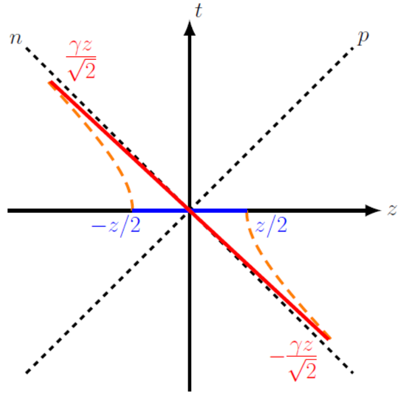
Recent Progress in LaMET



Xiangdong Ji

CNF, SURA & U. of Maryland

QCD Evolution Workshop 2021, UCLA

May 11, 2021

Outline

- What is a parton?
- LaMET: an effective theory for calculating partons
- Recent progress
  - Precision x-dependence calculations
  - GPDs
  - TMDs
  - Light-Front WFs
- Outlook

What is a parton?

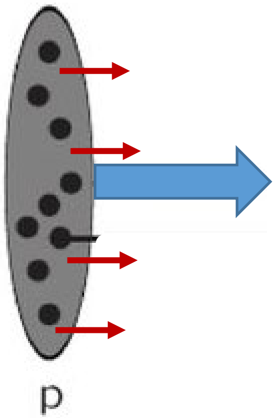
Feynman's parton model

- When a high-energy proton travels at $v \to c$, one can assume the proton travels exactly at v=c, or the proton momentum is

$$p=E=\infty$$

(Infinite momentum frame, IMF)

- The proton may be considered as a collection of interaction-free particles: partons



PHYSICAL REVIEW VOLUME 185, NUMBER 5 25 SEPTEMBER 1969

Inelastic Electron-Proton and γ-Proton Scattering and the Structure of the Nucleon*

J. D. Bjorken and E. A. Paschos
Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 10 April 1969)

A model for highly inelastic electron-nucleon scattering at high energies is studied and compared with existing data. This model envisages the proton to be composed of pointlike constituents ("partons") from which the electron scatters incoherently. We propose that the model be tested by observing γ rays scattered inelastically in a similar way from the nucleon. The magnitude of this inelastic Compton-scattering cross section can be predicted from existing electron-scattering data, indicating that the experiment is feasible, but difficult, at presently available energies.

I. INTRODUCTION

ONE of the most interesting results emerging from the study of inelastic lepton-hadron scattering at high energies and large momentum transfers is the possibility of obtaining detailed information about the structure, and about any fundamental constituents, of hadrons. We discuss here an intuitive but powerful model, in which the nucleon is built of fundamental pointlike constituents. The important feature of this model, as developed by Feynman, is its emphasis on the infinite-momentum frame of reference.

Partons in QFT are effective DOFs

- Partons are not just the quarks and gluons in the usual QCD lagrangian.
- Partons are a special type of IR collinear modes with momentum

$$k^\mu = \left(k^0, k^z, \vec{k}_\perp\right)$$

with $k^z \to \infty, k^0 \to \infty, k_\perp \sim \Lambda_{QCD}, k_\mu^2 \sim \Lambda_{QCD}^2$

Collins, Soper and Sterman, QCD factorization, 70'-80's

Bauer, Stewart et al, Soft-Collinear EFT, 00's

Partons are an idealized concept

- Partons do not exist in the real world, as there is no hadron traveling at $P^z = \infty$, but a useful concepts like “ideal gas” or “perfect fluid”
- A similar useful concept or effective d.o.f in QCD is “infinite mass quark”

$$m_Q = \infty$$

a good approximation for c, b, t quarks.

EFT: heavy-quark effective theory(HQET)

Partonic Effective Theory

- Hamiltonian Formulation

S. Weinberg (66')
S. J. Chang and S. K. Ma (69')
J. Kogut and D. Soper ('70) …

} Dirac's Light-Front Quantization

Much work has been devoted to LFQ due to the leadership by S. Brodsky and late K. Wilson.

- Lagrangian Formulation
  - QCD factorization: light-front correlations
  - Using the soft-collinear modes in SCET can construct the bound state of the proton

LaMET: an effective theory for calculating partons

X. Ji, PRL110 (2013), Sci. China PMA57 (2014)

EFT for partons: Full Euclidean QCD

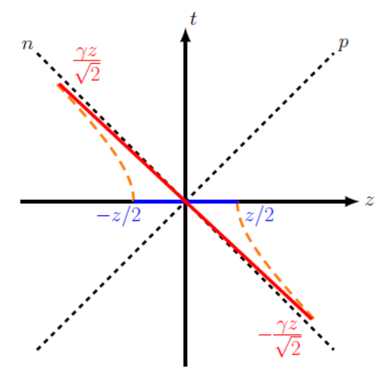
High-energy scattering in Minkowski QCD

Large-momentum Hadrons in Euclidean QCD

QCD factorization
Or Soft-Collinear EFT

Partonic Theory (LFQ…)

