Gluon distribution in the nucleon from Lattice QCD

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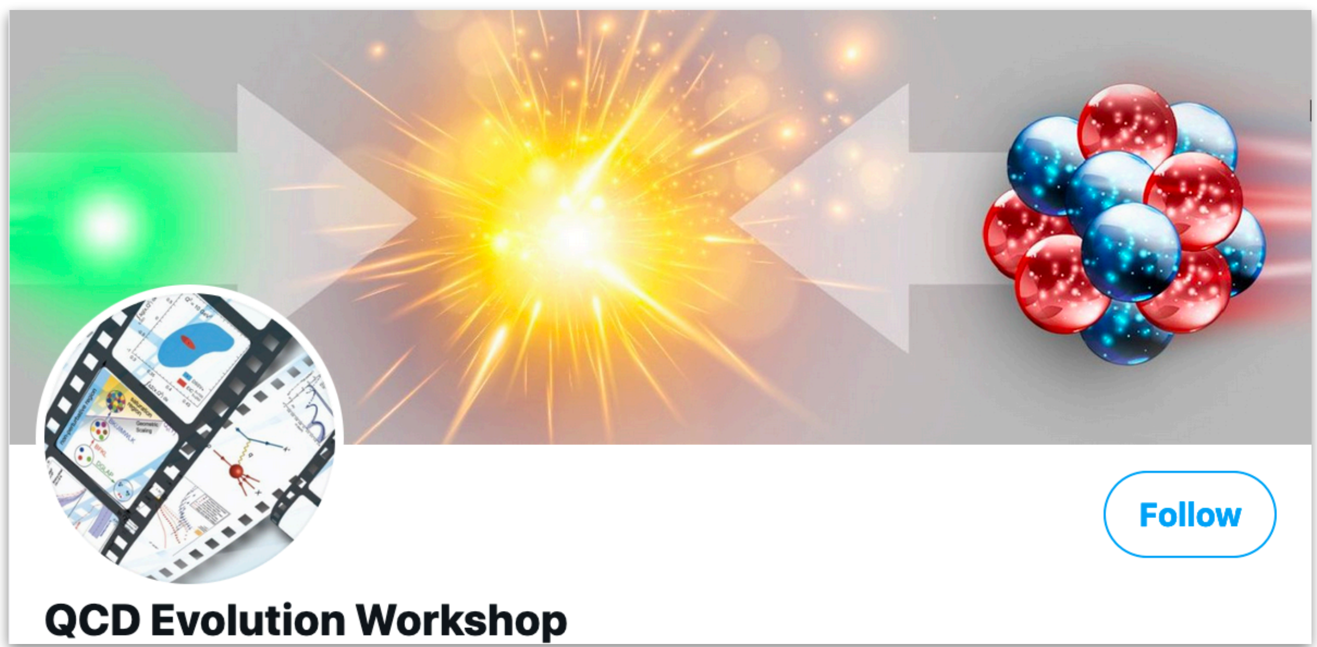
(For the HadStruc Collaboration)



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x-dependent hadron structure on lattice : Formalisms

- Hadronic tensor (Liu & Dong, PRL 1994)
- Position-space correlators (Braun & Müller, EPJ 2008)
- Quasi-PDFs & LaMET(Ji [PRL 2013, Sci. China Phys 2014])
- "Good" Lattice Cross Sections (Ma & Qiu, 2014, PRL 2018)

  Lattice calculation: *RSS*, Egerer, Karpie, Ma, Qiu, et al (PRD 2019, 2020)

- Pseudo-PDFs (Radyushkin, PLB 2017)

Today's talk

Gluon distribution in Pseudo-PDF approach

- On the lattice, calculate spatial correlation in coordinate space

$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, [z,0]\, G_{\lambda\beta}(0)|p\rangle$$

X. Ji [PRL 2013]

  - Extra linear UV divergence, $z/a \ (a \to 0)$ from Wilson line
  - Multiplicative renormalizability (coordinate space)

Ishikawa, Ma, Qiu, Yoshida [PRD 2017]

- Pseudo-PDF : Based on QCD short-distance factorization

$$\mathcal{P}(x,z^2) = \int_{-\infty}^{\infty} d\nu\, e^{-ix\nu} \mathcal{M}(\nu, z^2)$$

Radyushkin [PLB 2017]

$$\mathcal{P}(x,z^2) \overset{z^2=0}{\longrightarrow} f(x,\mu^2)$$

  - Ioffe time, $\nu = p_z z$ (convention from Braun, et al [PRD 1995])

B. L. Ioffe [PLB 1969]

Gluon distribution in Pseudo-PDF approach

- To determine unpolarized gluon distribution

$$M_{0i;0i}(\nu, z^2) + M_{ji;ij}(\nu, z^2) = 2p_0^2 M_{pp}(\nu, z^2)$$

Balitsky, Morris, Radyushkin [PLB 2020]

- **Renormalization**: UV divergences have no ν -dependence in leading log, and if $\mathcal{M}(\nu, z^2)$ is multiplicatively renormalizable,

$$\mathcal{M}^{\mathrm{unpol}}(\nu, z^2) = \frac{M_{pp}(\nu, z^2)}{M_{pp}(0, z^2)}$$

A. Radyushkin [PLB 2017]

- Also eliminates z/a $(a \to 0)$ UV divergences from Wilson line
- See a recent publication arXiv: 2103.02965 (Huo, Su, Gui, Ji, *et. al.*)

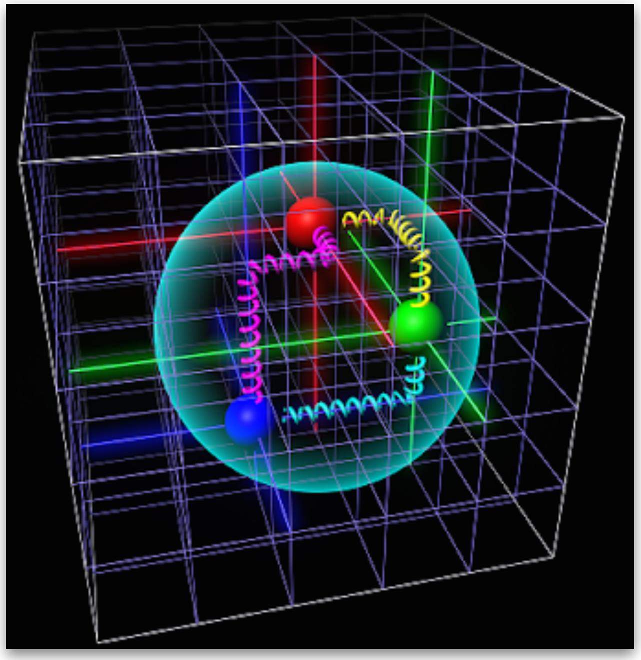
Gluon distribution in Pseudo-PDF approach

- For lattice QCD calculation, define reduced Ioffe time distribution (rITD)

$$\mathcal{M}(\nu, z^2) = \left(\frac{M_{pp}(\nu, z^2)}{M_{pp}(\nu, 0)|_{z=0}} \right) / \left(\frac{M_{pp}(0, z^2)|_{p=0}}{M_{pp}(0, 0)|_{p=0, z=0}} \right)$$

- ▶ Cancels multiplicative renormalization factors
- ▶ Cancels overall kinematic factors
- ▶ Reduces correlated errors in LQCD matrix elements

Lattice QCD calculation



- 2+1 flavor clover Wilson fermions
- Lattice size, $L \times T = 32^3 \times 64$
- Lattice spacing, $a \approx 0.094$ fm
- Pion mass, $m_\pi = 358$ MeV
- 351 configurations
- $z = [0, 0.56]$ fm
- Hadron boosted along z-direction, $p = \frac{2\pi n}{La} = [0, 2.46]$ GeV

