Transverse spin sum rules of the proton and twist-3 GPDs

Unveiling a partonic picture

Kyle Shiells
May 11, 2021

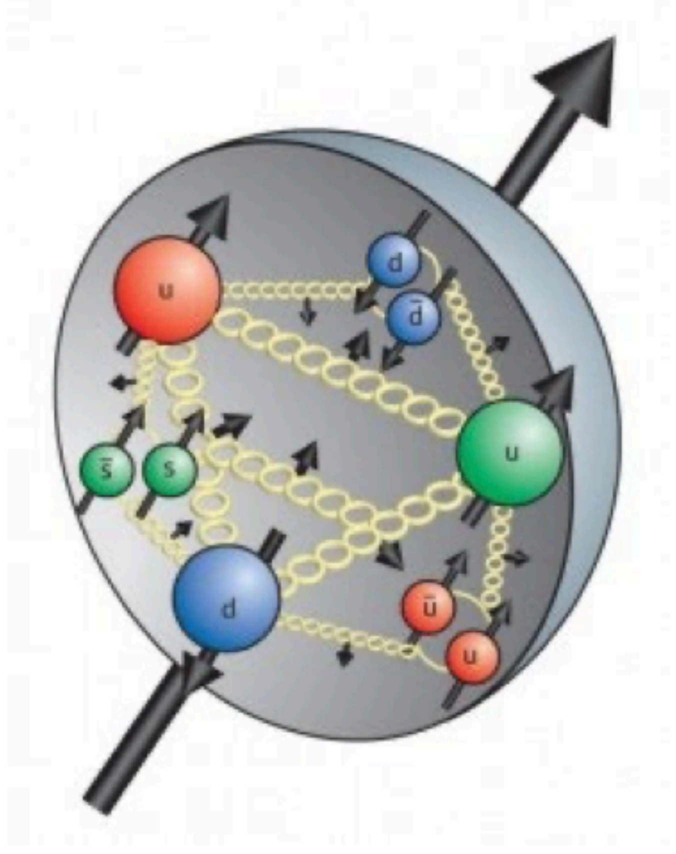
QCD Evolution Workshop 2021

CENTER *for* NUCLEAR FEMTOGRAPHY

Overview

- Proton spin overview
- Twist-2 Transverse Spin Sum Rule
- Twist-3 Transverse Spin Sum Rule
- Measuring AM contributions in the proton

Proton spin structure



- Has been an unsolved problem for over 30 years

  X. Ji, F. Yuan & Y. Zhao Nat. Rev. Phys. 3 (2021) 1, 27

- Solving this has led to the study of **“spin sum rules”**

  R. Jaffe & A. Manohar Nuc Phys B 337 (1990) 163

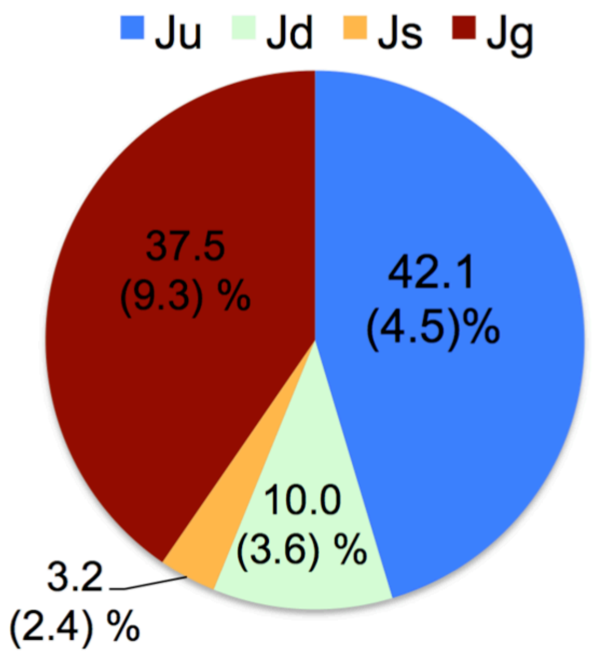
  X. Ji, PRL 78 (1997) 610

- Sum rule contributions to the proton can be expressed in terms of Generalized Parton Distributions (GPDs)

