x-dependent GPDs from Lattice QCD

(twist-2 & twist-3)

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Collaborators

Twist-2 GPDs

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Twist-3 GPDs

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  Temple University
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- **A. Scapellato**
  Temple University
- **F. Steffens**
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Relevant publications

Twist-2 GPDs

- *Unpolarized and helicity generalized parton distributions of the proton within lattice QCD*
  C.Alexandrou, K.Cichy, M.C, K. Hadjiyiannakou, K.Jansen, A.Scapellato, F.Steffens, PRL 125 (2020), [arXiv:2008.10573]

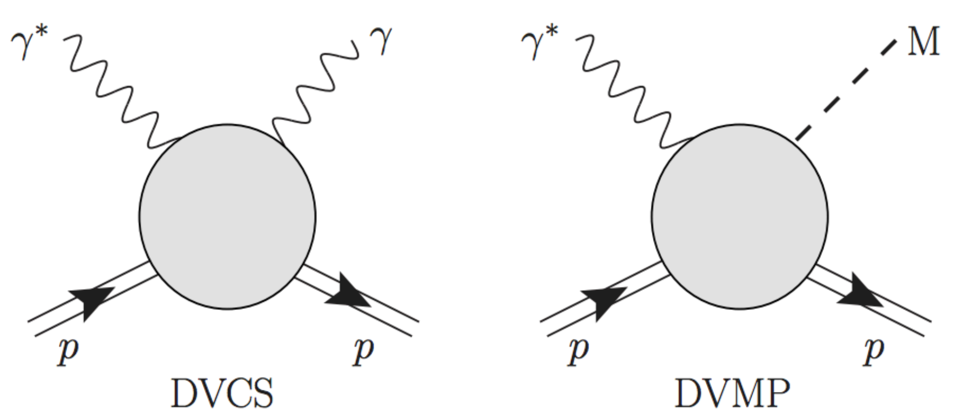
Twist-3 PDFs

- *New insights on proton structure from lattice QCD: the twist-3 parton distribution function $g_T(x)$*
  S.Bhattacharya, K.Cichy, M.C, A.Metz, A.Scapellato, F.Steffens, PRD Editors' Suggestion, [arXiv:2004.04130]
- *One-loop matching for the twist-3 parton distribution $g_T(x)$*
  S.Bhattacharya, K.Cichy, M.C, A.Metz, A.Scapellato, F.Steffens, PRD 102 (2020) 3, 034005, [arXiv:2005.10939]
- *The role of zero-mode contributions in the matching for the twist-3 PDFs $e(x)$ and $h_L(x)$*
  S. Bhattacharya, K. Cichy, M.C, A. Metz, A. Scapellato, F. Steffens, PRD 102 (2020) 114025, [arXiv:2006.12347]

Why GPDs?

★ GPDs provide information on spatial distribution of partons inside the hadron, and its mechanical properties (OAM, pressure, etc.)
[M. Burkardt, Phys.Rev.D62 071503 (2000), hep-ph/0005108]
[M. V. Polyakov, Phys. Lett. B555 (2003) 57, hep-ph/0210165]

★ Experimentally accessed in DVCS and DVMP
[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]
(Halls A,B,C (JLab), PHENIX, STAR, HERMES, COMPASS, GSI, BELLE, J-PARC)



★ Experimentally, GPDs are not well-constrained:

- independent measurements to disentangle GPDs
- limited coverage of kinematic region
- data on certain GPDs
- indirectly related to GPDs through the Compton FFs
- GPDs phenomenology more complicated than PDFs (multi-dimensionality)

Why twist-3 GPDs?

$$f_i = f_i^{(0)} + \frac{f_i^{(1)}}{Q} + \frac{f_i^{(2)}}{Q^2} \cdots$$

Twist-2 Twist-3

Higher-twist distributions:

★ **Lack density interpretation, but can be sizable**

★ **Sensitive to soft dynamics**

★ **Challenging to probe experimentally and isolate from leading-twist**

[Defurne et al., PRL 117, 26 (2016); Defurne et al., Nature Commun. 8, 1 (2017)]

★ **Needed for proton tomography**

★ **Related to certain spin-orbit correlations** [C. Lorce, PLB 735 (2014) 344, arXiv:1401.7784]

★ **Estimation on power corrections in hard exclusive processes (DVCS)**

★ $[\widetilde{H} + \widetilde{G}_2](x,\xi,t)$ **related to tomography of $F_\perp$ acting on the active q in DIS off a transversely polarized N right after the virtual photon absorbing**

[M. Burkardt, PRD 88 (2013) 114502, arXiv:0810.3589]

★ $G_2(x,\xi,t)$ **related to L_q :**

$$L_q = -\int_{-1}^{1} dx\, x\, G_2^q(x,\xi,t=0)$$

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249], [M. Penttinen et al., PLB 491 (2000) 96, arXiv:hep-ph/0006321]

GPDs from lattice QCD

(quasi-distributions method)

Useful Reviews:

[K. Cichy, M. Constantinou, Advances in HEP, Volume 2019, Article ID 3036904, arXiv:1811.07248]

[X. Ji, Y.-S. Liu, Y. Liu, J.-H. Zhang, and Y. Zhao (2020), 2004.03543]

[M. Constantinou (invited review) Eur. Phys. J. A 57 (2021) 2, 77, arXiv:2010.02445]

Access of GPDs on a Euclidean Lattice

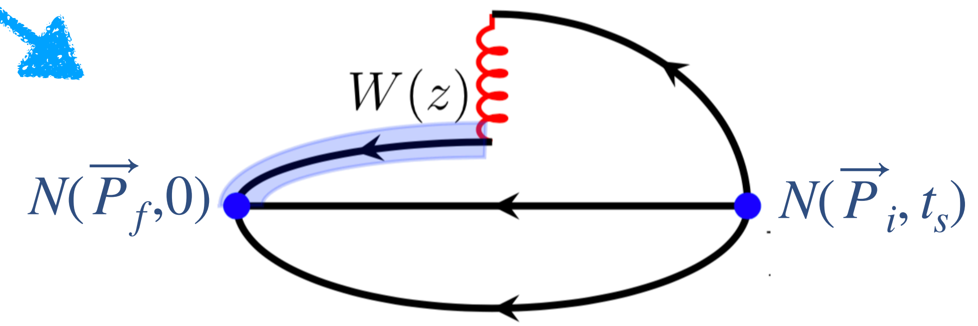
★ Relies on Large Momentum Effective Theory (LaMET) to reconstruct GPDs

[X. Ji, Phys. Rev. Lett. 110 (2013) 262002; X. Ji, Sci. China Phys. Mech. Astron. 57, 1407 (2014)]

★ Matrix elements of spatial operators with fast moving hadrons

$$\tilde{q}_{\Gamma}^{\rm GPD}(x,t,\xi,P_3,\mu) = \int \frac{dz}{4\pi} e^{-i x P_3 z} \underline{\langle N(P_f)\,|\,\bar{\Psi}(z)\,\Gamma\,\mathscr{W}(z,0)\Psi(0)\,|\,N(P_i)\rangle_\mu}$$

$$\Delta = P_f - P_i$$
$$t = \Delta^2 = -Q^2$$
$$\xi = \frac{Q_3}{2P_3}$$



Variables:

- length of the Wilson line (z)
- nucleon momentum boost (P_3)

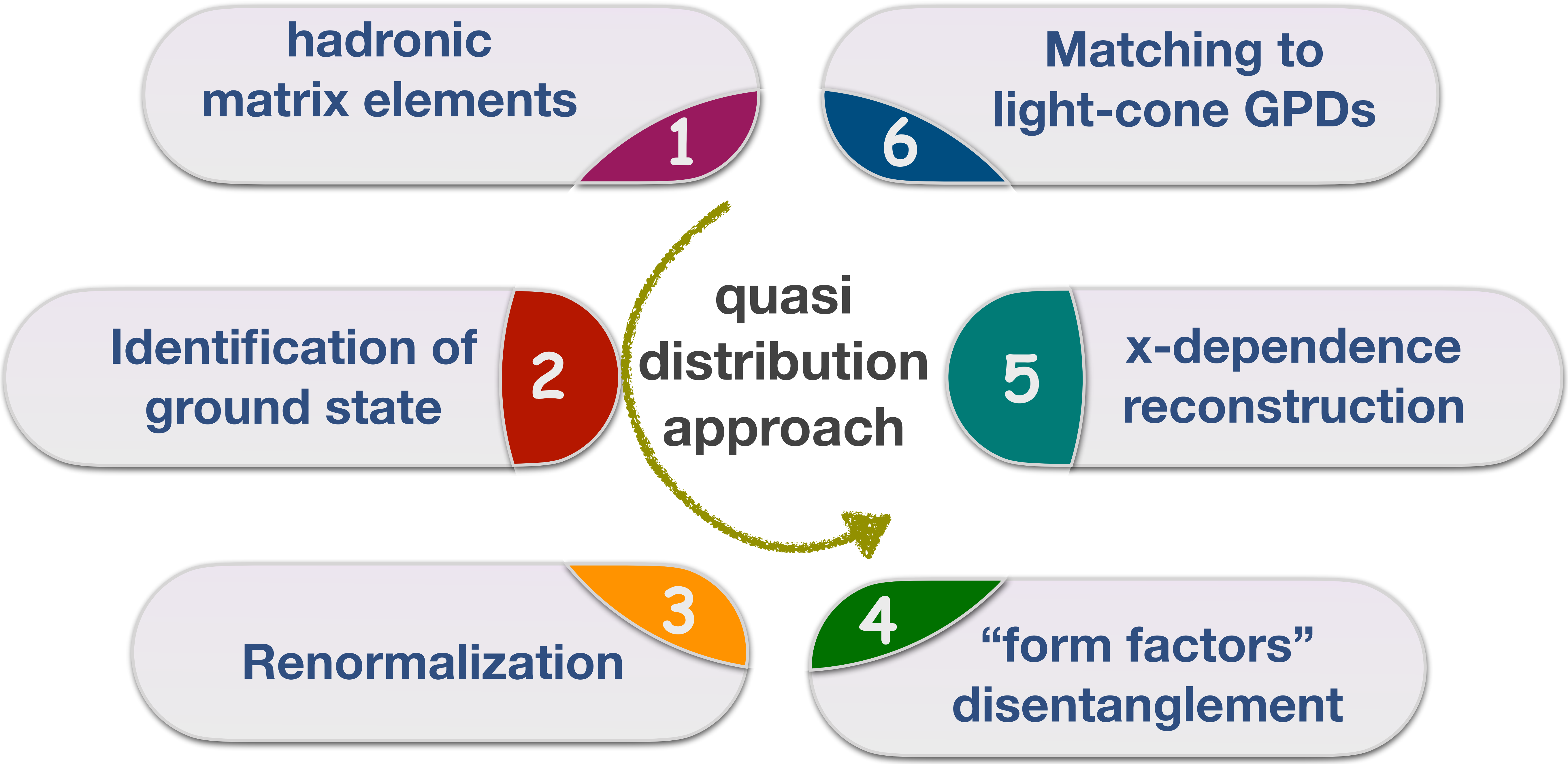
} PDFs & GPDs

- momentum transfer (t)
- skewness (ξ)

} GPDs

★ Challenges of calculation:

- Increased statistical uncertainties due to momentum transfer
- Need for multiple matrix elements to disentangle GPDs
- Frame dependence
- Matching for nonzero skewness, twist-3



High-complexity calculation

Computational and theoretical challenges

each step may introduce systematic uncertainties

Parameters of calculation
(u-d flavor combination)

- Nf=2+1+1 twisted mass fermions & clover term

- Ensemble parameters:

Pion mass:	260 MeV
Lattice spacing:	0.093 fm
Volume:	32^3 x 64
Spatial extent:	3 fm

- Kinematical setup:

Twist-2

P_3 [GeV]	$\vec{Q} \times \frac{L}{2\pi}$	$-t$ [GeV^2]	ξ	$N_{\rm confs}$	$N_{\rm meas}$
0.83	(0,2,0)	0.69	0	519	4152
1.25	(0,2,0)	0.69	0	1315	42080
1.67	(0,2,0)	0.69	0	1753	112192
1.25	(0,2,2)	1.39	1/3	417	40032
1.25	(0,2,-2)	1.39	-1/3	417	40032

zero and nonzero skewness

Twist-3

P_3 [GeV]	$\vec{Q} \times \frac{L}{2\pi}$	$-t$ [GeV^2]	ξ	$N_{\rm meas}$
0.83	(2,0,0)	0.69	0	4288
1.25	(2,0,0)	0.69	0	4288
1.25	(2,2,0)	1.39	0	4288
1.67	(2,0,0)	0.69	0	4288

We utilize

$\pm P_3$

$\pm \vec{Q}$

$\gamma^j \gamma^5, \quad j = 1,2$

- Excited states: $T_{\rm sink}$ up to 1.13 fm

GPDs Decomposition

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

$$F^\mu = P^\mu \frac{h^+}{P^+} H + P^\mu \frac{e^+}{P^+} E$$

$$+ \Delta_\perp^\mu \frac{b}{2m} G_1 + h_\perp^\mu \,(H + E + G_2) + \Delta_\perp^\mu \frac{h^+}{P^+} G_3 + \tilde{\Delta}_\perp^\mu \frac{\tilde{h}^+}{P^+} G_4 \,,$$

$$\widetilde{F}^\mu = P^\mu \frac{\tilde{h}^+}{P^+} \widetilde{H} + P^\mu \frac{\tilde{e}^+}{P^+} \widetilde{E}$$

$$+ \Delta_\perp^\mu \frac{\tilde{b}}{2m} (\widetilde{E} + \widetilde{G}_1) + \tilde{h}_\perp^\mu \,(\widetilde{H} + \widetilde{G}_2) + \Delta_\perp^\mu \frac{\tilde{h}^+}{P^+} \widetilde{G}_3 + \tilde{\Delta}_\perp^\mu \frac{h^+}{P^+} \widetilde{G}_4 \,,$$

$$h^\mu = \bar{u}(p')\, \gamma^\mu\, u(p)\,, \qquad e^\mu = \bar{u}(p')\, \frac{i\sigma^{\mu\nu}\Delta_\nu}{2m}\, u(p)\,, \qquad b = \bar{u}(p')\, u(p)\,,$$

$$\tilde{h}^\mu = \bar{u}(p')\, \gamma^\mu \gamma_5\, u(p)\,, \qquad \tilde{e}^\mu = \frac{\Delta^\mu}{2m}\, \tilde{b}\,, \qquad \tilde{b} = \bar{u}(p')\, \gamma_5\, u(p)$$