Large-momentum effective theory



IMF formulation of partons

- Euclidean correlation functions, e. g.

$$C(\lambda) = \langle P^z = \infty | \overline{\psi}(z) \Gamma \psi(0) | P^z = \infty \rangle$$

Correlation distance: $\lambda = \lim_{P^z \to \infty, z \to 0} (z P^z).$

- PDFs: Fourier Transformation of the spatial correlations

$$f(x) = \frac{1}{2P^+} \int \frac{d\lambda}{2\pi} e^{ix\lambda} C(\lambda) \ .$$

Partons from a large-P expansion of Euclidean observables

- Assuming $P^z \to \infty$ limit exists, parton physics is obtained by expansion (Feynman, 1969)

$$f(k^z, P^z) = f(x) + f_2(x)(M/P^z)^2 + \ldots$$

- Account for subtlety of non-commuting limits of $P^z \to \infty$ & $\Lambda_{QCD} \to \infty$ in QFT (Ji, 2013)

$$\tilde{f}(y, P^z) = \int Z(y/x, xP^z/\mu) f(x, \mu) dx + \mathcal{O}\Big(\frac{\Lambda^2_{\rm QCD}}{y^2(P^z)^2}, \frac{\Lambda^2_{\rm QCD}}{(1-y)^2(P^z)^2}\Big),$$

Alternative view on LaMET

- Approximate $P^z = \infty$ by a finite large P^z.

  We frequently do this in QCD

  Lattice QCD & HQET

- EFT expansion for PDFs in the spirit of Weinberg,

$$q(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} \tilde{C}\left(\frac{x}{y}, \frac{\mu}{yP^z}\right) \tilde{q}(y, P^z, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right).$$

x-dependence of PDF can be calculated from QCD!

Not by fitting as in extracting PDFs from exp. data.

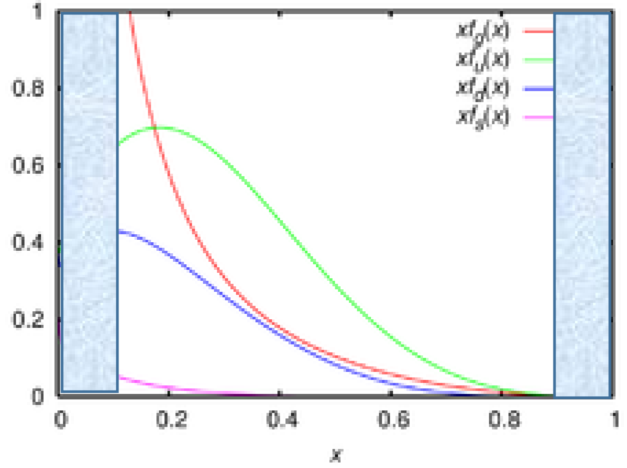
Limitations

- LaMET expansion breaks done near

$$xP^z \sim \Lambda_{QCD}; \quad (1-x)P^z \sim \Lambda_{QCD}$$

where the collinear modes end. These are soft modes or zero modes.

For $P^z \sim 2-3\ GeV$, max x-range: 0.1-0.9



Recent developments

Recent progress

- High precision calculations
- GPDs
- TMDs
- Light-Front WFs

How to get high-precision PDF

- Small lattice spacing limit, $a \to 0$
- Renormalization of linear divergences
- Matching up to two loops
- Physical point, $m_\pi \to m_\pi^{phys}$
- Larger momentum P
- …

Continuum limit

- For LaMET application, fine lattice spacing is extremely important
- Effective expansion parameter is

$$r = aP^z \sim a\gamma$$

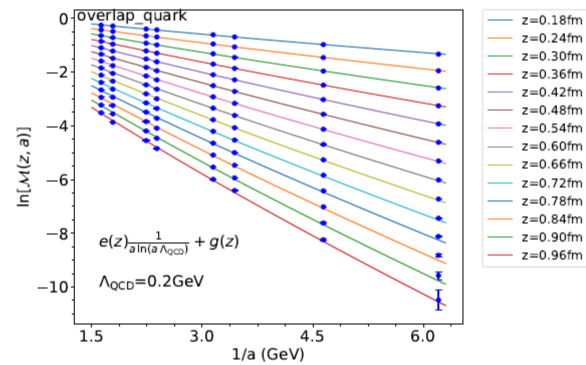
which can be large.

