Precision Calculation of the *x*-dependence of PDFs from Lattice QCD

QCD Evolution Workshop 2021
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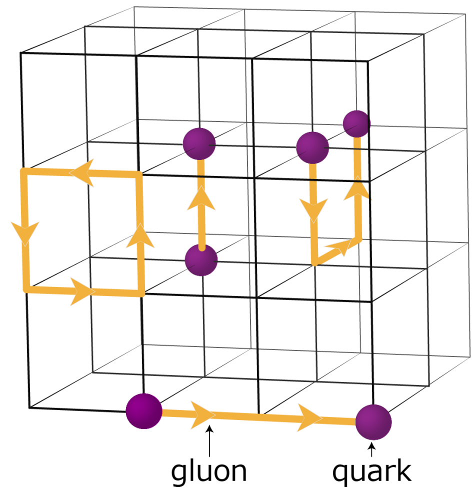
Argonne NATIONAL LABORATORY

In collaboration with Xiang Gao, Andrew Hanlon, Nikhil Karthik, Swagato Mukherjee, Peter Petreczky, Philipp Scior, Sergey Syritsyn, in preparation.

3D Tomography of the Proton (Hadron)

Lattice QCD



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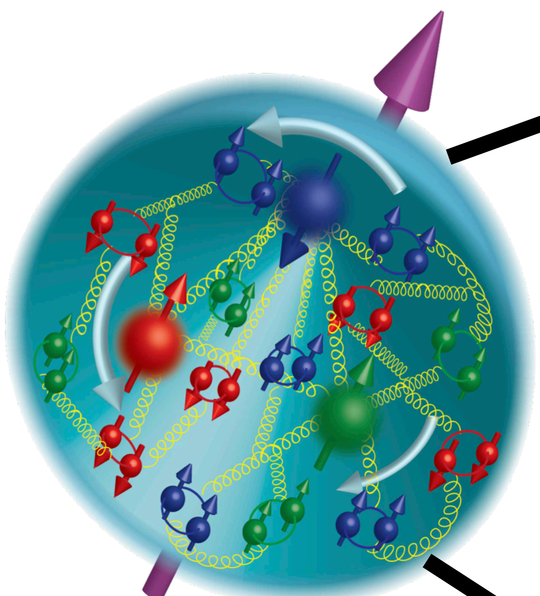
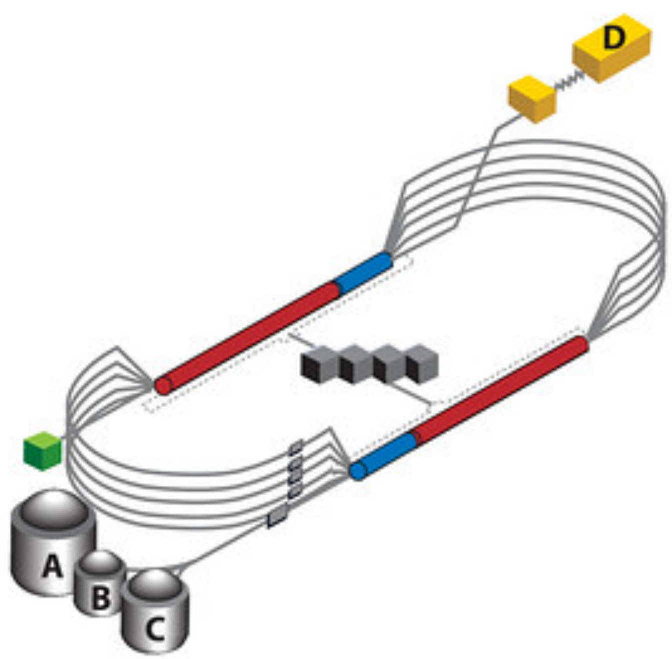
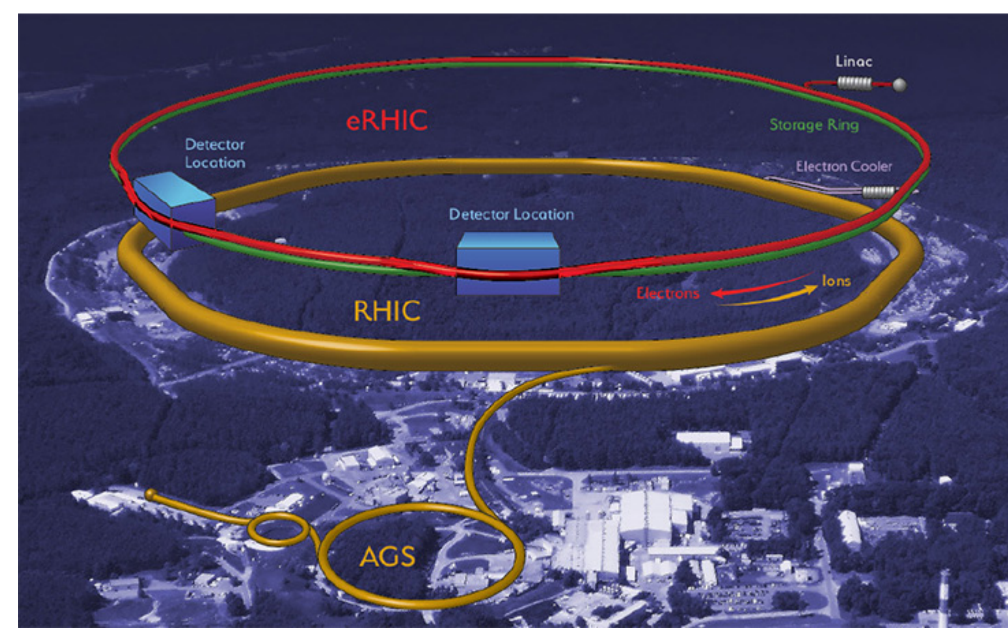


Precision is the key!

Hard Scattering



Jefferson Lab 12 GeV



The Electron-Ion Collider

Outline

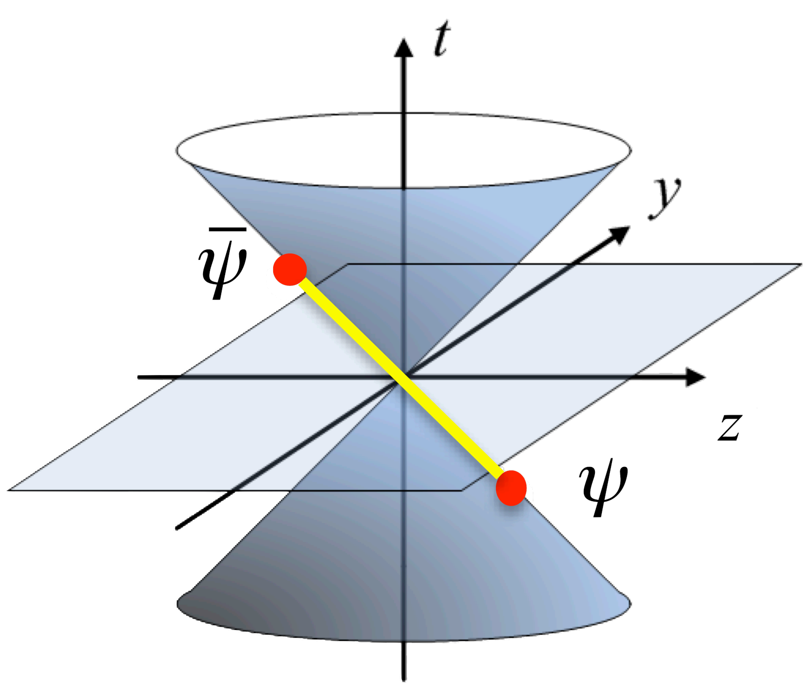
- Methodology
  - Large-momentum effective theory
  - Hybrid renormalization scheme
- Lattice calculation
  - Wilson line mass renormalization
  - Fourier transform and perturbative matching
  - Final results

