

valance PDF of pion from lattice QCD:
radial excitation, improved matching, physical mass, chiral quarks

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based on: 2007.06590 2101.11632 2102.01101 preliminary

role of gluons in hadron mass generation ... the enigma of pion mass

- significant contributions of gluons (trace-anomaly) to proton mass, even in the chiral limit
- contribution of gluons (trace-anomaly) to pion mass ?

$$m_q \rightarrow 0 : -\langle T_{\mu\mu} \rangle = \langle H_m \rangle + \langle H_a \rangle = M_\pi \rightarrow 0$$

a puzzling dichotomy !!

understanding the role of gluons in hadron mass generation will remain incomplete without explaining the *absence* of pions mass

where are the gluons? ... valance quark PDF of pion

different for different models/fits

data: Fermilab E615 Phys. Rev. D39, 92 (1989)

pQCD: $b \simeq 2$ dominated by one-gluon exchange
Berger, Brodsky: Phys. Rev. Lett. 42, 940

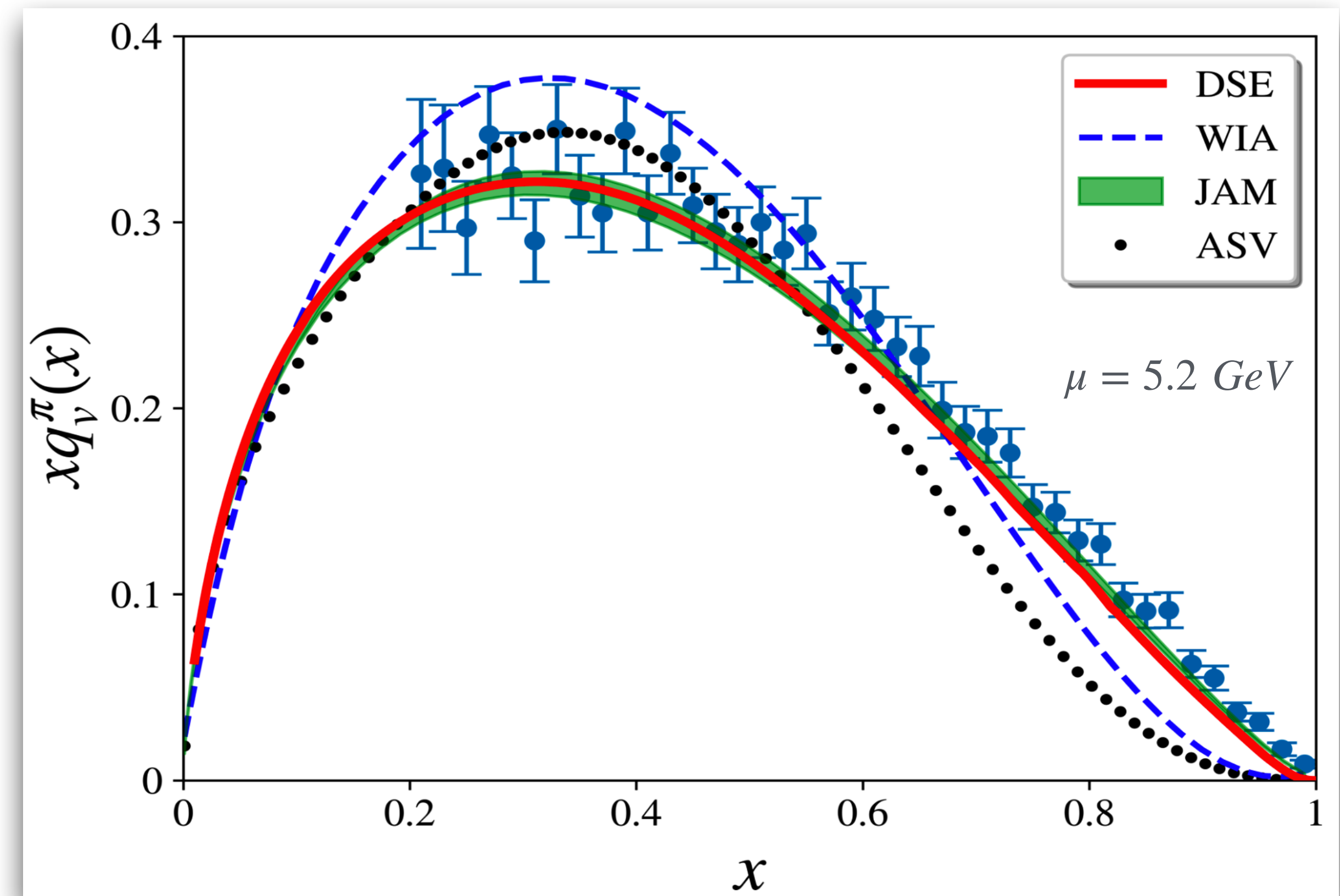
JAM: $b \simeq 1$ Phys. Rev. Lett. 121, 152001 (2018)

ASV: $b \simeq 2$ NLL threshold resummation
Phys. Rev. Lett. 105, 252003 (2010)

WIA: $b \simeq 2$ DSE, rainbow-ladder truncation
Nguyen et al: Phys. Rev. C83, 062201 (2011)

DSE: $b \simeq 2$ another DSE w/ different quark dressing,
different contributions of gluons
Phys. Rev. Lett. 124, 042002 (2020)

$$q_v^\pi(x) = Ax^\alpha(1-x)^\beta [1 + \text{sub leading}]$$



Bednar, Cloet, Tandy: Phys. Rev. Lett. 124, 042002 (2020)

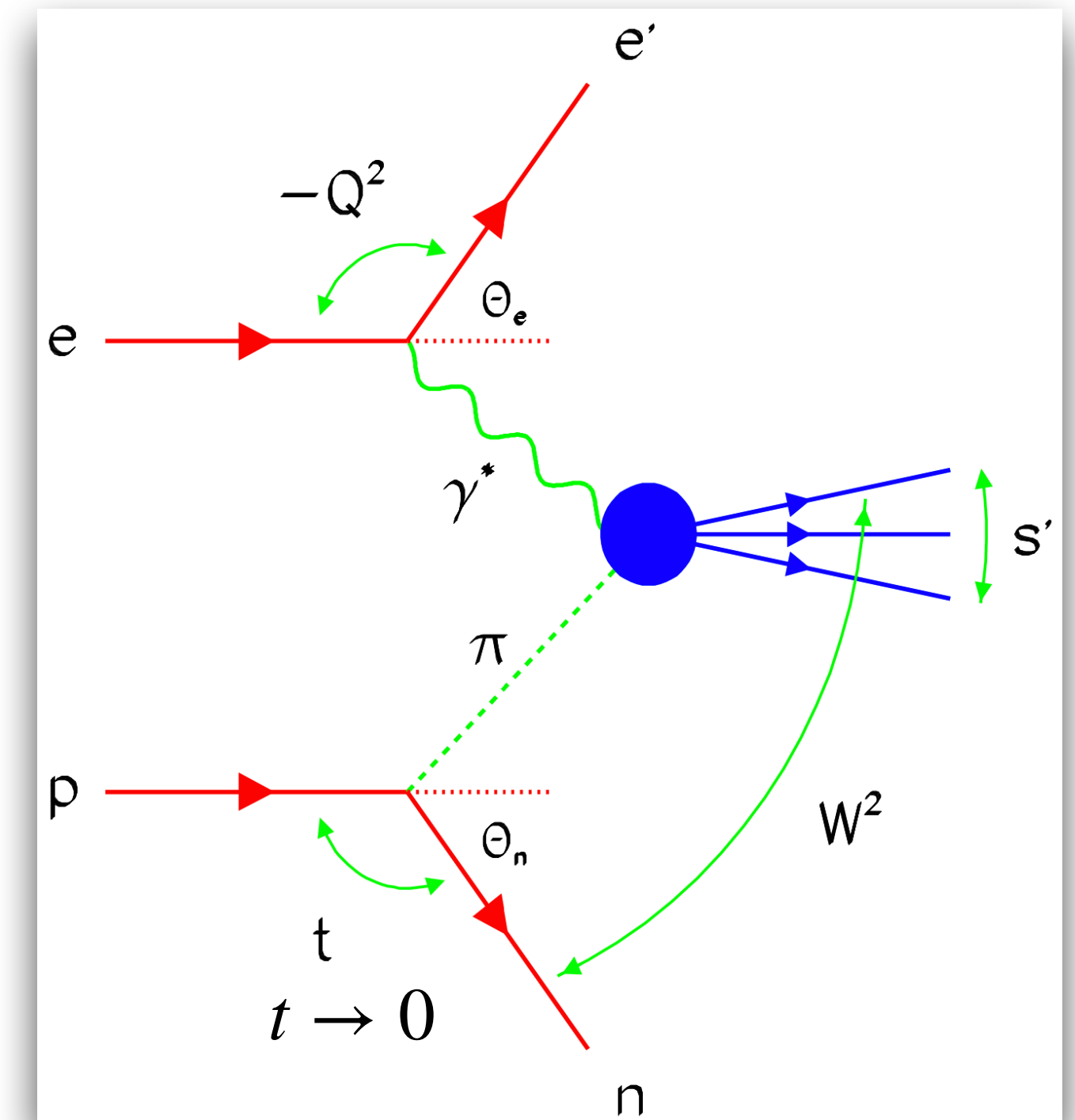
whitepaper: Eur. Phys. J. A55,190 (2019)

Pion and Kaon Structure at the Electron-Ion Collider

abstract

1 GeV mass-scale that characterizes atomic nuclei appear; why does it have the observed value; and, enigmatically, why are the composite Nambu-Goldstone (NG) bosons in quantum chromodynamics (QCD) abnormally light in comparison? In this perspective, we provide an analysis of the mass budget of the pion and proton in QCD; discuss the special role of the kaon, which lies near the boundary between dominance of strong and Higgs mass-generation mechanisms; and explain the need for a coherent effort in QCD phenomenology and continuum calculations, in exa-scale computing as provided by lattice QCD, and in experiments to make progress in understanding the origins of hadron masses and the distribution of that mass within them. We compare the unique capabilities foreseen at the electron-ion collider (EIC) with those at the hadron-electron ring accelerator (HERA), the