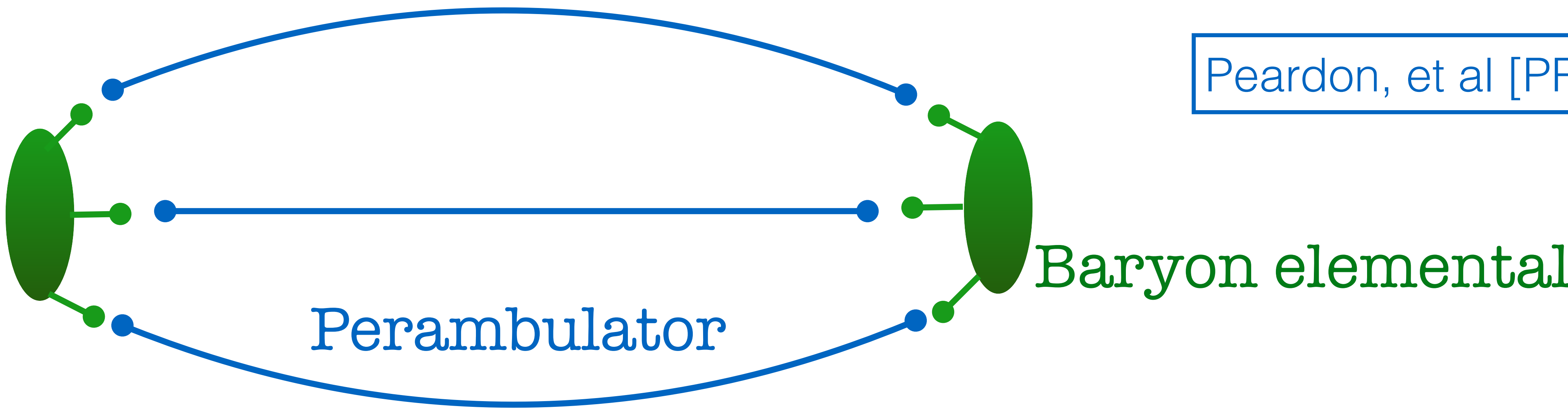
Some features of this calculation

- Gluonic operator using "Wilson flow"
  - Flow of gauge field, $B_\mu(\tau, x_\mu)$ so that $B_\mu|_{\tau=0} = A_\mu$
  - Diffusion length in x is $\sqrt{8\tau}$ $\quad (\tau \sim a^2)$

M. Luscher, JHEP 2010

- Nucleon correlation function using "Distillation"

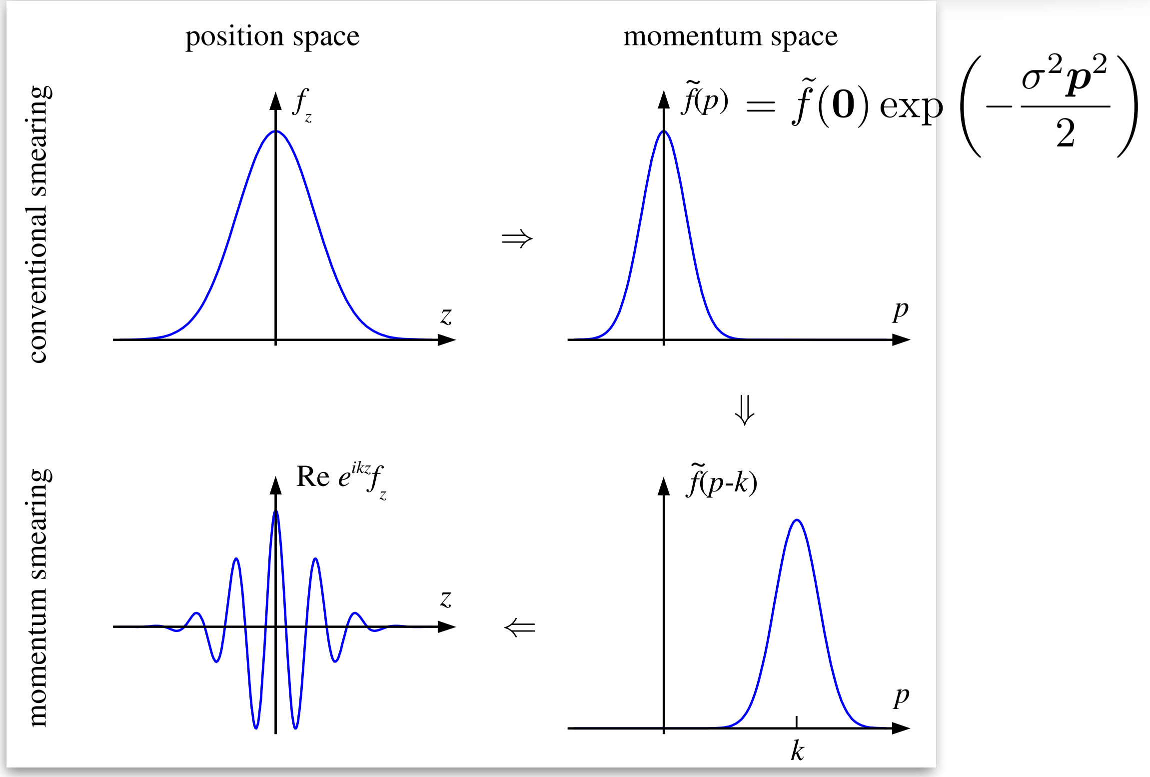
Peardon, et al [PRD 2009]



  - Basis of operators
  - Optimized operators reduce excited-state contaminations
  - Perambulators are independent of baryon elementals

Some features of this calculation

- Momentum smearing enhances overlap of the nucleon interpolating operators onto the lowest lying states of the boosted hadron



Bali, et al(PRD 2016)

- Correlation matrix analysis using variational technique
  - System of generalized eigenvalue equations for correlation matrix

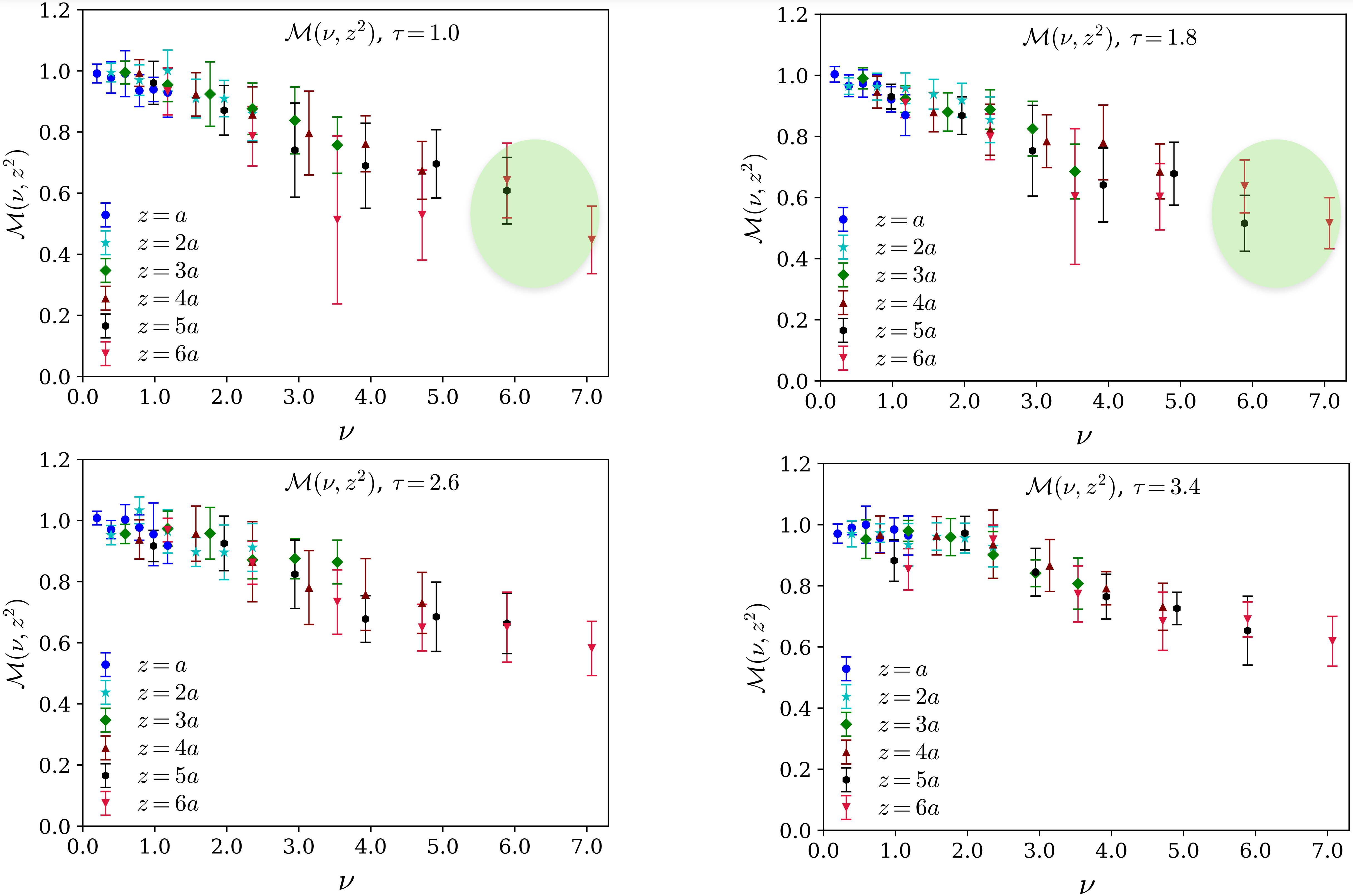
$$C(t)v^n(t) = \lambda_n(t)C(t_0)v^n(t)$$