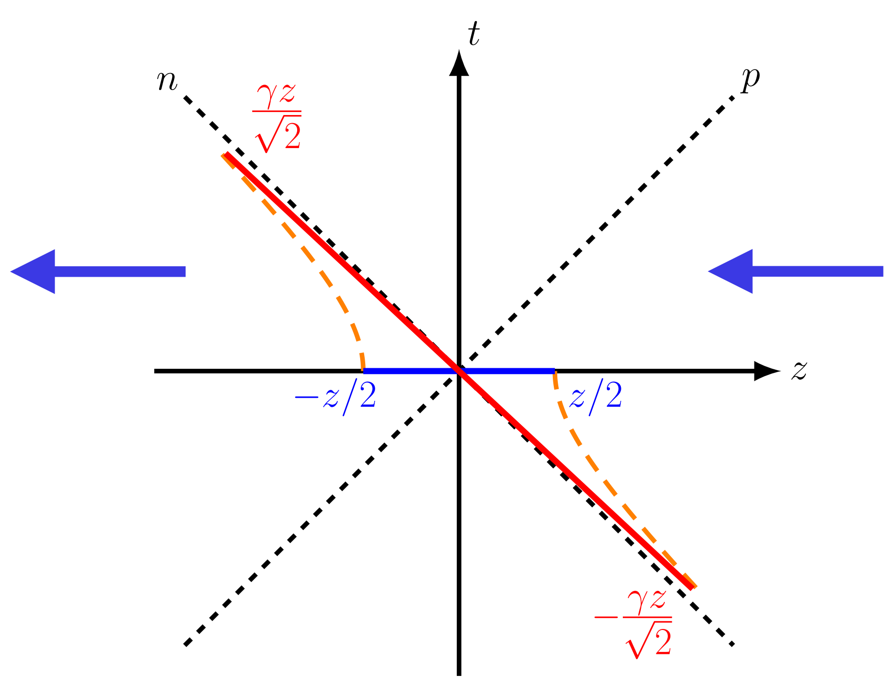
Large-Momentum Effective Theory (LaMET)

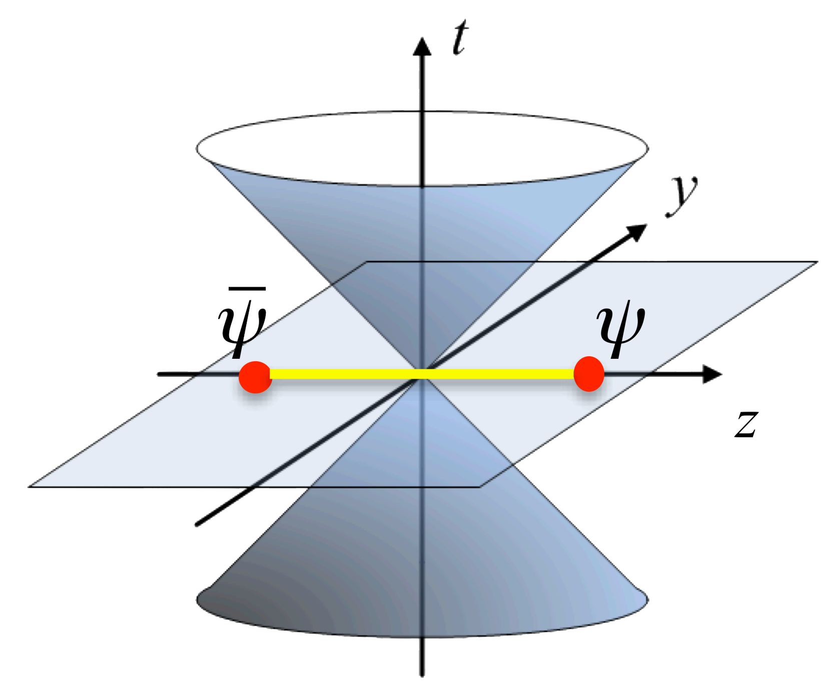
$z + ct = 0, \;\; z - ct \neq 0$

Related by Lorentz boost

$t = 0, \;\; z \neq 0$







PDF $f(x)$:
Cannot be calculated on the lattice

$$f(x) = \int \frac{dz^-}{2\pi} e^{-ib^-(xP^+)} \langle P | \bar{\psi}(z^-) \times \frac{\gamma^+}{2} W[z^-,0]\psi(0) | P \rangle$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, arXiv: 2004.03543.

Quasi-PDF $\tilde{f}(x, P^z)$:
Directly calculable on the lattice

$$\tilde{f}(x, P^z) = \int \frac{dz}{2\pi} e^{iz(xP^z)} \langle P | \bar{\psi}(z) \times \frac{\gamma^z}{2} W[z,0]\psi(0) | P \rangle$$

See X. Ji's talk for an overview.

Large-Momentum Effective Theory (LaMET)

- Factorization formula:

$$\tilde{f}(y, P^z, \tilde{\mu}) = \int_{-1}^{1} \frac{dx}{|x|} C\left(\frac{y}{x}, \frac{\mu}{xP^z}, \frac{\tilde{\mu}}{\mu}\right) f(x, \mu) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(yP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-y)P^z)^2}\right)$$

- X. Ji, PRL 110 (2013); SCPMA57 (2014);
- X. Xiong, X. Ji, J.-H. Zhang and YZ, PRD 90 (2014);
- Y.-Q. Ma and J. Qiu, PRD98 (2018), PRL 120 (2018);
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018).
- X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, 2004.03543.

Consider the discrete matrix form:

$$C_{x,y} = I_{x,y} + \frac{\alpha_s}{2\pi} C^{(1)}_{x,y} + \left(\frac{\alpha_s}{2\pi}\right)^2 C^{(2)}_{x,y} + \cdots$$

- The matching kernel can be inverted order by order in α_s:

$$f(x, \mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) + \mathcal{O}'\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

Controlled power expansion for $x \in [x_{\rm min}, x_{\rm max}]$ at finite P^z

Precision calculation of the x-dependence of PDFs

- Control of perturbative corrections at leading power:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

State-of-the-art: next-to-next-to-leading order (NNLO) matching for the non-singlet quark quasi-PDF.

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021).