Sullivan process w/ off-shell pion

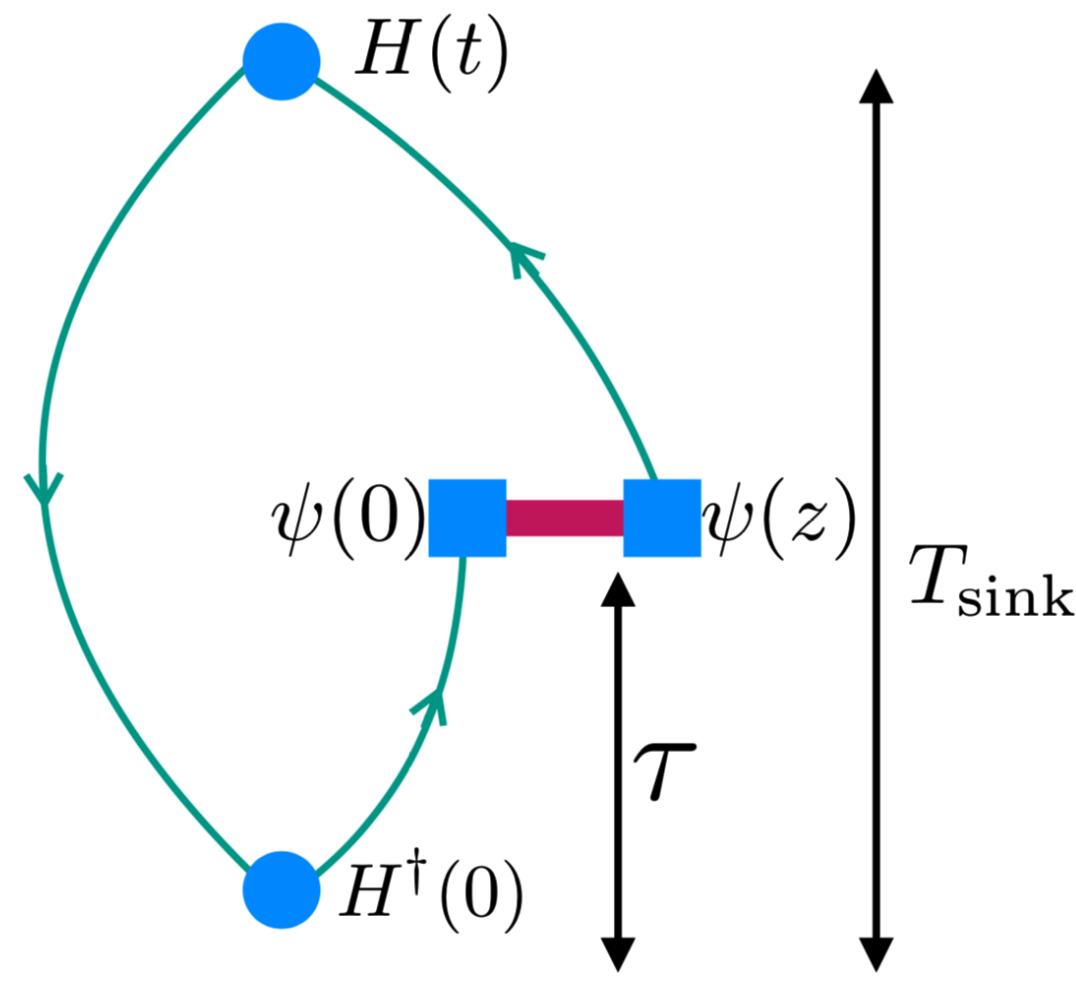


journey from Euclidean time to light-cone ...

● bare matrix element:

$$Q_b(z, p_z) = \langle H(p_z, 0) | \bar{\psi}(0, \tau) \gamma_0 W(0, z) \psi(z, \tau) | H(p_z, T_{\text{sink}}) \rangle$$

equal-time, non-local operator within boosted hadron



$W(0, z)$: Wilson line from 0 to z

● renormalization group invariant ratio

$$\mathcal{M}(z, p_z, p_z^0) = \frac{Q_b(z, p_z)}{Q_b(z, p_z^0)}$$

$p_z^0 = 0$ Radyushkin: Phys. Rev. D96, 034025 (2017)

$p_z^0 > 0$ BNL et. al.: Phys. Rev. D102, 074504 (2020)

- leading twist factorization in **momentum space**

X. Ji: Phys. Rev. Lett. 110, 262002 (2013)

Yong Zhao's talk

- leading twist factorization in **coordinate space**

Radyushkin: Phys. Rev. D96, 034025 (2017)
Phys. Lett. B781, 433 (2018)

short distance expansion

Karpie, Orginos, Zafeiropoulos: JHEP 11, 178 (2018)

BNL et.al.: Phys. Rev. D102, 074504 (2020)

$$\mathcal{M}(z, p_z, p_z^0) = \frac{\sum_{n=0}^{\infty} \frac{(-izp_z)^n}{n!} c_n(z^2\mu^2) \langle x^n \rangle(\mu)}{\sum_{n=0}^{\infty} \frac{(-izp_z^0)^n}{n!} c_n(z^2\mu^2) \langle x^n \rangle(\mu)} + \mathcal{O}\left(z^2 m_h^2, z^2 \Lambda_{\text{QCD}}\right)$$

coefficient function

PDF moments

target mass, higher twist

NLO: Izubuchi et. al.: Phys. Rev. D 98, 056004 (2018)

