Two-loop coefficient function for DVCS

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Based on 2007.06348 V. Braun, A. Manashov, S. Moch, J.S.

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- Deeply Virtual Compton Scattering, Müller 1994, Ji 1996, Radyushkin 1996

$$\gamma^*(q)\; N(p) \longrightarrow \gamma(q')\; N(p')$$

Goeke et al. 2001, Diehl 2003, Belitsky, Radyushkin 2005, Müller 2014

- The DVCS amplitude

$$\mathcal{A}_{\mu\nu}(q,q',p) = i \int d^4x\, e^{-iqx} \langle N(p')|T\{j^{\text{em}}_\mu(x) j^{\text{em}}_\nu(0)\}|N(p)\rangle\,.$$

- The leading twist approximation

$$\mathcal{A}_{\mu\nu} = -g^\perp_{\mu\nu} V + \epsilon^\perp_{\mu\nu} A + \ldots$$

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- **Vector flavor nonsinglet amplitude**

$$V(\xi, Q^2) = \sum_q e_q^2 \int_{-1}^{1} \frac{dx}{\xi} C(x/\xi, Q^2/\mu^2)\, F_q(x, \xi, t, \mu)\,.$$

C is the coefficient function, F_q - the generalized parton distribution (GPD)

$$\xi = -(\Delta, q)/2(P, q), \qquad t = \Delta^2, \qquad \Delta = p' - p, \qquad P = (p + p')/2,$$

$$\langle N(p')|\mathcal{O}_q(z_1, z_2)|N(p)\rangle = 2P_+ \int_{-1}^{1} dx\, e^{-iP_+\xi(z_1+z_2)+iP_+x(z_1-z_2)}\, F_q(x, \xi)\,.$$

$$\mathcal{O}_q(z_1, z_2) = \bar{q}(z_1 n)\gamma_+ q(z_2 n) \qquad \text{(\textbf{Light-ray operator})}$$

$$C(x/\xi, Q^2/\mu^2) = C^{(0)}(x/\xi) + a_s C^{(1)}(x/\xi, Q^2/\mu^2) + a_s^2 C^{(2)}(x/\xi, Q^2/\mu^2) + O(a_s^3).$$

LO and NLO coefficient functions **Ji, Osborne, 1998, Belitsky, Müller, 1998**

$$C^{(0)}(x/\xi) = \frac{\xi}{\xi - x} - \frac{\xi}{\xi + x},$$

$$C^{(1)}(x/\xi, Q^2/\mu^2) = \frac{2C_F\xi}{\xi - x}\left[\ln\frac{Q^2}{2\mu^2}\left(\frac{3}{2} + \ln\left(\frac{\xi - x}{2\xi}\right)\right) - \frac{9}{2} - \frac{1}{2}\ln^2 2 + \left[\frac{1}{2}\ln\left(1 - \frac{x}{\xi}\right) - \frac{3}{2}\frac{\xi - x}{\xi + x}\right]\ln\left(1 - \frac{x}{\xi}\right)\right] - (x \leftrightarrow -x).$$

Some properties of C:

- C is (up to scale dependence) a function of one variable, the ratio x/ξ.
- Therefore we lose no information by setting $\xi = 1$.
- C is antisymmetric.

$$C(-x/\xi, Q^2/\mu^2) = -C(x/\xi, Q^2/\mu^2).$$

- C is real in the ERBL region $|x/\xi| < 1$ and has branch points at $|x/\xi| = 1$.
- C can be analytically continued to the DGLAP region $|x/\xi| > 1$ by using the $\xi \to \xi - i0$ prescription.

The conformal group is an extension of the group of space-time symmetries (Poincare group). It includes dilatations and inversions.

The β function in $d = 4 - 2\epsilon$ dimensions.

$$\beta(a_s, \epsilon) = 2a_s(-\epsilon - \beta_0 a_s - \beta_1 a_s^2 + O(a_s^3)).$$

For some value of the coupling a_s we can fix $\epsilon = \epsilon_*$ such that $\beta(a_s, \epsilon_*) = 0$.

$$\epsilon_* = -(\beta_0 a_s + \beta_1 a_s^2 + ...).$$

At such values of the parameters QCD is symmetric under conformal transformations!