- NLO matching kernel:

$$C^{(1)}\left(\xi, \frac{\mu}{yP^z}\right) = -\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left[\frac{3}{2}\ln\frac{\mu^2}{4x^2P_z^2} + \frac{5}{2}\right]$$

$$\xi = \frac{x}{y}$$

$$-\frac{\alpha_s C_F}{2\pi}\begin{cases} \left(\frac{1+\xi^2}{1-\xi}\ln\frac{\xi}{\xi-1} + 1\right)_+ & \xi > 1 \\ \left(\frac{1+\xi^2}{1-\xi}\left[-\ln\frac{\mu^2}{y^2P_z^2} + \ln\left(4\xi(1-\xi)\right) - 1\right] + 1\right)_+ & 0 < \xi < 1 \\ \left(-\frac{1+\xi^2}{1-\xi}\ln\frac{-\xi}{1-\xi} - 1\right)_+ & \xi < 0 \end{cases}$$

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- NLO matching kernel:

$$\ln\frac{\mu^2}{y^2P_z^2} = \ln\frac{\mu^2}{x^2P_z^2} + \ln\frac{x^2}{y^2}$$

$$C^{(1)}\left(\xi, \frac{\mu}{yP^z}\right) = -\frac{\alpha_s C_F}{2\pi}\delta(1-\xi)\left[\frac{3}{2}\ln\frac{\mu^2}{4x^2P_z^2} + \frac{5}{2}\right]$$

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Similar to DGLAP evolution:

$$\frac{dC(\xi, \mu/(xP^z))}{d\ln(xP^z)} = \frac{\alpha_s C_F}{\pi}\left[P_{qq}^{(0)}(\xi) - \frac{3}{2}\delta(1-\xi)\right]$$

X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang and YZ, 2004.03543.

Precision calculation of the x-dependence of PDFs

- Control of perturbative corrections at leading power:

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Precision calculation of the *x*-dependence of PDFs

- Control of perturbative corrections at leading power:

$$f(x,\mu)=\int_{-\infty}^{\infty}\frac{dy}{|y|}C^{-1}\left(\frac{x}{y},\frac{\mu}{yP^z},\frac{\tilde{\mu}}{\mu}\right)\tilde{f}(y,P^z,\tilde{\mu})+\mathcal{O}\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2},\frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

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Threshold logarithms:

$$C(\xi,\mu/(yP^z))\sim\frac{\alpha_sC_F}{2\pi}\left[\frac{2\ln|1-\xi|}{|1-\xi|}-\frac{2}{1-\xi}\ln\frac{\mu^2}{P_z^2}-\frac{2}{1-\xi}\right]_+$$

Large-x behavior of the extracted PDF is sensitive to the large threshold logarithms.

X. Gao, YZ et al., 2102.01101.

Precision calculation of the *x*-dependence of PDFs

- Control of subleading power corrections:

$$f(x,\mu) = \int_{-\infty}^{\infty} \frac{dy}{|y|} C^{-1}\left(\frac{x}{y}, \frac{\mu}{yP^z}, \frac{\tilde{\mu}}{\mu}\right) \tilde{f}(y, P^z, \tilde{\mu}) \; + \; \mathcal{O}'\left(\frac{\Lambda_{\rm QCD}^2}{(xP^z)^2}, \frac{\Lambda_{\rm QCD}^2}{((1-x)P^z)^2}\right)$$

  - Starting at quadratic power of $1/P^z$ in the MSbar scheme;
  - Need to first obtain the quasi-PDF in the MSbar scheme.

$$\tilde{f}(x, P^z, \tilde{\mu}) = \int_{-\infty}^{\infty} \frac{dz}{2\pi}\, e^{iz(xP^z)}\, \tilde{h}(z, P^z, \tilde{\mu})$$

Lattice renormalization should not introduce uncontrolled nonperturbative effects at large distance!

Precision calculation of the *x*-dependence of PDFs

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{\delta m|z|}\, Z_{j_1}(a)Z_{j_2}(a)O_R^\Gamma(z)$$

- Lattice renormalization:

- Ji, Zhang and YZ, Phys.Rev.Lett. 120 (2018);
- Ishikawa, Ma, Qiu and Yoshida, Phys.Rev.D 96 (2017);
- Green, Jansen and Steffens, Phys.Rev.Lett. 121 (2018).

$$\tilde{h}(z,P^z,\tilde{\mu}) = \lim_{a\to 0} \frac{\tilde{h}(z,P^z,a)}{Z_X(z,\tilde{\mu},a)}$$

Any ratio-type scheme that uses a matrix element of the same nonlocal operator as renormalization factor does introduce nonperturbative effects at large z.

- RIMOM: $Z_X = \langle q\,|\,O^\Gamma(z)\,|\,q\rangle$
- Hadron matrix element: $Z_X = \langle P_0^z = 0\,|\,O^\Gamma(z)\,|\,P_0^z = 0\rangle$
- Vacuum expectation value: $Z_X = \langle \Omega\,|\,O^\Gamma(z)\,|\,\Omega\rangle$

- Stewart and YZ, PRD 97 (2018);
- Constantinou and Panagopoulos, PRD 96 (2017);
- Radyushkin, PRD 96 (2017);
- Orginos et al., PRD 96 (2017);
- Braun, Vladimirov and Zhang, PRD 99 (2019);
- Li, Ma and Qiu, PRL 126 (2021).

Hybrid renormalization scheme

$$O_B^\Gamma(z,a) = \bar{\psi}_0(z)\Gamma W_0[z,0]\psi_0(0) = e^{\delta m|z|}\, Z_{j_1}(a)Z_{j_2}(a)O_R^\Gamma(z)$$