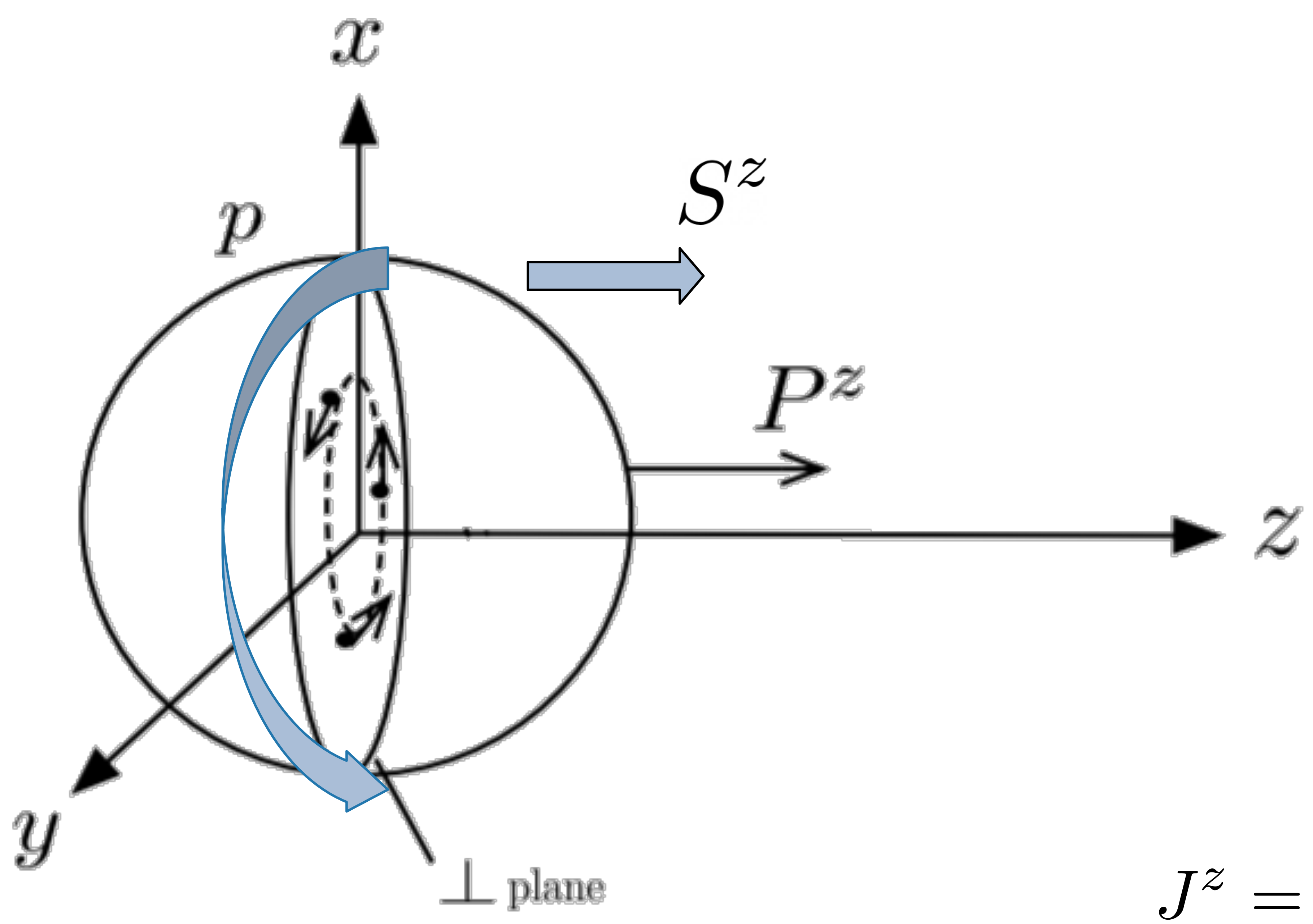
- Efforts for measuring different spin sum rule components (SIDIS, p-p collisions, DVCS) at RHIC, HERMES, COMPASS, JLab 6GeV

- First-principles calculations can been done but challenging

  Alexandrou, C. *et al.* , *PRD*101, 094513, (2020)
  Yang, Y.-B. *et al.* , PRL 118, 102001, (2017)



Longitudinally Polarized Proton:



$$J^z = \sum_i r_i^x p_i^y - r_i^y p_i^x$$

$$[J^z, K^z] = 0$$

- This means that J^z is independent of P^z

$$J^z = \sum_i J_i^z \rightarrow \boxed{\sum_i \langle J_i^z \rangle = \frac{1}{2}}$$

QCD spin sum rule

- Longitudinal AM inherently a twist-3 quantity
- We know that $\langle J^z \rangle$ is independent of P^z, but are $\langle J_i^z \rangle$?

→ Yes, so long as they obey the proper transformation properties!

X. Ji, X. Xiong, and F. Yuan, PLB 717, 214 (2012)

Longitudinal Spin SUM RULES

$$\langle P, S|J^i|P, S\rangle = \frac{\hbar}{2}$$ → **SPIN SUM RULE**

Jaffe & Manohar 1990:

$$\vec{J}_{QCD} = \int d^3x \left[\psi_f^\dagger \frac{\vec{\sigma}}{2}\psi_f + \psi_f^\dagger \vec{x}\times(-i\vec{\partial})\psi_f + \vec{E}_a \times \vec{A}_a + E_a^i(\vec{x}\times\vec{\partial})A_a^i\right]$$

Quark canonical OAM | Gluon spin | Gluon canonical OAM

$$\frac{1}{2}\Delta q + l_q^z + \Delta G + l_g^z = \frac{\hbar}{2}$$

- Last 3 terms are gauge-dependent
- Used in infinite momentum frame (IMF) in light cone gauge ⇒ connected to observables
- All terms possess a partonic density

Ji 1995:

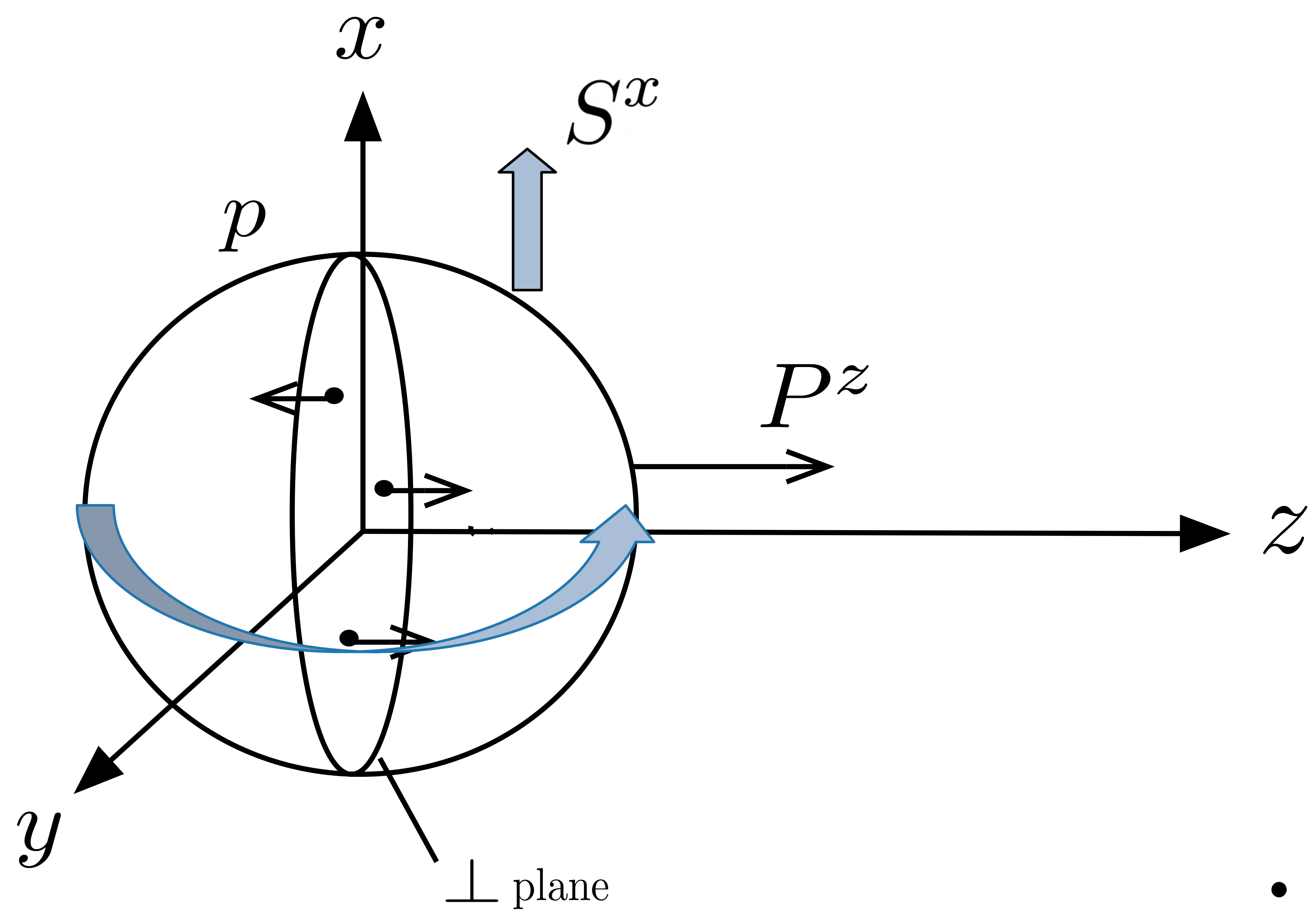
$$= \int d^3x \left[\psi_f^\dagger \frac{\vec{\sigma}}{2}\psi_f + \psi_f^\dagger \vec{x}\times(-i\vec{\nabla} - g\vec{A})\psi_f + \vec{x}\times(\vec{E}\times\vec{B})\right]$$

Quark spin | Quark kinetic OAM | Gluon total AM

$$\frac{1}{2}\Delta q + L_q^z + J_g^z = \frac{\hbar}{2}$$

- Each term is gauge-independent
- Frame-independent
- Not all terms have a partonic density in the IMF

Transversely Polarized Proton:



$$J^x = \sum_i r_i^y p_i^z - r_i^z p_i^y$$

Kinematics: $\sim \gamma$ (Tw-2) $\quad \sim \gamma^{-1}$ (Tw-4)

$$\gamma = (1-\beta^2)^{-1/2}$$

$$[J^{x,y}, K^z] \neq 0$$

- This means that J^x is P^z (frame)-dependent ☹

- Less studied than Longitudinal case because:
    1. It is frame-dependent with non-trivial boost properties
    2. A key issue is separating intrinsic contributions from CM ones
- This has led to some controversy in previous works

Separating CM contributions:

- We need a systematic way of only keeping the intrinsic contributions to AM

- The Pauli-Lubanski vector is an intrinsic relativistic spin operator   J.K. Lubanski, *Physica* 9 (3), 310 (1942)

$$W^\mu = -\frac{1}{2}\epsilon^{\mu\alpha\lambda\sigma}\frac{J_{\alpha\lambda}P_\sigma}{M}$$

$$-W^\mu S_\mu|P,S\rangle = \frac{\hbar}{2}|P,S\rangle$$

- W^μ reduced to the usual spin operator in the rest frame

- However, it tells us that: $J_{\alpha\lambda} \sim P_\alpha, P_\lambda \Rightarrow \epsilon^{\mu\alpha\lambda\sigma}P_{\{\alpha,\lambda\}}P_\sigma = 0$

If any P^α, P^λ terms found in $J^{\alpha\lambda}$ are CM contributions

X. Ji and F. Yuan, PLB 810, 135786 (2020)

- Key point: this does not require one to actually define the CM!

Transverse AM from EMT Form Factors:

- We want transverse AM expectation value — attainable from spacial moments of the Energy Momentum Tensor (EMT):

$$\langle J^x \rangle = \langle \int \xi^y T^{0z} d^3\xi \rangle - \langle \int \xi^z T^{0y} d^3\xi \rangle$$

- The matrix elements of the EMT can be expressed in terms of 4 gravitational for factors, $A, B, C, \tilde{C}$:

$$\left\langle P' \left| T^{\mu\nu}_{q,g,\mathrm{Bel}} \right| P \right\rangle = \bar{u}\left(P'\right)\Big[A_{q,g}\left(\Delta^2\right)\gamma^{(\mu}\bar{P}^{\nu)} + B_{q,g}\left(\Delta^2\right)\bar{P}^{(\mu} i\sigma^{\nu)\alpha}\Delta_\alpha/2M$$

$$+ C_{q,g}\left(\Delta^2\right)\left(\Delta^\mu\Delta^\nu - g^{\mu\nu}\Delta^2\right)/M + \bar{C}_{q,g}\left(\Delta^2\right) g^{\mu\nu} M \Big] u(P) \ .$$