  - Orthogonality conditions on the eigenvectors of different states

$$v^{n'\dagger}C(t_0)v^n = \delta_{n,n'}$$

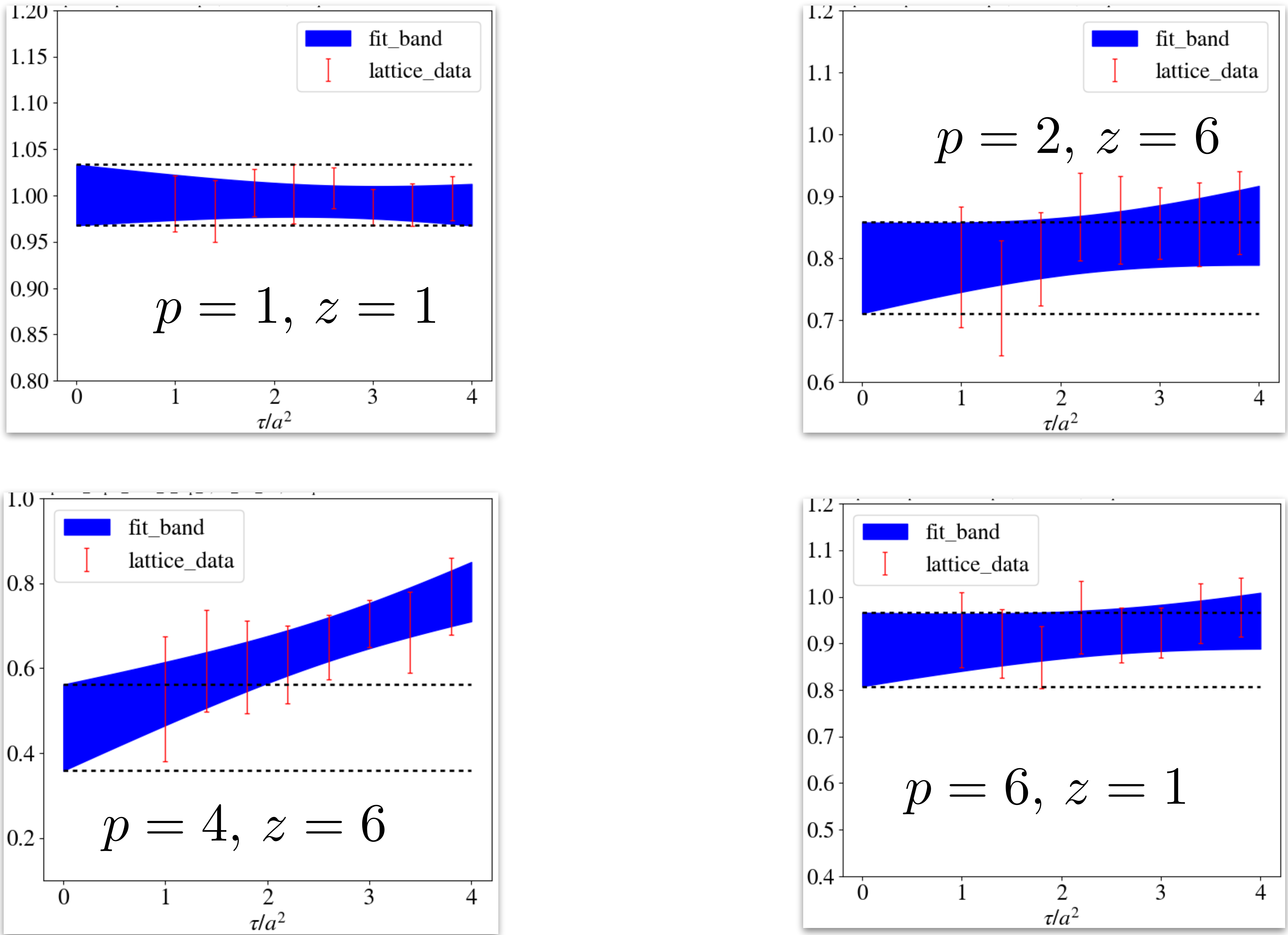
Difficult to distinguish degenerate states by their time-dependence alone