Twist-2 GPDs

Decomposition of matrix elements (twist-2)

Unpolarized

$$\Pi_{\gamma^0}(\mathcal{P}_0;P_f,P_i) = C\Bigg[F_H(Q^2)\left(\frac{E_f E_i}{2m^2}+\frac{E_f+E_i}{4m}+\frac{P_{f\,\rho}P_{i\rho}}{4m^2}+\frac{1}{4}\right)$$
$$+F_E(Q^2)\left(P_{f\,\rho}P_{i\rho}\left(\frac{E_f+E_i}{8m^3}+\frac{1}{4m^2}\right)-\frac{(E_f-E_i)^2}{8m^2}+\frac{E_f+E_i}{8m}+\frac{1}{4}\right)\Bigg],$$

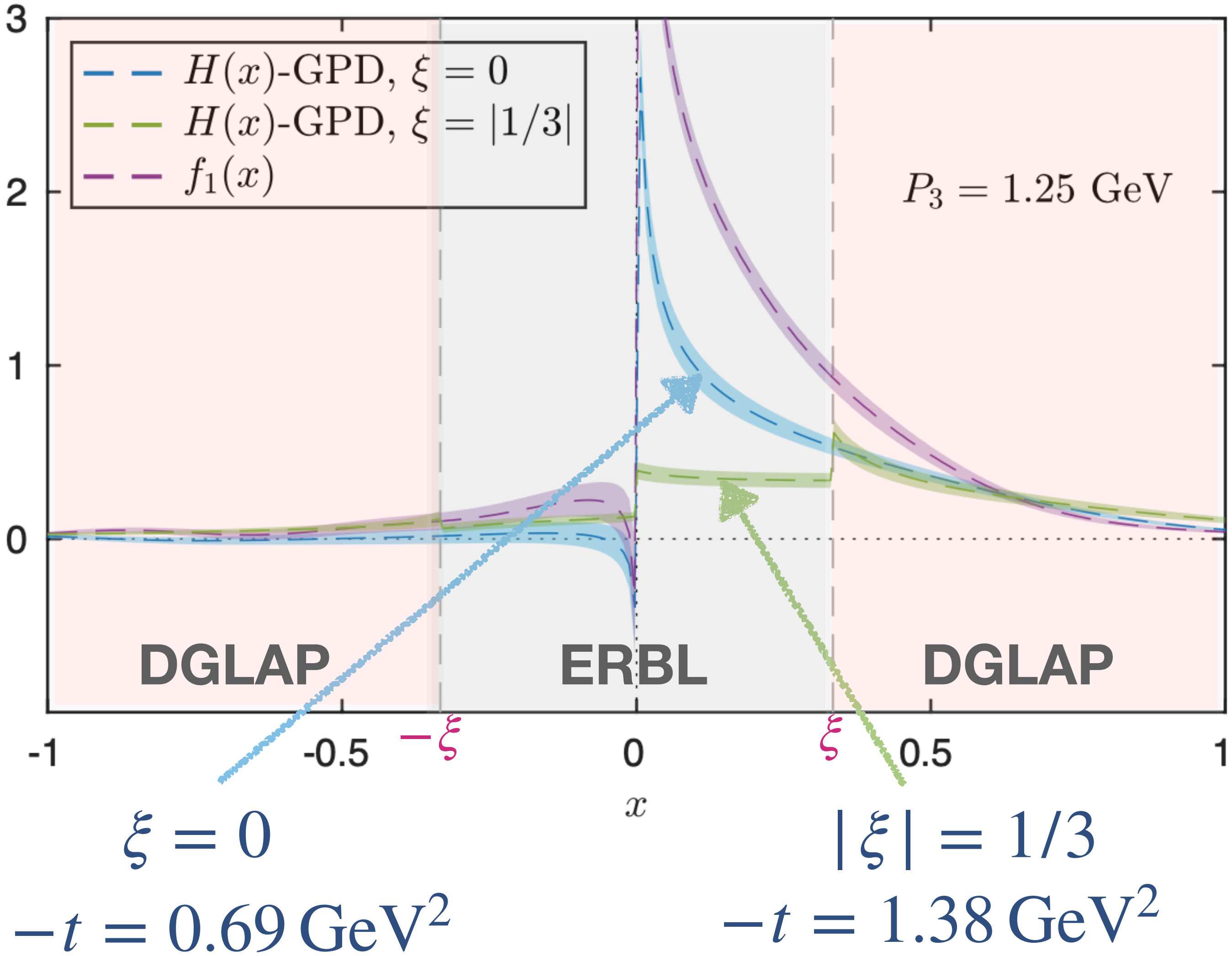
$$\Pi_{\gamma^0}(\mathcal{P}_j;P_f,P_i) = i\,\epsilon_{j\,0\,\rho\,\tau}C\left[F_H(Q^2)\frac{P_{f\,\rho}P_{i\tau}}{4m^2}+F_E(Q^2)\left(\frac{(E_f+E_i)P_{f\,\rho}P_{i\tau}}{8m^3}+\frac{P_{f\,\rho}P_{i\tau}-P_{f\,\tau}P_{i\rho}}{8m^2}\right)\right],$$

Helicity

$$\Pi_{\gamma^3\gamma^5}(\mathcal{P}_j;P_f,P_i) = i\,C\Bigg[F_{\widetilde{H}}(Q^2)\left(\delta_{j3}\left(\frac{E_f+E_i}{4m}-\frac{P_{f\,\rho}P_{i\rho}}{4m^2}+\frac{1}{4}\right)+\frac{P_{i3}\,P_{f\,j}+P_{f\,3}\,P_{ij}}{4m^2}\right)$$
$$-F_{\widetilde{E}}(Q^2)\frac{(P_{f\,3}-P_{i3})(-E_fP_{ij}+E_iP_{f\,j}+m(P_{f\,j}-P_{ij}))}{8m^3}\Bigg],$$

- **Kinematic factors defined by calculation setup**
- **Decomposition similar to the decomposition of form factors**
- **Zero skewness:** $F_{\tilde{E}}(Q^2)=0$

Comparison of zero and nonzero skewness



✦ ERBL region ($|x| < \xi$):