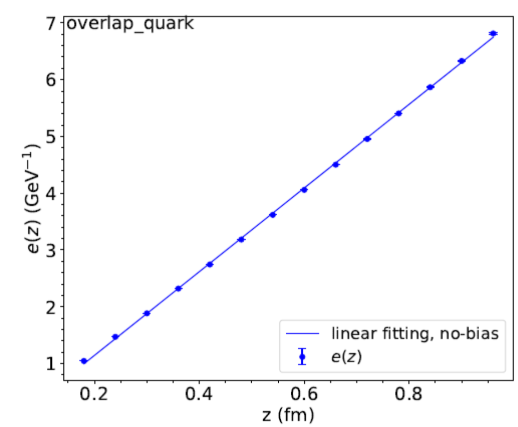
- Controlled extrapolation to continuum…

MSU, EMTC, BNL/ANL, LPC

Renormalization of linear divergences

- Renormalization of linear divergences causes large numerical uncertainties.
- The standard approaches (RI/MOM,ratio) could have large non-pert. effects
- New approaches
  - Auxiliary field approach (Green et al, Ji et al, 2018, C. Alexandrou et al, 2021)
  - Hybrid renormalization (Ji et al, 2021)

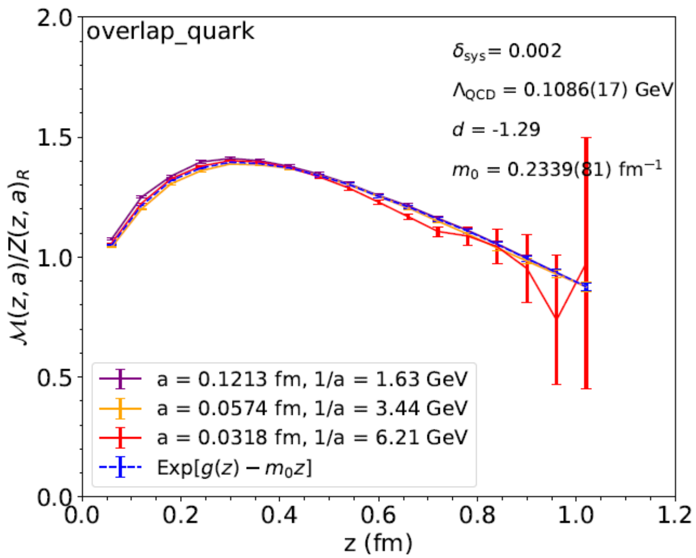




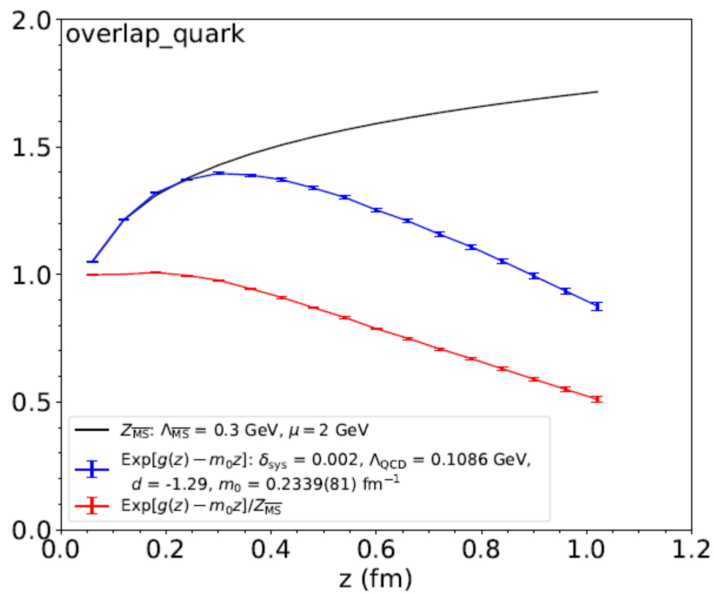
Hybrid Renormalization

Lattice Parton Collaboration (LPC)

