Renormalization schemes for parton densities

Ted Rogers
Jefferson Lab and Old Dominion University

With John Collins and Nobuo Sato: In preparation

QCD Evolution: May 10, 2021

Historical: Two approaches to pdfs and factorization

- Track A:
  - Define pdf in terms of ultraviolet renormalization of bare number density operator.

Historical: Two approaches to pdfs and factorization

- Track A:
  - Define pdf in terms of ultraviolet renormalization of bare number density operator.
- Track B:
  - Calculate higher order hard scattering amplitudes. "Absorb" collinear divergences into pdf.

Track A:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
  - Deal with them using standard UV renormalization techniques.

Track A:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
  - Deal with them using standard UV renormalization techniques.
- Factorization (e.g., DIS):
  - Obtained from general region analysis.
  - Beyond parton model: Higher order hard scattering constructed from nested subtractions.

Track A:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
  - Deal with them using standard UV renormalization techniques.

$$f^{\mathrm{bare,a}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-}\ \langle p|\, \bar{\psi}_0(0, w^-, \mathbf{0}_\mathrm{T}) \frac{\gamma^+}{2} W[0, w^-] \psi_0(0, 0, \mathbf{0}_\mathrm{T})\, |p\rangle$$

Track A:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
  - Deal with them using standard UV renormalization techniques.

$$f^{\mathrm{bare,a}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-}\ \langle p|\, \bar{\psi}_0(0, w^-, \mathbf{0}_{\mathrm{T}}) \frac{\gamma^+}{2} W[0, w^-] \psi_0(0, 0, \mathbf{0}_{\mathrm{T}})\, |p\rangle$$

$$f^{\mathrm{renorm,a}}(\xi) \equiv Z^a \otimes f^{\mathrm{bare,a}}$$

$$Z^a = \delta(1-\xi) + \sum_{j=1}^{\infty} C_j \left(\frac{S_\epsilon}{\epsilon}\right)^j$$

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

Massless partonic

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

  Massless partonic

- Collinear divergences! $\mathrm{d}\hat{\sigma} = \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\mathrm{finite}}$

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

  Massless partonic

- Collinear divergences! $\mathrm{d}\hat{\sigma} = \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

- So… $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

Massless partonic

- Collinear divergences! $\mathrm{d}\hat{\sigma} = \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\mathrm{finite}}$

- So… $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\mathrm{finite}}$

- Absorb: $f = f_{\text{“bare,b”}} \otimes \mathcal{C}$

Track B:

- Assert(?): $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

  Massless partonic

- Collinear divergences! $\mathrm{d}\hat{\sigma} = \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

- So… $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathcal{C} \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

- Absorb: $f = f_{\text{“bare,b”}} \otimes \mathcal{C}$

- Then: $\mathrm{d}\sigma = f \otimes \mathrm{d}\hat{\sigma}_{\text{finite}}$

Track B:

- Issues:
  - We have not found a derivation of factorization for step 1 ($\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$) in existing literature
  - Bare pdf ($f_{\text{“bare,b”}}$) of step 1 is undefined
  - Collinear pdfs viewed as physical?
  - Can we reverse engineer $f_{\text{“bare,b”}}$?

Track A vs. Track B Logic

- Do the differences have practical consequences?

Track A vs. Track B Logic

- Do the differences have practical consequences?

- Example: Track-B leads to arguments that pdf positivity is an absolute property of pdfs in certain schemes (MS-bar).

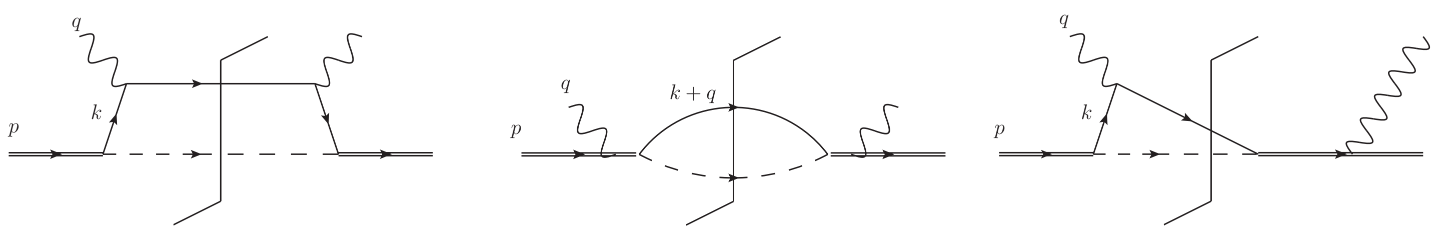
$f(x;\mu) \geq 0$ *A. Candido, S. Forte, and F. Hekhorn (2020), 2006.07377, JHEP 11 (2020) 129*

Positivity example

- Stress-test assertions about DIS factorization in other finite-range renormalizable theories.