Results: Lattice QCD rITD as a function of flow time

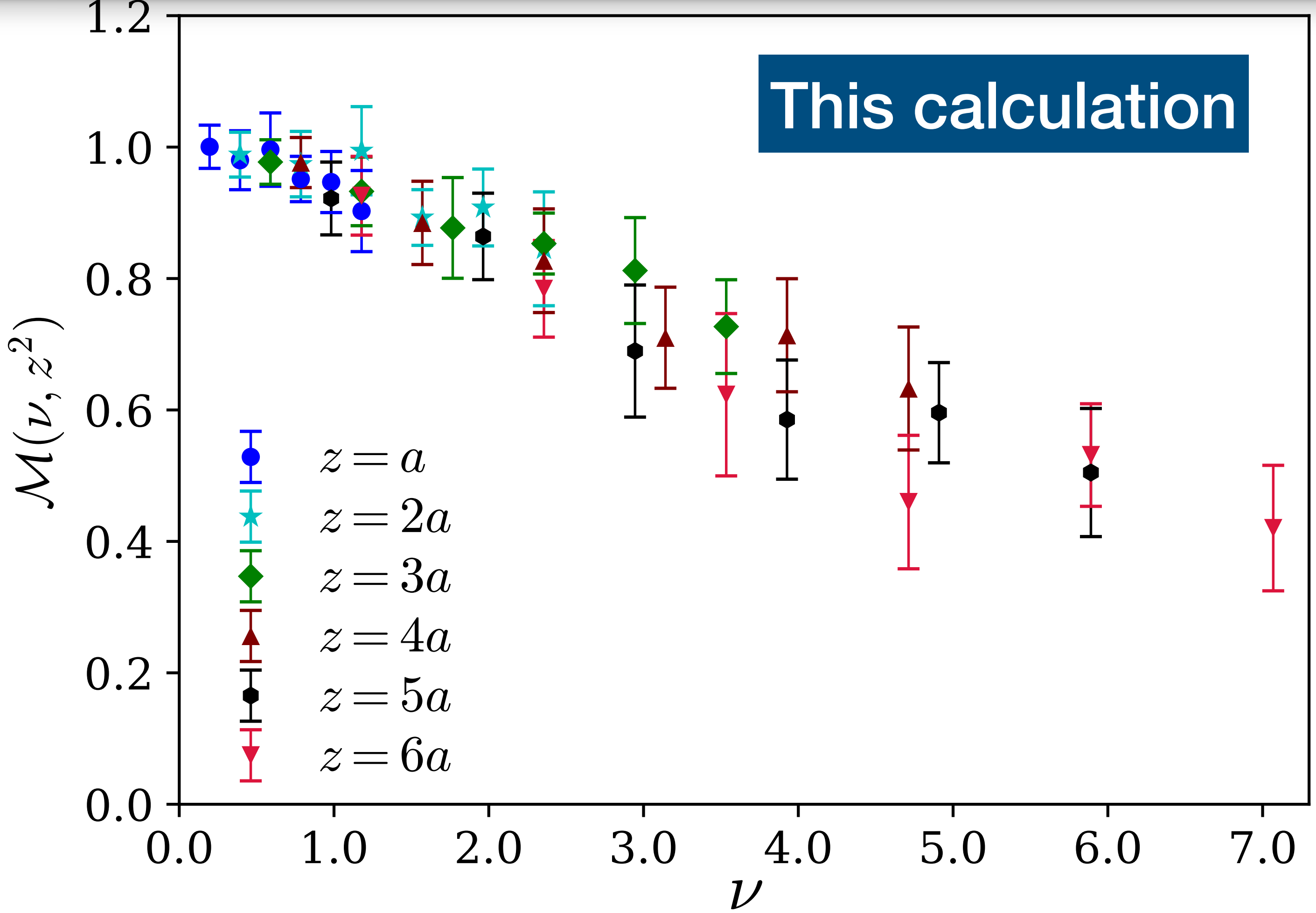


- Flow time dependence is minimized in the double ratio

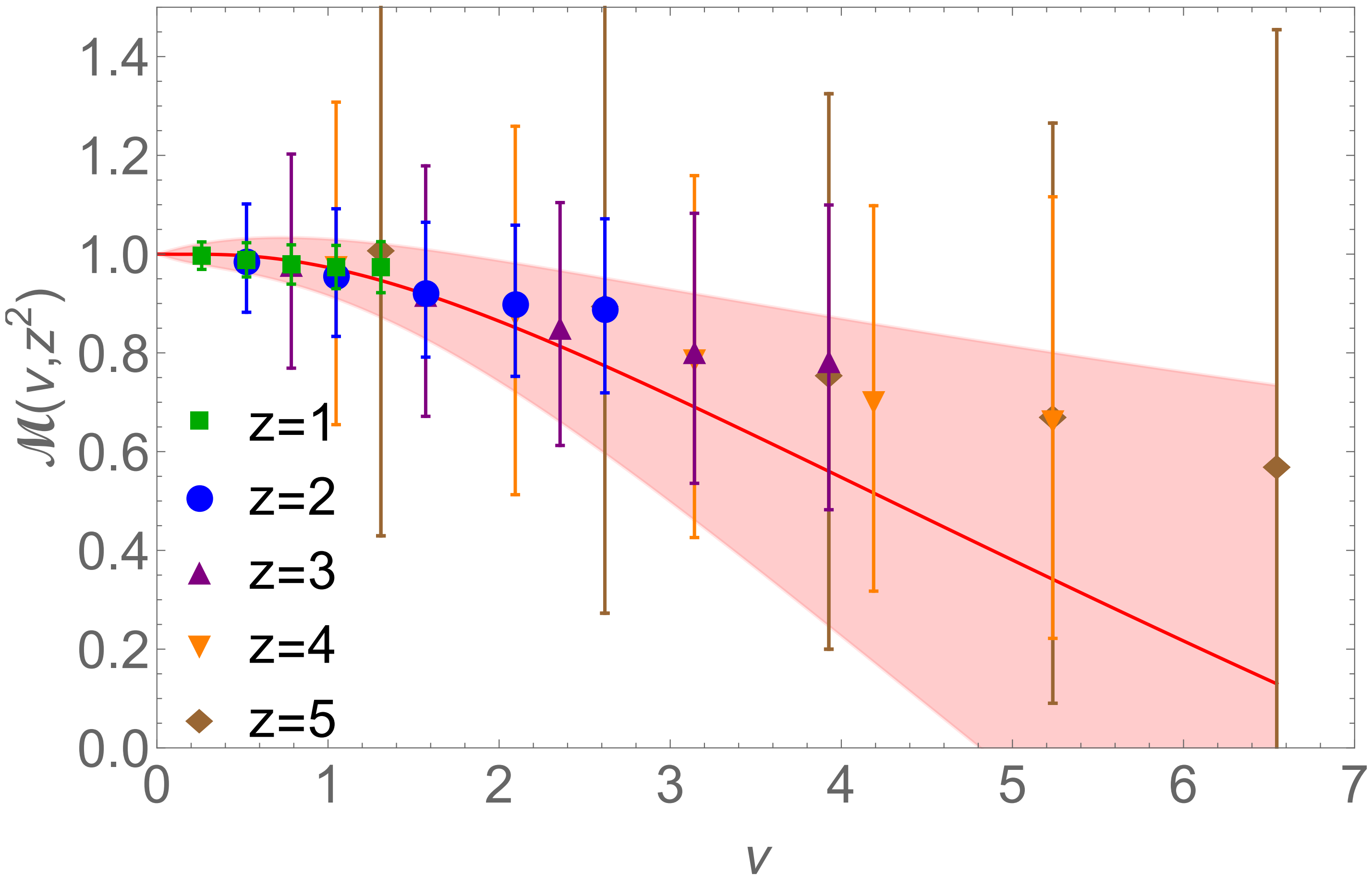
- For fixed p & z, fit forms: $A + B\tau$, $A + B\tau + C\tau^2$, etc.



Ioffe time pseudo-distribution in the zero flow time limit



Most precise
LQCD determination
to date
(in preparation)



Fan, Zhang, Lin (2007.16113)

$$m_\pi = 310\,\mathrm{MeV}$$
$$a = 0.12\,\mathrm{fm}$$

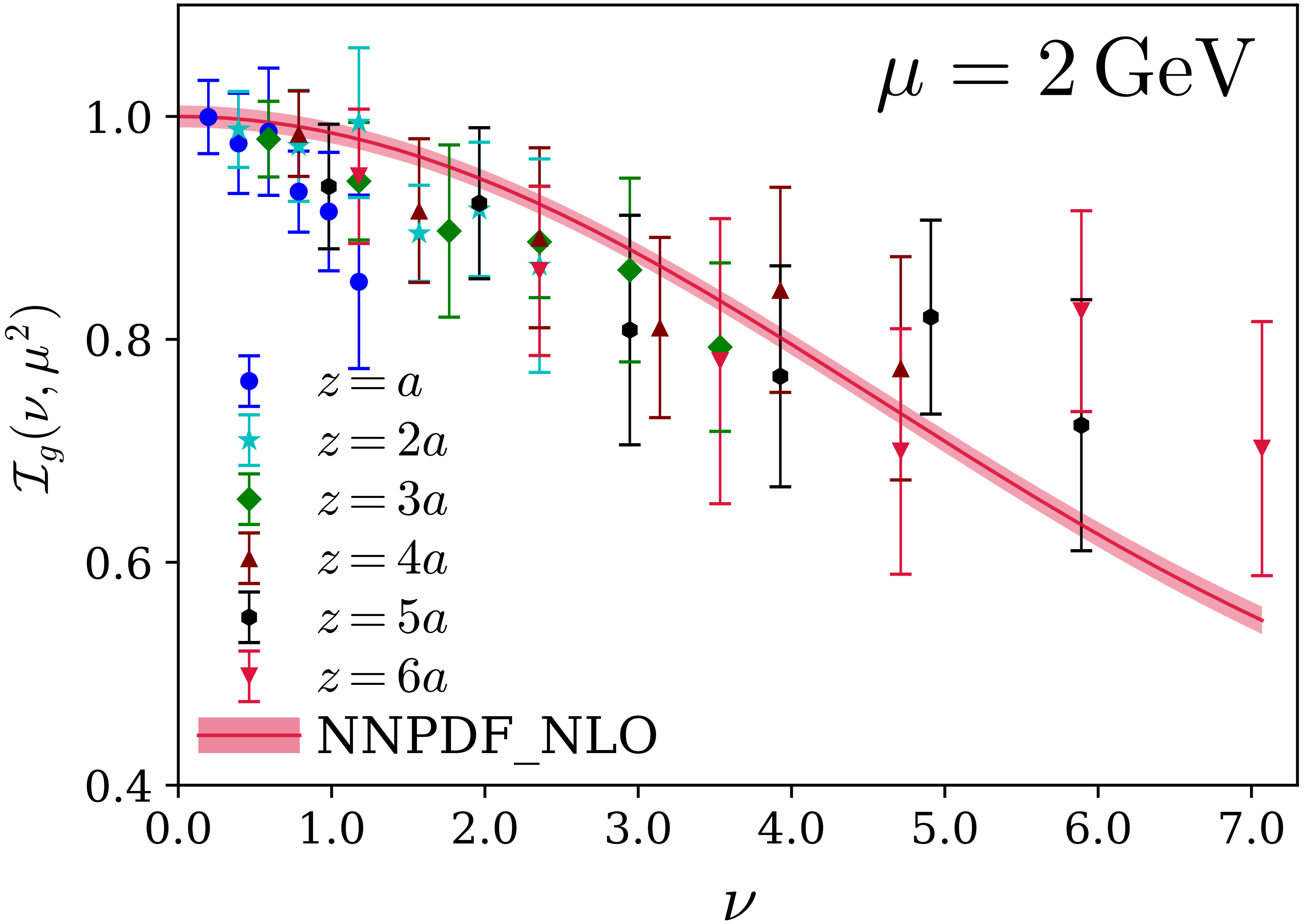
From pseudo-distribution to light-cone distribution

- One loop matching

Balitsky, Morris, Radyushkin [PLB 2020]

$$\mathcal{M}(\nu, z_3^2) = \frac{\mathcal{I}_g(\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} - \frac{\alpha_s N_c}{2\pi} \int_0^1 du\, \frac{\mathcal{I}_g(u\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \left\{ \ln\left(\frac{z_3^2 \mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u + ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1 + 6u - 6u^2 - u^3\right]_+ \right\} - \frac{\alpha_s C_F}{2\pi} \ln\left(\frac{z_3^2 \mu^2 e^{2\gamma_E}}{4}\right) \int_0^1 dw\, \frac{\mathcal{I}_S(w\nu, \mu^2)}{\mathcal{I}_g(0, \mu^2)} \mathfrak{B}_{gq}(w)$$

- Quark-gluon mixing not considered in this calculation for now



ITD provides a clear comparison between LQCD data and global fits

Future precise LQCD data can be used as inputs in global fits [Ma & Qiu (2014)]