X.Ji, PRL 78, 610 (1997)

$$\Rightarrow \quad J^{ij}_{q,g} = -\frac{\epsilon^{ij\alpha\beta}}{2M} S_\alpha P_\beta \left(A_{q,g}(0) + B_{q,g}(0)\right) - \frac{\boldsymbol{P}^i(\boldsymbol{P}\times\boldsymbol{s})^j - \boldsymbol{P}^j(\boldsymbol{P}\times\boldsymbol{s})^i}{2M(P^0+M)} A_{q,g}(0) \ , \quad \Rightarrow$$

CM contribution

Total Intrinsic transverse AM

$$J^x_{q,g} = \frac{\gamma}{2}\left[A_{q,g}(0) + B_{q,g}(0)\right]$$

True in any frame

Transverse Polarization Sum Rules:

- Let's look again at transverse AM, but **split in terms of its 2 contributions**:

$\xi^y T^{0z}(\xi)$ ↓ $\qquad$ $\xi^z T^{0y}(\xi)$ ↓

$$J^x = \left(\gamma - \frac{1}{2\gamma}\right) J^{x(2)} + \frac{1}{2\gamma} J^{x(3)} = \gamma \frac{\hbar}{2}$$

Note: equal in rest frame by rotational symmetry

$J^{x(2)}$ & $J^{x(3)}$ ~ proton matrix elements of q and g fields

- Solution: $J^{x(2)} = J^{x(3)} = \frac{\hbar}{2}$

Twist-2 Ji Sum Rule

Novel Twist-3 Sum Rule

Spin sum rules are γ-independent!

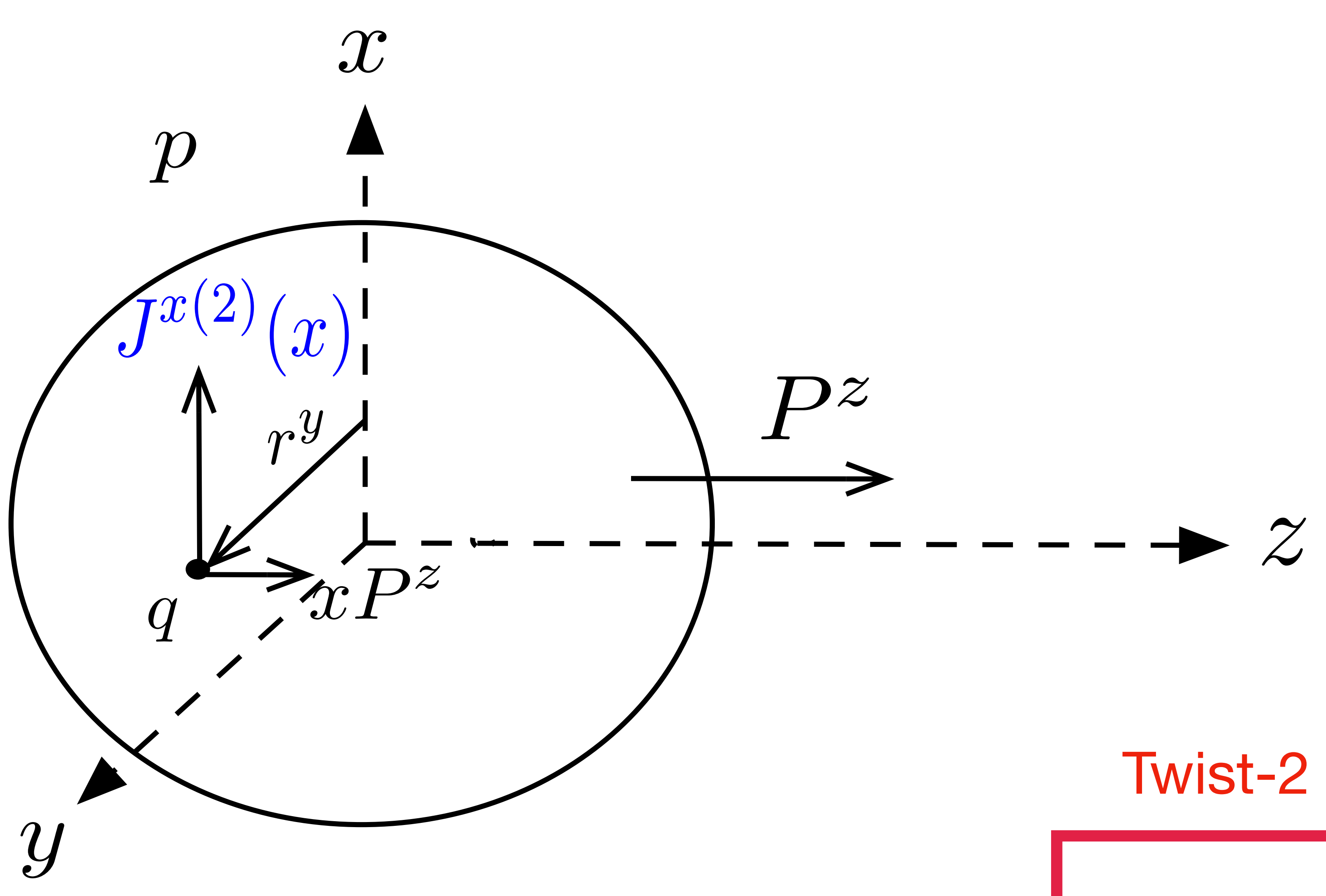
2 Sum Rules:

$$J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2}$$ Twist-2

$$J_q^{x(3)} + J_g^{x(3)} = \frac{\hbar}{2}$$ Twist-3

Twist-2 Contribution

Readily expressed in terms of partonic density in IMF: $J_q^{x(2)}(x) = \int \frac{\mathrm{d}\lambda \mathrm{d}^2\boldsymbol{\xi}^\perp}{2\pi} e^{i\lambda x} \left\langle P,S \left| \bar{\psi}\left(-\frac{\lambda n}{2}, \boldsymbol{\xi}^\perp\right) \gamma^+ \xi^y i \overleftrightarrow{\partial}^+ \psi\left(\frac{\lambda n}{2}, \boldsymbol{\xi}^\perp\right) \right| P,S \right\rangle$



xP^z = parton momentum fraction

Relate LF operator to twist-2 GPDs

Twist-2 Sum Rule:

$$J_q^{x(2)} + J_g^{x(2)} = \frac{\hbar}{2}$$

$$J_q^{x(2)}(x) = \frac{x}{2}\Big(H_q(x) + E_q(x)\Big)$$

$$J_g^{x(2)}(x) = \frac{x}{2}\Big(H_g(x) + E_g(x)\Big)$$

X. Ji, PRL 78 (1997) 610

X. Ji, X. Xiong, and F. Yuan, PLB 717, 214 (2012)

Twist-2 Sum Rule involves 4 twist-2 GPDs

Twist-3 Contribution

The Quarks:

- In order to get a partonic picture, now we need to decompose J^x into spin and canonical OAM contributions

$$J_q^{x(3)} \to l_q^{x(3)} + \frac{1}{2}\Delta q_T$$

Transverse quark spin $\Delta q_T = \int dx g_T(x)$ R. Jaffe & X. Ji Nuc.Phys.B 375, 527 (1992)

$$g_T(x) = \frac{1}{2M}\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle PS_\perp|\psi_+^\dagger(0)\gamma_0\gamma^\perp\gamma^5\psi_-(\lambda n) + \psi_-^\dagger(0)\gamma_0\gamma^\perp\gamma^5\psi_+(\lambda n)|PS_\perp\rangle$$

In principle measurable and calculable on lattice!

There is also a “potential” term

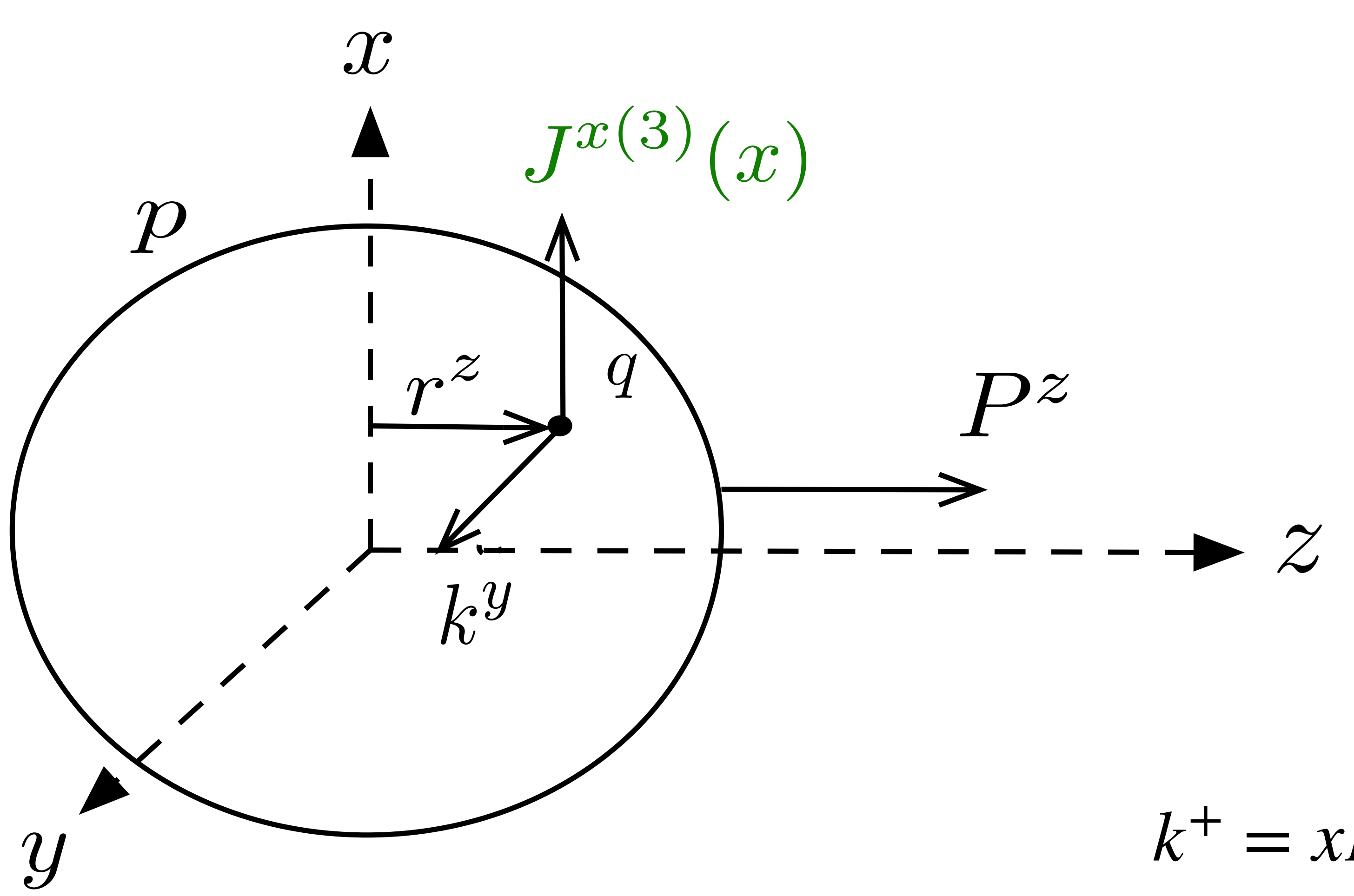
$$l_q^{x(3)}(x) = -\frac{1}{P^+}\int \frac{\mathrm{d}\lambda \mathrm{d}^2\boldsymbol{\xi}^\perp}{2\pi} \mathrm{e}^{i\lambda x}\langle PS|\,\bar\psi(0,\boldsymbol{\xi}^\perp)\gamma^+\xi^z i\partial^y\psi(\lambda n,\boldsymbol{\xi}^\perp)\,|PS\rangle$$