Positivity example

- Stress-test assertions about DIS factorization in other finite-range renormalizable theories.



(a) (b) (c)

$$\mathcal{L}_{\rm int} = -\lambda \, \overline{\Psi}_N \, \psi_q \, \phi \;+\; \text{H.C.}$$

- Exact $O(\lambda^2)$ DIS cross section is easy to calculate exactly.

Track A:

- Operator definition of the pdf from the beginning.
  - The only divergences are ultraviolet.
  - Deal with them using standard UV renormalization techniques.

$$f^{\mathrm{bare,a}}(\xi) \equiv \int \frac{\mathrm{d}w^-}{2\pi}\, e^{-i\xi p^+ w^-} \, \langle p|\, \bar{\psi}_0(0, w^-, \mathbf{0}_\mathrm{T}) \frac{\gamma^+}{2} W[0, w^-] \psi_0(0, 0, \mathbf{0}_\mathrm{T}) \,|p\rangle$$

$$f^{\mathrm{renorm,a}}(\xi) \equiv Z^a \otimes f^{\mathrm{bare,a}}$$

$$Z^a = \delta(1-\xi) + \sum_{j=1}^{\infty} C_j \left(\frac{S_\epsilon}{\epsilon}\right)^j$$

Positivity example

Collinear Factorization

$$F_1(x,Q) = \sum_f \int_x^1 \frac{\mathrm{d}\xi}{\xi}$$

$$\times \underbrace{\frac{1}{2}\left\{\delta\left(1-\frac{x}{\xi}\right)\delta_{qf} + a_\lambda(\mu)\left(1-\frac{x}{\xi}\right)\left[\ln(4) - \frac{\left(\frac{x}{\xi}\right)^2 - 3\frac{x}{\xi} + \frac{3}{2}}{\left(1-\frac{x}{\xi}\right)^2} - \ln\frac{4x\mu^2}{Q^2(\xi - x)}\right]\delta_{pf}\right\}}_{\hat{F}_{1,q/f}(x/\xi,\mu/Q;a_\lambda(\mu))} \times$$

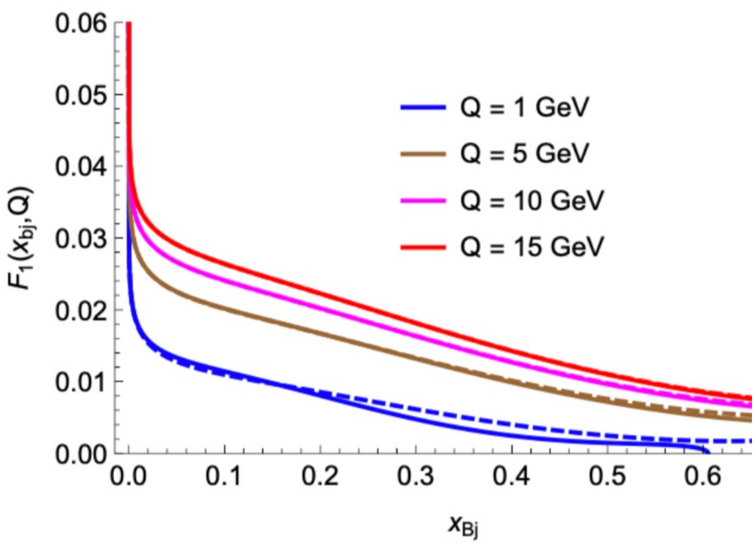
$$\times \underbrace{\left\{\delta(1-\xi)\,\delta_{fp} + a_\lambda(\mu)(1-\xi)\left[\frac{(m_q + \xi m_p)^2}{\Delta(\xi)^2} + \ln\left(\frac{\mu^2}{\Delta(\xi)^2}\right) - 1\right]\delta_{fq}\right\}}_{f_{f/p}(\xi;\mu)} .$$