X. Ji, YZ, et al., NPB 964 (2021).

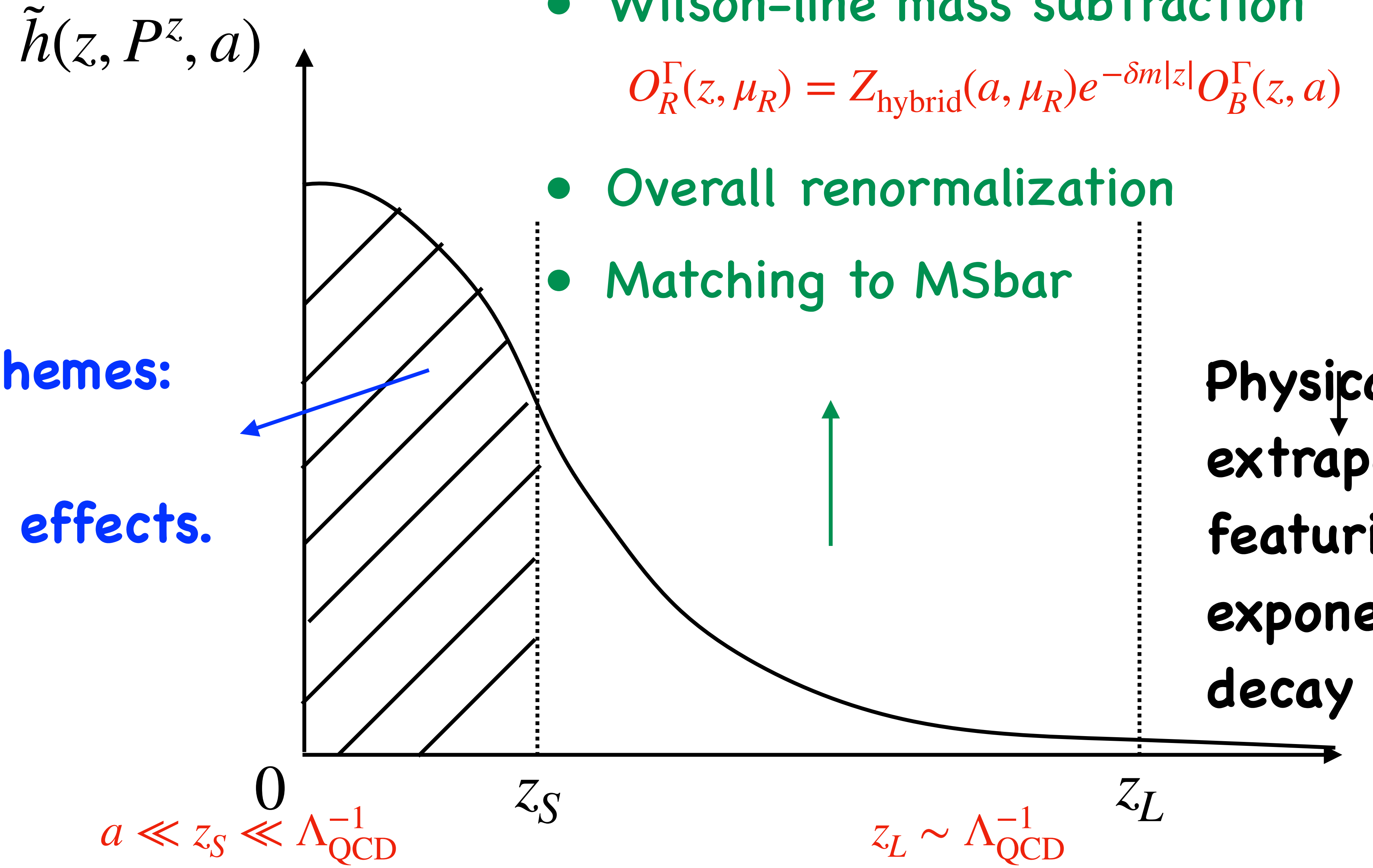
A minimal subtraction:

- Wilson-line mass subtraction

$$O_R^\Gamma(z,\mu_R) = Z_{\text{hybrid}}(a,\mu_R)e^{-\delta m|z|}O_B^\Gamma(z,a)$$

- Overall renormalization
- Matching to MSbar

Ratio-type schemes: cancel the discretization effects.



Physical extrapolation featuring exponential decay

Outline

- Methodology
  - Large-momentum effective theory
  - Hybrid renormalization scheme

- Lattice calculation
  - Wilson line mass renormalization
  - Fourier transform and perturbative matching
  - Final results

Lattice data for the pion valence PDF

- Wilson-clover fermion on 2+1 flavor HISQ configurations.

n_z	P_z (GeV)		ζ
	$a = 0.06$ fm	$a = 0.04$ fm	
0	0	0	0
1	0.43	0.48	0
2	0.86	0.97	1
3	1.29	1.45	2/3
4	1.72	1.93	3/4
5	2.15	2.42	3/5
	$48^3 \times 64$	$64^3 \times 64$	

$m_\pi = 300$ MeV

$a = 0.076$ fm

$P_z = 0$ GeV	1.27 GeV
0.25 GeV	1.53 GeV
0.51 GeV	1.78 GeV
0.76 GeV	2.04 GeV
1.02 GeV	2.29 GeV

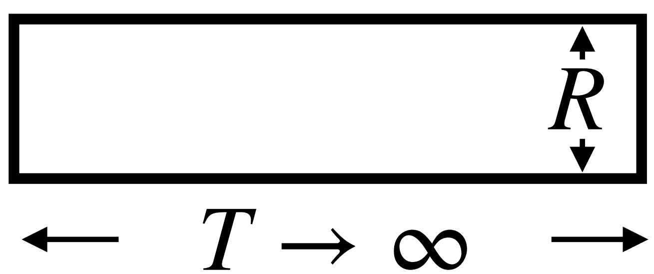
$64^3 \times 64$

$m_\pi = 140$ MeV

- X. Gao, YZ, et al., PRD102 (2020).
- X. Gao, YZ, et al., 2102.01101.

Hybrid scheme renormalization

- Use P^z=0 matrix element for short-distance renormalization.
- Wilson-line mass renormalization.
  - Polyakov loop

$\langle\Omega|$  $|\Omega\rangle \quad \propto \exp[-V(R)T]$

  - Renormalization condition:

$$V^{\mathrm{lat}}(r,a)\Big|_{r=r_0} + 2\delta m(a) = 0.95/r_0$$

$$\delta m(a) = \frac{1}{a}\sum_n c_n \alpha_s^n(1/a) + \delta m_0^{\mathrm{lat}}$$