Quark transverse AM partonic density:

$$l_q^{x(3)}(x) = \int \mathrm{d}y\Big[\left(G_{q,D,3}(x,y) + G'_{q,D,4}(x,y)\right) + \mathrm{P}\frac{1}{y-x}\left(G_{q,F,3}(x,y) + G'_{q,F,4}(x,y)\right)\Big]$$

~ 3 field twist-3 GPDs

X. Ji, Y. Guo & K. Shiells 2101.05243 (2021)



$k^+ = xP^z \to xP^+$

The Gluons:

- Again, we decompose into canonical OAM + potential + spin:

$$J_g^{x(3)} \to l_g^{x(3)} + \frac{1}{2}\Delta G_T$$

Gluon transversity (spin) distribution X. Ji, PLB 289, 137 (1992)

$$\Delta G_T(x) = \frac{i}{xP^+}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \left\langle P,S \left| 2\mathrm{Tr}\left\{ G^{+\alpha}\left(-\frac{\lambda n}{2}\right) W_{-\frac{\lambda}{2},\frac{\lambda}{2}} \tilde{G}^{\perp}{}_{\alpha}\left(\frac{\lambda n}{2}\right)\right\}\right| P,S\right\rangle$$

More difficult to measure, but well-known

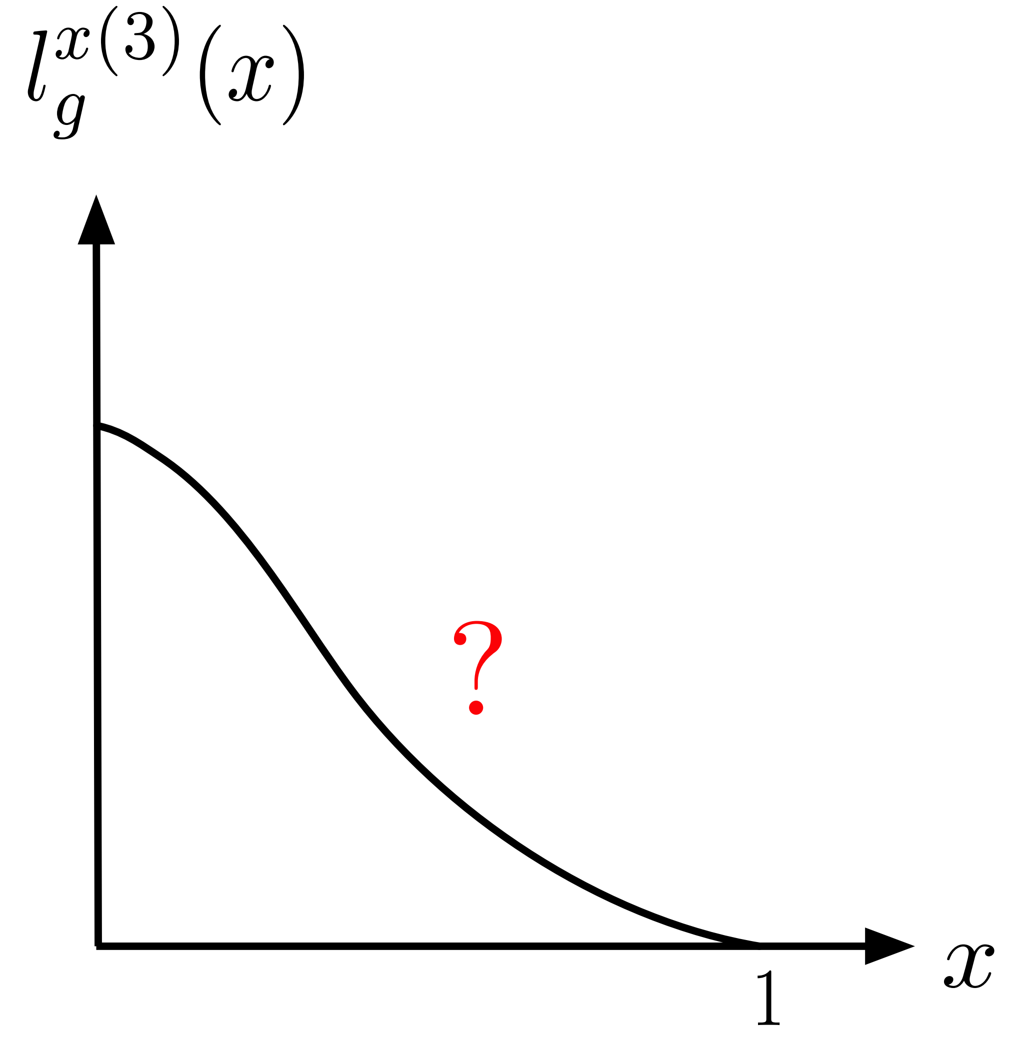
The "potential" term cancels that in quark AM!

$$l_g^{x(3)}(x) = \int \frac{d\lambda d^2\boldsymbol{\xi}^\perp}{2\pi} e^{i\lambda x} \langle P,S|\, 2\mathrm{Tr}\left\{G^{+\alpha}(0,\boldsymbol{\xi}^\perp)\xi^z \partial^y A_\alpha(\lambda n,\boldsymbol{\xi}^\perp)\right\}|P,S\rangle$$

Gluon transverse AM partonic density:

$$l_g^{x(3)}(x) = \int dy \Bigg[\mathrm{P}\frac{1}{x+y}\left(G_{g,D,3}(x,y) + G'_{g,D,4}(x,y)\right) + \mathrm{P}\frac{1}{y^2-x^2}\left(G_{g,F,3}(x,y) + G'_{g,F,4}(x,y)\right)\Bigg]$$