- amplitude of removing a quark-antiquark pair with momentum -Δ

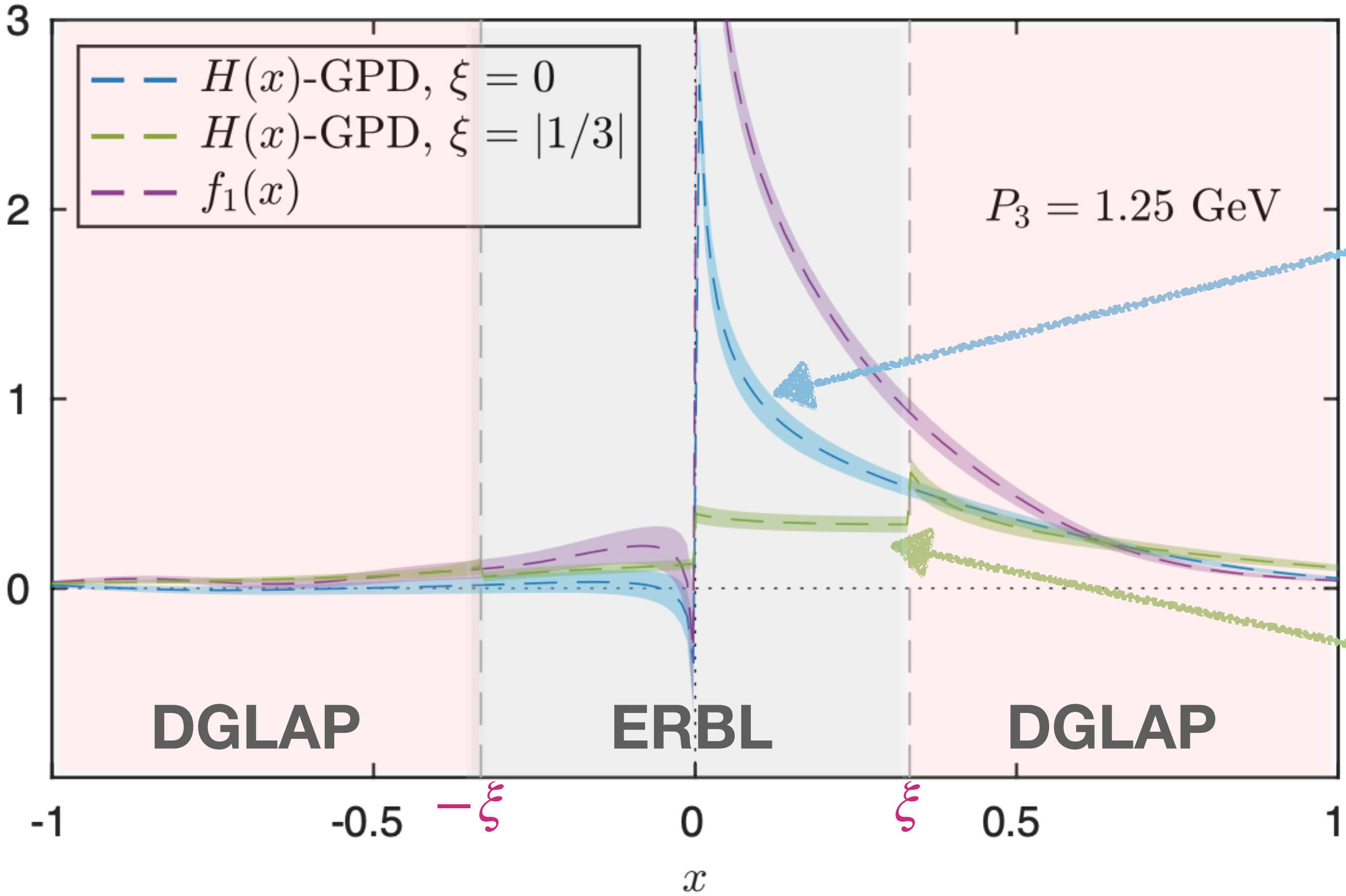
✦ DGLAP region ($\xi < |x| < 1$):

- amplitude of removing a quark/antiquark from the hadron and inserting it back with momentum increase Δ

✦ ERBL region:
- $f_1(x)$ dominant
- $H(x,1/3)$ suppressed compared to $H(x,0)$

✦ DGLAP region:
- $H(x,1/3)$ similar to $H(x,0)$

Comparison of zero and nonzero skewness



$\xi = 0, \quad -t = 0.69\,\mathrm{GeV}^2$

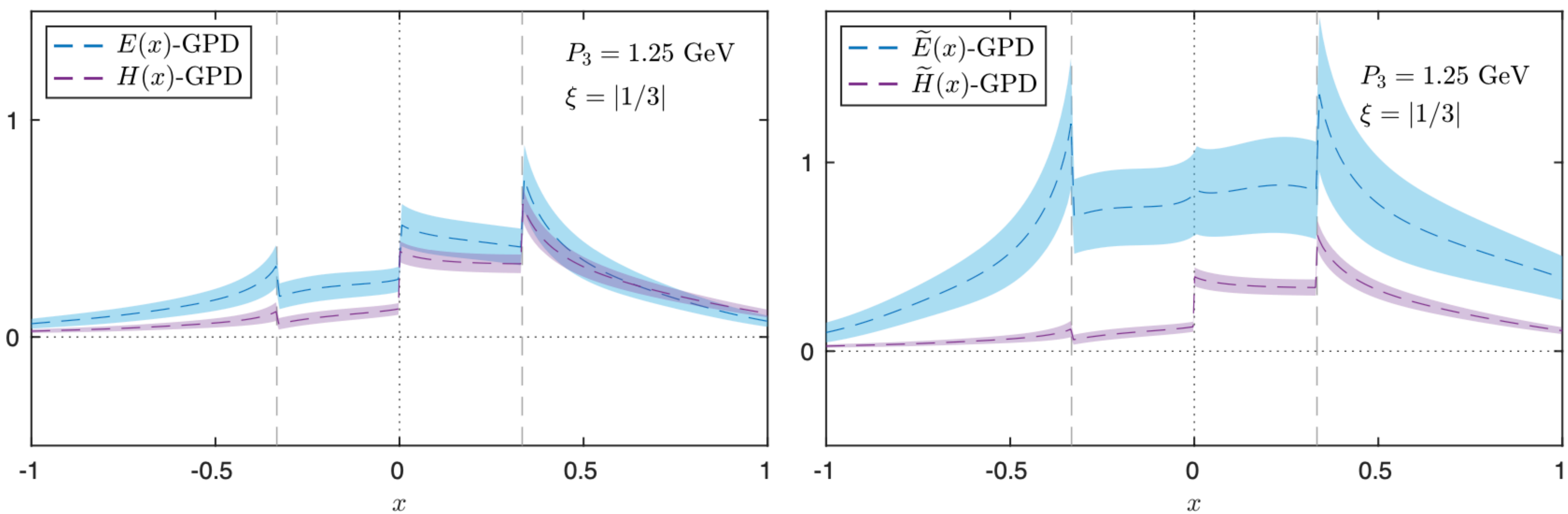
$|\xi| = 1/3, \quad -t = 1.38\,\mathrm{GeV}^2$

$x \to 1$ region: qualitatively comparison with power counting analysis

[F. Yuan, Phys.Rev. D69 (2004) 051501, hep-ph/0311288]

- t-dependence vanishes at large-x
- $H(x,0)$ asymptotically equal to $f_1(x)$
- $1/(1-\xi)^2$ dependence predicts GPDs larger than PDFs

GPDs comparison



- H-GPD and E-GPD are similar in magnitude

  important contribution in the proton spin

$$\int_{-1}^{+1} dx\, x^2 H^q(x,\xi,t) = A_{20}^q(t) + 4\xi^2 C_{20}^q(t)\,, \qquad \int_{-1}^{+1} dx\, x^2 E^q(x,\xi,t) = B_{20}^q(t) - 4\xi^2 C_{20}^q(t)$$