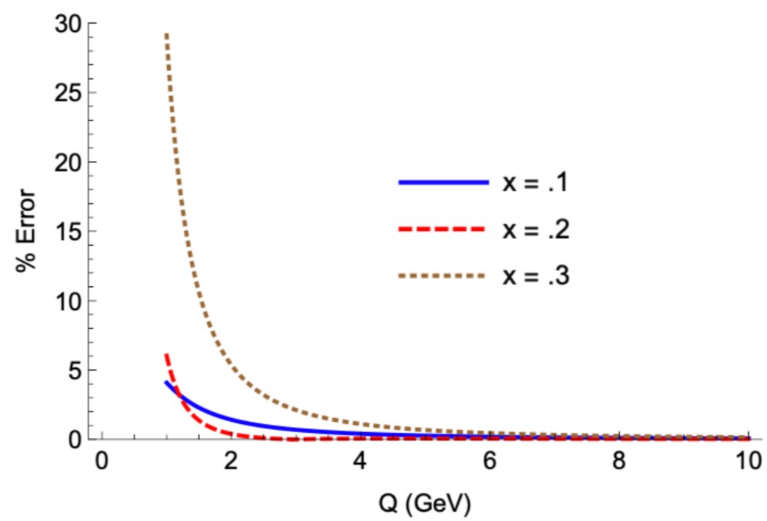
Partonic structure function

Parton Distribution

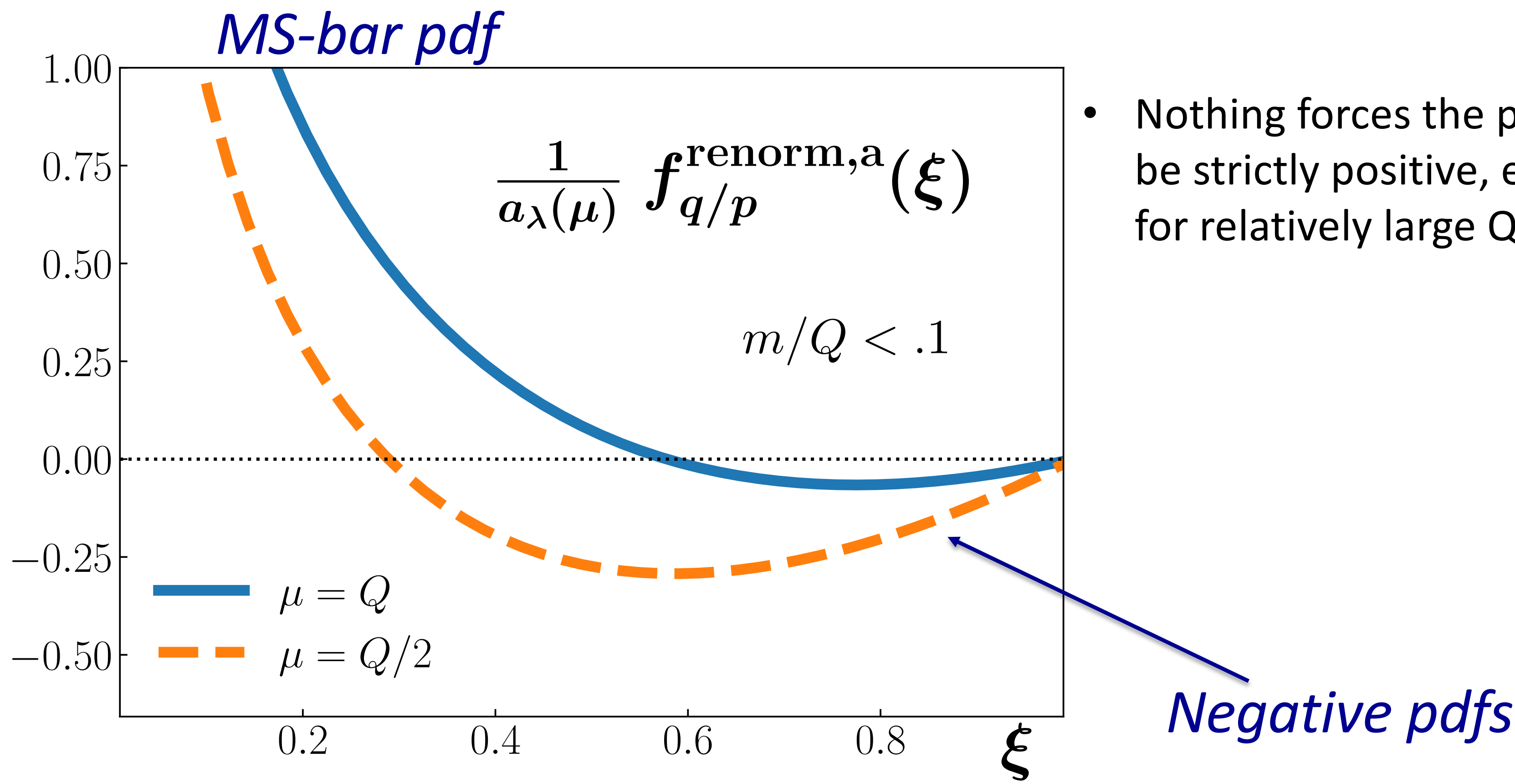
Positivity example

Collinear Factorization





Positivity example



- Nothing forces the pdf to be strictly positive, even for relatively large Q.

Track B:

- Issues:
  - We have not found a derivation of factorization for step 1 ($\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$) in existing literature
  - Bare pdf ($f_{\text{“bare,b”}}$) of step 1 is undefined
  - Collinear pdfs viewed as physical?
  - Can we reverse engineer $f_{\text{“bare,b”}}$?

Track B bare pdf: Can we reverse engineer it?

- In renormalized QCD, neither is UV divergent:

$$\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$$

- So, $f_{\text{“bare,b”}}$ cannot be UV divergent.

- $\Longrightarrow$ $f_{\text{“bare,b”}}$ must be a type of renormalized pdf (?)

- Follow Wilson-Zimmerman style of OPE derivation

Track B bare pdf: Can we reverse engineer it?

- $f_{\text{“bare,b”}}$ must be a renormalized pdf, not a bare one

$$d\sigma = f_{\text{“bare,b”}} \otimes d\hat{\sigma}$$

?

- Factorization is target-independent:

- Take the target to be a massless, on shell parton

- Infer the renormalization scheme for $f_{\text{“bare,b”}}$

Track B bare pdf: Can we reverse engineer it?

- $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma} \implies \mathrm{d}\hat{\sigma} = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

