QCD factorization for twist-three parton quasi(pseudo)distributions

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based on: 2103.12105

Quantum mechanics of partons

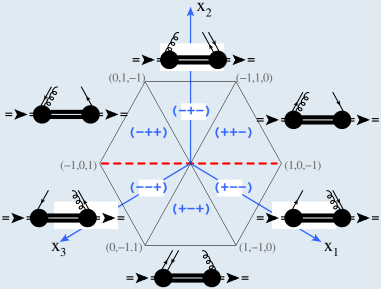
- Quark-antiquark-gluon correlation function

$$\mathbb{S}(a,b,c) = g\,\bar{q}(an)\gamma_+\Big[iF_{+\perp}\gamma_5 + \widetilde{F}_{+\perp}\Big](bn)q(cn), \qquad n^2 = 0$$

$$\langle p,s|\mathbb{S}(a,b,c)|p,s\rangle = -4s_\perp p_+^2 \int [dx]\, e^{-ip_+(ax_1+bx_2+cx_3)} S(x_1,x_2,x_3)$$

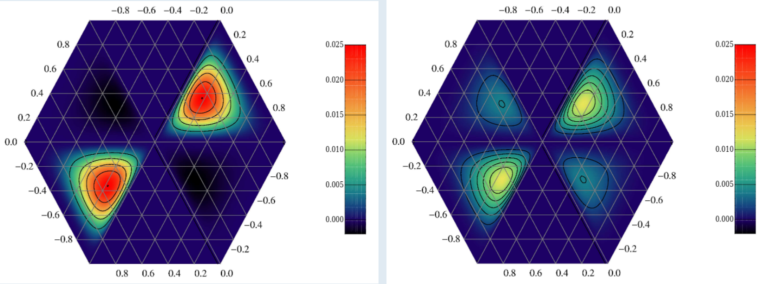
$$\int [dx] = \int_{-1}^{1} dx_1 dx_2 dx_3\, \delta(x_1+x_2+x_3)$$

- Domain interpretation



Light-cone WF overlap model

VB, A. Manashov, B. Pirnay: 1103.1269



- $S_u(x_1,x_2,x_3)$ and $S_ud(x_1,x_2,x_3)$ at 1 GeV

Particular projections:

- Structure function $g_2(x, Q^2)$ in polarized DIS
- SIDIS, Qiu-Sterman function
- Lattice QCD?

In this talk: What can we learn from parton quasi(pseudo)distributions?

$$\langle p, s|\bar{q}(z)\gamma^\mu\gamma^5[z,0]q(0)|p,s\rangle$$

$$[z,0] = \mathrm{P}\exp\left(ig\int_0^1 d\sigma\, z^\mu A_\mu(\sigma z)\right)$$

study motivated by: S. Bhattacharya *et al.*: 2004.04130, 2005.10939

Parton distributions

for light-like separations $z^2 = 0$

$$\langle p, s|\bar{q}(z)\not{z}\gamma^5[z,0]q(0)|p,s\rangle = 2M(sz)\int_{-1}^{1} dx\, e^{ix(pz)}\Delta q(x) \equiv 2M(sz)\,\widehat{\Delta q}(pz),$$

$$\langle p, s|\bar{q}(z)\gamma_T^{\mu}\gamma^5[z,0]q(0)|p,s\rangle = 2Ms_T^{\mu}\int_{-1}^{1} dx\, e^{ix(pz)}g_T(x) \equiv 2s_T^{\mu}M\,\widehat{g}_T(pz).$$