~ 3 field twist-3 GPDs



Rotational symmetry ⇒ $\int l_g^z(x)dx = \int l_g^{x(3)}dx$

Twist-3 Transverse Partonic Spin Sum Rule

- Requires the decomposition $J = L + S$ (OAM + spin)
- Parton densities depicts a physical picture of the spin structure inside the nucleon
- Rotated version of the Jaffe Manohar Longitudinal spin sum rule

NEW

Transverse Polarization:

$$\frac{1}{2}\Delta q_T + \Delta G_T + l_q^{x(3)} + l_g^{x(3)} = \frac{\hbar}{2}$$

Twist-3
Our 2021 result

Δq_T , ΔG_T Involve measurable PDFs in DIS and correspond to spin

$l_q^{x(3)}$, $l_g^{x(3)}$ Involve twist-3 GPDs and correspond to canonical OAM

X. Ji, Y. Guo & K. Shiells 2101.05243 (2021)

Experimental testing spin sum rules

	Longitudinal	Transverse
Frame-independent sum rule:	$\frac{1}{2}\Delta q + L_q^i + J_g^i = \frac{\hbar}{2}$	$\longrightarrow J_q^x + J_g^x = \frac{\hbar}{2}$
		$\sim \int dx x$ moments of: $H_{q,g}(x), E_{q,g}(x)$ 4 twist-2 GPDs extractable from DVCS & DVMP!
IMF sum rule:	$\frac{1}{2}\Delta q + l_q^z + \Delta G + l_g^z = \frac{\hbar}{2}$	$\frac{1}{2}\Delta q_T + l_q^x + \Delta G_T + l_g^x = \frac{\hbar}{2}$

$\int dx g_1(x)$

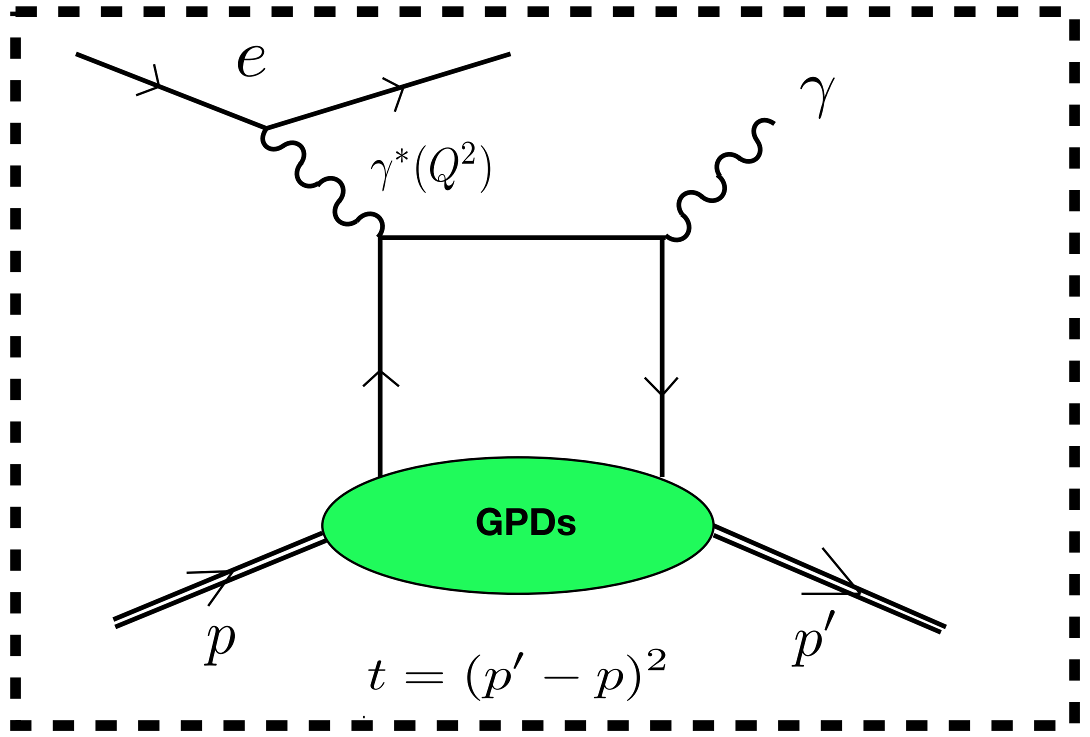
Quark helicity distributions probed in SIDIS

