Determination of the Collins-Soper Kernel from Lattice QCD

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QCD Evolution Workshop
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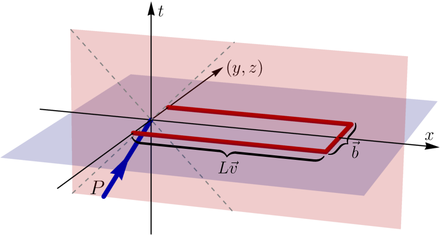
Outline

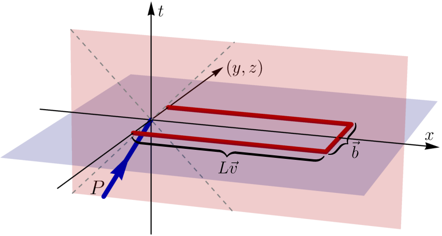


Musch, et. al., 1011.1213

- Position space-bilocal nucleon matrix element, with momentum P and spin S which is sensitive to transverse momentum

$$W_f^{[\Gamma]}(b; L, v; P, S) = \frac{1}{2}\langle P, S|\bar{q}_f(b)\Gamma\,[b, b + Lv]\,[b + Lv, Lv]\,[Lv, 0]\,q(0)|P, S\rangle$$

- b^μ is orthogonal to v^μ and P^μ therefore $b \cdot v = b \cdot P = 0$
- Time equal matrix element $v^0 = b^0 = 0$



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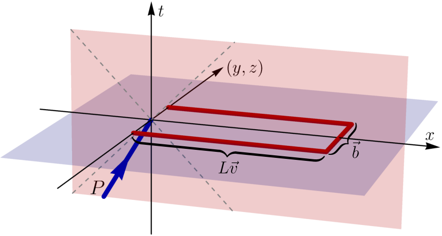
Matrix element



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Factorization of qTMDs

For energetic hadrons and large L we can factorize the matrix element

Vladimirov, Schäfer, 2002.07527

$$W_f^{[\Gamma]}(b; L, v; P, S) = \frac{1}{P^+}\int dx \left| C_H\left(\frac{|x|P^+}{\mu}\right)\right|^2 \Phi_{f\leftarrow h}^{[\Gamma']}(x, b; \mu, \zeta)\Psi(b; \mu, \bar{\zeta})$$

- $C_H(...)$: perturbative coefficient function (NLO)
- $\Phi_{f\leftarrow h}^{[\Gamma']}(x, b; \mu, \zeta)$: physical TMD distribution with $\Gamma' = \frac{\gamma^+\gamma^-\Gamma\gamma^-\gamma^+}{4}$
- $\Psi(b; \mu, \bar{\zeta})$: combination of soft factors
- μ, ζ, $\bar{\zeta}$: factorization scales

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$$W_f^{[\Gamma]}(b;L,v;P,S) = \frac{1}{P^+}\int dx \left| C_H\left(\frac{|x|P^+}{\mu}\right)\right|^2 \Phi^{[\Gamma']}_{f\leftarrow h}(x,b;\mu,\zeta)\Psi(b;\mu,\bar{\zeta})$$

The universal Collins-Soper-Kernel $K(b,\mu)$ relates different scales ζ of the physical TMDs:

$$\Phi^{[\Gamma]}_{f\leftarrow h}(x,b;\mu,\zeta) = \left(\frac{\zeta}{\zeta_0}\right)^{K(b,\mu)/2} \Phi^{[\Gamma]}_{f\leftarrow h}(x,b;\mu,\zeta_0)$$

and the factorization scales ζ and $\bar{\zeta}$ satisfy the following relation:

$$\zeta\bar{\zeta} = (2xP^+v^-)^2\mu^2$$

Extraction of the CS kernel from ratios P^+

Using the relations we can evolve two of these matrix elements at different P to ζ_0 and μ_0:

$$R^{[\Gamma]}(b;L,v;P_1,P_2,S) = \frac{W_f^{[\Gamma]}(b;L,v;P_1,S)}{W_f^{[\Gamma]}(b;L,v;P_2,S)} = \left(\frac{P_2^+}{P_1^+}\right)^{K(b,\mu)} r^{[\Gamma]} + \mathcal{O}(\lambda)$$