Continuum limit



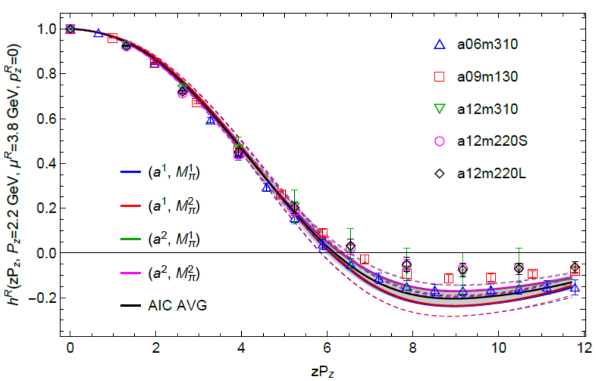
Pert.vs Non-pert Z



Continuum extrapolations

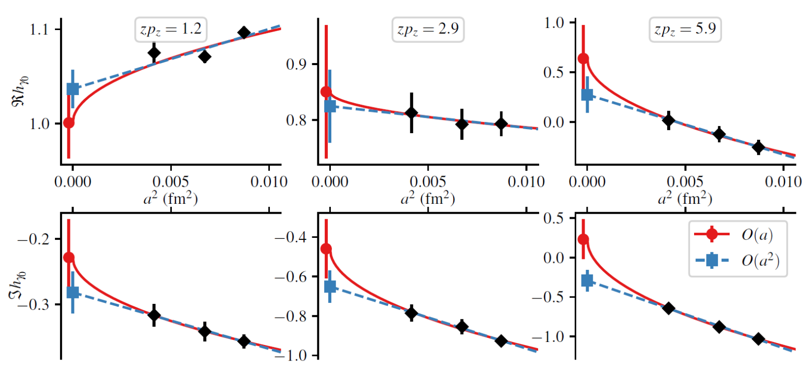
MSU

2011.14971
a=0.06,0.09,0.12



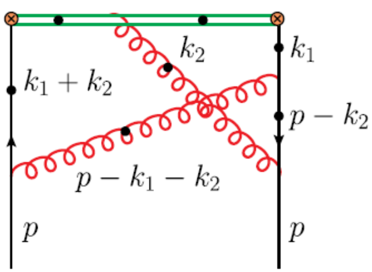
ETMC

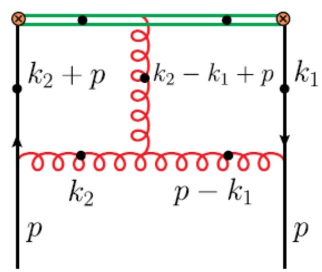
2011.00964
a= 0.064,0.082,0.093



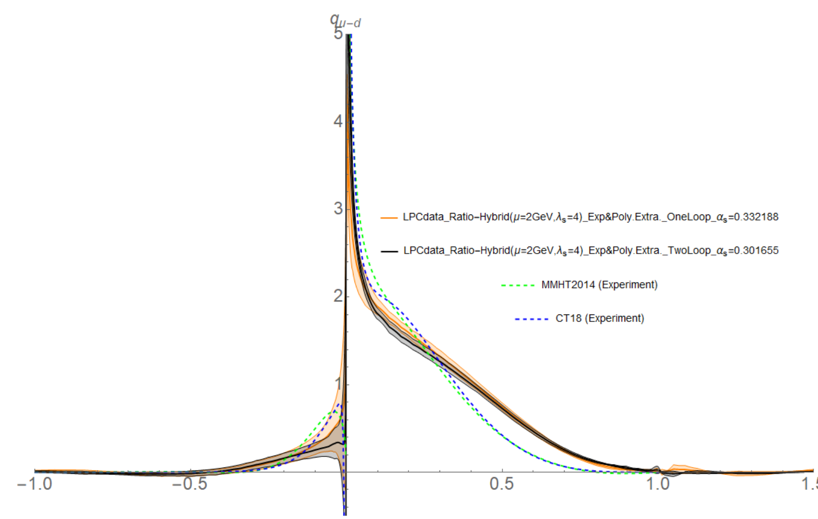
Two-loop matching

- Two-loop matching in the non-singlet sector has been calculated recently, L. Chen et al, PRL 126 (2021); Z. Li et al, PRL 126 (2021);