$\int dx \Delta G(x)$

Gluon helicity distribution probed in p-p collisions at RHIC

$\int dx dy \mathcal{F}^{(3)}_{\rm GPD}(x,y)$

Twist-3 OAM quark and Gluon GPDs extractable From DVCS & DVMP

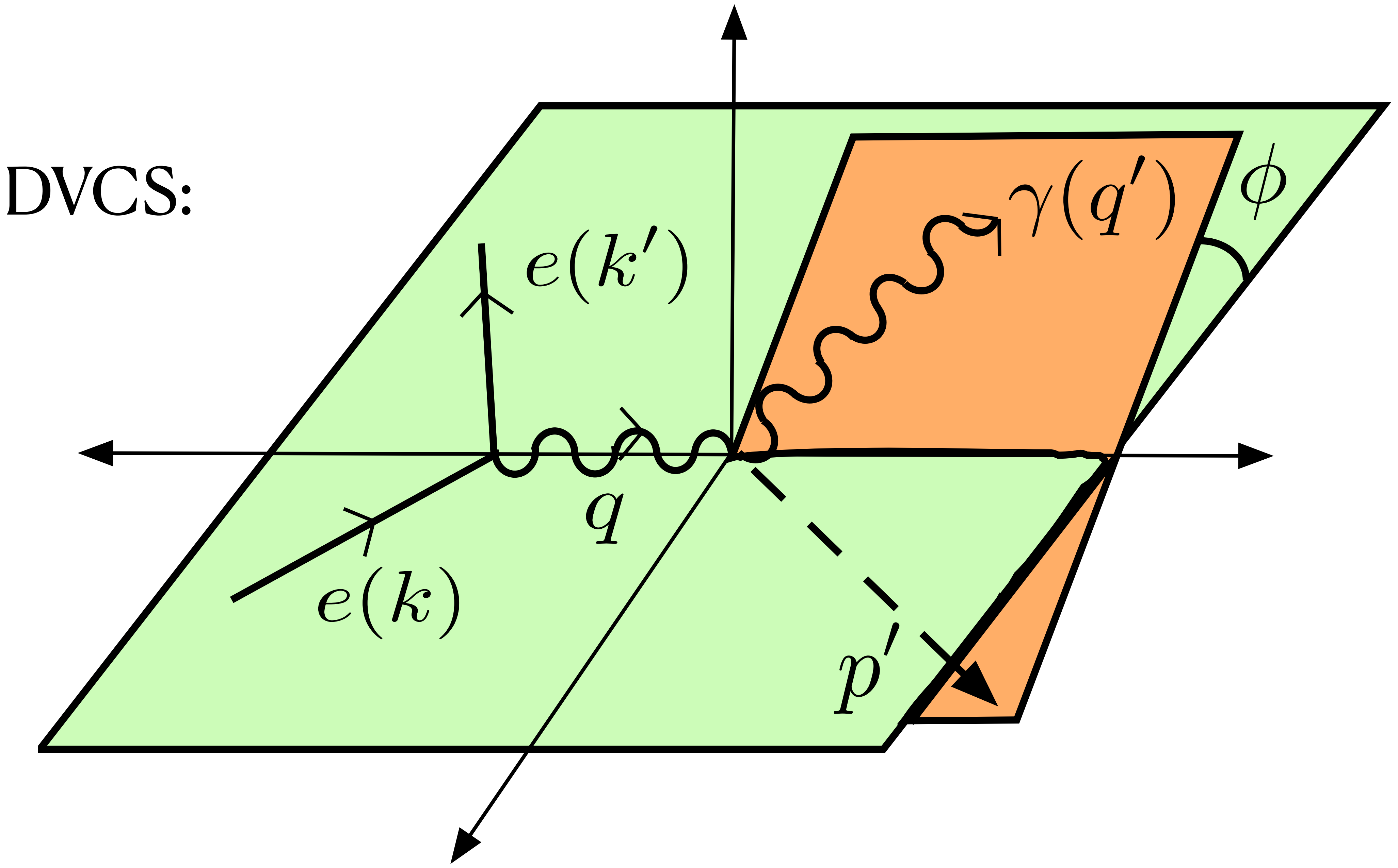


Less experimental constraints

DVCS Measurements of GPDs

- We need an extensive DVCS (& DVMP) observable measurement program

DVCS:



Cross section asymmetries: UU, UT, UL, LU, LL, LT

$$\sigma_{P_b,P_t} = \sigma_{P_b,P_t}(\{\text{kin. scalars}\}, \text{Re}\mathcal{F}, \text{Im}\mathcal{F})$$

Compton Form Factors (CFFs)

$$\mathcal{F} = \mathcal{H}, \mathcal{E}, \cdots$$

GPDs

Determines spin contributions of the proton

- Includes a full set of polarization combinations: beam and target
- Together with angular dependence allows one to isolate different combinations of twist-2 and twist-3 CFFs

K. Shiells

Twist-3 GPDs and Angular Momentum

Longitudinal Jaffe & Manohar Sum rule:

$$\frac{1}{2}\Delta q + l_q^z + \Delta G + l_g^z = \frac{\hbar}{2}$$

$$l_q^z = \int dx\, l_q^z(x)$$

S. Liuti et al PRD (2016,2018):

$$\tilde{E}_{2T} = -\int_x^1 \frac{dy}{y}(H+E) - \left[\int_x^1 \frac{dy}{y^2}\tilde{H} - \frac{\tilde{H}}{x}\right]$$

$L \qquad J \qquad S$

X. Ji, Y. Guo & K. Shiells 2101.05243 (2021):

Twist-3 quark OAM density: $l_q^z(x) = \int dy G_{q,D,3}(x,y) + \int dy \mathcal{P}\frac{1}{y-x}G_{q,F,3}(x,y)$

Twist-3 OAM GPD: $\tilde{E}_{2T}(x)$

Connection under investigation!

GPD	Twist	$P_q P_p$	TMD	$P_{Beam} P_p$ (DVCS)	$P_{Beam} P_p$ ($\mathcal{I}$)
$\mathbf{2\widetilde{H}_{2T} + E_{2T}} - \xi \widetilde{E}_{2T}$	3	UU	$f^{\perp}$	$UU^{\cos\phi}, LU^{\sin\phi}$	UU, LU
$\mathbf{\widetilde{E}_{2T}} - \xi E_{2T}$	3	UL	$f_L^{\perp\,(*)}$	$UU^{\cos\phi}, UL^{\cos\phi}, LU^{\sin\phi}, LL^{\cos\phi}$	UU, LU, UT

B. Kriesten et al., Phys Rev D 101 (2020) 054021

Ideal beam and target polarizations to extract quark OAM GPD!

K. Shiells

Closing Remarks

- We have derived a clear partonic twist-3 spin sum rule for a transversely-polarized nucleon
- Correctly deriving the sum rule requires a removal of the CM contributions — systematically achieved via the PL vector
- The sum rule involves well-known PDFs and well-defined twist-3 GPDs
- It is a rotated version of the Jaffe & Manohar sum rule from 1990, true in the IMF in the light cone gauge
- Alternatively, there is a simpler twist-2 sum rule involves twist-2 GPDs (Ji sum rule), which are also partonic
- Measurement of the involved PDFs and the twist-2 GPDs potentially within reach (JLab 12 GeV)
- Possible measurement of the twist-3 OAM GPDs involved is now being investigated