- $\widetilde{E}$-GPD dominant compared to $\widetilde{H}$-GPD

Twist-3 GPDs

Helicity flip twist-3 GPDs

★ **Transverse matrix element of axial operator (proton boost: $\overrightarrow{P} = (0,0,P_3)$)**

$$\widetilde{F}^\mu = P^\mu \frac{\tilde{h}^+}{P^+} \widetilde{H} + P^\mu \frac{\tilde{e}^+}{P^+} \widetilde{E}$$

$$+ \Delta_\perp^\mu \frac{\tilde{b}}{2m} (\widetilde{E} + \widetilde{G}_1) + \tilde{h}_\perp^\mu (\widetilde{H} + \widetilde{G}_2) + \Delta_\perp^\mu \frac{\tilde{h}^+}{P^+} \widetilde{G}_3 + \tilde{\Delta}_\perp^\mu \frac{h^+}{P^+} \widetilde{G}_4$$

[D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105, arXiv:hep-ph/0212372]

[F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

★ **Sum Rules (generalization of Burkhardt-Cottingham)**

$$\int_{-1}^{1} dx\, \widetilde{H}(x,\xi,t) = G_A(t)\,, \qquad \int_{-1}^{1} dx\, \widetilde{E}(x,\xi,t) = G_P(t)\,, \qquad \int_{-1}^{1} dx\, \widetilde{G}_i(x,\xi,t) = 0$$

[X. D. Ji, Phys. Rev. Lett. 78, 610 (1997), hep-ph/9603249]

★ **Sum Rules (generalization of Efremov-Leader-Teryaev)**

[A. Efremov, O. Teryaev , E. Leader, PRD 55 (1997) 4307, hep-ph/9607217]

$$\int_{-1}^{1} dx\, x\, \widetilde{G}_1(x,\xi,t) = \frac{1}{2}\left[F_2(t) + \left(\xi\frac{\partial}{\partial\xi} - 1\right)\int_{-1}^{1} dx\, x\, \widetilde{E}(x,\xi,t)\right], \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_2(x,\xi,t) = \frac{1}{2}\left[\xi^2 G_E(t) - \frac{t}{4m^2}F_2(t) - \widetilde{A}_{20}(t)\right],$$

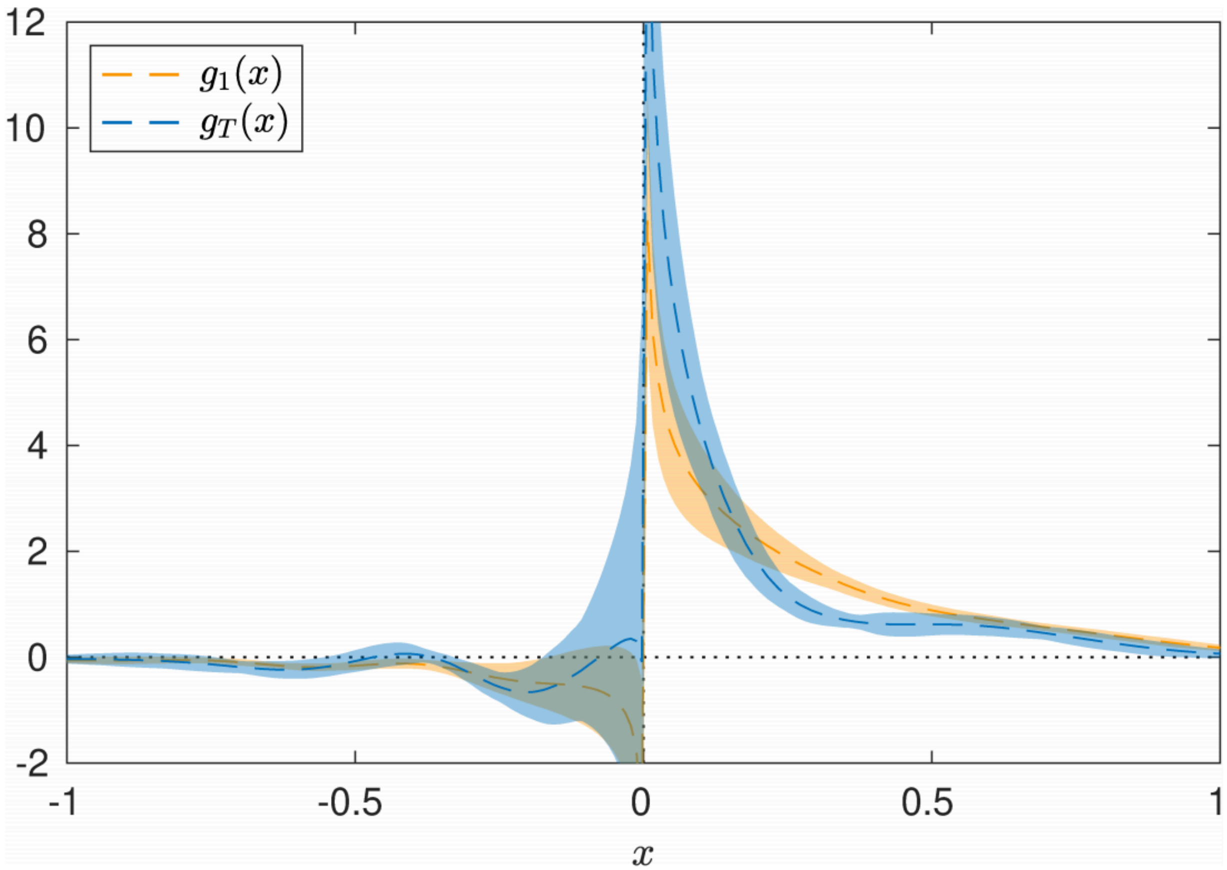
$$\int_{-1}^{1} dx\, x\, \widetilde{G}_3(x,\xi,t) = \frac{\xi}{4}G_E(t)\,, \qquad \int_{-1}^{1} dx\, x\, \widetilde{G}_4(x,\xi,t) = \frac{1}{4}G_E(t)$$