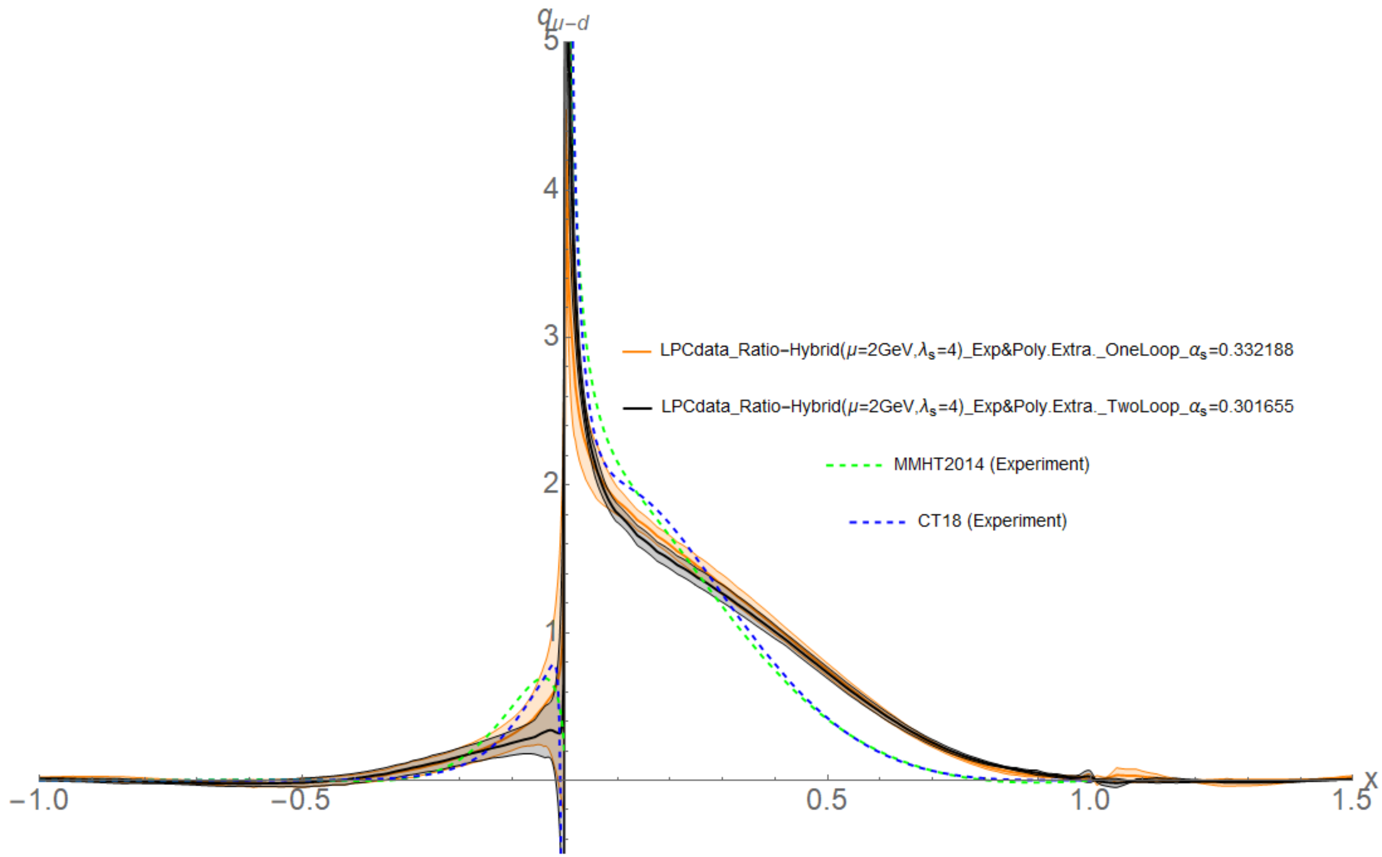




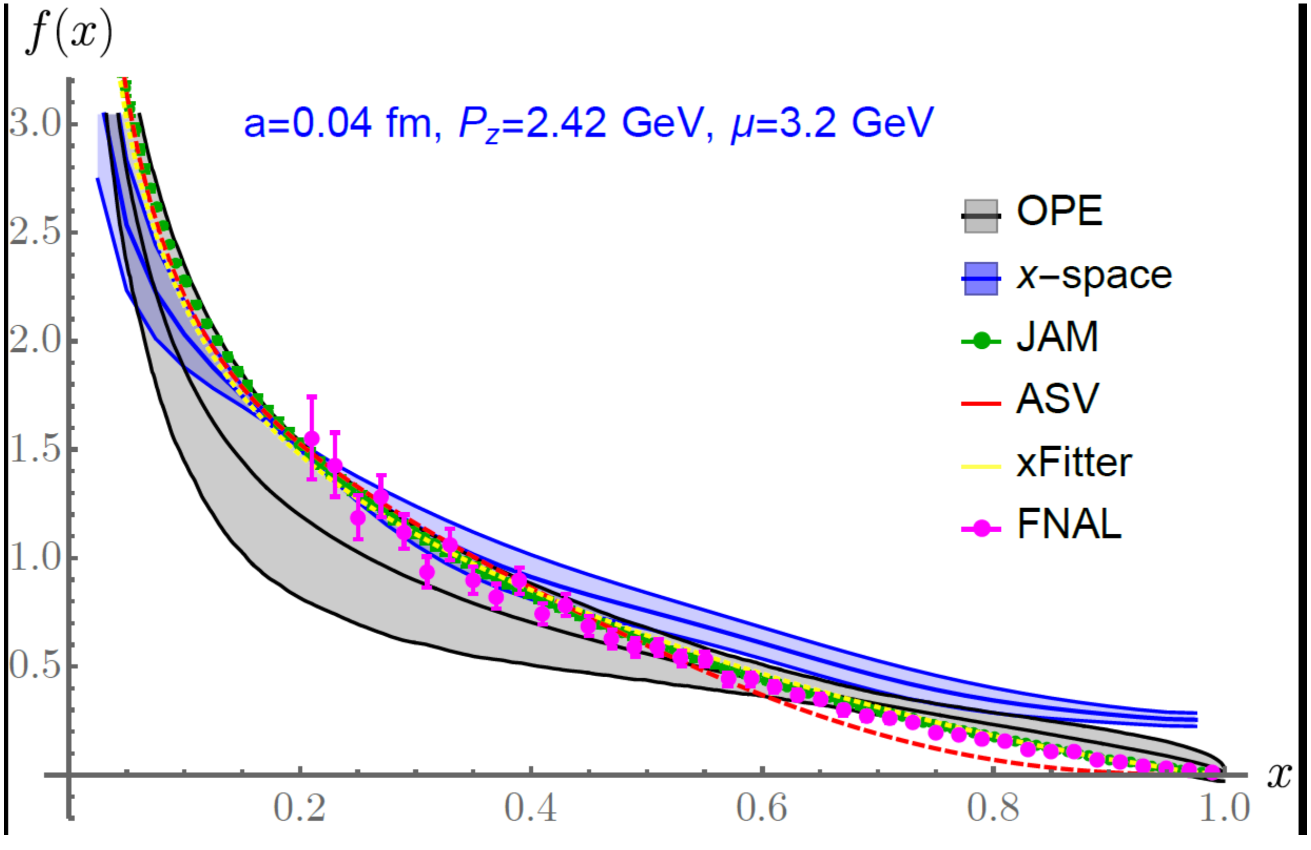
- Singlet part?