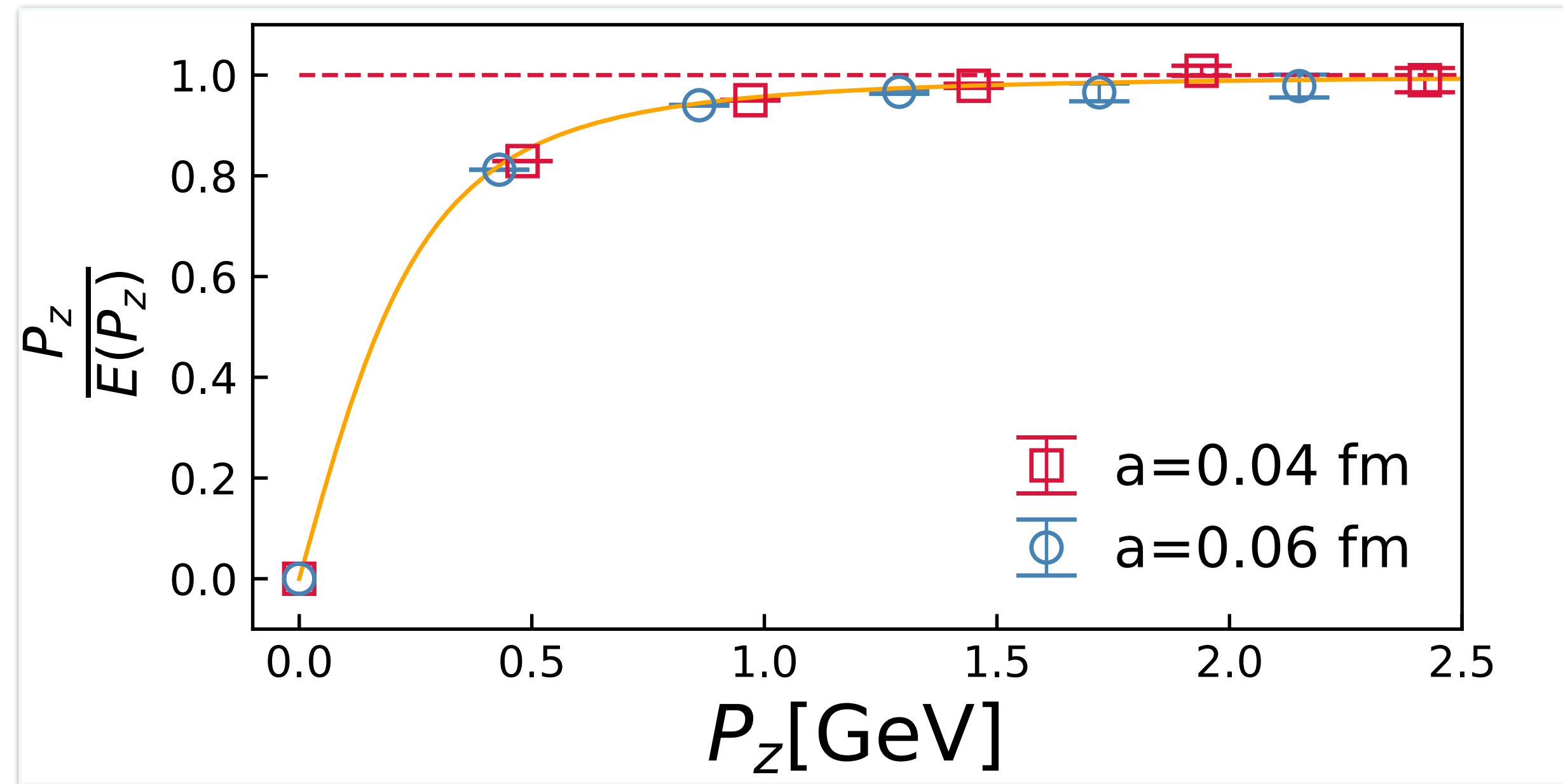
NNLO: Li, Ma, Qiu: 2006.12370

NLO+NLL: BNL-ANL: 2102.01101

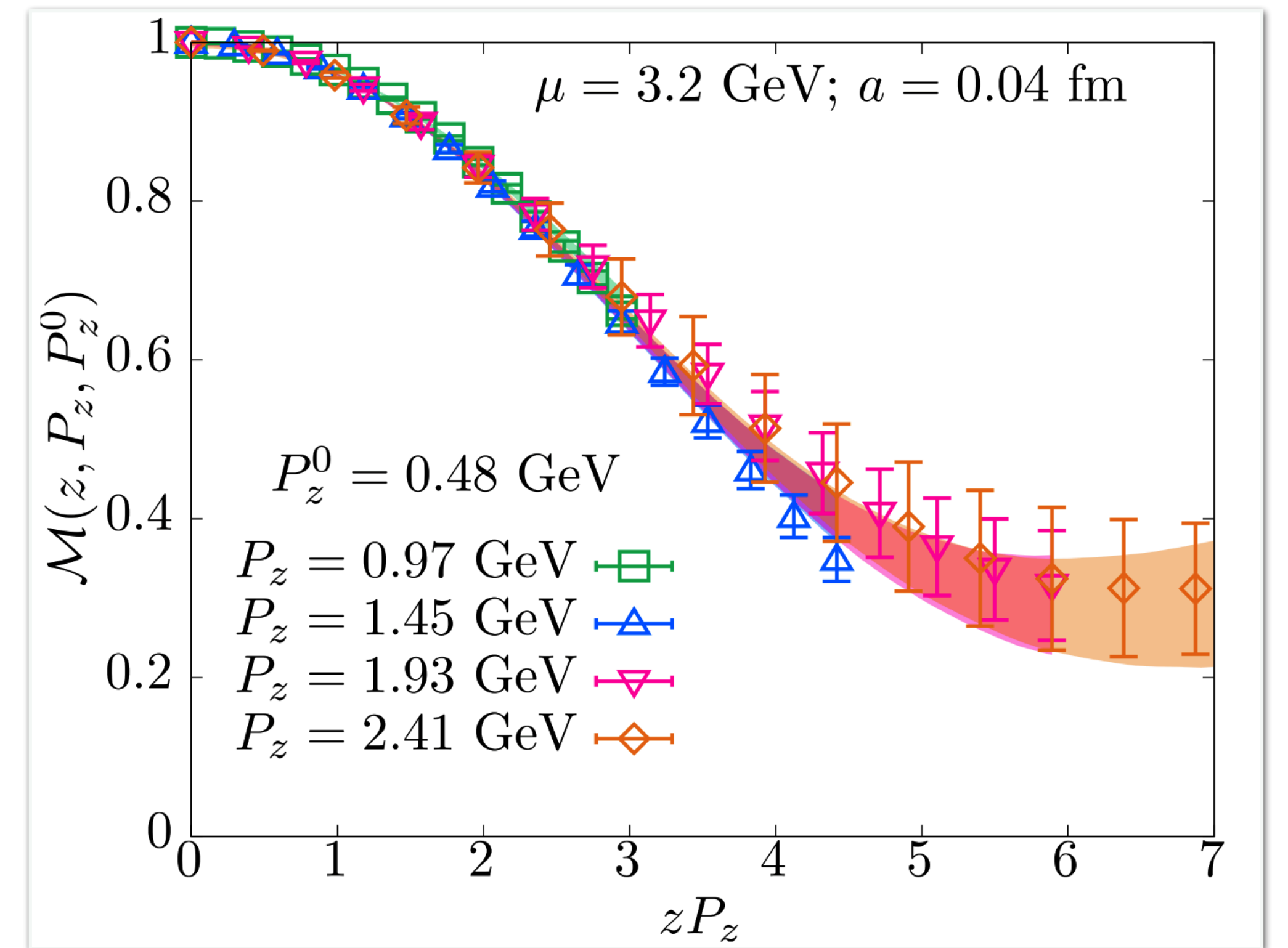
$$\langle x^n \rangle = \int_{-1}^1 dx x^n f(x, \mu)$$

valance PDF of pion ...

$$m_\pi = 300 \text{ MeV}$$



BNL: Phys. Rev. D102, 094513 (2020)



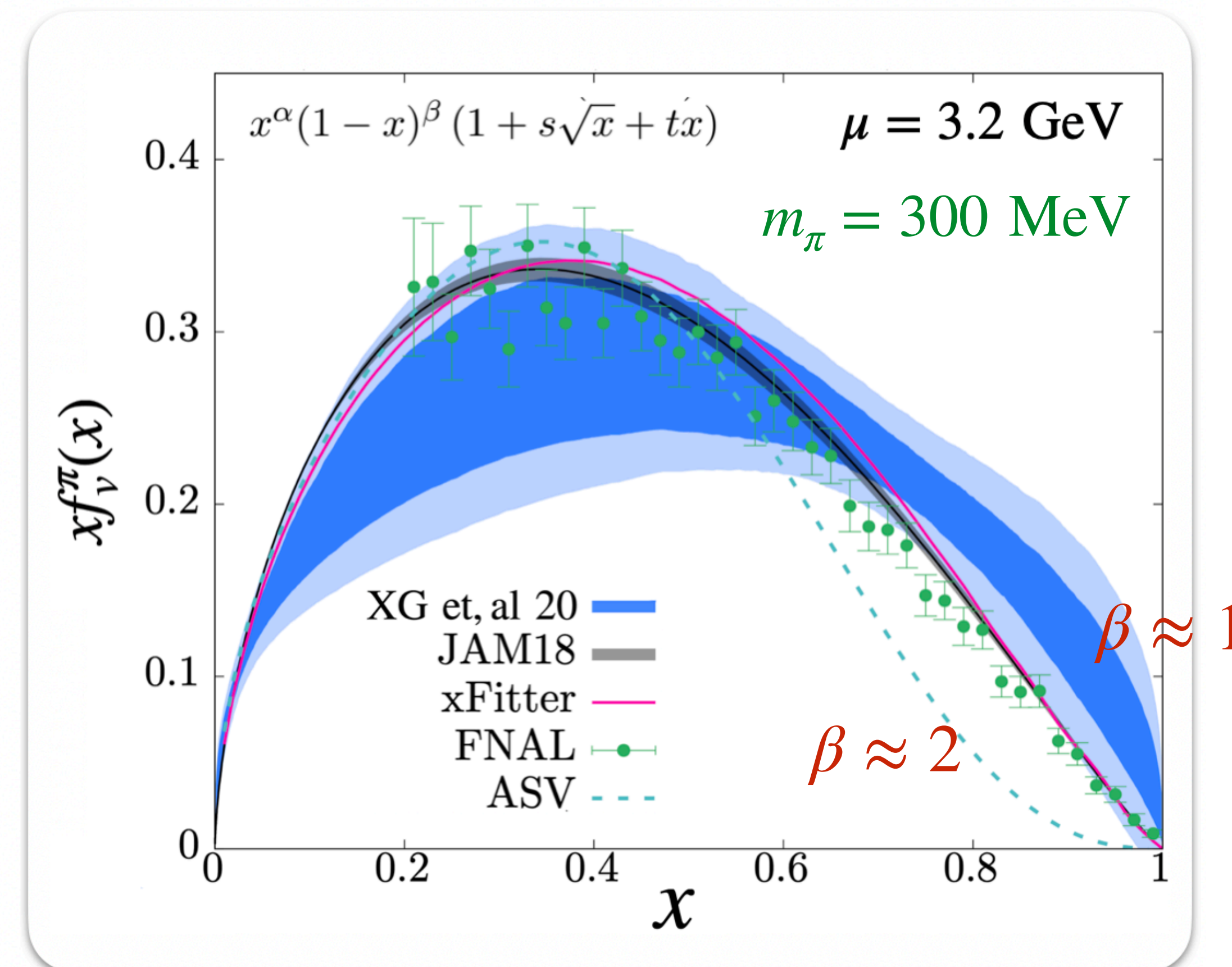
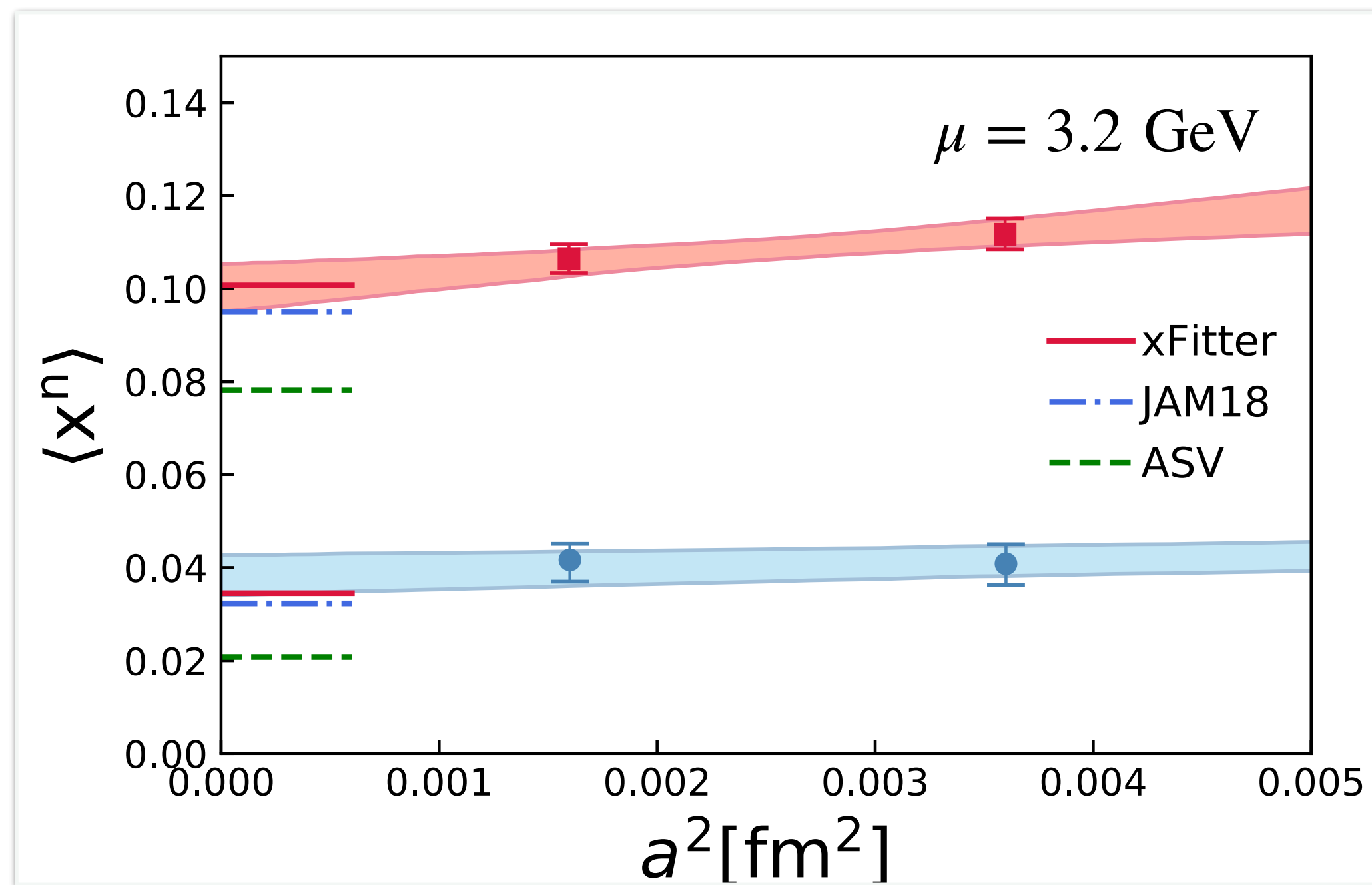
valance quarks: Wilson-Clover