Hereafter

$$\boxed{\zeta = (p\cdot z)}$$

Twist decomposition $g_T(x) = g_T^{\text{tw2}}(x) + g_T^{\text{tw3}}(x)$

Exact relations based on operator identities:

$$\widehat{g}_T^{\text{tw2}}(\zeta) = \int_0^1 d\alpha\, \widehat{\Delta q}(\alpha\zeta),$$

$$g_T^{\text{tw2}}(x) = \theta(x)\int_x^1 \frac{dy}{y}\Delta q(y) - \theta(-x)\int_{-1}^x \frac{dy}{y}\Delta q(y)$$

$$\widehat{g}_T^{\text{tw3}}(\zeta) = 2\zeta^2 \int_0^1 d\alpha \int_\alpha^1 d\beta\, \bar\beta\, \widehat{S}(\zeta,\beta\zeta,\alpha\zeta) \qquad \boxed{\bar\beta = 1-\beta}$$

$$g_T^{\text{tw3}}(x) = 2\int [dx] \int_0^1 d\alpha \Big(\frac{\delta(x+\alpha x_1)}{x_1 x_3} + \frac{\delta(x+x_1+\alpha x_2)}{x_2 x_3} + \frac{\delta(x+x_1)}{x_1 x_2}\Big) S(x_1,x_2,x_3)$$

- momentum space expressions are more cumbersome due to multitude of partonic domains

Parton quasi(pseudo)distributions

for non-light-like separations $z^2 \neq 0$

$$\langle p,s|\bar{q}(z)\not{z}\gamma^5[z,0]q(0)|p,s\rangle = 2M(sz)\,\widehat{\mathcal{G}}_1(\zeta,z^2)$$

$$\langle p,s|\bar{q}(z)\gamma_T^\mu\gamma^5[z,0]q(0)|p,s\rangle = 2Ms_T^\mu\,\widehat{\mathcal{G}}_T(\zeta,z^2)$$

At tree level

$$\widehat{\mathcal{G}_1}(\zeta) = \widehat{\Delta q}(\zeta) \qquad\qquad \widehat{\mathcal{G}_T}(\zeta) = \widehat{g}_T(\zeta)$$

To all orders of perturbation theory (factorization theorem)

$$\boxed{\begin{aligned}\mathcal{G}_1(\zeta,z^2) &= \int_0^1 d\alpha\, C_1(\alpha,z^2\mu_F^2)\,\widehat{\Delta q}(\alpha\zeta;\mu_F) + \mathcal{O}(z^2)\\ \mathcal{G}_T(\zeta,z^2) &= \int_0^1 d\alpha\, C_2(\alpha,z^2\mu_F^2)\,\widehat{\Delta q}(\alpha\zeta;\mu_F)\\ &\qquad + 2\zeta^2\int_0^1 d\alpha\int_0^1 d\beta\, C_3(\alpha,\beta,z^2\mu_F^2)\widehat{S}(\zeta,\beta\zeta,\alpha\zeta;\mu_F) + \mathcal{O}(z^2)\end{aligned}}$$

- all-order proof = operator product expansion
- one-loop coefficient functions calculated

Three-to-two-particle reduction?

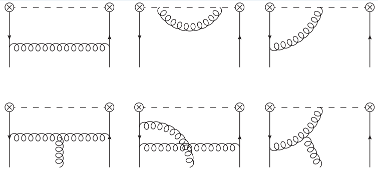
Alternative representation

$$\mathcal{G}_T(\zeta, z^2) = \int_0^1 d\alpha\, \mathbf{C}_T(\alpha, z^2\mu_F^2)\, \widehat{g}_T^{\mathrm{tw2}}(\alpha\zeta;\mu_F) + \int_0^1 d\alpha\, \mathbf{C}_{2\mathrm{pt}}(\alpha, z^2\mu_F^2)\, \widehat{g}_T^{\mathrm{tw3}}(\alpha\zeta;\mu_F)$$
$$+ 2\zeta^2 \int_0^1 d\alpha \int_0^1 d\beta\, \mathbf{C}_{3\mathrm{pt}}(\alpha, \beta, z^2\mu_F^2) \widehat{S}(\zeta, \beta\zeta, \alpha\zeta;\mu_F)$$

with $\mathbf{C}_T^{(0)} = \delta(1-\alpha)$

- $\mathbf{C}_T \neq \mathbf{C}_{2\mathrm{pt}}$
- Separation between $\mathbf{C}_{2\mathrm{pt}}$ and $\mathbf{C}_{3\mathrm{pt}}$ not unique