where $r^{[\Gamma]}$ in NLO is given by

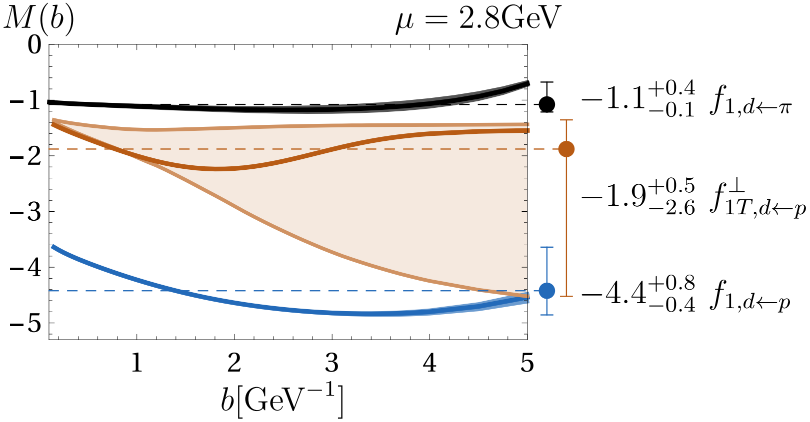
$$r^{[\Gamma]} = 1 + 4C_F\frac{\alpha_s(\mu)}{4\pi}\ln\left(\frac{P_1^+}{P_2^+}\right)\left[1 - \ln\left(\frac{4P_1^+P_2^+|v^-|^2}{\mu^2}\right) - 2\mathbf{M}_{f\leftarrow h}^{[\Gamma]}(b,\mu)\right] + \mathcal{O}(\alpha_s^2)$$

with non-perturbative contribution $\mathbf{M}_{f\leftarrow h}^{[\Gamma]}(b,\mu)$

Vladimirov, Schäfer, 2002.07527

Where we assume $\mathbf{M}_{f\leftarrow h}^{[\Gamma]}(b,\mu)$ to be a constant function in b and μ

$$\mathbf{M}_{f\leftarrow h}^{[\Gamma]}(b,\mu) = \frac{\int dx_1 \ln|x_1| x_1^{K(b,\mu)} \Phi^{[\Gamma]}(x_1, b; \mu, \zeta_0)}{\int dx_2 x_2^{K(b,\mu)} \Phi^{[\Gamma]}(x_2, b; \mu, \zeta_0)}$$



Outline

Extract matrix element and mass

- For our analysis we require $b^+ = 0$ and $\mathbf{P}_T = \mathbf{v}_T = \mathbf{0}$ thus

$$S = (0,0,0,1), \qquad P = (P_0, P_1, 0, 0)$$
$$b = (0,0,b_2,b_3), \qquad v = (0, v_1, 0, 0)$$

- To improve the lattice signal, HYP- and quark momentum smearing is applied
- u-d Quark channel to cancel disconnected diagrams
- We use the CLS ensembles with $N_f = 2+1$ flavors and dynamical Wilson-Clover fermions

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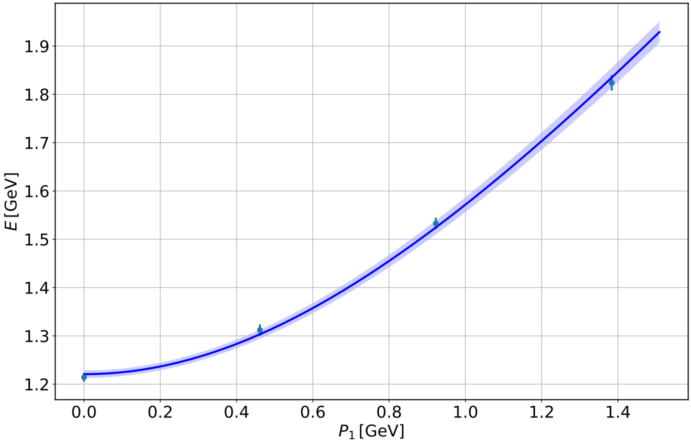
CLS ensemble H101

β	$L^3 \times T$	a	m_π	#cnfg
3.4	$32^3 \times 96$	0.084fm	422 MeV	2000



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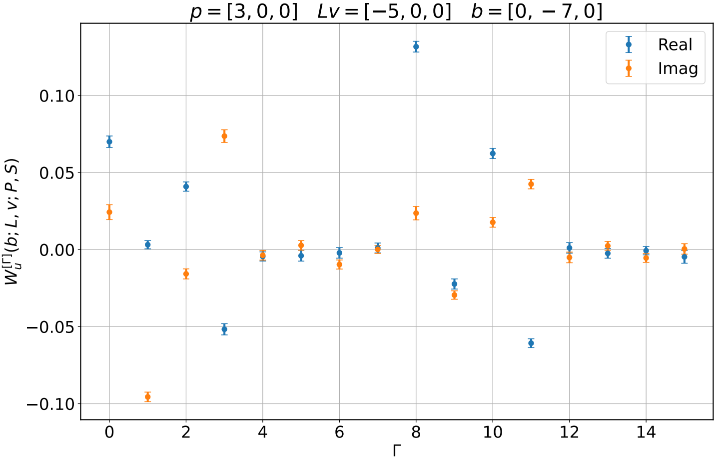