sea quarks: 2+1 flavor HISQ, $m_\pi^{sea} \simeq 160$ MeV, $m_K^{sea} \simeq 500$ MeV

model-dependent fit:

$$f_v^\pi(x) = Ax^\alpha(1-x)^\beta(1+s\sqrt{x}+tx)$$

model-independent moments

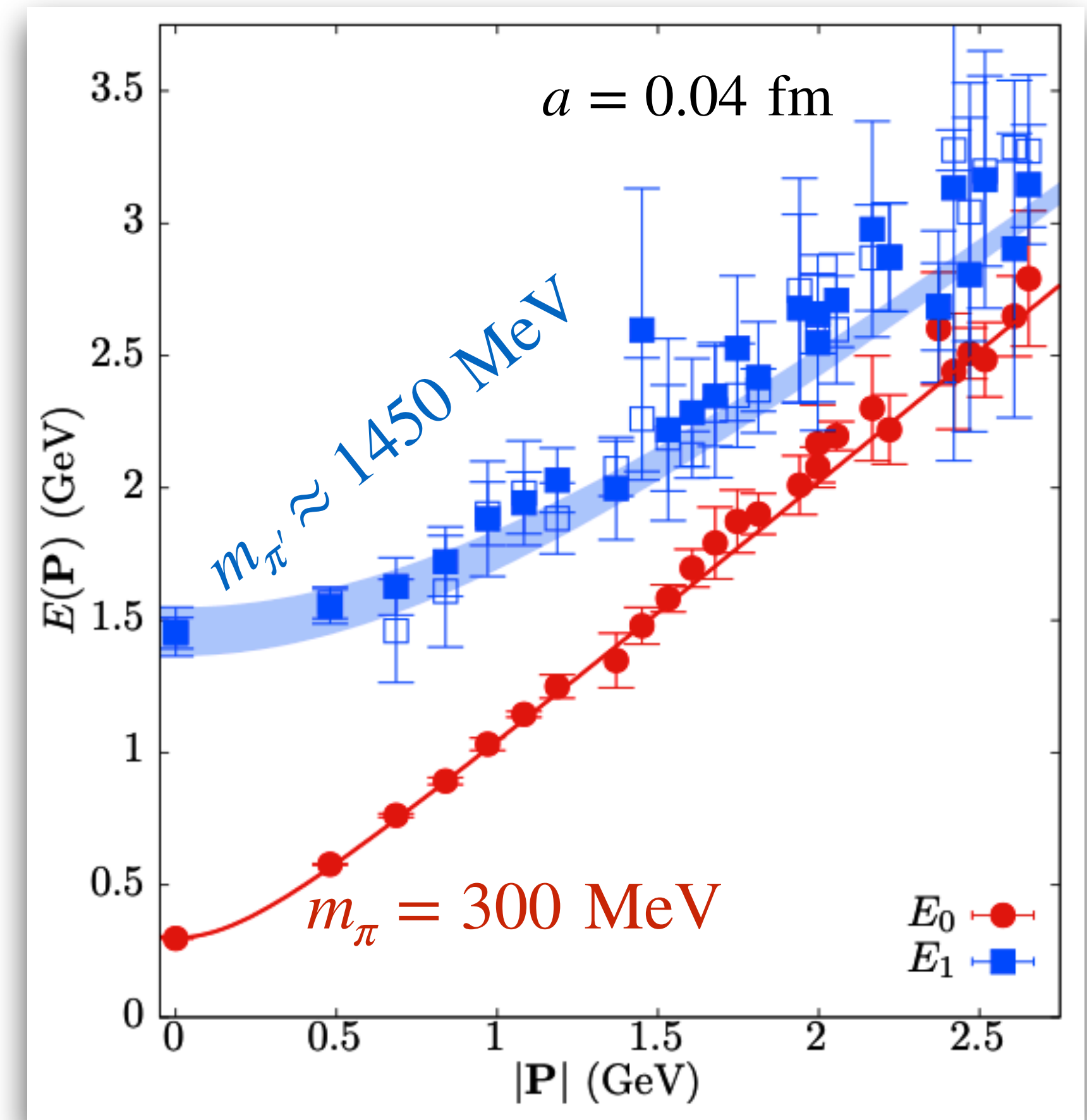
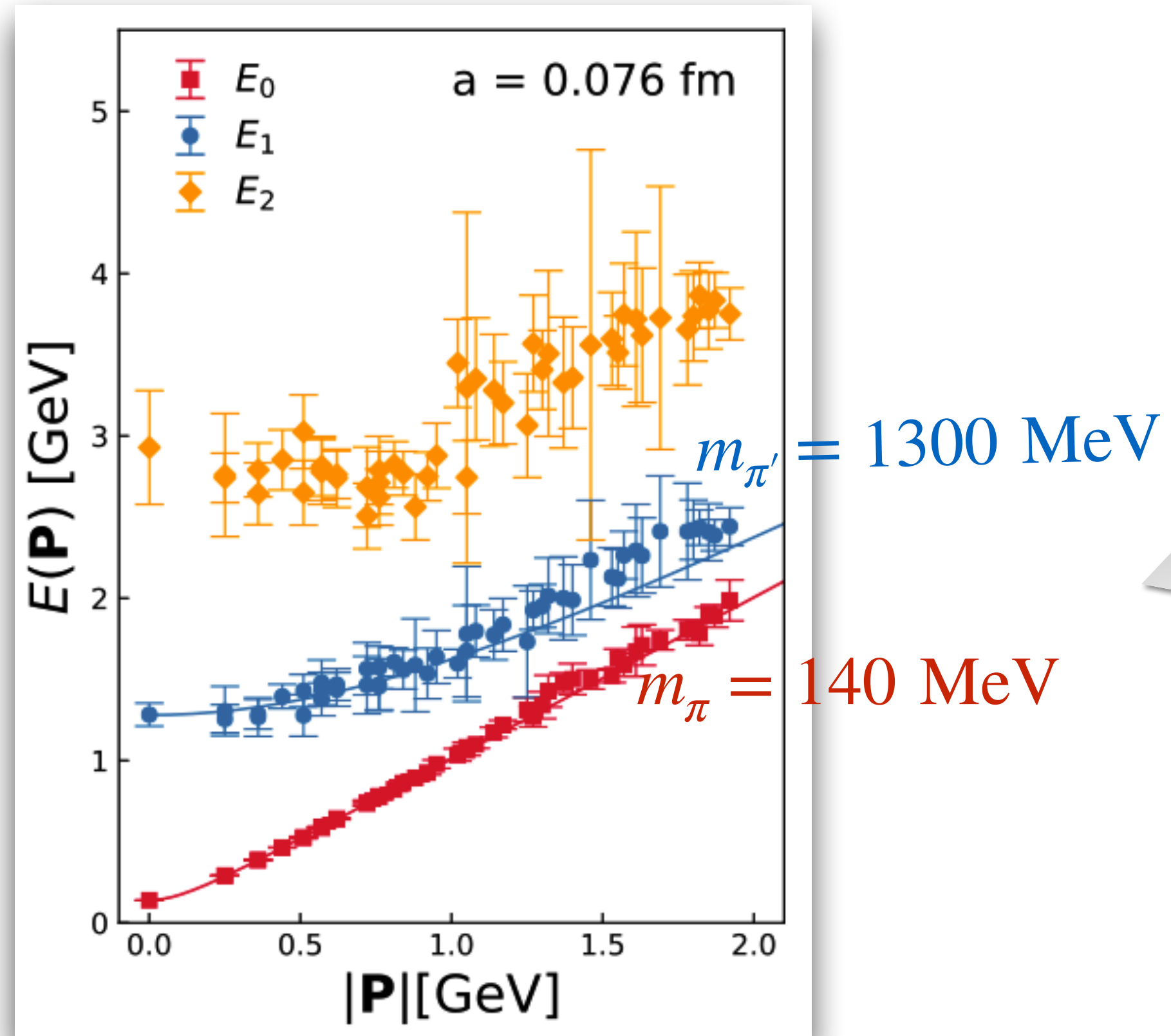


BNL: Phys. Rev. D102, 094513 (2020)