An example of precision PDF (LPC)

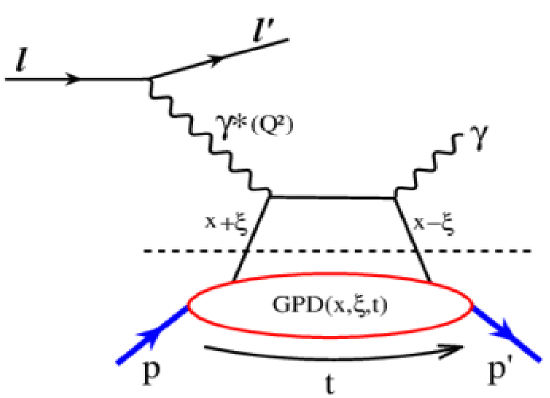


Pion PDF (BNL&Argonne)



Generalized parton distributions (GPD)

- GPD are hybrids of form factors and parton distributions: spin & mass structure, 3D imaging (Mueller et al.'94, Ji'96, Burkardt'00)
- Experimental processes that can be used to measure GPD: DVCS (Ji, 1996) & DVMP (Radyushkin, 1996, Collins et al, 1998)



Kinematic variables: x, ξ, t

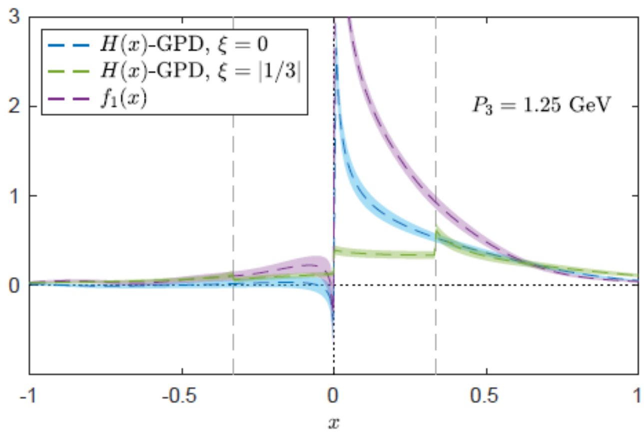
GPDs via LaMET

ETMC

PRL125 (2020)

MSU

2008.12474 (2020)



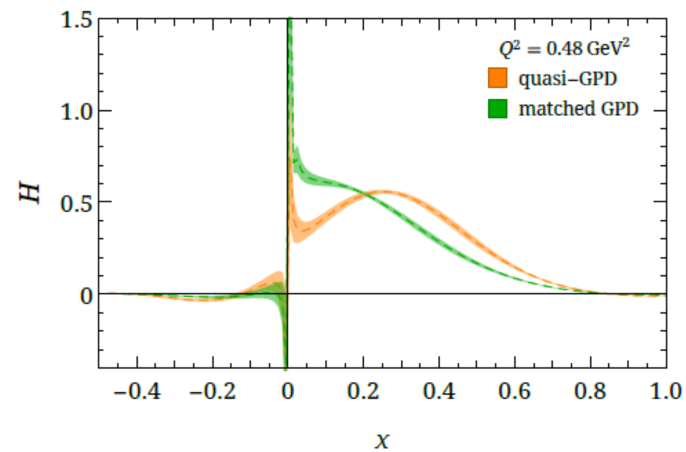


Figure 2: Examples of the first GPD data from lattice QCD. The left plot shows the matched GPD H at $\xi = 0$ and $|\xi| = 1/3$ with momentum transfer $Q^2 = 0.69$ GeV^2 [18]. The right one shows the quasi-GPD and matched GPD H at momentum transfer $Q^2 = 0.48$ GeV^2 and $\xi = 0$ [19].

TMDPDF & Lattice calculations

- Started from A. Schafer et al., invariant functions, matching with continuum quantities?

  Hagler et al, Much et al, Yoon et al. PRD96,094508 (2017)…

- A number of LaMET formulations:

  Ji et al., PRD91,074009 (2015); PRD99,114006(2019)

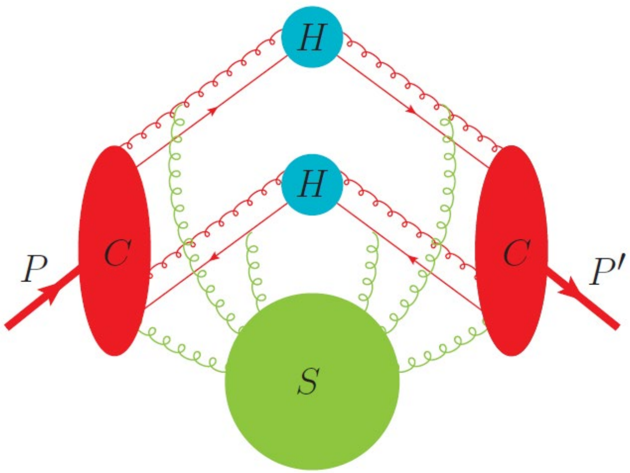
  Ebert, Stewart, Zhao, PRD99,034505 (2019), JHEP09,037(2019)