$$\delta m_0^{\mathrm{lat}} \sim \Lambda_{\mathrm{QCD}}$$

C. Bauer, G. Bali and A. Pineda, PRL108 (2012).

$$a\delta m(a = 0.04\ \mathrm{fm}) = 0.1508(12)$$

$$a\delta m(a = 0.06\ \mathrm{fm}) = 0.1586(8)$$

$$a\delta m(a = 0.076\ \mathrm{fm}) = 0.1597(16)$$

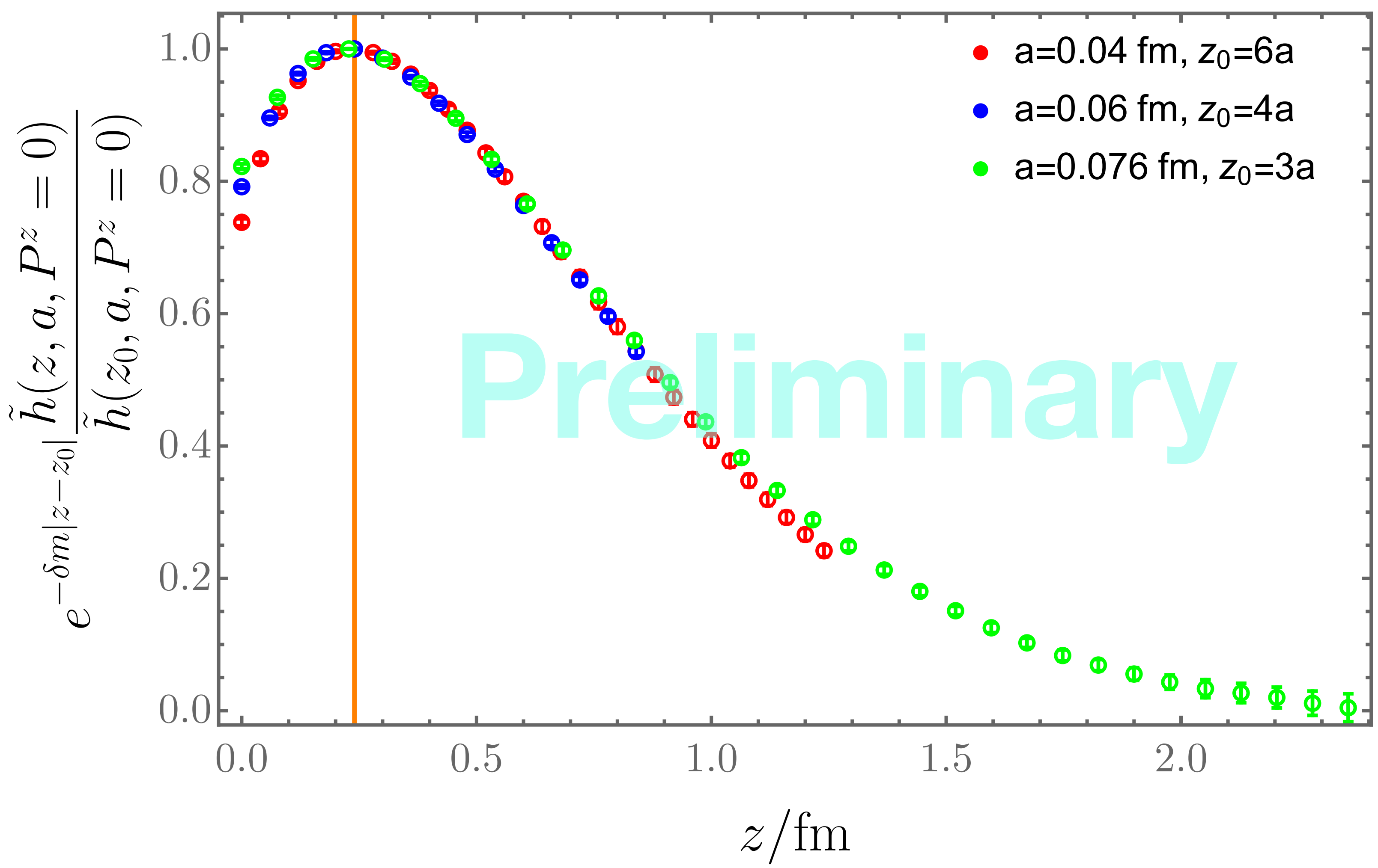
A. Bazavov et al., TUMQCD, PRD98 (2018).

Hybrid scheme renormalization

- Check of continuum limit:

$$O_B^\Gamma(z,a) = e^{\delta m|z|}\, Z_{j_1}(a) Z_{j_2}(a) O_R^\Gamma(z)$$

$$\lim_{a\to 0} e^{-\delta m(z-z_0)} \frac{\tilde{h}(z,a,P^z=0)}{\tilde{h}(z_0,a,P^z=0)} = \frac{\tilde{h}(z,P^z=0,\ \mu)}{\tilde{h}(z_0,P^z=0,\ \mu)} \qquad z, z_0 \gg a$$



Hybrid scheme renormalization

- Matching to the MSbar scheme:

$$e^{\delta m_0^{\overline{\rm MS}}(z-z_0)} \lim_{a\to 0} e^{-\delta m(z-z_0)} \frac{\tilde{h}(z,a,P^z=0)}{\tilde{h}(z_0,a,P^z=0)} = \frac{\tilde{h}^{\overline{\rm MS}}(z,P^z=0,\ \mu)}{\tilde{h}^{\overline{\rm MS}}(z_0,P^z=0,\ \mu)} \qquad z, z_0 \gg a$$

Renormalon in the Wilson-line mass correction.

C. Bauer, G. Bali and A. Pineda, PRL108 (2012);
C. Alexandrou et al. (ETMC), 2011.00964.

OPE:

$$\tilde{h}^{\overline{\rm MS}}(z,P^z=0,\ \mu) = \left[C_0(\alpha_s(\mu), z^2\mu^2) + \mathcal{O}(z^2\Lambda_{\rm QCD}^2)\right] \qquad z \ll \Lambda_{\rm QCD}^{-1}$$

Perturbative:

Known to NNLO with 3-loop anomalous dimension

- L.-B. Chen, R.-L. Zhu and W. Wang, PRL126 (2021);
- Z.-Y. Li, Y.-Q. Ma and J.-W. Qiu, PRL126 (2021);
- V. Braun and K. G. Chetyrkin, JHEP 07 (2020).

Non-perturbative:

Leading infrared renormalon contribution is quadratic

$$\propto z^2\Lambda_{\rm QCD}^2$$

- V. Braun, A. Vladimirov and J.-H. Zhang, PRD99 (2019).

Hybrid scheme renormalization

- Matching to the MSbar scheme:

$$e^{-\delta m(z-z_0)} \frac{\tilde{h}(z, a, P^z = 0)}{\tilde{h}(z_0, a, P^z = 0)} = e^{-\delta m_0^{\overline{\rm MS}}(z-z_0)} \frac{C_0(\alpha_s(\mu), z^2\mu^2) + \Lambda z^2}{C_0(\alpha_s(\mu), z_0^2\mu^2) + \Lambda z_0^2} \qquad z, z_0 \gg a$$

  - The two-parameter model can fit the data very well up to z=0.72 fm with NNLO Wilson coefficient C_0;
  - The a-dependences of the parameters are very small.

Hybrid scheme renormalization

- At short distance, take advantage of that the NNLO OPE with leading IR renormalon contribution can fit the data well:

$$\lim_{a\to 0}\frac{\tilde{h}(z,a,P^z)}{\tilde{h}(z,a,P^z=0)}=\frac{\tilde{h}^{\overline{\rm MS}}(z,P^z,\ \mu)}{\tilde{h}^{\overline{\rm MS}}(z,P^z=0,\ \mu)}=\frac{\tilde{h}^{\overline{\rm MS}}(z,P^z,\ \mu)}{C_0(\alpha_s(\mu),z^2\mu^2)+\Lambda z^2}$$

$$\downarrow$$

$$\lim_{a\to 0}\frac{\tilde{h}(z,a,P^z)}{\tilde{h}(z,a,P^z=0)}\frac{C_0(\alpha_s(\mu),z^2\mu^2)+\Lambda z^2}{C_0(\alpha_s(\mu),z^2\mu^2)}=\frac{\tilde{h}^{\overline{\rm MS}}(z,P^z,\ \mu)}{C_0(\alpha_s(\mu),z^2\mu^2)}$$