Determination of unpolarized gluon distribution

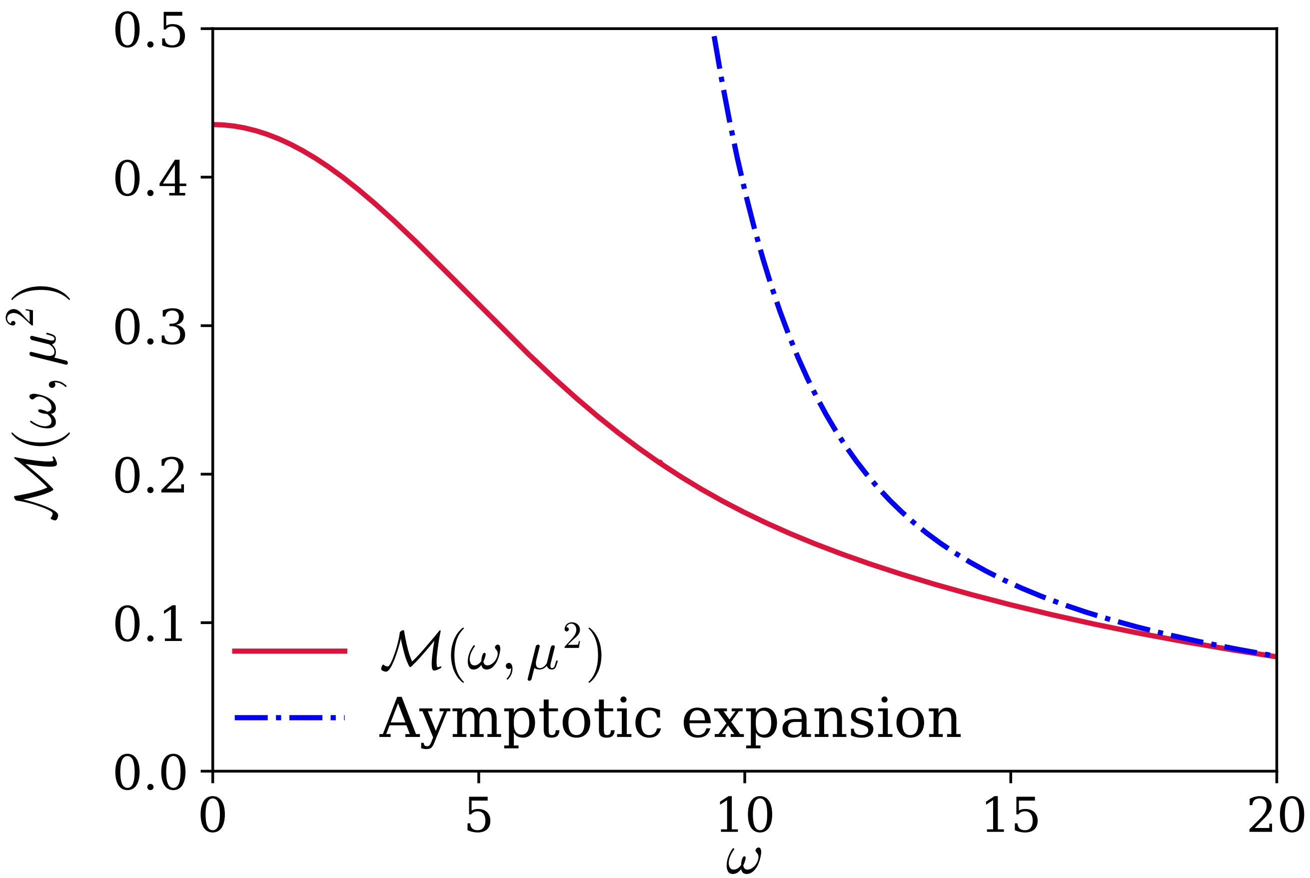
- Access to Quasi/Pseudo PDFs matrix elements are limited by available z & p

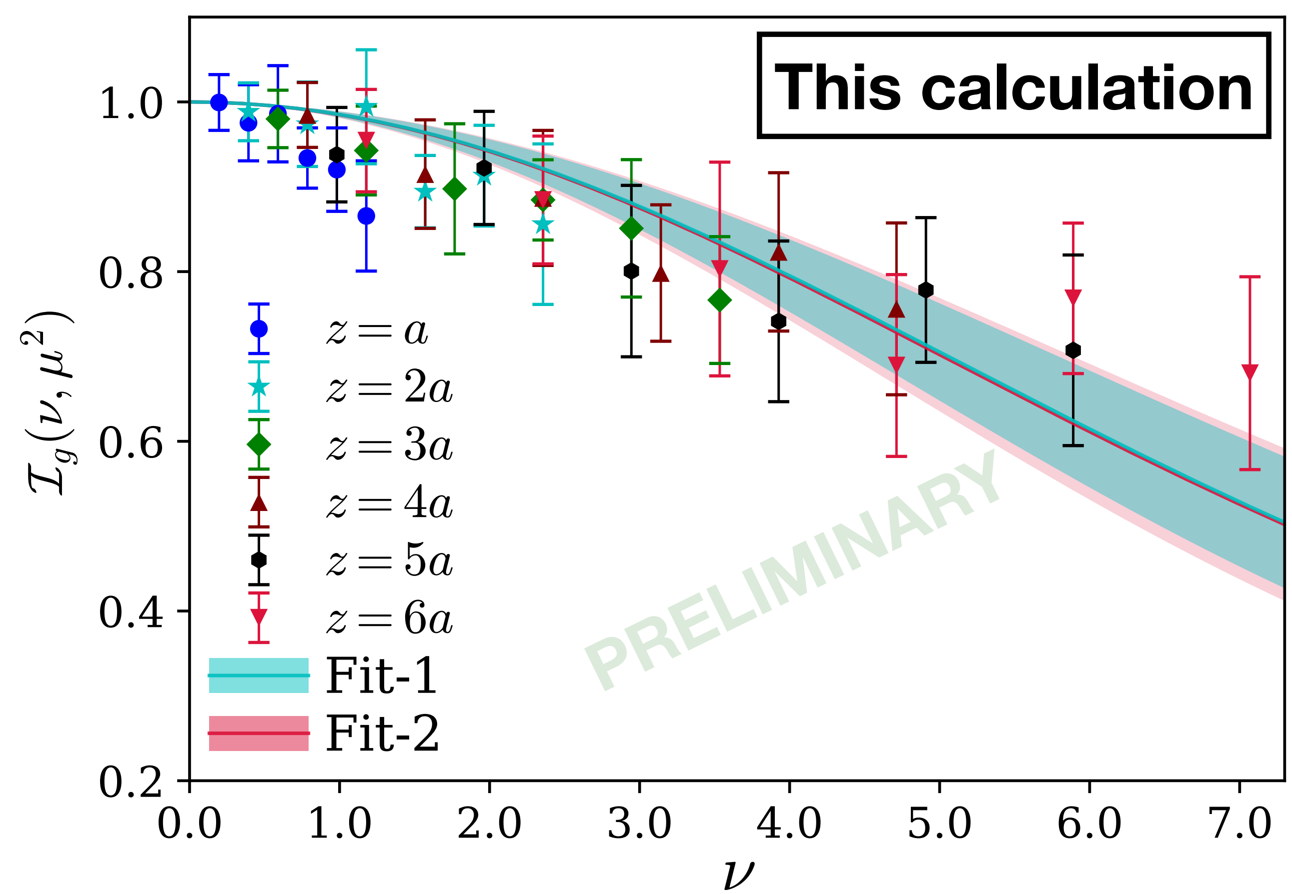
$$\mathcal{I}_g(\nu, \mu^2) = \int_0^1 dx\ \cos(x\nu)\, xg(x)$$

- Similar challenge in experiment comes from available kinematics

- **Fit-1** $$\mathcal{I}_g(\nu) = \int_0^1 dx \cos(\nu x)\, N_1 x^\alpha (1-x)^\beta$$

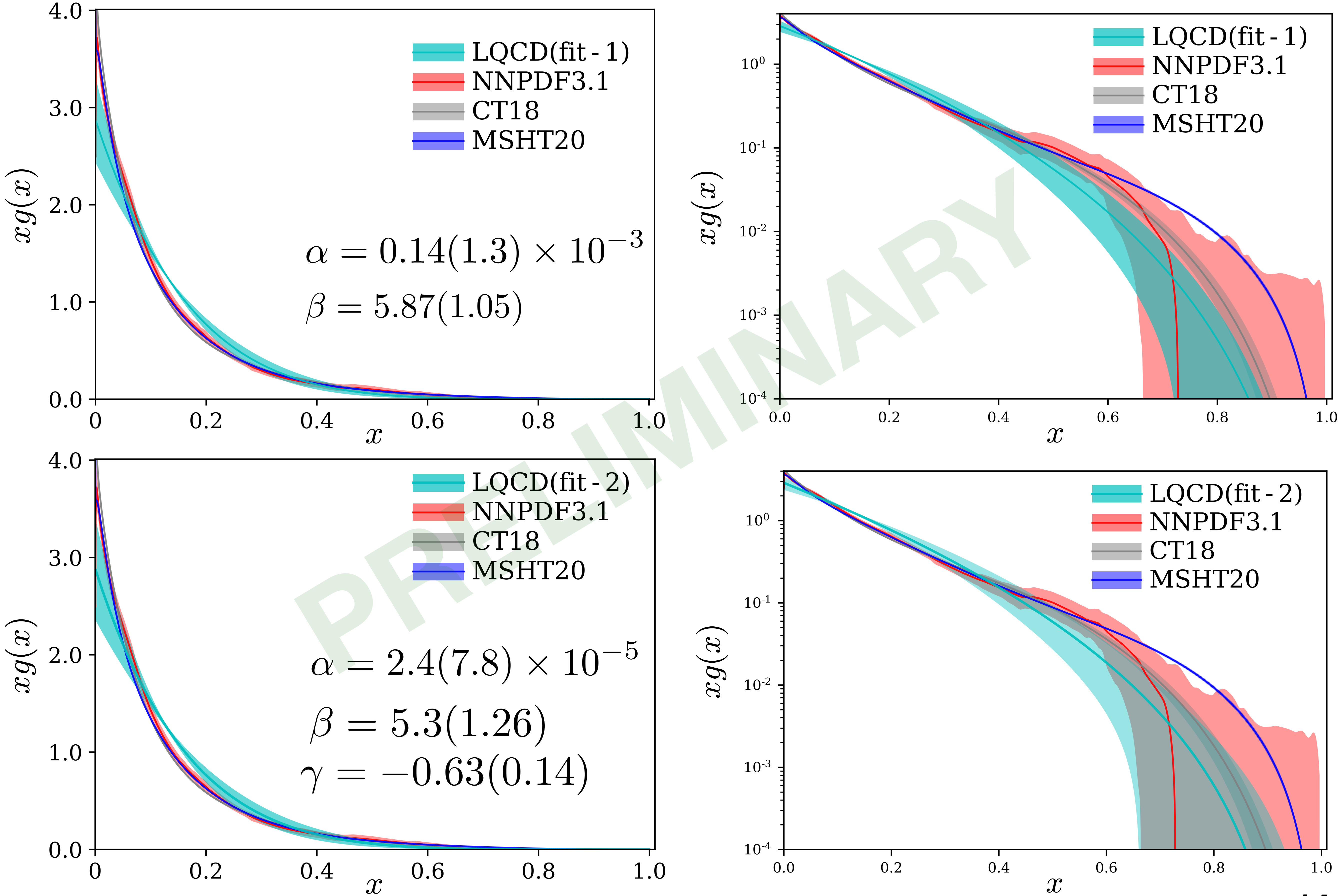
Fit-2 $$\mathcal{I}_g(\nu) = \int_0^1 dx \cos(\nu x)\, N_2 x^\alpha (1-x)^\beta (1+\gamma x)$$