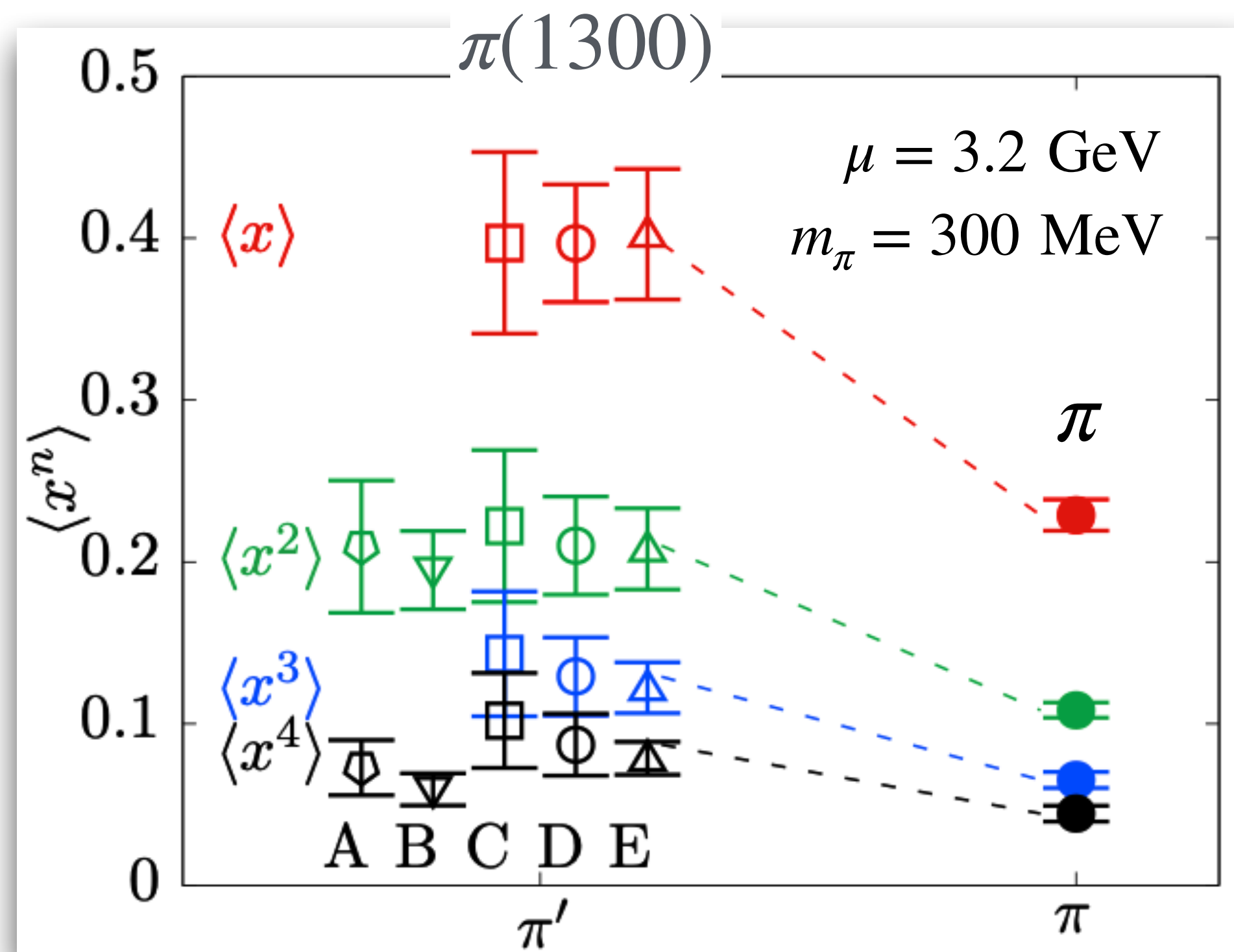
radial excitation: valance PDF

$\pi(1300)$

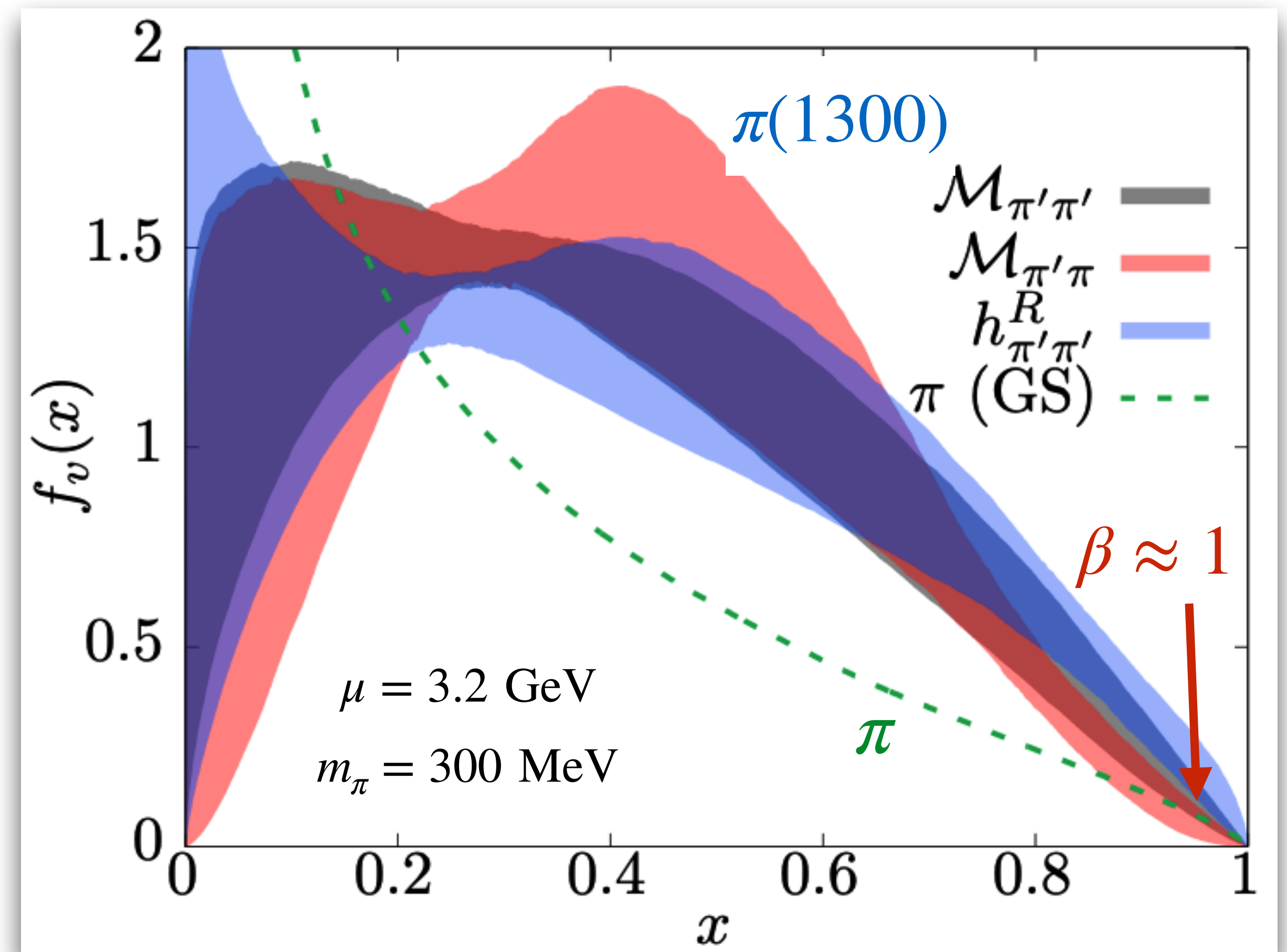
BNL: 2101.11632



BNL-ANL: 2102.06047



moments

model fit: $f_v^\pi(x) = Ax^\alpha(1-x)^\beta$

Melin space:

$$\lim_{N \rightarrow \infty} C_N^{\text{NLO}} = \frac{\alpha_s(\mu) C_F}{2\pi} \left[2 \ln N' \ln(z_0^2 \mu^2) - 2 \ln^2 N' + 2 \ln N' - \frac{\pi^2}{3} \right]$$

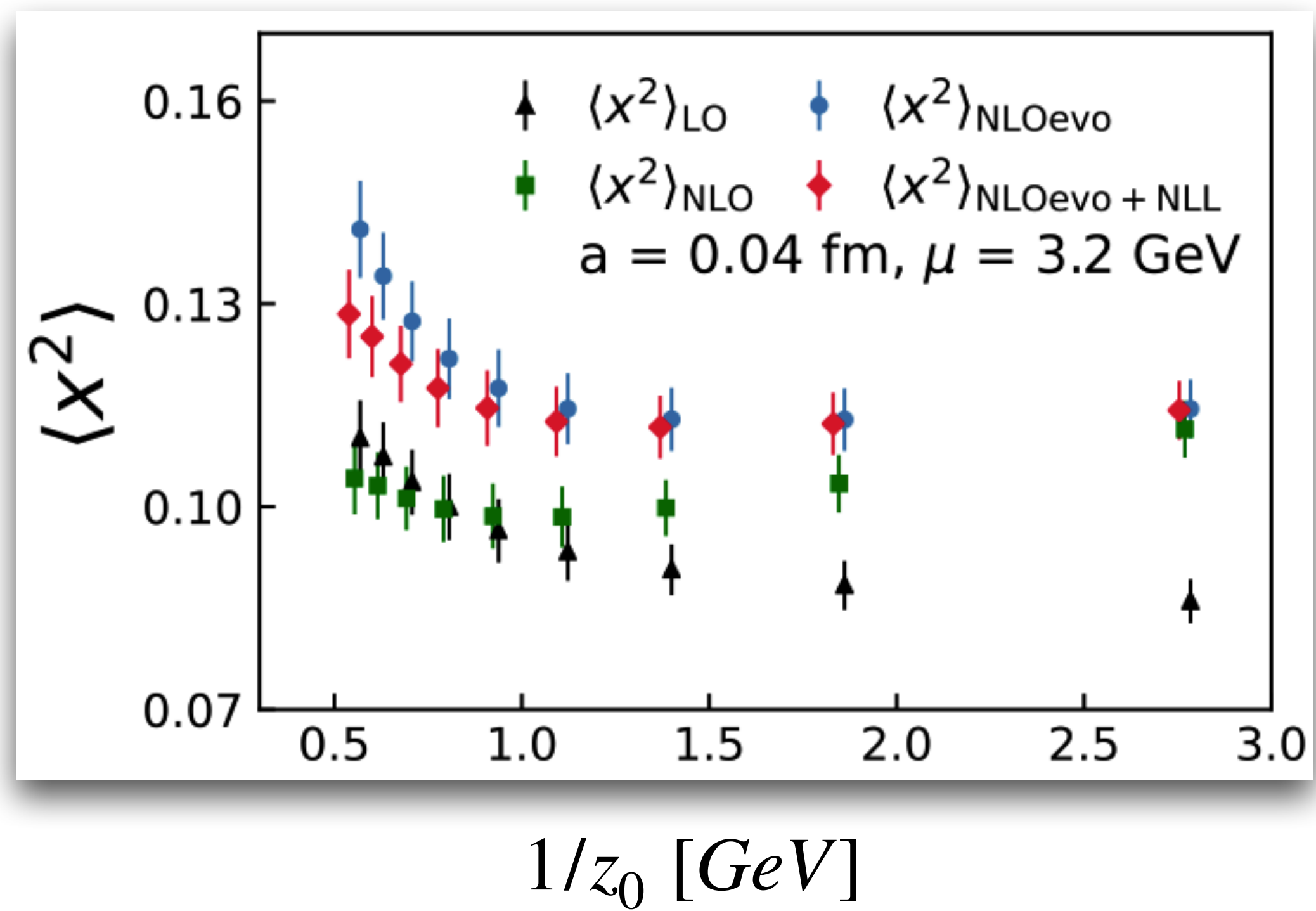
resum: $\alpha_s \ln^2 N'$ and $\alpha_s \ln N'$

$$C_n(z^2 \mu^2) = 1 + \frac{\alpha_s(\mu) C_F}{2\pi} \left[\left(\frac{3 + 2n}{2 + 3n + n^2} + 2H_n \right) \ln(z_0^2 \mu^2) + \dots \right] + \dots$$