Lattice correlator parametrization

The correlators are grouped into Lorentz-invariant products of b, P, Lv: $(P^2, b^2, (Lv)^2, Lv \cdot P)$ and parameterized, e.g. for the vector channel γ^μ as

$$\begin{aligned}\widetilde{W}^{[\gamma^\mu]}(b; L; v, P, S) = {}& P^\mu \tilde{a}_2 + m_N^2 (Lv^\mu)\tilde{b}_1 - i m_N \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha S_\beta \tilde{a}_{12} \\ & - i m_N^3 \epsilon^{\mu\nu\alpha\beta} b_\nu (Lv_\alpha) S_\beta \tilde{b}_8 - i m_N^2 b^\mu \tilde{a}_3 \\ & + m_N \epsilon^{\mu\nu\alpha\beta} P_\nu (Lv_\alpha) S_\beta \tilde{b}_7 - m_N^3 (b \cdot S) \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha (Lv_\beta) \tilde{b}_9 \\ & - i m_N^3 ((Lv) \cdot S) \epsilon^{\mu\nu\alpha\beta} P_\nu b_\alpha (Lv_\beta) \tilde{b}_{10}\end{aligned}$$

with nucleon mass m_N.

Musch, et al., 1111.4249

Obtaining moments of TMDs in b-space

All a_i and b_i depend on $(P^2, b^2, (Lv)^2, Lv \cdot P)$

$$W^{[f_1]}(P^2, b^2, (Lv)^2, Lv \cdot P) = 2\frac{a_2 + rb_1}{S(b^2, (Lv)^2, Lv \cdot P)}$$

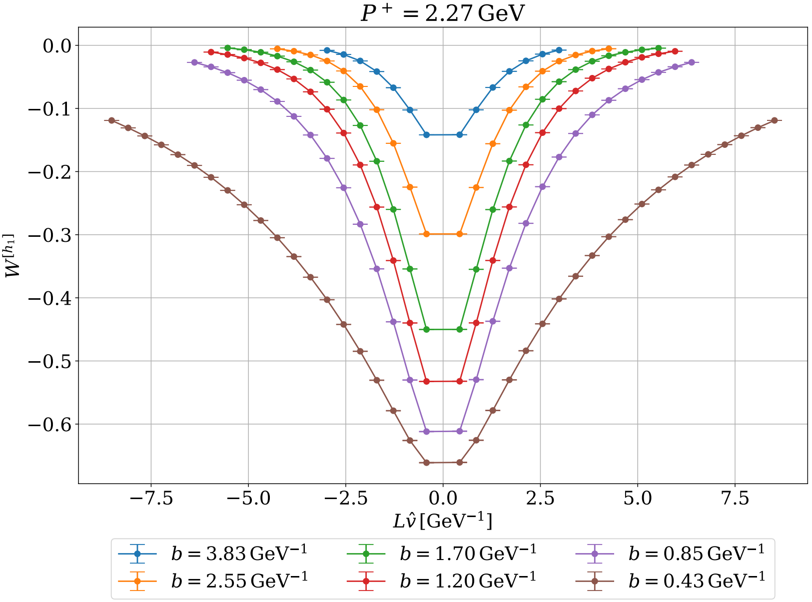
$$W^{[g_{1T}]}(P^2, b^2, (Lv)^2, Lv \cdot P) = 2\frac{a_6 + (1-r)(b_{11} + rb_{14})}{S(b^2, (Lv)^2, Lv \cdot P)}$$

$$W^{[h_1]}(P^2, b^2, (Lv)^2, Lv \cdot P) = 2\frac{a_9 + rb_{15} - \frac{1}{2}m_N^2 b^2(a_{11} - rb_{17})}{S(b^2, (Lv)^2, v \cdot P)}$$

