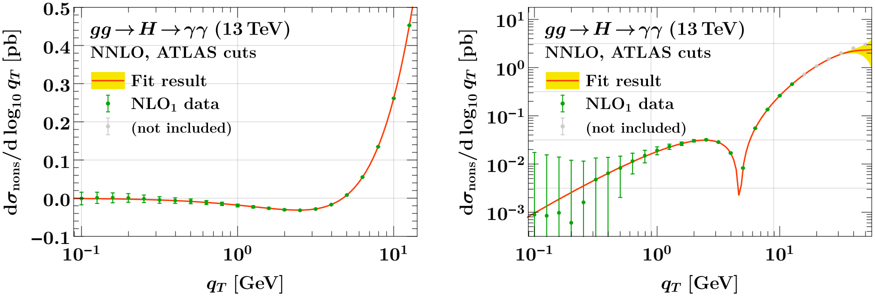

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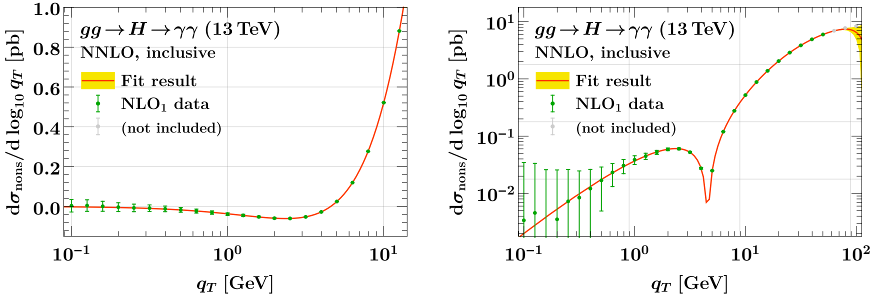

Fit procedure:

- Perform separate χ^2 fits of $\{a_k^{\rm incl,fid}\}$ to inclusive and fiducial nonsingular data
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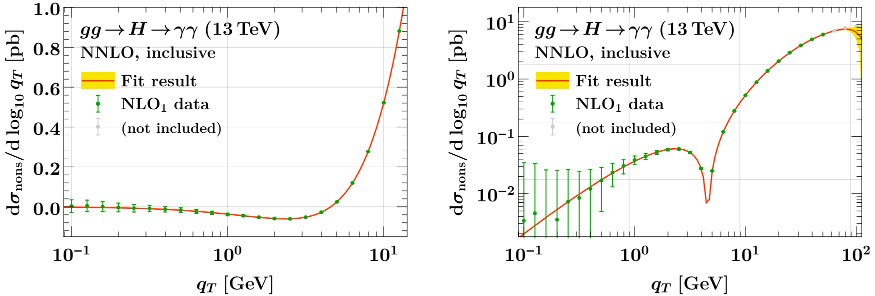
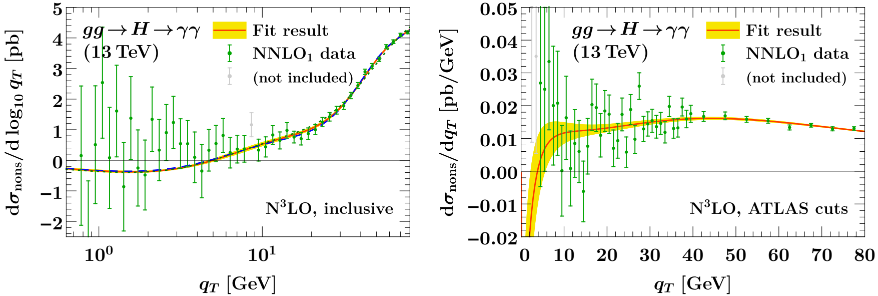


- Check the purely hadronic $a_k^{\rm fid}$ by directly fitting them to

$$q_T \int dY\, A^{(0)}(Y;\Theta)\big[W - W^{(0)}\big] = \frac{q_T^2}{m_H^2} \sum_{k=0}^{2n-1} \Big(a_k^{\rm fid} + c_k' \frac{q_T^2}{m_H^2} + \cdots \Big) \ln^k \frac{q_T^2}{m_H^2} \quad ✓$$

- Recover analytic (N)NLO coefficient of $\sigma_{\rm incl}$ at 10^{-5} (10^{-4}) ✓
- Analytic implementation gives us awesome precision on *all* NLP coefficients (all logs at NLO *and* NNLO, also differential in Y, broken down by color structure, …)
  - Can serve as benchmark for q_T factorization & resummation of $W^{(2)}$

Fit results at N^{3}LO



Setup:

- Perform a combined fit to all inclusive and fiducial data
  [NNLO$_1$: Chen, Cruz-Martinez, Gehrmann, Glover, Jaquier '15-16; as used in Chen et al. '18; Bizoń et al. '18]
  [Incl. N^{3}LO: Mistlberger '18]
- Empirically find $0.4 \leq a_k^{\rm fid}/a_k^{\rm incl} \leq 0.55$ at (N)NLO $\Rightarrow$ use as weak 1σ constraint
  - Makes sense, $a_k^{\rm fid,incl}$ are same underlying $W^{(2)}$ in slightly different Y range
  - Note that we are *not* just rescaling any part of the cross section by an acceptance
- Add $\sigma_{\rm incl}(q_T \leq q_T^{\rm cut}) = \sigma_{\rm incl}^{\rm N^3LO} - \sigma_{\rm incl}(q_T > q_T^{\rm cut})$ as additional incl. data point