Rigorous ratio scheme in

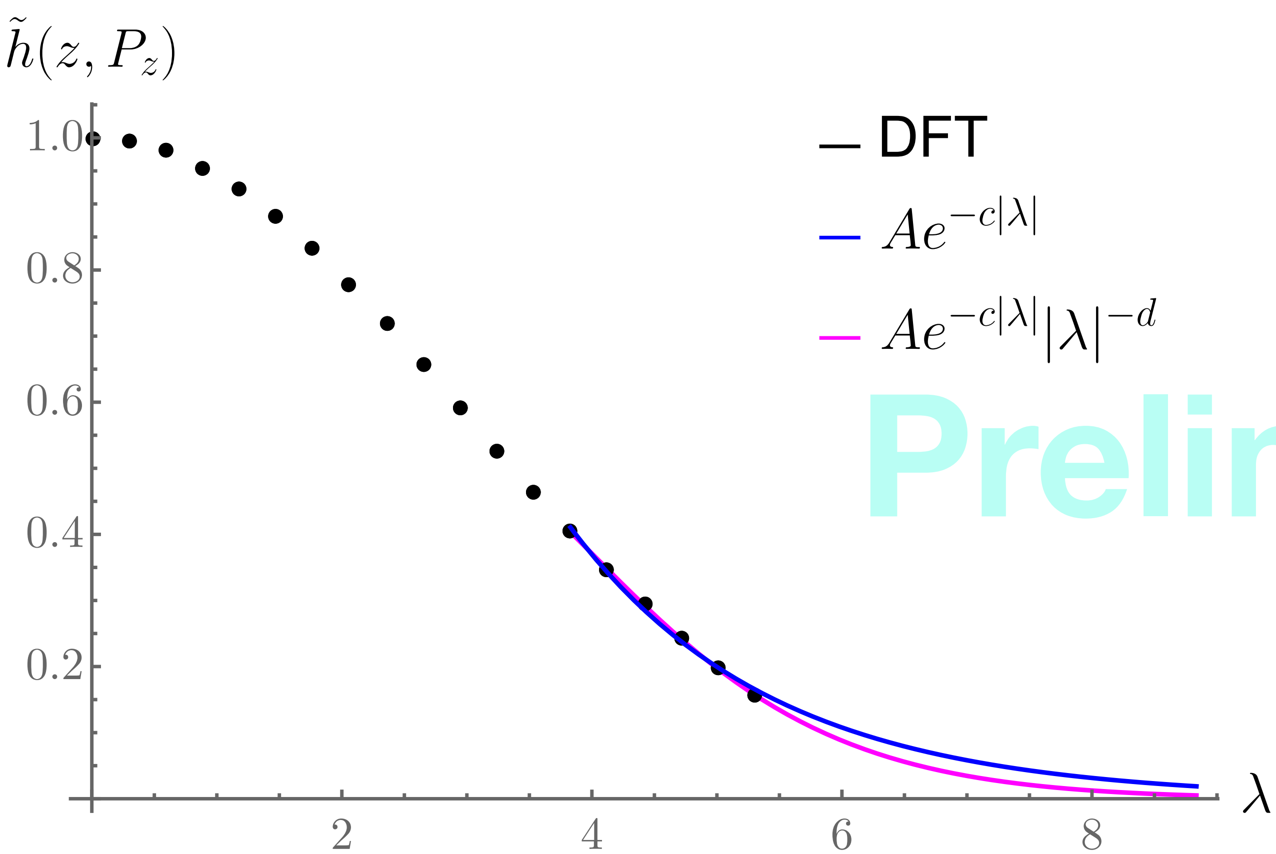
- T. Izubuchi, X. Ji, L. Jin, I. Stewart, and YZ, PRD98 (2018);

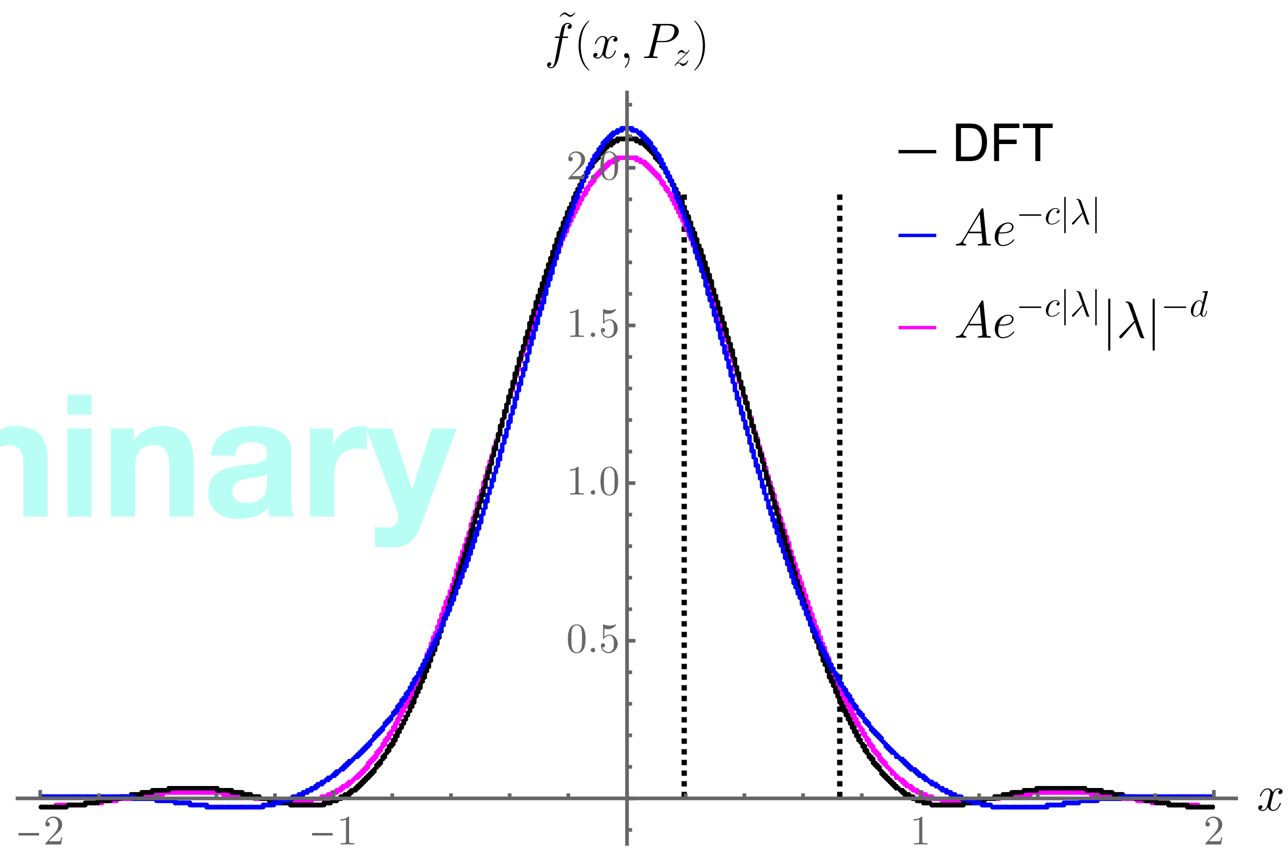
as compared to the original ratio in

- Radyushkin, PRD 96 (2017);
- Orginos et al., PRD 96 (2017).