- $f_{\text{“bare,b”},ij} = \delta_{ij}\delta(1-x)$

Track B bare pdf: Can we reverse engineer it?

- $\mathrm{d}\sigma = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma} \implies \mathrm{d}\hat{\sigma} = f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$

- $f_{\text{“bare,b”},ij} = \delta_{ij}\delta(1-x)$

- Analogy with Bogoliubov-Parasiuk-Hepp-Zimmermann (BPH(Z)) renormalization *(used by Wilson-Zimmermann for UV divergences)*
  - BPH(Z): Subtract at zero external momentum
  - BPH(Z)0: Subtract at x = 1, masses = 0

Summary

- Historically two alternative ways of viewing divergences and their role in pdf definitions.
  - Track A: UV renormalization – no collinear divergences
  - Track B: Collinear absorption – absorb collinear divergences
- Track A is the more logically consistent approach.
- Minimal requirement for track B argument: The BPHZ0 renormalization scheme
  - Open questions: Factorization?
- Positivity is not a general property of MSbar renormalized parton densities
- Most practical calculations are unaffected, but there are interesting exceptions:
  - Positivity *(Constraints on pdfs at low-ish Q?)*
  - Soffer bound
  - Heavy quarks

Not discussed today

Backup

What is the track B bare pdf?

- Apply BPHZ(0) to a general pdf.
- Now has no ultraviolet divergences, but is sensitive to collinear regulator
- Question: Does the factorization hold with this pdf?

$$\mathrm{d}\sigma \overset{??}{=} f_{\text{“bare,b”}} \otimes \mathrm{d}\hat{\sigma}$$