Sum rules simplify for zero skewness

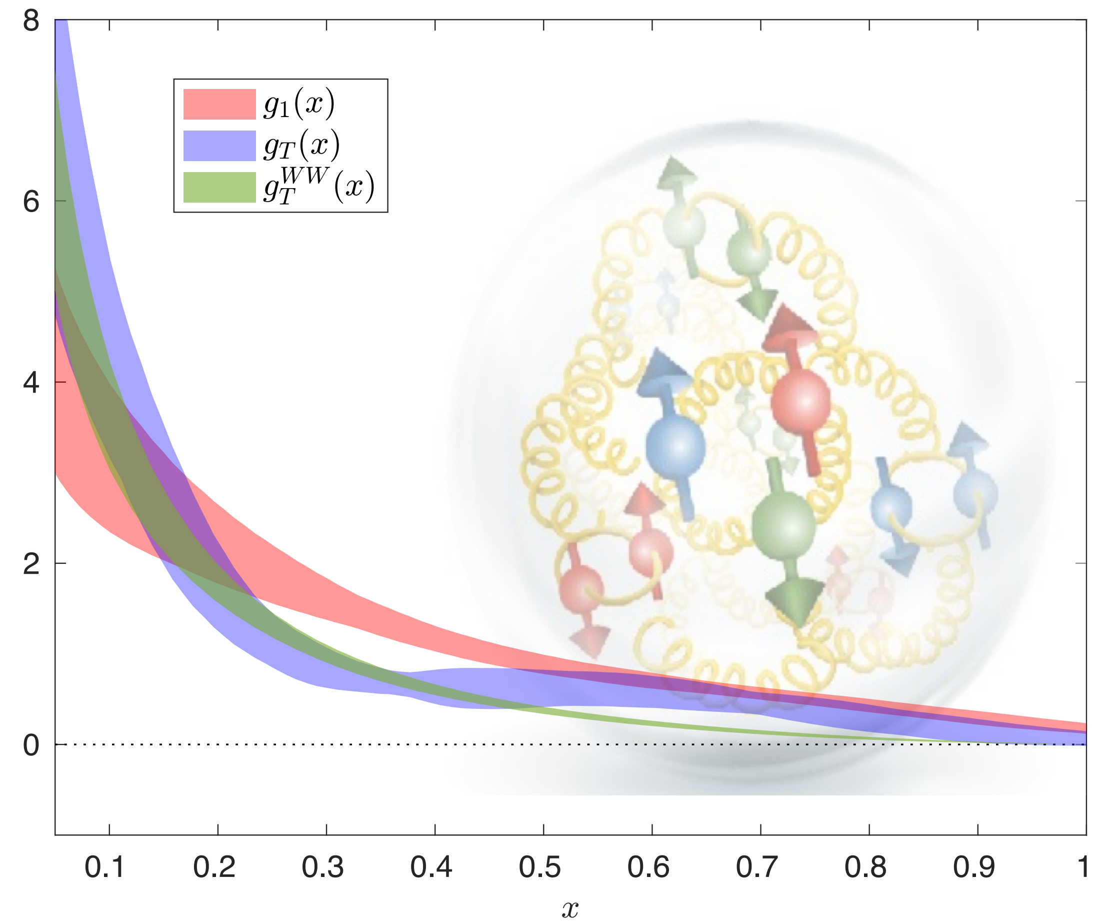
F_2 : Pauli FF
G_E : electric FF
$\widetilde{A}_{20}$, $\widetilde{B}_{20}$: axial GFFs

Results on twist-3 PDFs

★ Helicity flip ($g_T(x)$) and transversity ($h_L(x)$) twist-3 PDFs



PRD Editors' Suggestion Highlight



[S. Bhattacharya et al., PRD 102 (2020) 11, arXiv:2004.04130]

See talks @ GHP 2021 by:

S. Bhattacharya (on matching)

A. Scapellato (on lattice results)

Decomposition of matrix elements

- **Decomposition in Minkowski space** [F. Aslan et a., Phys. Rev. D 98, 014038 (2018), arXiv:1802.06243]

$$\widetilde{F}^{\mu} = P^{\mu}\frac{\tilde{h}^{+}}{P^{+}}\widetilde{H} + P^{\mu}\frac{\tilde{e}^{+}}{P^{+}}\widetilde{E} + \Delta_{\perp}^{\mu}\frac{\tilde{b}}{2m}(\widetilde{E}+\widetilde{G}_1) + \tilde{h}_{\perp}^{\mu}(\widetilde{H}+\widetilde{G}_2) + \Delta_{\perp}^{\mu}\frac{\tilde{h}^{+}}{P^{+}}\widetilde{G}_3 + \tilde{\Delta}_{\perp}^{\mu}\frac{h^{+}}{P^{+}}\widetilde{G}_4$$

$$h^{\mu} = \bar{u}(p')\,\gamma^{\mu}\,u(p)\,, \qquad e^{\mu} = \bar{u}(p')\,\frac{i\sigma^{\mu\nu}\Delta_{\nu}}{2m}\,u(p)\,, \qquad b = \bar{u}(p')\,u(p)\,,$$

$$\tilde{h}^{\mu} = \bar{u}(p')\,\gamma^{\mu}\gamma_5\,u(p)\,, \qquad \tilde{e}^{\mu} = \frac{\Delta^{\mu}}{2m}\,\tilde{b}\,, \qquad \tilde{b} = \bar{u}(p')\,\gamma_5\,u(p)$$

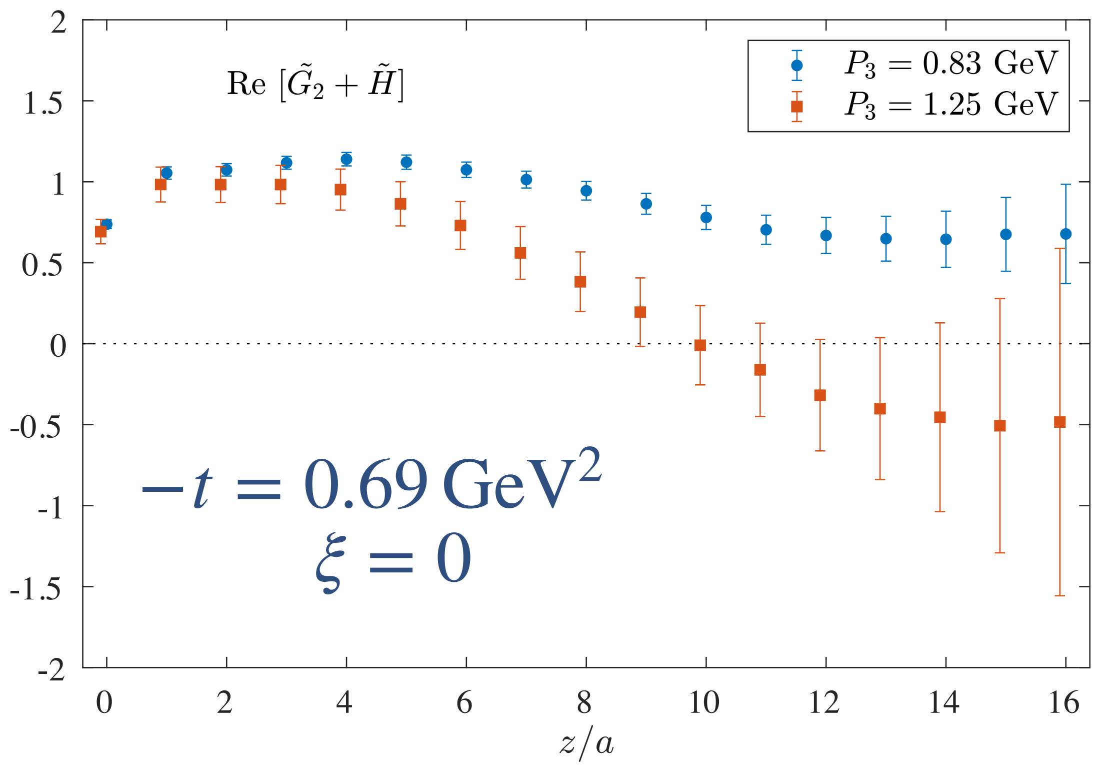
- **Kinematic factors defined by calculation setup**
- **All twist-3 helicity GPDs:** **4 x computational cost compared to PDFs !**
- **For $\vec{Q} = (Q_x, 0, 0)$ the following matrix elements contribute**
  - $\Pi(\gamma^2\gamma^5, \Gamma_0): \widetilde{H} + \widetilde{G}_2, \quad \widetilde{G}_4$
  - $\Pi(\gamma^2\gamma^5, \Gamma_2): \widetilde{H} + \widetilde{G}_2, \quad \widetilde{G}_4$
  - $\Pi(\gamma^1\gamma^5, \Gamma_1): \widetilde{H} + \widetilde{G}_2, \quad \widetilde{E} + \widetilde{G}_1$
  - $\Pi(\gamma^1\gamma^5, \Gamma_3): \widetilde{G}_3$