Fourier transform with physical extrapolation

- Extrapolation with models featuring an exponential decay:





$\lambda = zP^z$

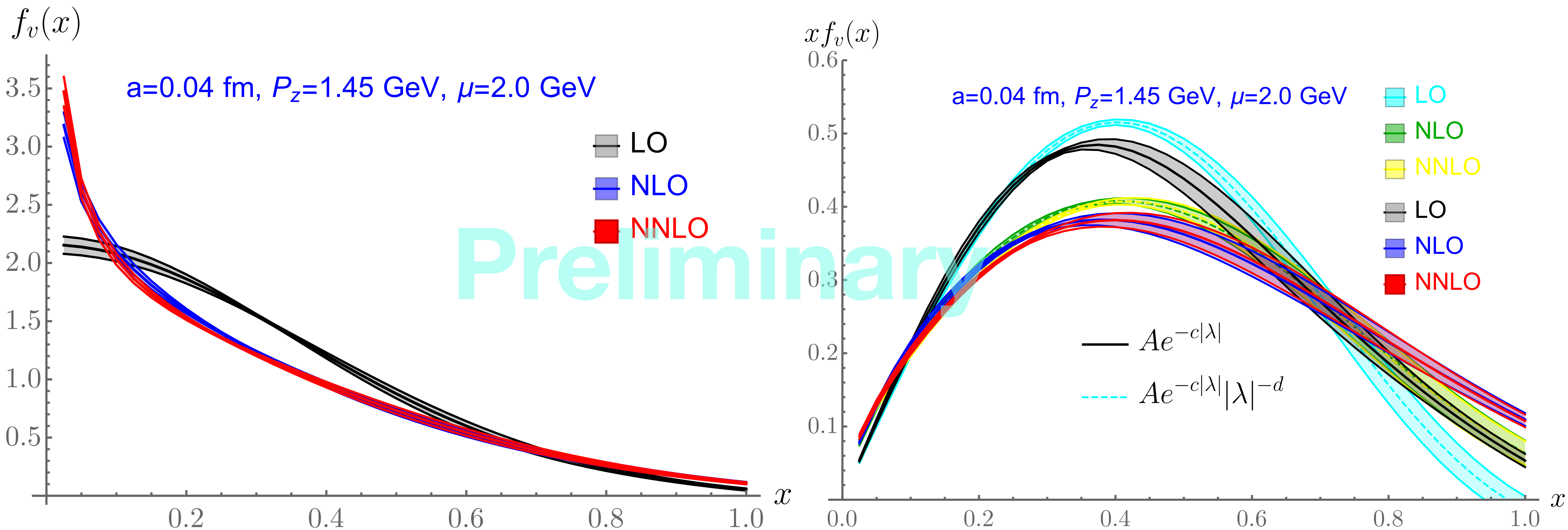
$a = 0.04$ fm, $P_z = 1.45$ GeV, $z_S = 6a$, $z_L = 19a$.

Extrapolation barely affects the moderate x region, as expected.

Caveat: we are still in the process of finishing the hybrid-scheme matching. Here we choose z_S=z_L<=0.72 fm with ratio scheme matching, which is in principle in doubt at large z.