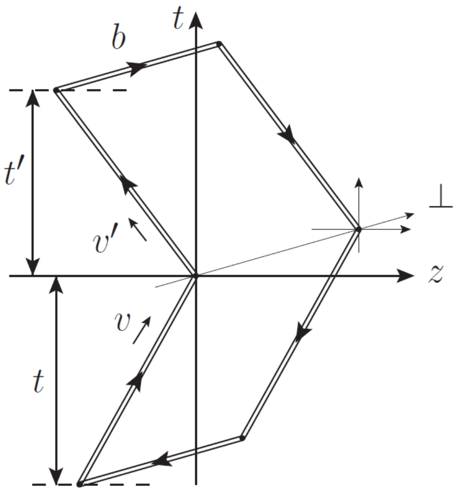
- Collins-Soper kernel can be calculated on lattice

  Ji et al., PRD91,074009 (2015); Ebert, Stewart and YZ, PRD 99(2019).M. Ebert, JHEP 03 (2020) 099

Soft Function

Ji, Liu, Liu, Nucl. Phys. B955 (2020)



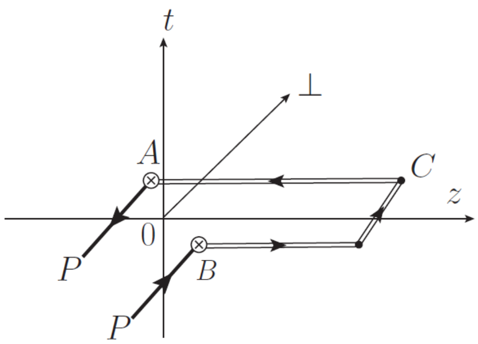


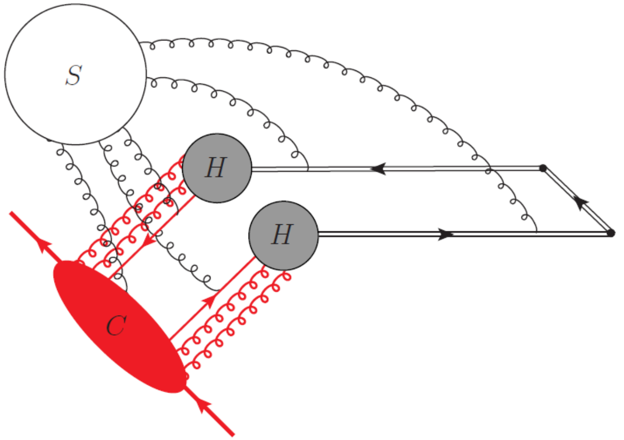
Factorization of form-factor of Light meson

Form-factors of heavy-quark pair

Quasi-TMDPDF Factorization

Ji, Liu, Liu, Phys.Lett.B 811 (2020)