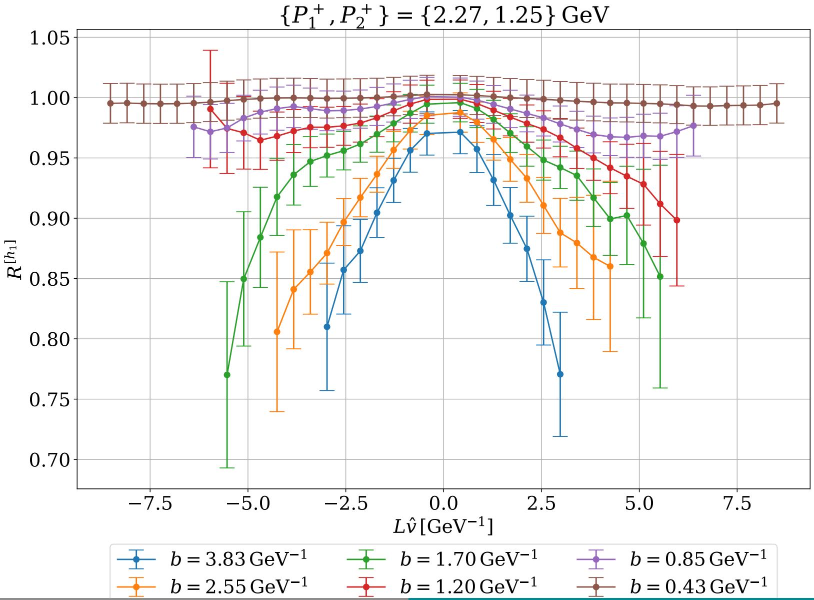
$r(P^2, Lv \cdot P) = \frac{m^2 Lv^+}{P^+}$

Musch, et al., 1111.4249

Obtaining moments of TMDs in b-space

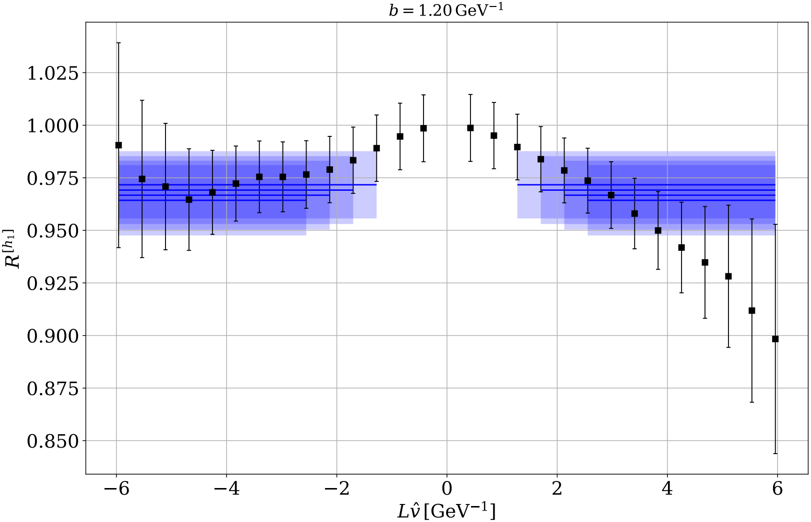


Ratios at different momentum P



Extrapolate $L \to \infty$

$\{P_1^+, P_2^+\} = \{2.27, 1.25\}\,\mathrm{GeV}$



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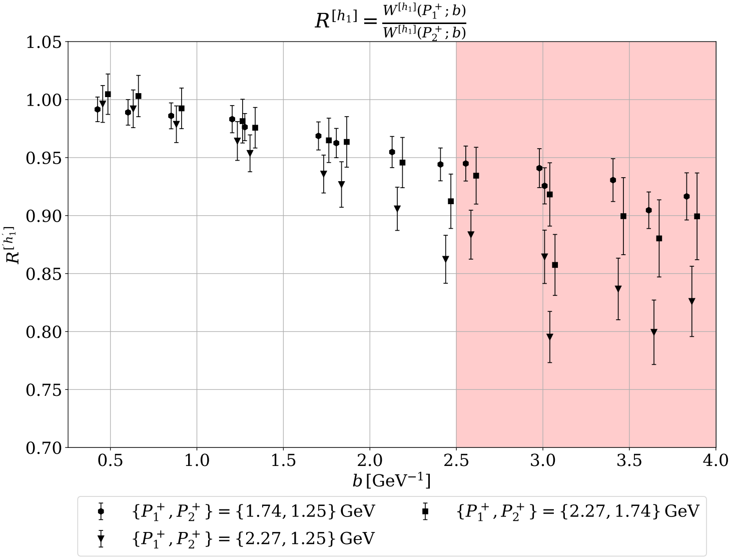
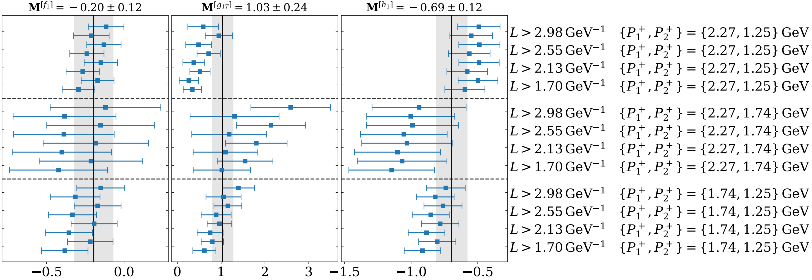
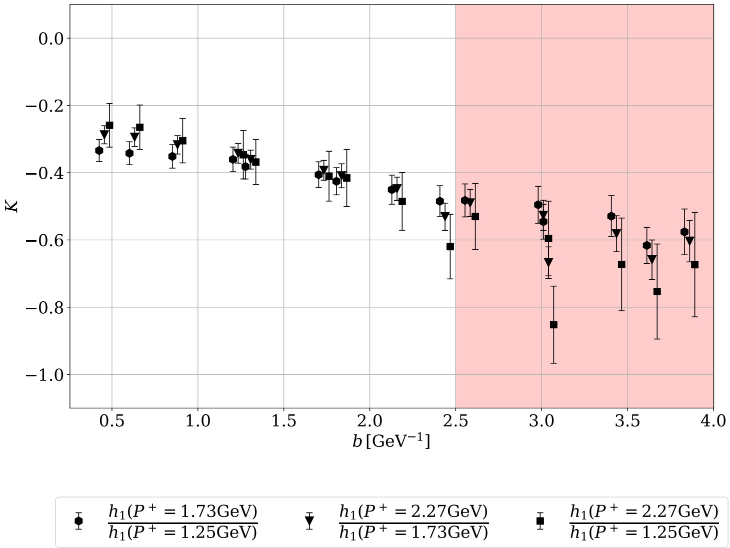