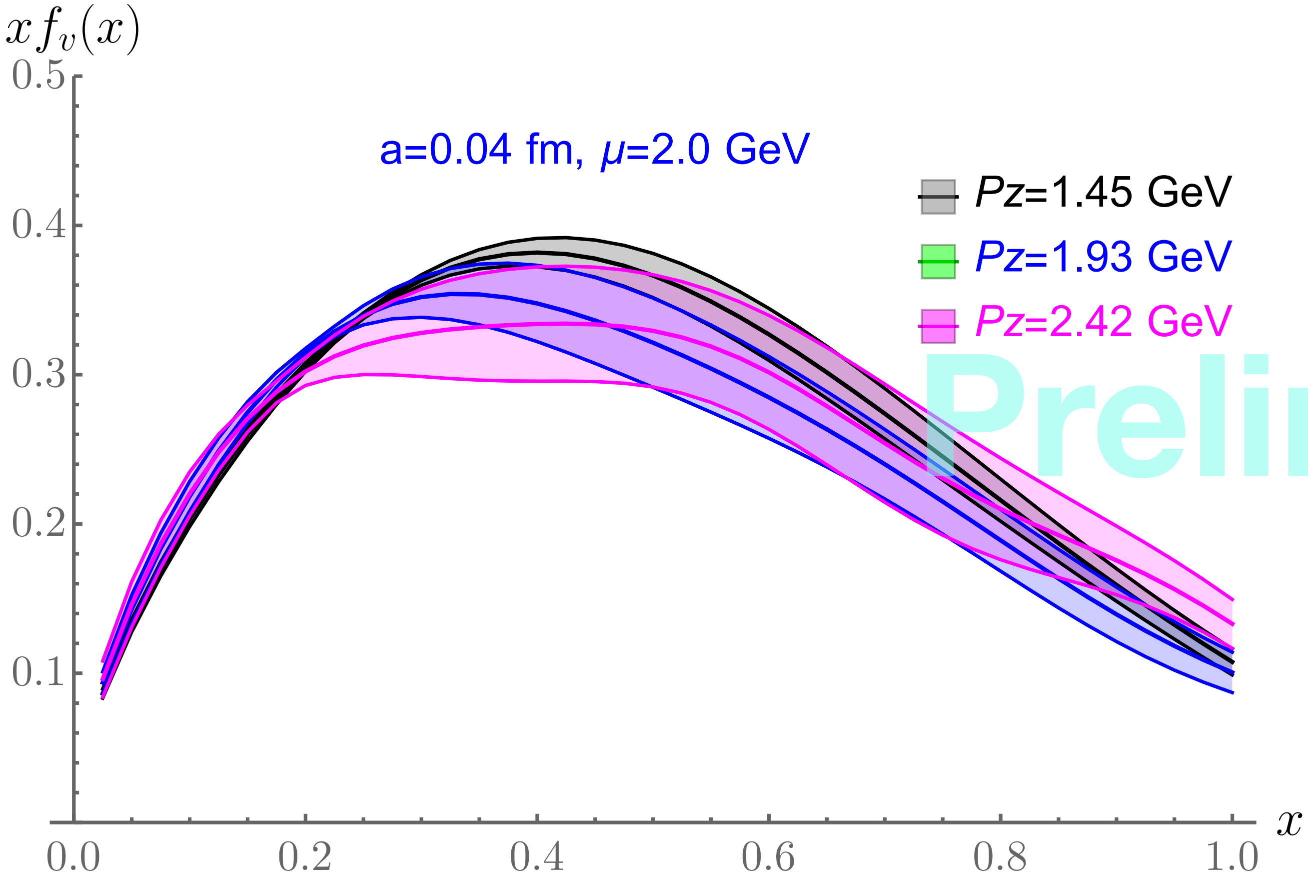
Perturbative matching at NNLO

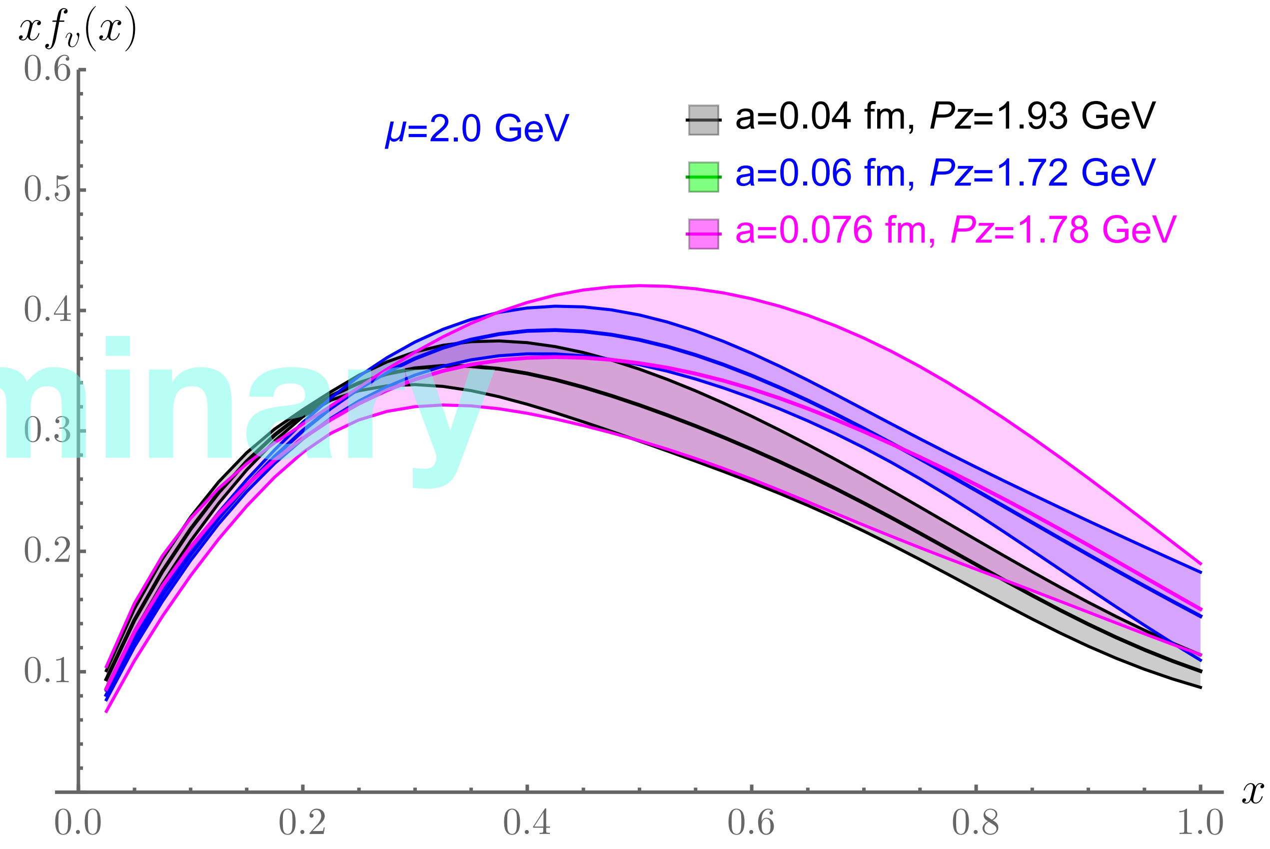
- Perturbative correction shows good convergence.



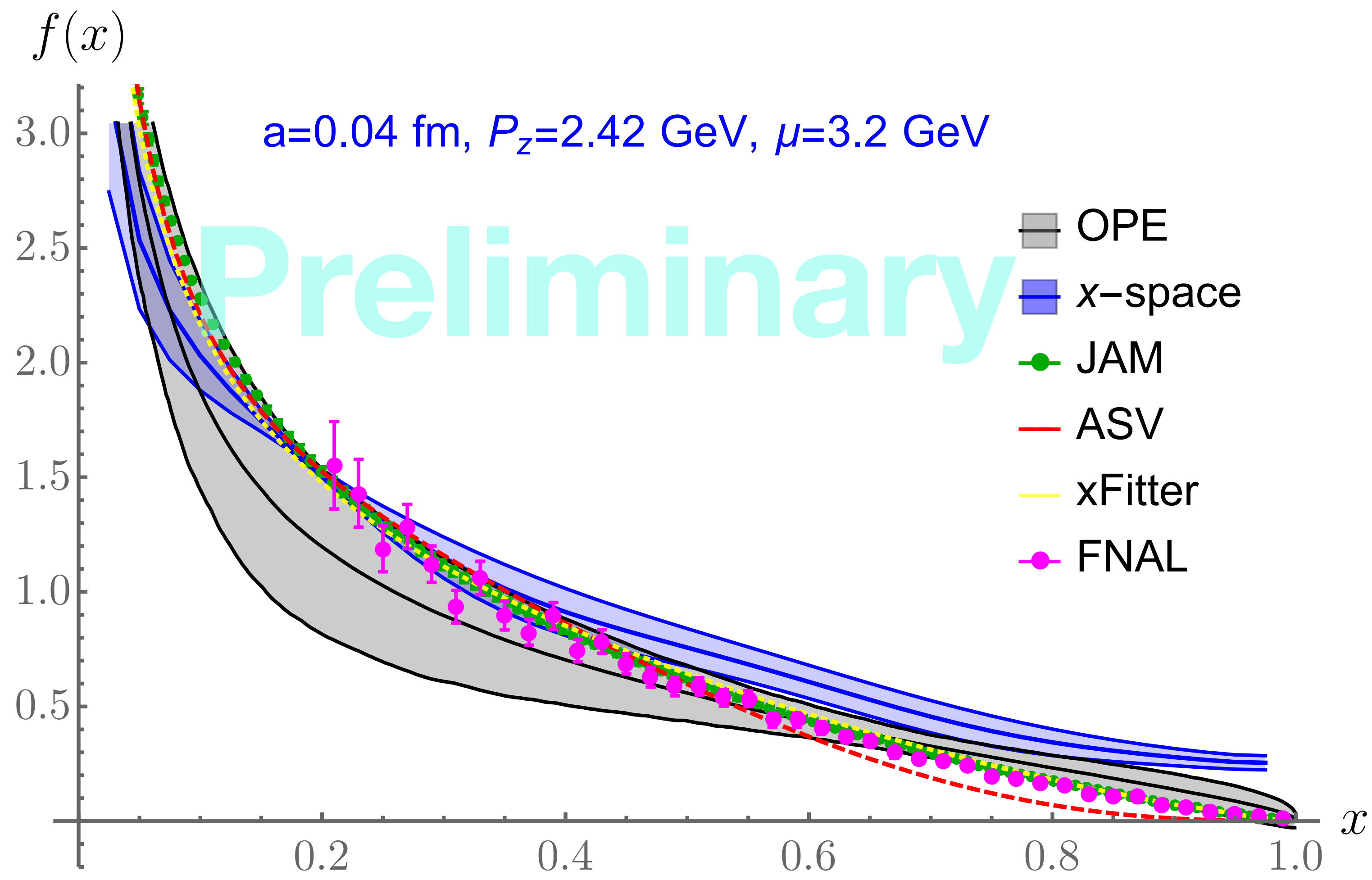
Error band only includes statistical uncertainty.

Dependence on P_z and a





Comparison with previous analysis and phenomenology



Better agreement with experimental fits for 0.1 < x < 0.45 compared to our previous analysis using OPE and model fitting of the PDF in coordinate space.

Conclusion

- LaMET allows for model-independent lattice calculation of the x-dependence of the PDFs with controlled systematics;
- The Wilson-line mass in the hybrid scheme can be well determined from lattice and matched to the MSbar scheme, which is key to control the power corrections in LaMET;
- NNLO matching shows good perturbative convergence;
- Full NNLO hybrid-scheme matching results will be available soon.