One consequence are constraints of the local operator product expansion. This gives the conformal operator product expansion (COPE).

DVCS Amplitude (COPE) Müller, 1995

$$V(\xi, Q^2) = \sum_{N,\text{even}} f_N(\xi) \left(\frac{1}{2\xi}\right)^N c_{1,\text{DIS}}\left(N, \frac{Q^2}{\mu^2}, a_s, \epsilon_*\right) \frac{\Gamma(d/2-1)\Gamma(2j_N)}{\Gamma(j_N)\Gamma(j_N+d/2-1)}$$

$c_{1,\text{DIS}}$: OPE coefficients in DIS (known to NNNLO)

f_N: parametrize matrix elements of local conformal operators $\mathcal{O}_N$

$j_N = N + 1 - \epsilon_* + \frac{1}{2}\gamma_N(a_s)$ is the conformal spin of the $\mathcal{O}_N$ and γ_N are their anomalous dimensions.

Strategy: Compare the COPE for V to the factorized form (omitting quark charges)

$$V(\xi, Q^2) = \int_{-1}^{1} \frac{dx}{\xi} C(x/\xi, Q^2/\mu^2)\, F_q(x, \xi, t, \mu)\,.$$

Expand also the light-ray operator in terms of local conformal operators and compare coefficients of the f_N.

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Conformal symmetry approach

Ansatz for the CF

$$C = C^{(0)} \otimes K \otimes e^{\mathbb{X}}.$$

- $\otimes$ denotes convolution.
- $e^{\mathbb{X}}$ is a known integral operator that rotates from the so-called conformal scheme to be $\overline{\text{MS}}$ scheme.
- K is an $SL(2)$-invariant integral operator, i.e. it commutes with the generators of conformal transformations.

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- The eigenvalues of K are obtained from matching the COPE with the factorizated form.

$$K_N = c_{1,\mathrm{DIS}}\left(N, \frac{Q^2}{\mu^2}, a_s, \epsilon_*\right) \sigma_N^{-1} \frac{\Gamma(d/2-1)\Gamma(j_N+\lambda_N-\frac{1}{2})}{\Gamma(\lambda_N-\frac{1}{2})\Gamma(j_N+d/2-1)}.$$

σ_N: Eigenvalues of $e^{\mathbb{X}}$, $\lambda_N = \frac{3}{2} - \epsilon_* + \frac{1}{2}\gamma_N$.

- A consequence of K being an invariant operator is that it is uniquely determined by its spectrum.
- K has reciprocity property: Eigenvalues K_N are symmetric under the exchange $j_N \to 1 - j_N$. It provides a highly non-trivial check for the calculation.

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- For calculating the convolutions we used the formula

$$\int dx' C^{(0)}(x') T(x', x) = \int_0^1 d\alpha \int_0^{\bar{\alpha}} d\beta \left(\frac{\boldsymbol{t}(\alpha, \beta)}{\bar{\alpha}(1-x) + \beta(1+x)} - (x \leftrightarrow -x) \right) ,$$

where the integral operator T is defined in terms of $\boldsymbol{t}(\alpha, \beta)$ and its action in position space is given by

$$(Tf)(z_1, z_2) = \int_0^1 d\alpha \int_0^{1-\alpha} d\beta \, \boldsymbol{t}(\alpha, \beta) f(z_1(1-\alpha) + z_2\alpha, z_2(1-\beta) + z_1\beta) \, .$$

These integrals can be calculated in a short amount of time by the **HyperInt** maple package. **1403.3385**

Results

To get the final result for CF in $d=4$ one should replace occurences of ϵ with $\epsilon_* = -\beta_0 a_s + O(a_s^2)$, so you should also add a contribution from the $O(\epsilon)$ term of the one-loop CF.

$$\underbrace{C^{(2)}(x)}_{\text{CF in } d=4} = \underbrace{C_*^{(2)}(x)}_{\text{CF at } d=4-2\epsilon_*} + \beta_0 \underbrace{C^{(1,1)}(x)}_{\epsilon^1 \text{ piece of one-loop CF}} .$$