One-loop: the calculation



- Background field technique, L.Abbott. '81; I.Balitsky, V.B, '89

One-loop results: twist two in position space

Notations: $\boxed{a_s = \frac{\alpha_s}{4\pi}}$ $\boxed{\mathrm{L}_z = \ln\left(\frac{-z^2\mu^2}{4\,e^{-2\gamma_E}}\right)}$

$$\mathcal{G}_1(\zeta, z^2;\mu) = \widehat{\Delta q}(\zeta;\mu) + a_s \int_0^1 d\alpha\, \mathbf{C}_1^{(1)}(\alpha, \mathrm{L}_z;\mu)\, \widehat{\Delta q}(\alpha\zeta;\mu)\,,$$

$$\mathbf{C}_1^{(1)}(\alpha, \mathrm{L}_z) = 2C_F\left[\left(-\mathrm{L}_z\frac{1+\alpha^2}{1-\alpha} + \frac{3-8\alpha+3\alpha^2-4\ln\bar\alpha}{1-\alpha}\right)_+ + \delta(\bar\alpha)\left(\frac{3}{2}\mathrm{L}_z + \frac{7}{2}\right)\right]$$

$$\mathcal{G}_T^{\text{tw2}}(\zeta, z^2;\mu) = \widehat{g}_T^{\text{tw2}}(\zeta;\mu) + a_s \int_0^1 d\alpha\, \mathbf{C}_T^{(1)}(\alpha, \mathrm{L}_z;\mu)\, \widehat{g}_T^{\text{tw2}}(\alpha\zeta;\mu)$$

$$\mathbf{C}_T^{(1)}(\alpha, \mathrm{L}_z;\mu) = 2C_F\left[\left(-\mathrm{L}_z\frac{1+\alpha^2}{1-\alpha} + \frac{1-4\alpha+\alpha^2-4\ln\bar\alpha}{1-\alpha}\right)_+ + \delta(\bar\alpha)\left(\frac{3}{2}\mathrm{L}_z + \frac{5}{2}\right)\right]$$

- $\mathbf{C}_1^{(1)}\neq\mathbf{C}_T^{(1)}$, thus WW relation violated for q(p)PDFs

One-loop results: twist three in position space

Notations: $\boxed{a_s = \frac{\alpha_s}{4\pi}}$ $\boxed{\mathrm{L}_z = \ln\left(\frac{-z^2\mu^2}{4\,e^{-2\gamma_E}}\right)}$ $\boxed{\zeta = (p\cdot z)}$

$$\mathcal{G}_T^{\text{tw3}} = \widehat{g}_T^{\text{tw3}}(\zeta;\mu) + a_s\int_0^1 d\alpha\,\mathbf{C}_T^{(1)}(\alpha,\mathrm{L}_z;\mu)\,\widehat{g}_T^{\text{tw3}}(\alpha\zeta;\mu)$$
$$+ 2a_sN_c\zeta^2\int_0^1 d\alpha\int_\alpha^1 d\beta\left[\bar\beta\left(\bar\alpha\mathrm{L}_z + 2 + \alpha\right) + 2\ln\beta\right]\widehat{S}(\zeta,\beta\zeta,\alpha\zeta) + \mathcal{O}(1/N_c)$$

or

$$\mathcal{G}_T^{\text{tw3}} = \widehat{g}_T^{\text{tw3}}(\zeta;\mu) + a_s\int_0^1 d\alpha\left[\mathbf{C}_T^{(1)}(\alpha,\mathrm{L}_z;\mu) + N_c\left(\mathrm{L}_z\left(\delta(\bar\alpha) - \alpha\right) + \alpha + 2\delta(\bar\alpha)\right)\right]\widehat{g}_T^{\text{tw3}}(\alpha\zeta;\mu)$$
$$+ 4a_sN_c\zeta^2\int_0^1 d\alpha\int_\alpha^1 d\beta\,\ln\beta\,\widehat{S}(\zeta,\beta\zeta,\alpha\zeta) + \mathcal{O}(1/N_c)$$