RSS, Liu, Paul (PRD 2021)

Determination of unpolarized gluon distribution



Summary & Outlook

- We have presented the most precise LQCD determination of unpolarized gluon Ioffe time distribution
- Near future calculation: A similar LQCD calculation with 2-3 times more statistics
- Near future calculation: Quarks singlet distributions
- Polarized gluon distribution using Lattice Cross Sections and Pseudo-PDFs formalism

The Big Picture

LQCD Community: Contribute to 3D imaging of the nucleon

Thank you!

EXTRA

$$\tilde{q}\left(x,\mu^2,P_3\right)=\int\frac{\mathrm{d}z}{4\pi}e^{-ixzP_3}\left\langle P\right|\bar{\psi}\left(z\right)\gamma^3\exp\left(-ig\int_0^z\mathrm{d}z'A^z\left(z'\right)\right)\psi\left(0\right)\left|P\right\rangle$$

$$\mathcal{P}\left(x,z_3^2\right)=\int_{-\infty}^{\infty}\mathrm{d}\nu e^{-ix\nu}\left\langle p\right|\phi\left(z\right)\phi\left(0\right)\left|p\right\rangle=\int_{-\infty}^{\infty}\mathrm{d}\nu e^{-ix\nu}\mathcal{M}\left(p_3z_3,z_3^2\right)$$

$$\mathcal{P}\left(x,0\right)=f\left(x\right)$$

$$\mathfrak{M}\left(\nu,z_3^3\right)=\frac{\mathcal{M}\left(\nu,z_3^2\right)}{\mathcal{M}\left(0,z_3^2\right)}$$

This leads to the evolution equation:

$$\frac{\mathrm{d}}{\mathrm{d}\log z_3^2}\mathfrak{M}\left(\nu,z_3^3\right)=-\frac{\alpha_s}{2\pi}C\int_0^1\mathrm{d}u\;B\left(u\right)\mathfrak{M}\left(u\nu,z_3^3\right)$$

Gluon distributions and lattice QCD

- Uncertainty in gluon distributions at large x is an avenue for LQCD to explore

- Gluon contribution to proton spin unconstrained from experiment

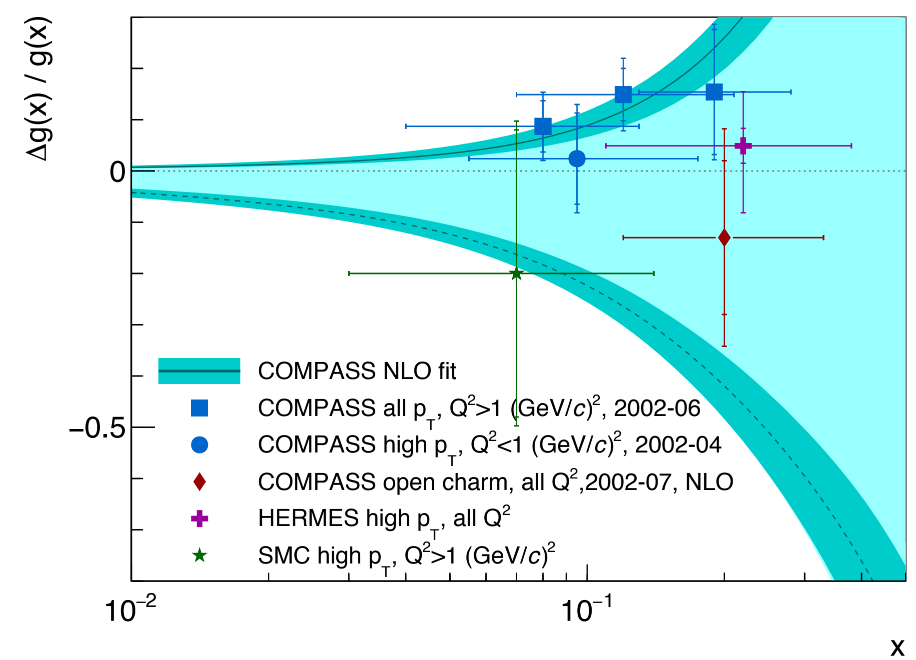
- LQCD determination of gluon spin

$$\Delta G(\mu^2 = 10\text{GeV}^2) = 0.251(47)(16)$$

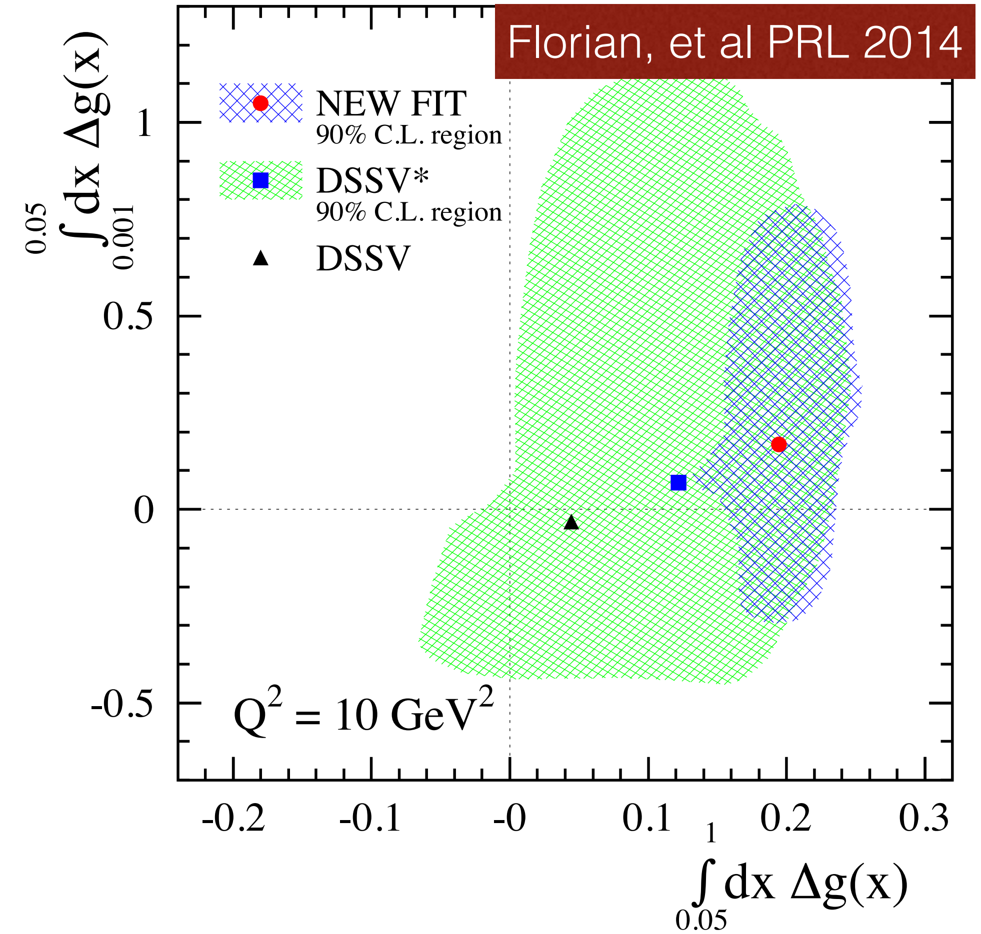
Yang, RSS, et al (PRL 2017)