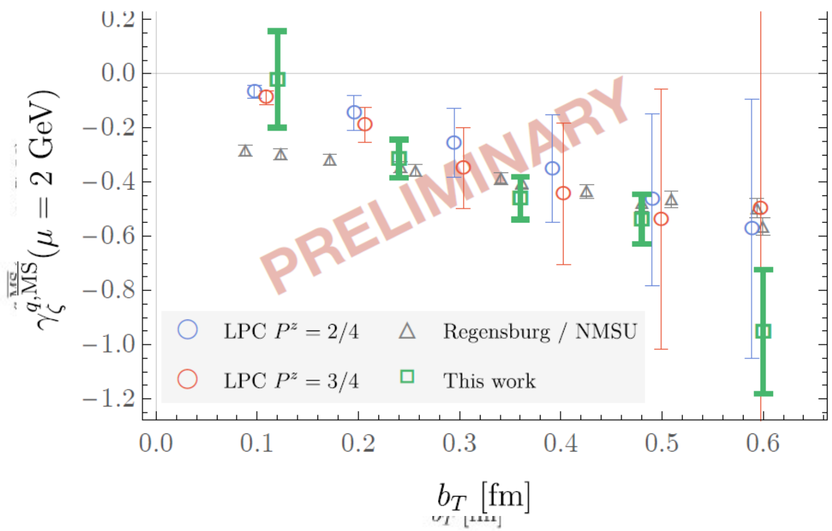


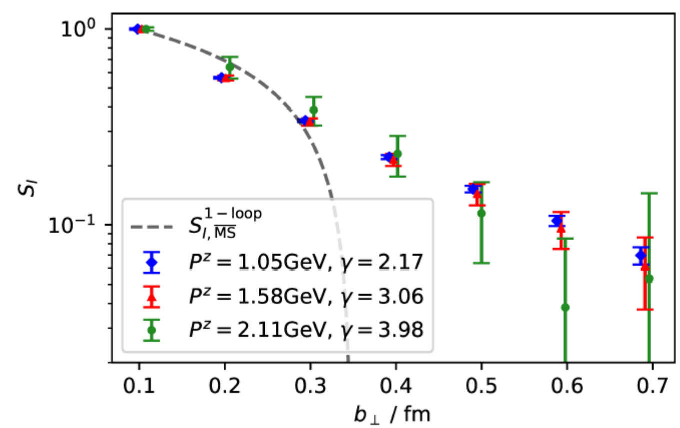


$$\tilde{f}(x, b_\perp, \mu, \zeta_z)\sqrt{S_r(b_\perp, \mu)}$$
$$= H\left(\frac{\zeta_z}{\mu^2}\right) e^{K(b_\perp,\mu)\ln(\frac{\zeta_z}{\zeta})} f^{\mathrm{TMD}}(x, b_\perp, \mu, \zeta) + \ldots$$

$$\mu\frac{d}{d\mu}\ln H\left(\frac{\zeta_z}{\mu^2}\right) = \Gamma_{\mathrm{cusp}}\ln\frac{\zeta_z}{\mu^2} + \gamma_C$$

Collins-Soper Kernel and Soft Factor





P. Shanahan et al, PRD 102 (2020) & unpublished
Q.A.Zhang et al, PRL125 (2020)
M.Schlemmer, 2103.16991 (2021)

Q. A. Zhang et al, PRL125 (2020) (LPC)

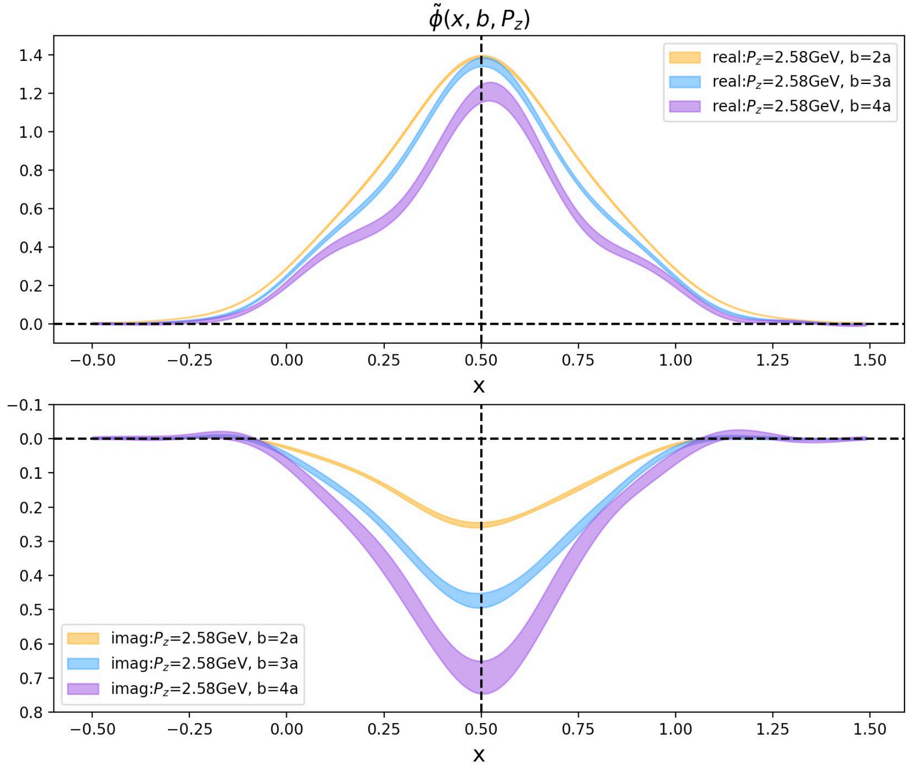
Light-Front Wave-Functions

- LF quantization focuses on the WFs, from which everything can be calculated: a very ambitious goal! Brodsky et al. Phys. Rept. 301 (1998)
- However, there are a number of reasons this approach has not been very successful.
- LaMET provides the practical way to calculate non-perturbative WF, at least for lowest few components. Ji & Liu, to be published.
- All WF can be computed as gauge-invariant matrix elements

$$\left\langle 0 \left| \hat{O}\left(z_1, \vec{b}_1, z_2, \vec{b}_2 \ldots, z_k, \vec{b}_k\right) \right| P \right\rangle$$

LPC result soon.

Meson TMD Light-Front Wave Function (LPC, preliminary)



Important for studying B-meson decays

Outlook

- LaMET is a systematic framework to calculate parton physics (indirectly)
- LaMET3.0 (~ 5% error?)
  - Improved non-pert renormalization at large z.
  - two-loop matching
  - $P \to 3$ GeV
  - Singlet quark and gluon
- GPDs & TMDs & Wigner Functions & LFWF
  - More to be done!