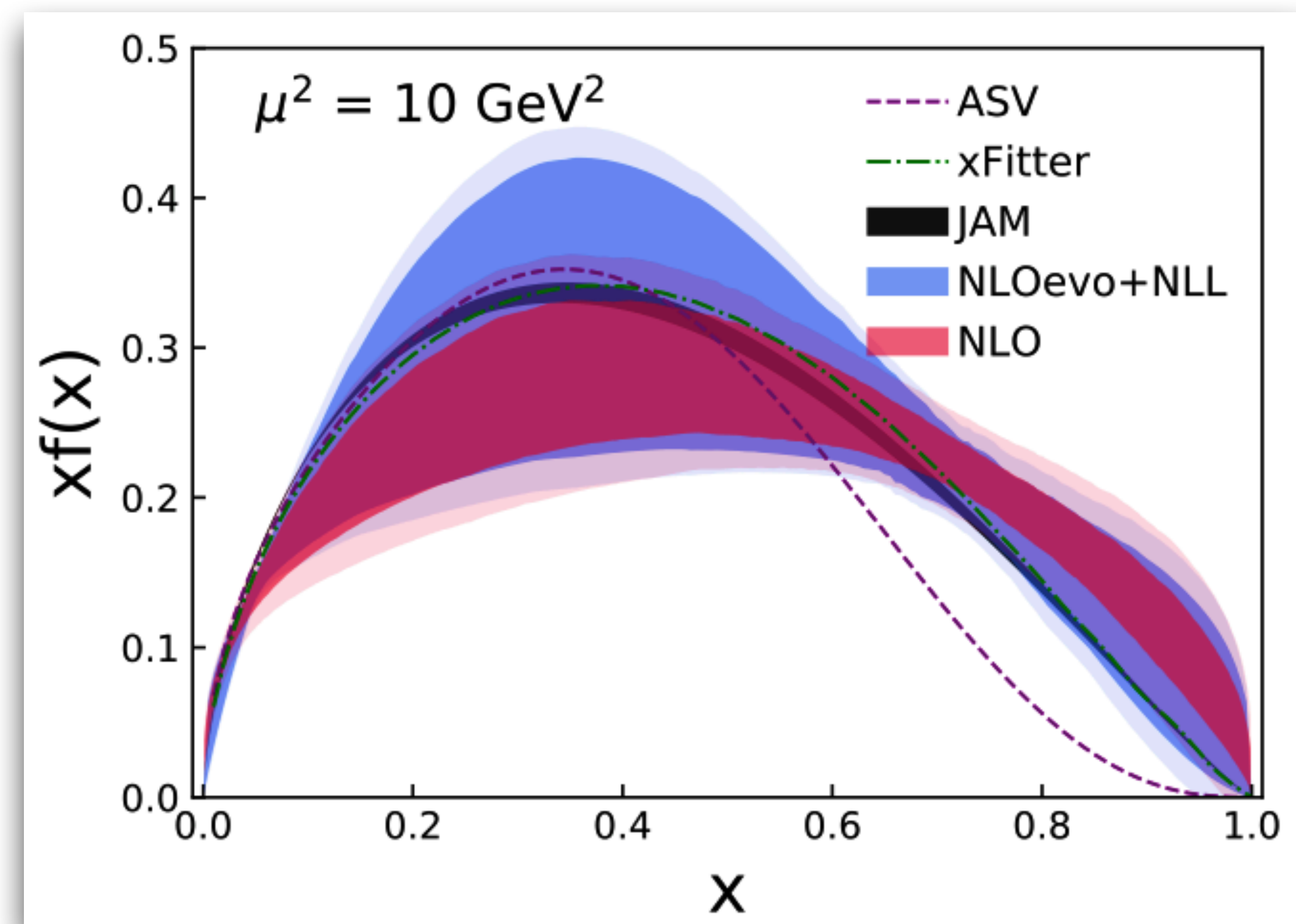
‘evo’: resum DGLAP logs

$$\left[\frac{\partial}{\partial \ln \mu^2} + \beta(\alpha_s(\mu)) \frac{\partial}{\partial \alpha_s} - \gamma_n \right] C_n^{\text{evo}} = 0$$

$$z_0^2 = z^2 e^{2\gamma_E} / 4$$

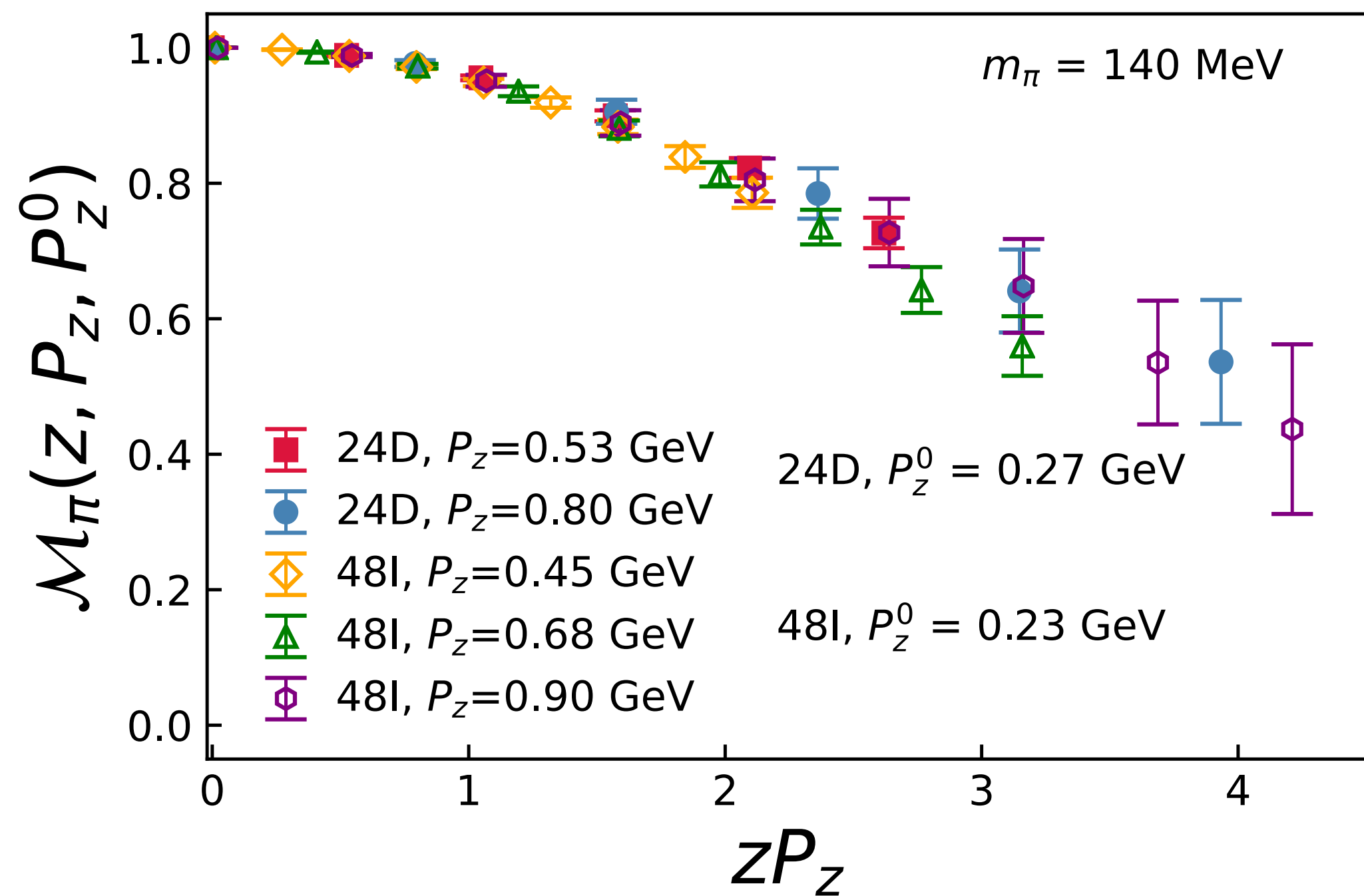


$$z_0^2 = z^2 e^{2\gamma_E}/4$$



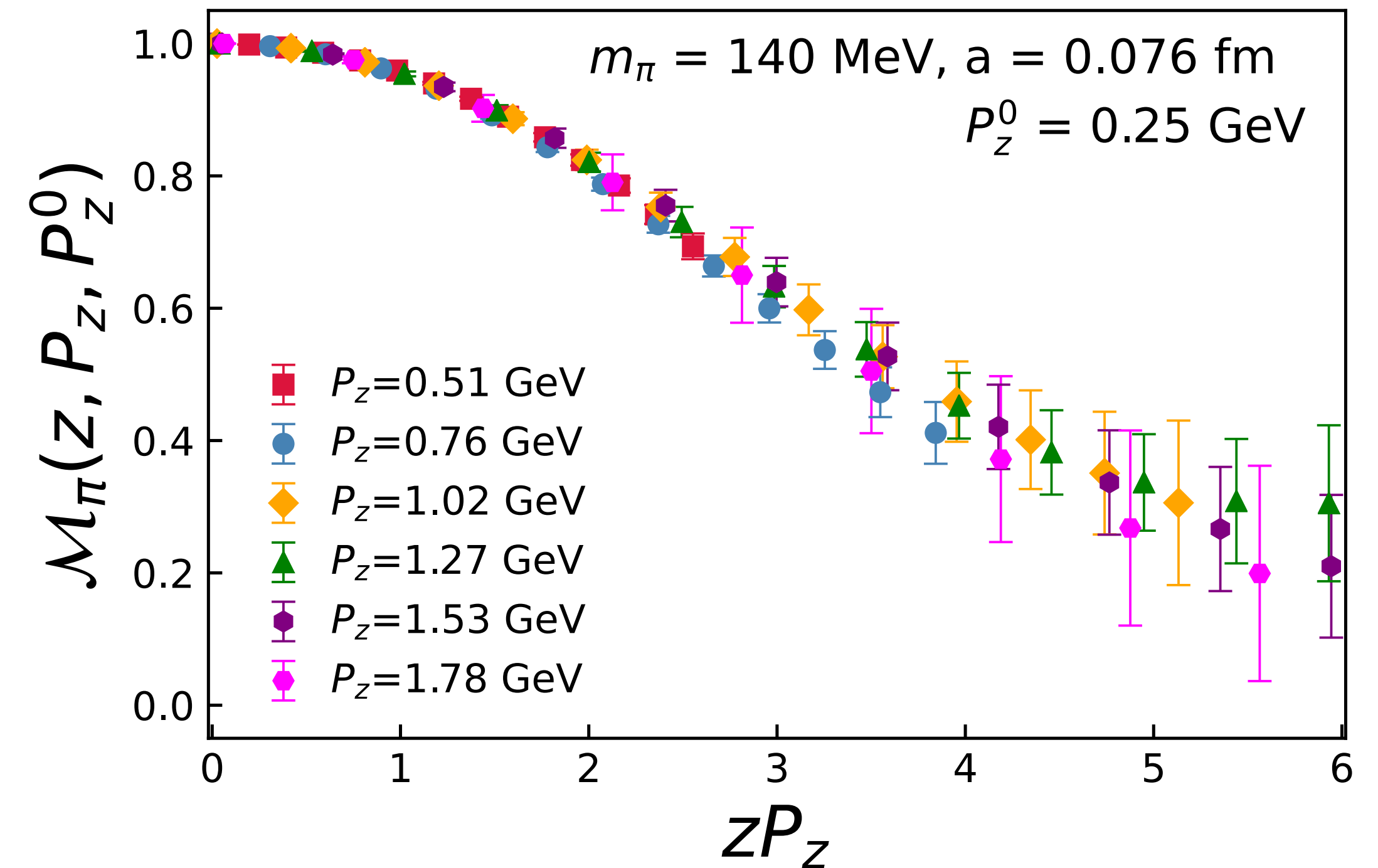
unitary chiral domain wall fermion (DWF)