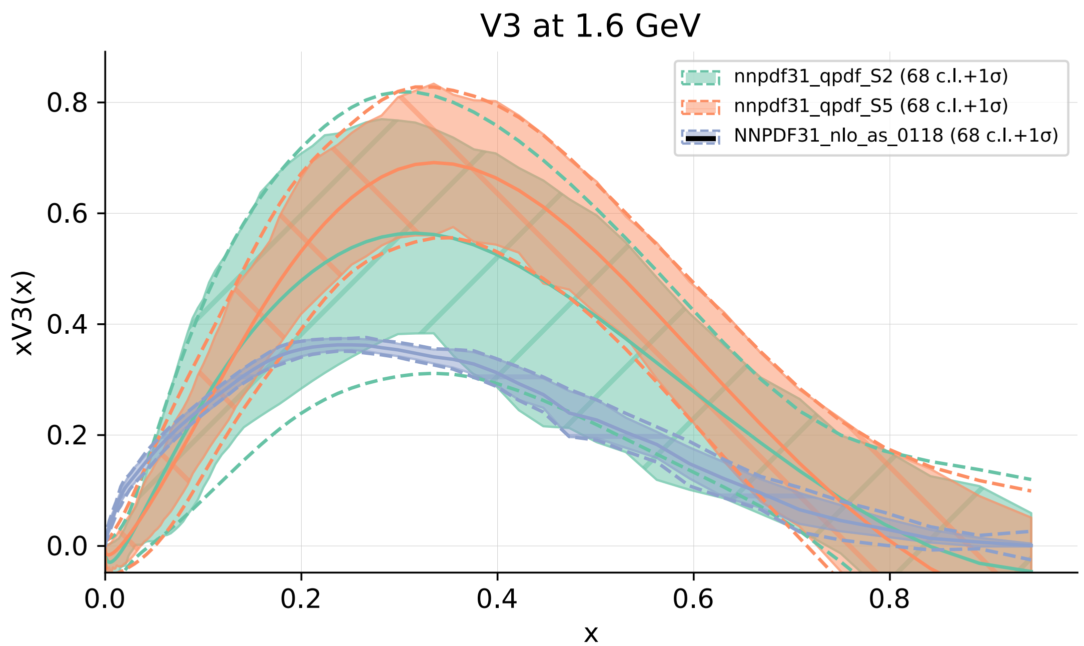
- Small-x gluon distributions from experiment (e.g. EIC) and large-x PDF from LQCD can be complementary

COMPASS (PLB 2016)