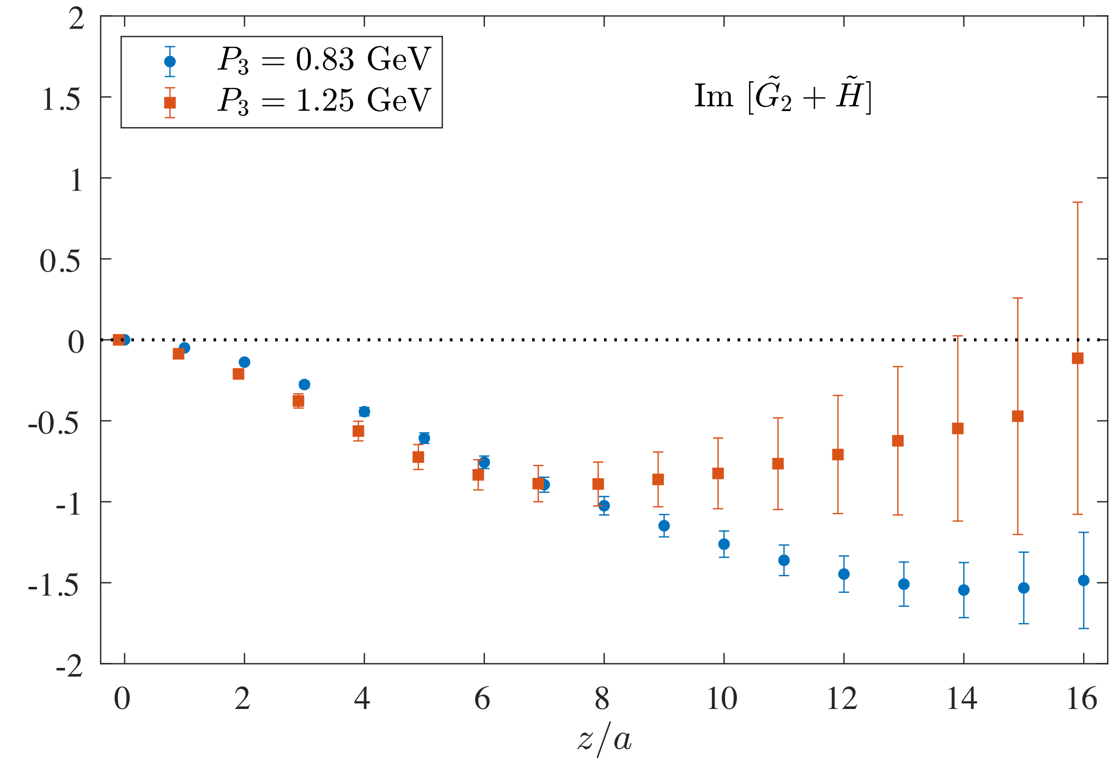
Parity projectors

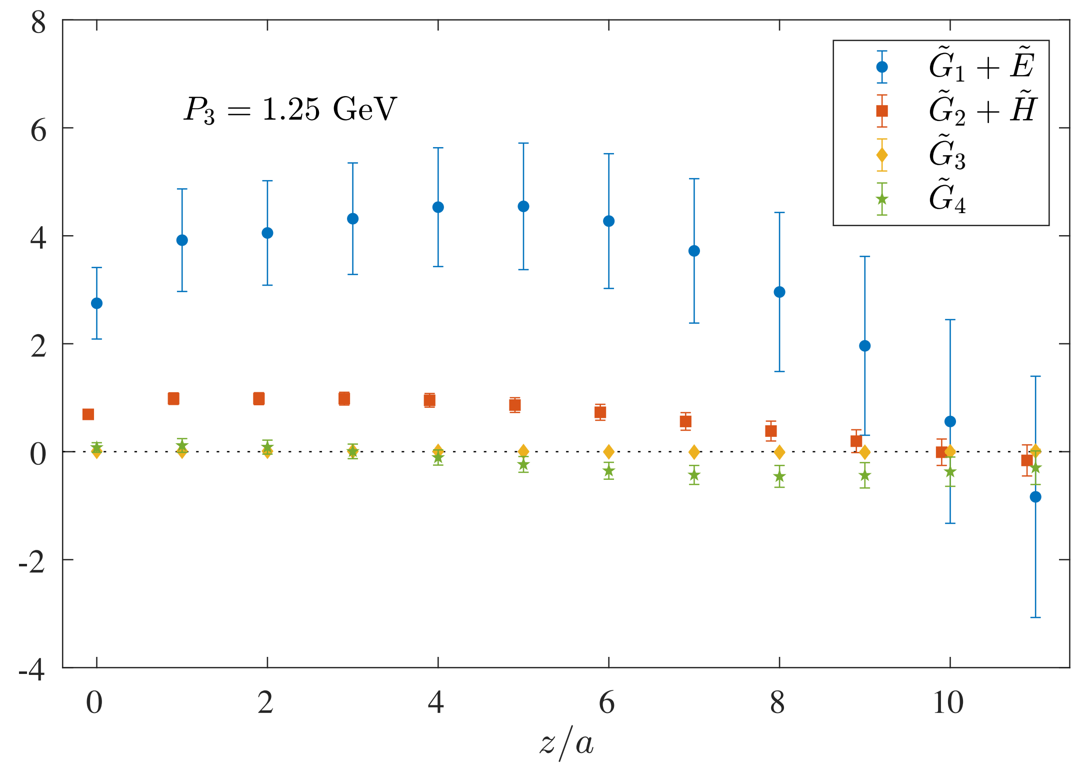
$$\Gamma_0 = \frac{1}{4}(1+\gamma^0)$$

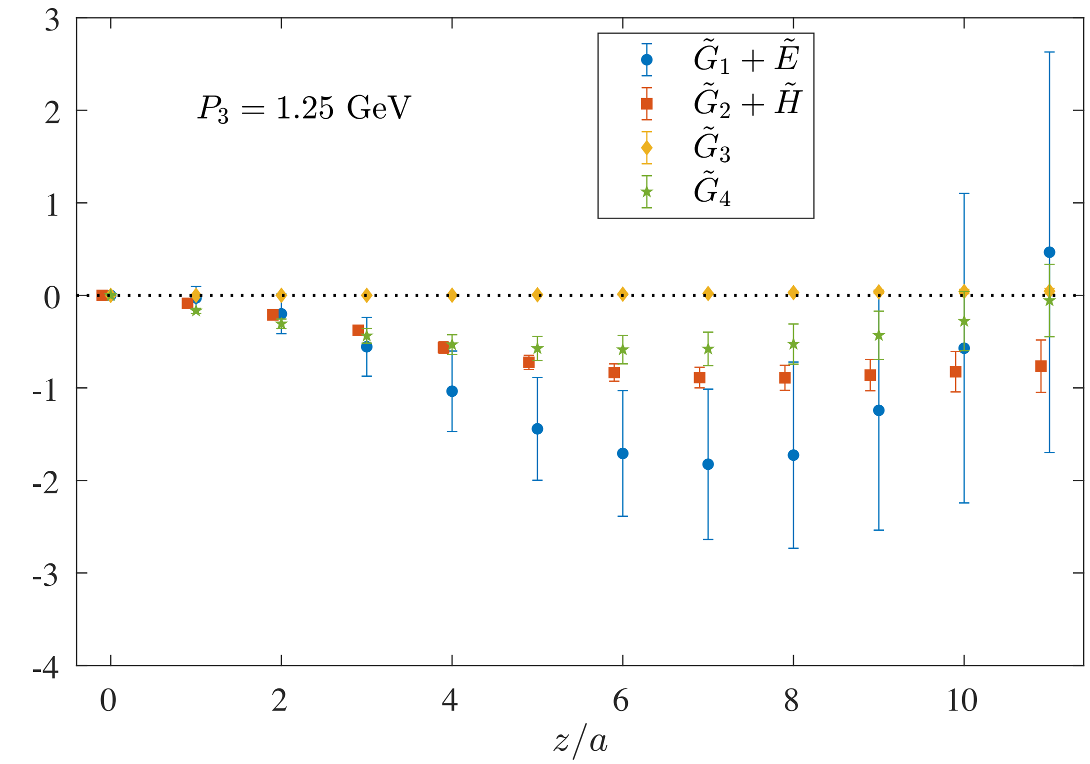
$$\Gamma_i = \frac{1}{4}(1+\gamma^0)\gamma^5\gamma^i$$

Matrix elements decomposition







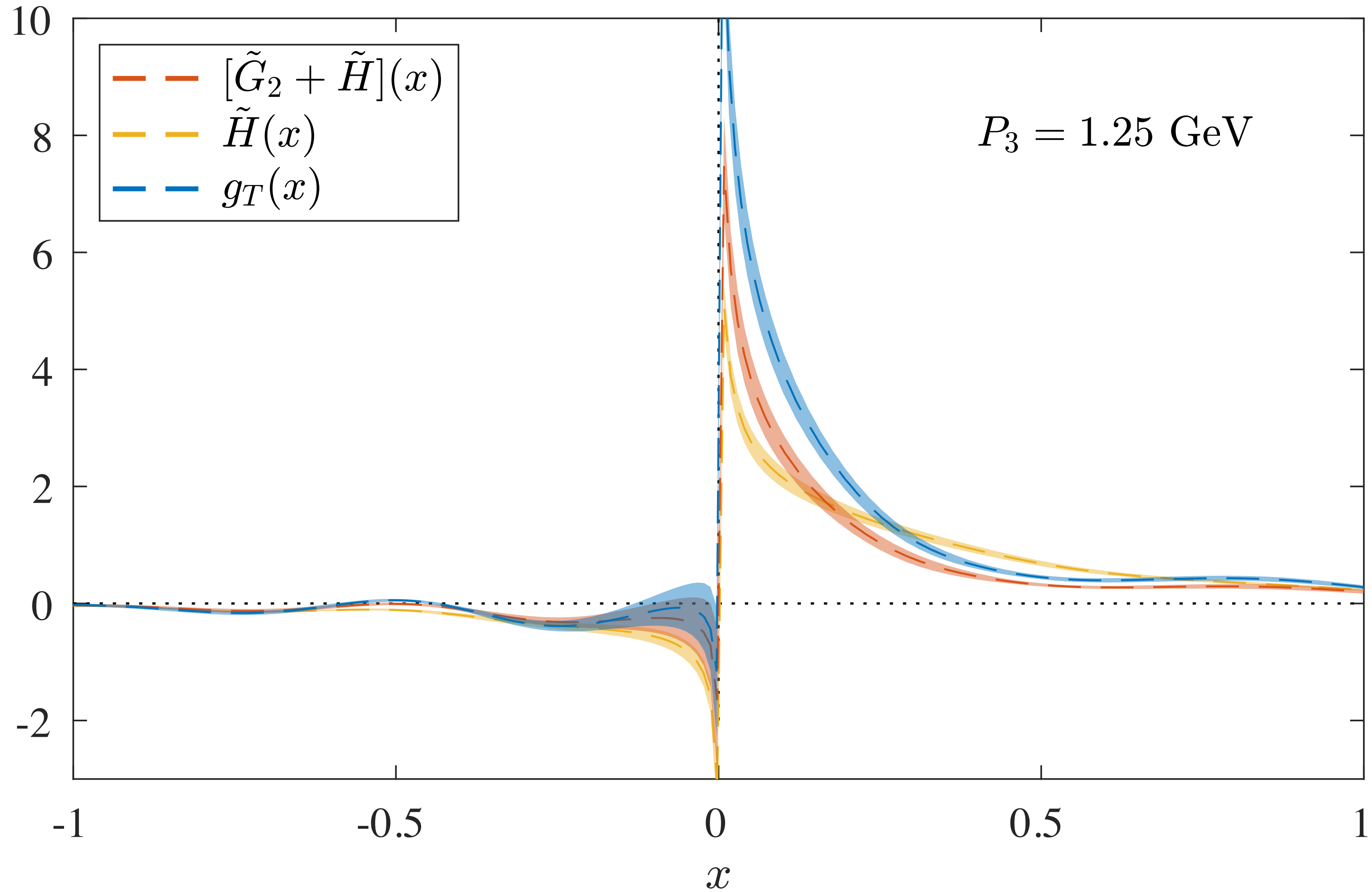


★ $\widetilde{E} + \widetilde{G}_1$ **: highest contribution**

★ $\widetilde{G}_3, \ \widetilde{G}_4$ **: suppressed**

x-dependence of twist-3 GPDs

$$-t = 0.69\,\mathrm{GeV}^2\,, \qquad \xi = 0$$



- ★ $g_T(x)$: **dominant distribution**
- ★ $\widetilde{H} + \widetilde{G}_2$ **similar in magnitude to** $\widetilde{H}$
- ★ $\widetilde{G}_2$ **is expected to be small**

Concluding Remarks

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- ★ GPDs multi-dimensionality poses computational challenges

- ★ Current status: exploratory and qualitative

- ★ At twist-3: 2-parton and 3-parton correlations, such as qgq
  - 0^{th} moments of twist-3 GPDs is zero [D. Kiptily and M. Polyakov, Eur. Phys. J. C37 (2004) 105]
  - 1^{st} moments of twist-3 GPDs have zero qgq contribution
  - alternative matching proposed in Braun, Ji, Vladimirov, arXiv: 2103.12105

- ★ Extraction of twist-3 GPDs is promising with several interesting investigations (WW-approximation, sum rules)

- ★ Nonzero skewness of particular interest: twist-3 GPDs (in models) exhibit discontinuities at $x = \pm\,\xi$

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Thank you

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