$m_\pi = 140$ MeV



$a = 0.11, 0.19$ fm

$P_z < 1$ GeV

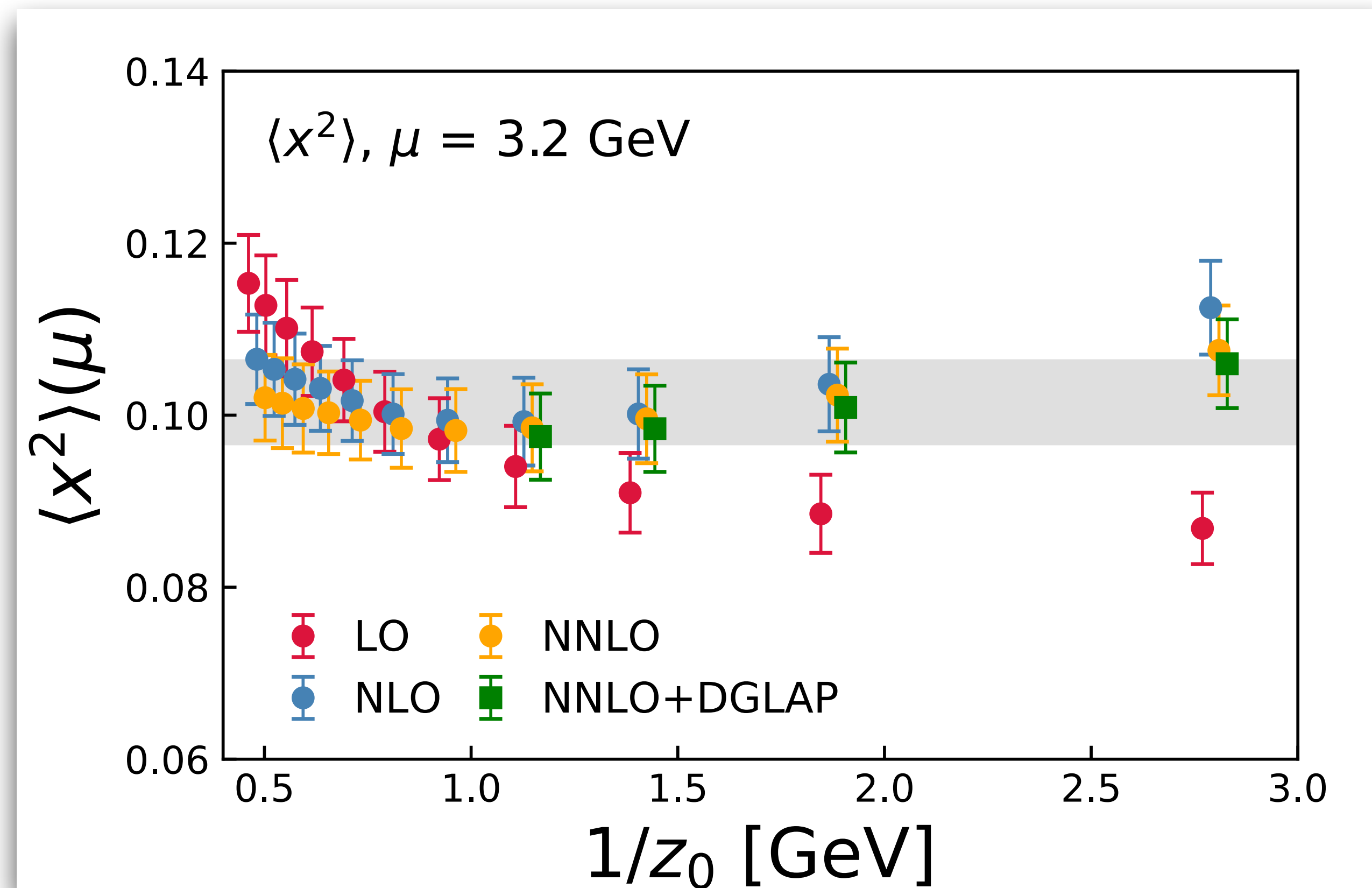


$P_z \leq 1.8$ GeV

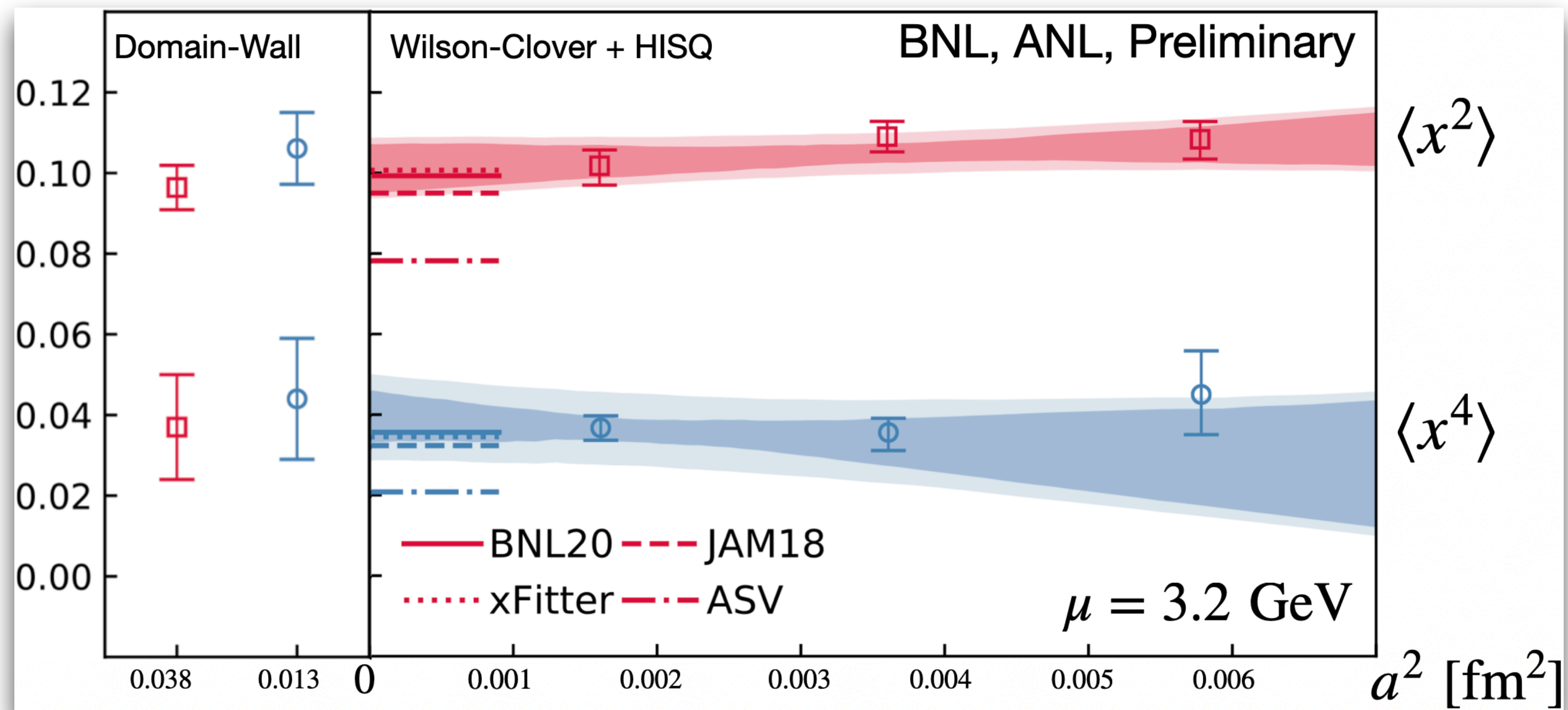
Wilson-Clover + HISQ

... with NNLO matching

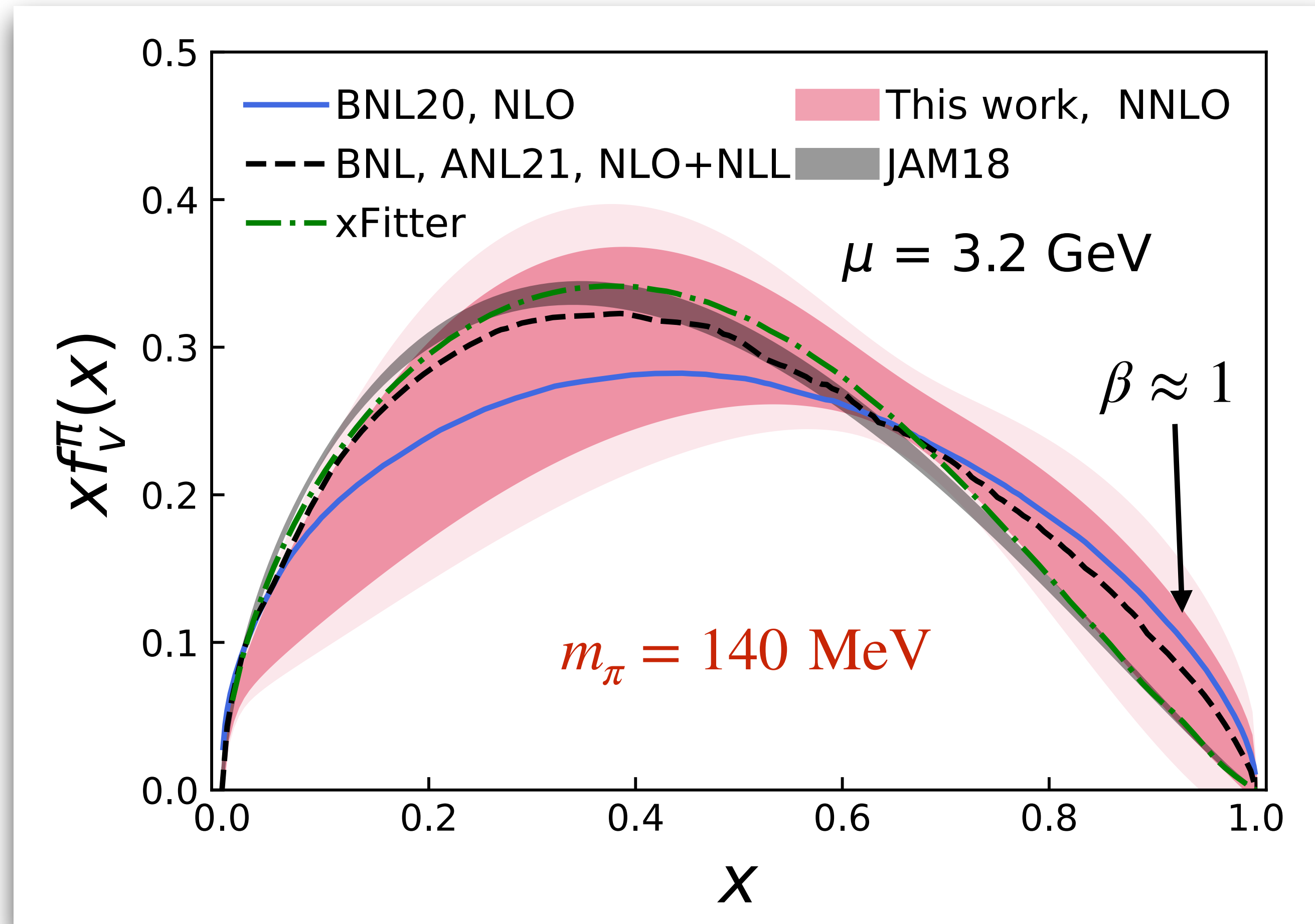
BNL-ANL: preliminary



model-independent moments
 NNLO matching, physical mass, chiral quarks



- pion mass dependence is very mild