- $\mathcal{O}(1/N_c)$ corrections see 2103.12105

One-loop results: twist three in position space (2)

1. $\widehat{g}_T^{\text{tw3}}$ and $\widehat{g}_T^{\text{tw2}}$ have different coefficient functions already at the leading log level (and at large N_c)
2. The main effect to the LO accuracy is the constant shift by N_c of the anomalous dimensions of the twist-three operators as compared to the leading twist ones [Ali:1991em]
3. A genuine three-particle contribution involving $\ln\beta$ appears. It cannot be rewritten in terms of $\widehat{g}_T^{\text{tw3}}$

The $1/N_c$ corrections have a more complicated structure and do not allow a separation of two-particle contributions in a natural way

Parton quasidistributions (qPDFs) at NLO

X. Ji, 1305.1539

$$
\begin{aligned}
\mathrm{g}_1(x,p_v) &= p_v \int \frac{dz}{2\pi} e^{-ixzp_v} \mathcal{G}_1(zp_v, z^2) \\
\mathrm{g}_T(x,p_v) &= p_v \int \frac{dz}{2\pi} e^{-ixzp_v} \mathcal{G}_T(zp_v, z^2)
\end{aligned}
$$

$$
\begin{aligned}
v^\mu &= z^\mu/|z| \\
p_v &= p\cdot v \\
\mathrm{L}_p &= \ln\left(\frac{\mu^2}{4y^2p_v^2}\right)
\end{aligned}
$$

Factorization theorem

$$
\begin{aligned}
\mathrm{g}_1(x,p_v) &= \Delta q(x) + a_s \int_{-1}^{1} \frac{dy}{|y|} \mathrm{C}_1^{(1)}\left(\frac{x}{y}, \mathrm{L}_p\right) \Delta q(y)\,, \\
\mathrm{g}_T(x,p_v) &= g_T(x) + a_s \int_{-1}^{1} \frac{dy}{|y|} \left(\mathrm{C}_T^{(1)}\left(\frac{x}{y}, \mathrm{L}_p\right) g_T^{\text{tw2}}(y) + \mathrm{C}_{2\mathrm{pt}}^{(1)}\left(\frac{x}{y}, \mathrm{L}_p\right) g_T^{\text{tw3}}(y)\right) \\
&\quad + 2a_s \mathrm{C}_{3\mathrm{pt}}^{(1)} \otimes S
\end{aligned}
$$