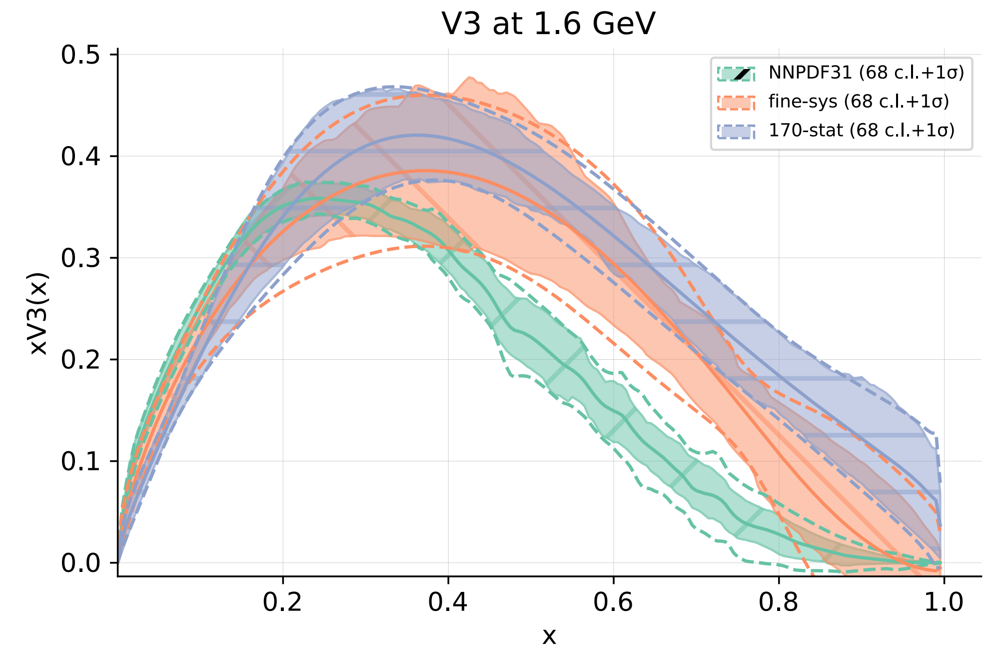


Florian, et al PRL 2014



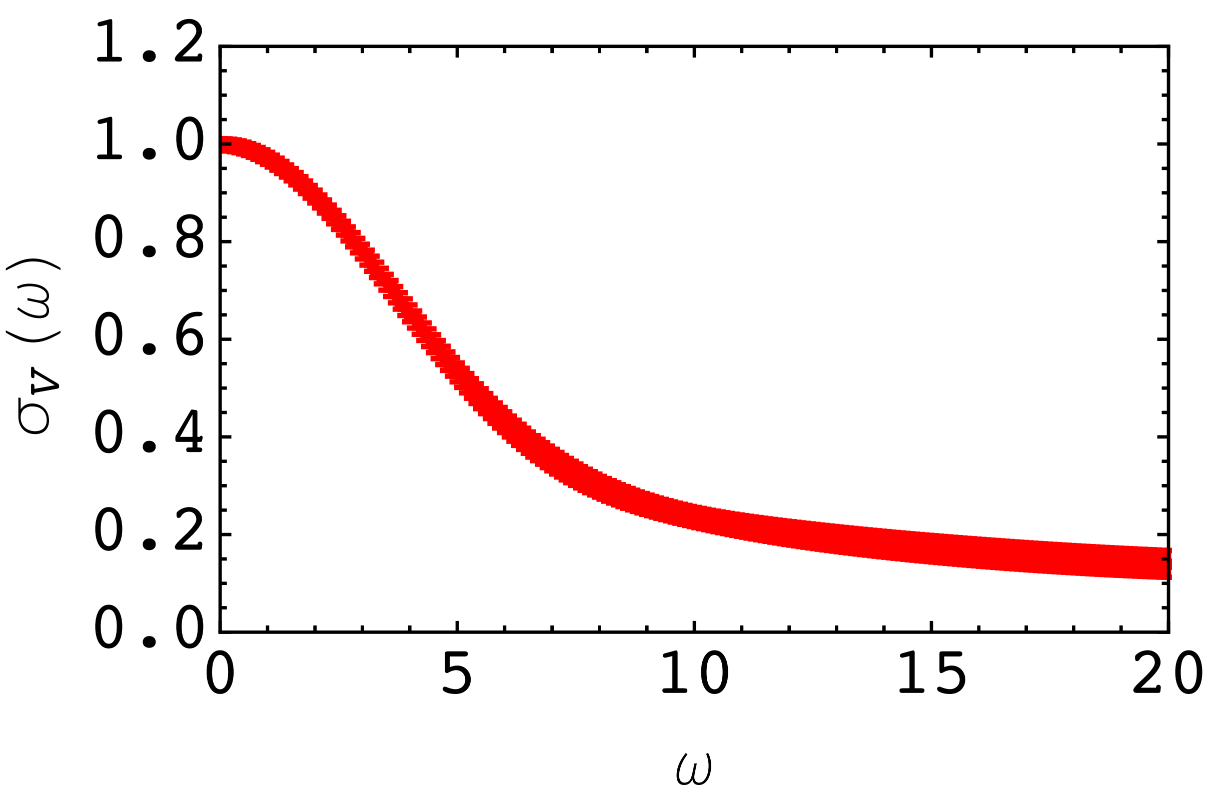


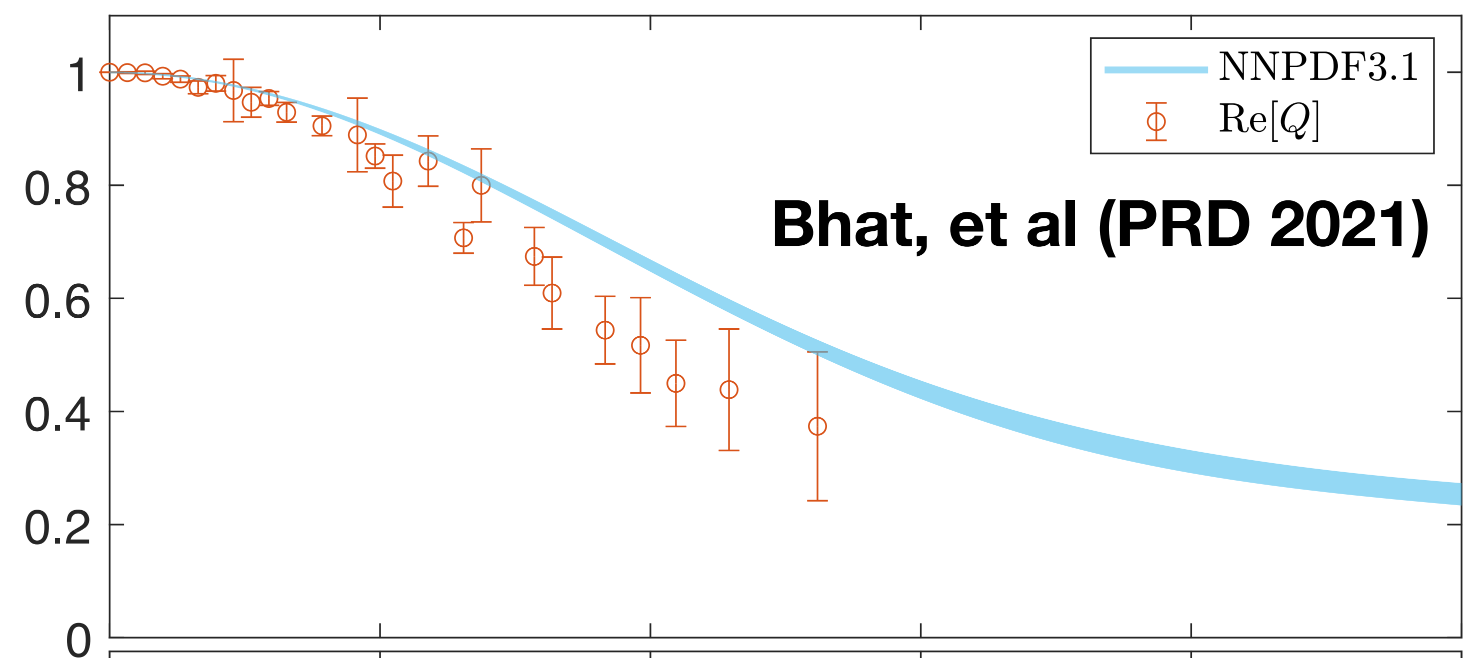
NNPDF fit from Alexandrou, et al (PRL 2018) [Quasi-PDF]

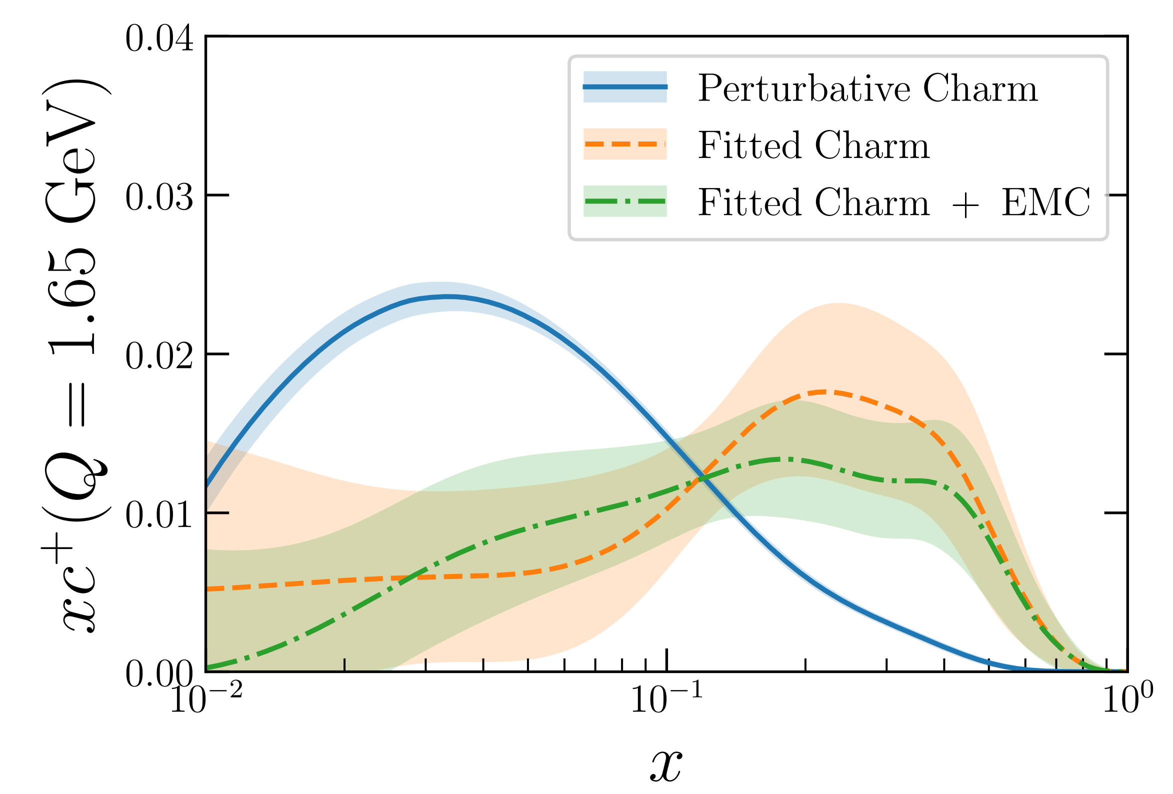


NNPDF fit from Joo, et al (PRL 2020) [Pseudo-PDF]

- **When NNPDF $q_{\rm v}(x)$ converted to Ioffe-time distribution**

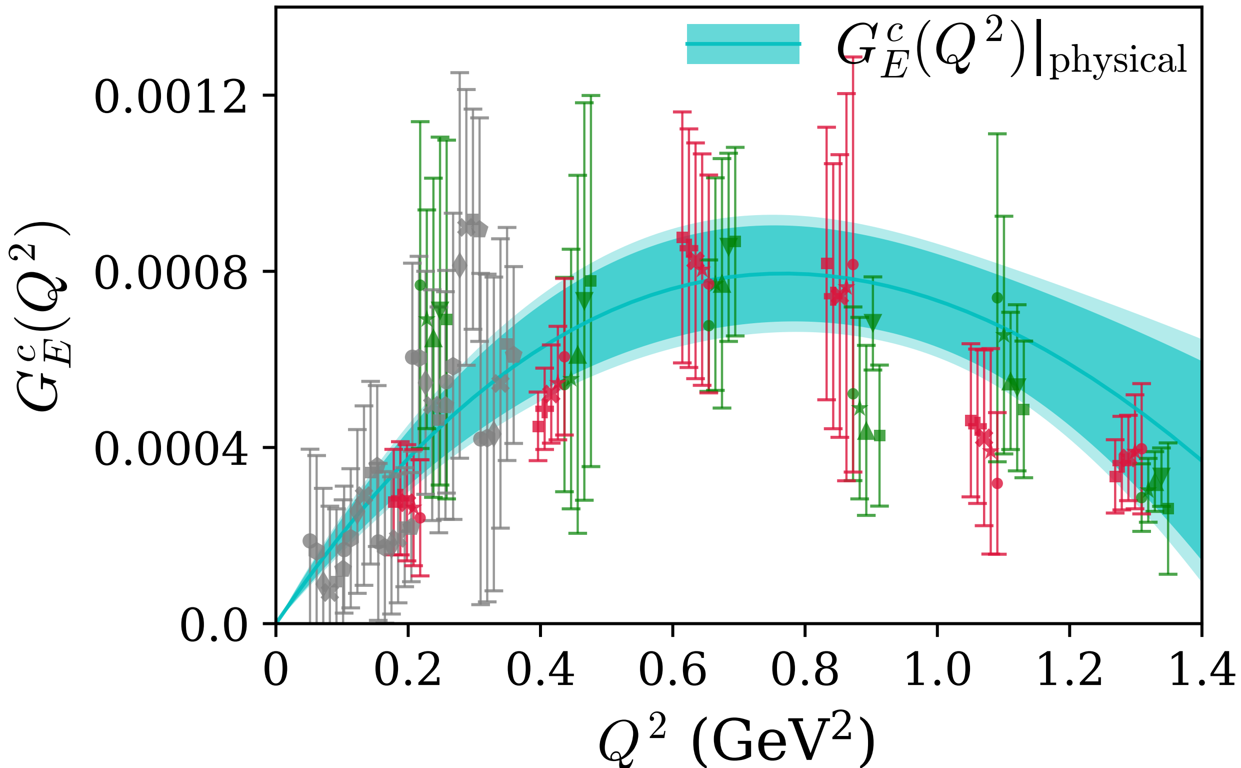






Upcoming NNPDF4.0 [arXiv: 2105.00006]

"This comparison highlights how current data favours a valence-like structure for the charm PDF at low-scales, which in turn is consistent with the hypothesis of an intrinsic charm component in the proton wave function."



RSS, et. al (PLB 2020)
Physical pion mass, 3 ensembles