- good agreement between DWF and Wilson-Clover + HISQ

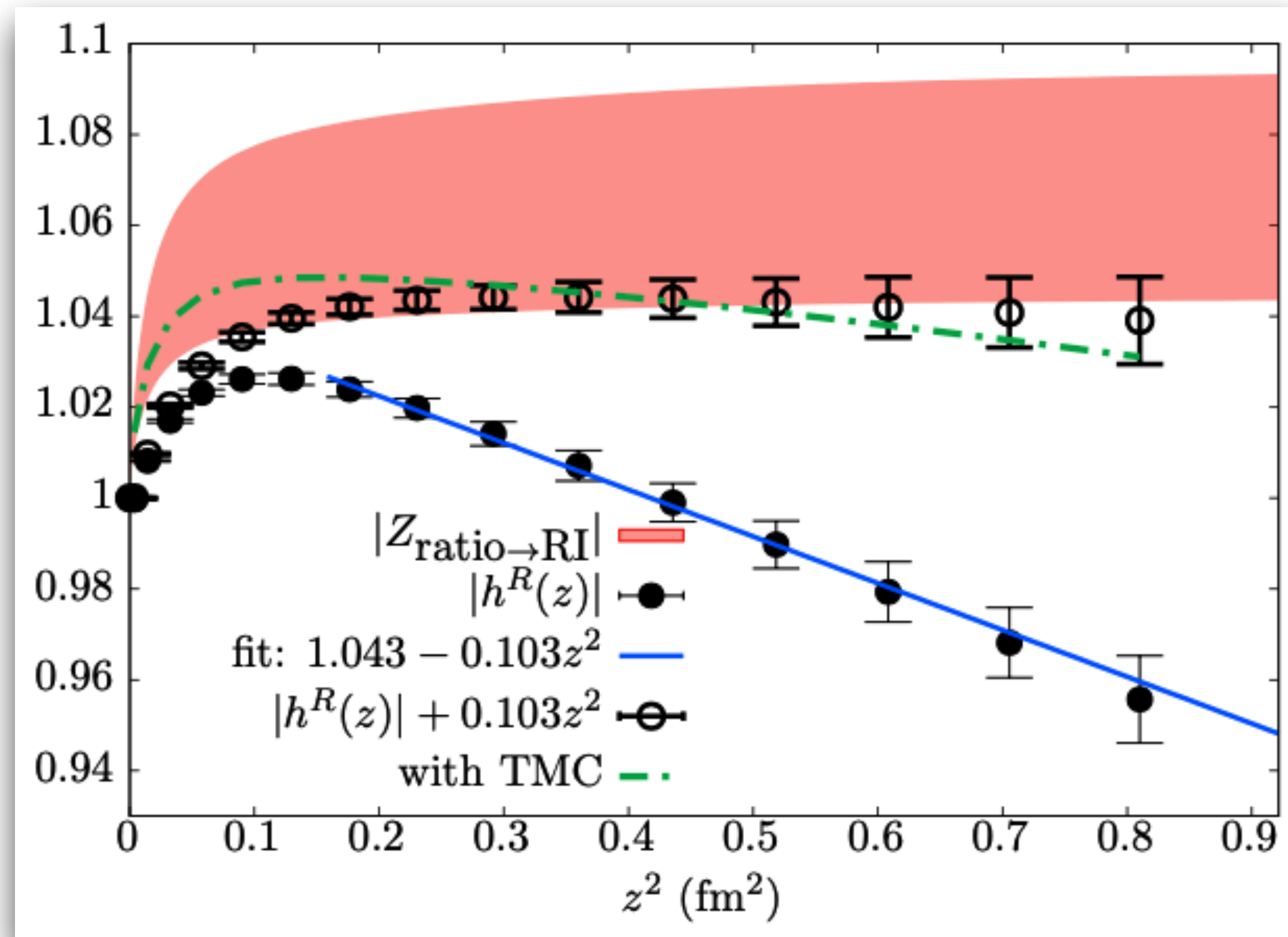


model fit: $f_v^\pi(x) = Ax^\alpha(1-x)^\beta(1+s\sqrt{x}+tx)$

back up ...

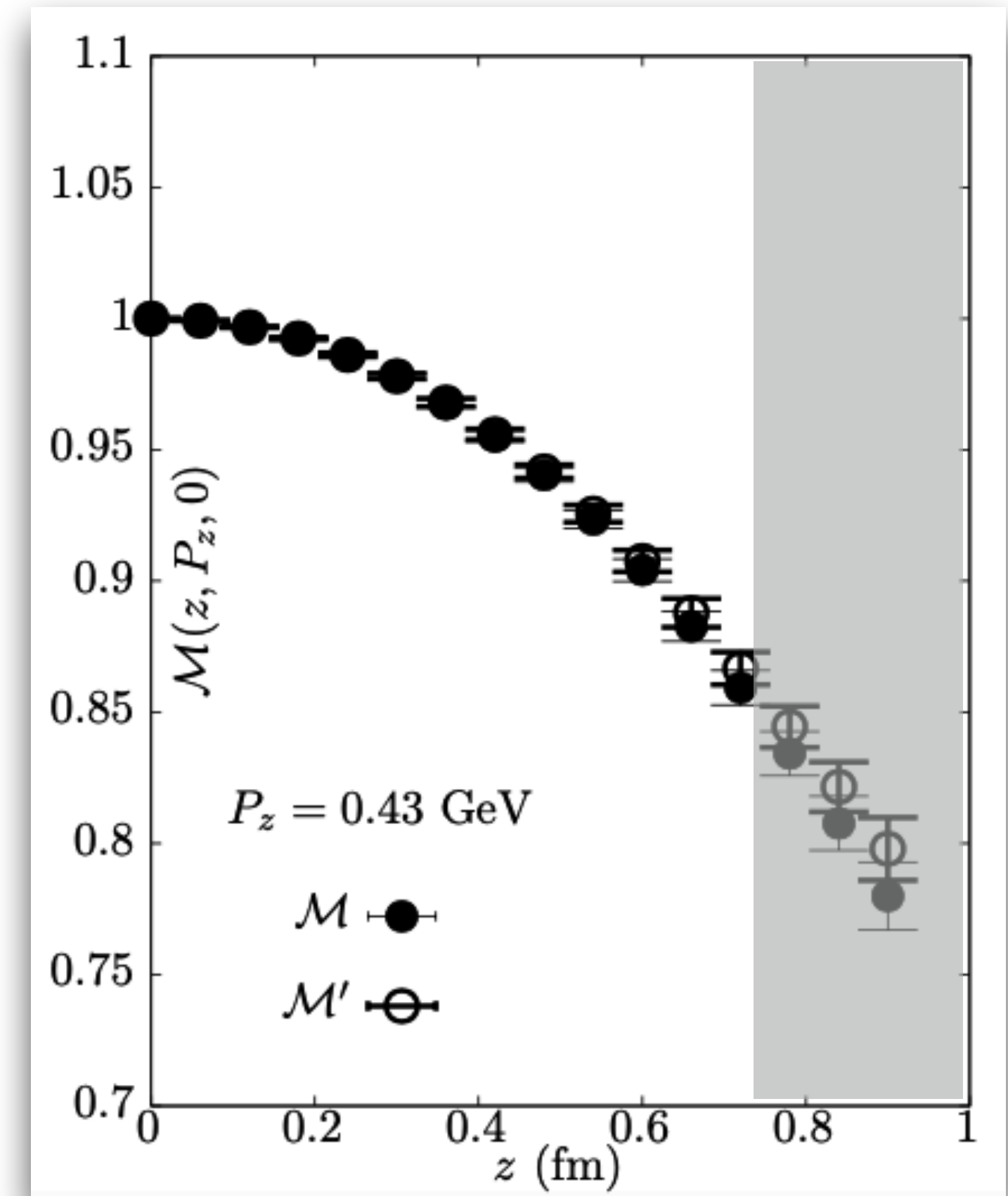
assessing higher twist corrections

RI-MOM renormalized



$$Q(z, p_z = 0; p_z^R = 1.29 \text{ GeV}, p_\perp^R = 2.98 \text{ GeV})$$

ratio scheme



$$\mathcal{M}(z, p_z = 0.43 \text{ GeV}, p_z^0 = 0)$$