qPDFs at NLO, twist two

$$
\begin{aligned}
\mathtt{C}_1^{(1)}(\alpha,\mathrm{L}_p) &= 2C_F\delta(\bar\alpha)\Big(\frac{3}{2}\mathrm{L}_p+\frac{7}{2}\Big)\\
&+2C_F\begin{cases}
\Big(\dfrac{1+\alpha^2}{1-\alpha}\ln\dfrac{\alpha}{-\bar\alpha}+1-\dfrac{3}{2\bar\alpha}\Big)_\oplus, & \alpha>1\\
\Big(\dfrac{1+\alpha^2}{1-\alpha}\Big[-\mathrm{L}_p+\ln(\alpha\bar\alpha)-1\Big]+3-2\alpha+\dfrac{3}{2\bar\alpha}\Big)_+, & 0<\alpha<1\\
\Big(\dfrac{1+\alpha^2}{1-\alpha}\ln\dfrac{\bar\alpha}{-\alpha}-1+\dfrac{3}{2\bar\alpha}\Big)_\ominus, & \alpha<0
\end{cases}\\
\mathtt{C}_T^{(1)}(\alpha,\mathrm{L}_p) &= 2C_F\delta(\bar\alpha)\Big(\frac{3}{2}\mathrm{L}_p+\frac{5}{2}\Big)\\
&+2C_F\begin{cases}
\Big(\dfrac{1+\alpha^2}{1-\alpha}\ln\dfrac{\alpha}{-\bar\alpha}+1-\dfrac{3}{2\bar\alpha}\Big)_\oplus, & \alpha>1\\
\Big(\dfrac{1+\alpha^2}{1-\alpha}\Big[-\mathrm{L}_p+\ln(\alpha\bar\alpha)-1\Big]+1+\dfrac{3}{2\bar\alpha}\Big)_+, & 0<\alpha<1\\
\Big(\dfrac{1+\alpha^2}{1-\alpha}\ln\dfrac{\bar\alpha}{-\alpha}-1+\dfrac{3}{2\bar\alpha}\Big)_\ominus, & \alpha<0
\end{cases}
\end{aligned}
$$

qPDFs at NLO, twist three: two-particle contributions

$$\mathtt{C}^{(1)}_{2\mathrm{pt}}(\alpha,\mathrm{L}_p) = \mathtt{C}_2(\alpha,\mathrm{L}_p) + N_c\Big[\delta(\bar\alpha)\Big(\frac{1}{2}\mathrm{L}_p+\frac{5}{2}\Big) + \theta(0<\alpha<1)\Big[\alpha(1-\mathrm{L_p})\Big]_+ + \mathtt{r}_1(\alpha)\Big]$$

$$\mathtt{r}_1(\alpha) = \begin{cases} \Big(\alpha\ln\dfrac{\alpha}{-\bar\alpha} - 1 + \dfrac{1}{2\bar\alpha}\Big)_\oplus, & \alpha>1,\\ \Big(1-2\alpha+\alpha\ln(\alpha\bar\alpha) - \dfrac{1}{2\bar\alpha}\Big)_+, & 0<\alpha<1,\\ \Big(\alpha\ln\dfrac{\bar\alpha}{-\alpha} + 1 - \dfrac{1}{2\bar\alpha}\Big)_\ominus, & \alpha<0, \end{cases}$$

qPDFs at NLO, twist three: three-particle contributions

$$\mathbb{C}^{(1)}_{3\mathrm{pt}} \otimes S = -2N_c\, \theta(|x|<1) \int [dx_i] \int_0^1 \frac{d\alpha}{\bar\alpha} \left[\frac{\delta(x+\alpha x_1)}{x_1 x_3} + \frac{\delta(x-x_3-\alpha x_2)}{x_2 x_3} + \frac{\delta(x+x_1)}{x_1 x_2} \right] S(x_1,x_2,x_3) + \mathcal{O}(1/N_c)$$

- $\mathcal{O}(1/N_c)$ corrections also present in $x<0$ and $x>1$ regions, see 2103.12105

Summary

- QCD factorization for qPDFs/pPDfs/qITDs etc., is valid to all orders and to all twists $\Leftarrow$ OPE
- Complete NLO results available to twist-three accuracy
- Three-particle terms at large N_c can partially be reabsorbed in terms of $g_T^{\rm tw3}$, but:
  — CFs of $g_T^{\rm tw2}$ and $g_T^{\rm tw3}$ are different; no factorization exists in terms of $g_T = g_T^{\rm tw2} + g_T^{\rm tw3}$
  — CFs cannot be recovered from quark-antiquark matrix elements alone
  — "genuine" three-particle contributions unavoidable at NLO and beyond
- Wandzura-Wilczek relation does not hold for qPDFs/pPDfs/qITDs etc. This is in difference to DIS, where it holds for structure functions
- Chiral-odd qPDFs/pPDfs/qITDs $h_L(x)$ and $e(x)$ — work in progress