Outline

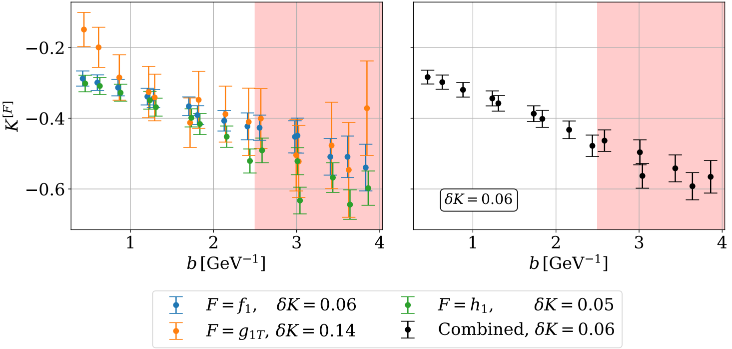
Determine $\mathbf{M}^{[\Gamma]}_{f\leftarrow h}$ at fixed $b \approx 1\mathrm{GeV}^{-1}$ by inverting $r^{[\Gamma]}$ for different cutoffs



Reconstruct $K(b, \mu = 2\,\mathrm{GeV})$ by assuming $\mathbf{M}^{[\Gamma]}_{f\leftarrow h}(b, \mu)$ constant for all b

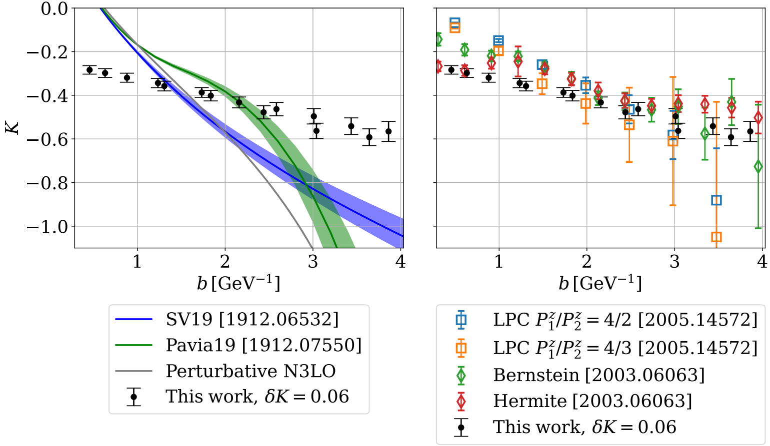


Average the different momenta combinations $\{P_1^+, P_2^+\}$ and the different TMDs



Comparison to other extractions

Comparison to phenomenological extractions and other lattice computations



Conclusion

- Agreement with other lattice extractions, lattice model computations and pertubation theory
- Systematic errors need to be reduced, i.e. better understood
- Larger lattices need to be analyzed such that the b range can be increased where the data can be trusted

Thank you for your attention
Questions?