The result looks like

$$C_*^{(2)}(x) = \beta_0 C_F\, C_*^{(2\beta)}(x) + C_F^2 C_*^{(2P)}(x) + \frac{C_F}{N_c} C_*^{(2A)}(x) .$$

$$\begin{aligned} C_*^{(2P)}(x) = \frac{1}{\omega}\Big(&6\mathrm{H}_{0,0,0,0} - \mathrm{H}_{1,0,0,0} - 2\mathrm{H}_{2,0,0} - \mathrm{H}_{1,1,0,0} - \mathrm{H}_{1,2,0} - \mathrm{H}_{2,1,0} + \mathrm{H}_{1,1,1,0}\Big) \\ &- \frac{1}{\bar\omega}\mathrm{H}_{0,0,0} - \left(\frac{4}{\omega} - \frac{2}{\bar\omega}\right)\mathrm{H}_{1,0,0} + \frac{1}{\bar\omega}\mathrm{H}_{2,0} + \frac{2}{\omega}\mathrm{H}_{1,1,0} \\ &- \left(\frac{13}{2\bar\omega} + \frac{19}{3\omega}\right)\mathrm{H}_{0,0} + \left(\frac{3}{\bar\omega} + \frac{11}{3\omega}\right)\mathrm{H}_{1,0} + \frac{1}{\omega}\zeta_2\Big(\mathrm{H}_{1,1} - \mathrm{H}_2 - \mathrm{H}_{1,0} - 4\mathrm{H}_{0,0}\Big) \\ &+ \left(\frac{1}{\bar\omega}\left(\frac{223}{12} + 5\zeta_2 - 2\zeta_3\right) + \frac{1}{\omega}\left(3\zeta_2 + 16\zeta_3 - \frac{32}{9}\right)\right)\mathrm{H}_0 \\ &+ \frac{1}{48\omega}\left(701 + 128\zeta_2 + 936\zeta_3 + 72\zeta_2^2\right) - (\omega \leftrightarrow \bar\omega) \qquad \boxed{\omega = (1-x)/2} \end{aligned}$$

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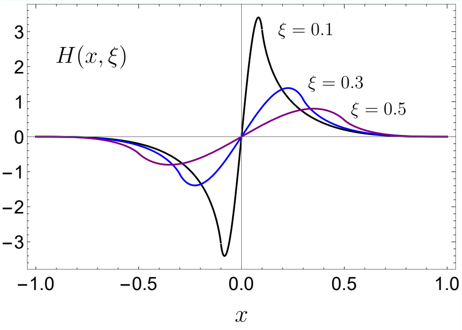
Physical observables in DVCS are Compton form factors, in particular

$$\mathcal{H}(\xi) = \int_{-1}^{1} \frac{dx}{\xi} C(x/\xi) H(x,\xi) = R(\xi) e^{i\Phi(\xi)}.$$

In order to estimate the size we used the GPD model from **0504030**



Results

$\mu = Q = 2$ GeV

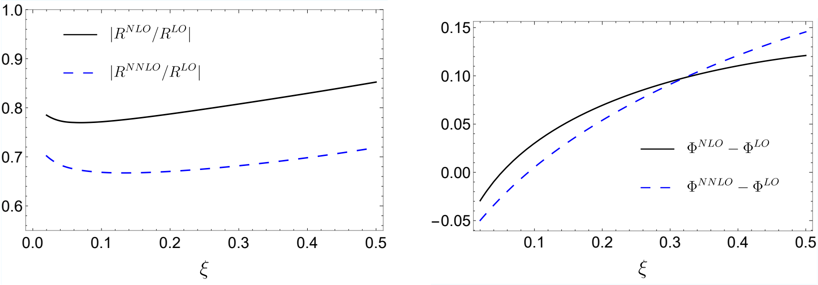


Figure: Higher-order QCD corrections to the Compton form factor $\mathcal{H}(\xi)$.The ratios of the Compton form factor calculated to the NNLO and NLO accuracy with respect to the tree-level are shown for the absolute value and the phase of $\mathcal{H}(\xi)$ on the left and the right panels, respectively.

Conclusions

- Using an approach based on conformal symmetry we have calculated the two-loop CF in DVCS in $\overline{\text{MS}}$ scheme for the flavor-nonsinglet vector contributions
- Numerical estimates show that the two loop corrections to the Compton Form Factors are relatively large
- Three-loop anomalous dimensions are known **1703.09532**